1. Suppose the debt holders of a cosmetics firm hold debt with a face value of $500,000. The rest of the firm is owned by stockholders. The firm’s profits will depend on the demand it will face in the market for cosmetic products. Case (1): If demand is very low, the firm’s profits will be $300,000. Case (2): If the demand is at its average value, the firm’s profits will be $500,000. Case (3): If demand is very high, the firm’s profits will be $1,000,000.

(a): Calculate the payoffs to the debt holders and the equity holders in each of the three cases.

(b): For each of the three cases, state whether the firm is bankrupt or solvent.

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market value</td>
<td>300,000</td>
<td>500,000</td>
<td>1,000,000</td>
</tr>
<tr>
<td>Debt holder payoff</td>
<td>300,000</td>
<td>500,000</td>
<td>500,000</td>
</tr>
<tr>
<td>Equity holder payoff</td>
<td>0</td>
<td>0</td>
<td>500,000</td>
</tr>
<tr>
<td>Bankrupt/Solvent</td>
<td>Bankrupt</td>
<td>Solvent</td>
<td>Solvent</td>
</tr>
</tbody>
</table>

2. You are among the OTC market makers in the stock of BioEngineering, Inc. and quote a bid of 102 1/4 and an ask of 102 1/2. Suppose that you have zero inventory. Remember that a market buy order means that the market maker is selling (the market is buying). Also, remember that a dealer sells at her ask price. Similar reasoning applies to a market sell order.

(a): On Day 1 you receive market buy orders for 10,000 shares and market sell orders for 4,000 shares. How much do you earn on the 4,000 shares that you bought and sold? What is the value of your inventory at the end of the day? (Hints: It is possible to have negative inventory. Further, there is more than one correct way to value an inventory, but please state what assumption your valuation is based on.)

Solution. You bought 4,000 shares at a price of 102 1/4 which cost $409,000. You sold 4,000 shares at a price of 102 1/2 which made $410,000. Your profits were $1,000. Assuming you executed the full buy order, you were short 6,000 shares at the end of the day. You could value your inventory at either the bid price or at the ask price. If you use the bid price, then the value of your inventory was -$613,500. If you use the ask price, then the value of your inventory was -$615,000. You might also use the average
between the bid and the ask which would result in an inventory value of $-614,250.

(b): Before trading begins on Day 2 the company announces trial testing of a cure for acne in mice. The quoted bid and ask jump to 110 1/4 - 110 1/2. During Day 2 you receive market sell orders for 8,000 shares and buy orders for 2,000 shares. What is your total profit or loss over the two-day period? What is the value of your inventory at the end of Day 2?

Solution. You sell 2,000 shares at a price of 110 1/2 for $221,000. You buy 8,000 shares at a price of 110 1/4 for $882,000. 6,000 of the shares you bought you had sold yesterday for a price of $615,000. Therefore your profits/losses on these 8,000 shares was 221,000 + 615,000 - 882,000 = -46,000. Your inventory at the end of the 2nd day is 0.

(c): Where did your profit or loss come from? What is a market maker’s objective, that is, how does a market maker seek to make money? Is there anything you could have done during Day 1, consistent with a market maker’s objective, that would have improved your performance over the two-day period?

Solution. The profits came from picking up the 1/4 point bid-ask spread. However this profit was over-shadowed by losses on the outstanding negative inventory held over-night when the price went up. A market-maker seeks to make money on the bid-ask spread and not from speculation about price movements. Therefore to have avoided any P&L from speculation, we could have only filled some of the buy order on the 1st day in order to avoid taking a short position overnight.

3. Which of the following investments do you prefer?
   (a): Purchase a zero-coupon bond, which pays $1000 in ten years, for a price of $550.
   (b): Invest $550 for ten years in Chase at a guaranteed annual interest rate of 4.5%.

Solution. In both options we pay $550 today. We can answer the question either by calculating the implied interest rate in option a, or by calculating the 10 year future value of option b. I’ll do both.
(a): The implied interest rate, \( r \), satisfies
\[
FV = PV(1 + r)^t
\]
\[\Rightarrow \$1000 = \$550(1 + r)^{10}\]
\[\Rightarrow \frac{20}{11} - 1 = r\]
\[\Rightarrow 6.16 = r\]

Which means option a provides the higher return and is superior to option b.

(b): The future value in option b satisfies
\[
FV = PV(1 + r)^t
\]
\[= \$550(1 + .045)^{10}\]
\[= \$854.13\]

Which, again, means option a is the better option.

4. Suppose you get for free one of following two securities: (a) an annuity that pays $10,000 at the end of each of the next 6 years; or (b) a perpetuity that pays $10,000 forever, but payments do not begin until 10 years from now (the first cash payment from this security is 11 years from today). Which security would you choose if the annual interest rate is 5%? Does your answer change if the interest rate is 10%? Explain why or why not.

Solution with interest rate of 5%. The value of the 6 year annuity with $10,000 coupons is given by
\[
PV = \frac{C}{r} \left(1 - \frac{1}{1 + r} \right)^t
\]
\[= \frac{\$10,000}{.05} \left(1 - \frac{1}{1+.05} \right)^6\]
\[= \$50,756.92\]

The value of the perpetuity which begins in 6 years is equal to the value of a perpetuity today discounted back 6 years:
\[
PV = \frac{C}{r} \left(\frac{1}{1 + r} \right)^t
\]
\[= \frac{\$10,000}{.05} \left(\frac{1}{1+.05} \right)^{10}\]
\[= \$122,782.65\]

Therefore the perpetuity which begins in 6 years is a better deal.
Solution with interest rate of 10%. The value of the 6 year annuity with $10,000 coupons is given by

\[ PV = \frac{C}{r} \left(1 - \frac{1}{1 + r} \right)^t \]

\[ = \frac{10,000}{.1} \left(1 - \frac{1}{(1 + .1)} \right)^6 \]

\[ = $43,552.61 \]

The value of the perpetuity which begins in 6 years is equal to the value of a perpetuity today discounted back 6 years:

\[ PV = \frac{C}{r} \left( \frac{1}{1 + r} \right) \]

\[ = \frac{10,000}{.1} \left( \frac{1}{(1 + .1)} \right)^{10} \]

\[ = $38,544.32 \]

Therefore the annuity is a better deal.

5. You are considering the following one-year investments: (i) Bank A promises to pay 8% on your deposit compounded annually. (ii) Bank B promises to pay 8% on your deposit compounded daily. (iii) Bank C promises to pay 8% on your deposit compounded continuously. Compare the effective annual rate (EAR) on these investments.

Solution. The best option should be clear without any calculation since the quoted rate for all three options is 8% – the option which compounds more frequently results in a higher effective annual rate. However, to demonstrate the point, I will calculate the effective annual rates for all three options.

(i): 8% compounded annually results in an EAR of 8%.
(ii): 8% compounded daily results in an EAR of \((1 + \frac{8}{365})^{365} - 1 = 8.328\%\).
(iii): 8% compounded continuously results in an EAR of \(e^{.08} - 1 = 8.329\%\).

6. Suppose that stock FFM is traded in the NYSE and all trading in that stock is through a limit order book. Further, assume that at July 1, 2008 at 10:48 am the limit order book of FFM was empty. Following are the trade orders for FFM after 10:48 am

(a): What is the Bid-Ask spread at the following points in time: 11:08 am, 11:13 am, 11:17 am, and 11:20 am?

(b): What is the last transaction price as of the following points in time: 11:13 am, 11:17 am, and 11:20 am?
(c): How much money did investor A spend until 11:20 am. What about investor D?

(d): Does the bid-ask spread vary greatly over time? Does the transaction price vary greatly over time? How does this relate to the concept of bid-ask bounce

**Solution. To answer this question, I used the following table:**

<table>
<thead>
<tr>
<th>Time</th>
<th>Trade?</th>
<th>Investor</th>
<th>Buy\Sell</th>
<th>Order Type</th>
<th>Quantity</th>
<th>Price</th>
<th>Post-Bid</th>
<th>Post-Ask</th>
</tr>
</thead>
<tbody>
<tr>
<td>10:49</td>
<td>No</td>
<td>A</td>
<td>Buy</td>
<td>Limit Order</td>
<td>1,000</td>
<td>$29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10:53</td>
<td></td>
<td>B</td>
<td>Buy</td>
<td>Limit Order</td>
<td>500</td>
<td>$30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10:57</td>
<td></td>
<td>C</td>
<td>Buy</td>
<td>Limit Order</td>
<td>3,000</td>
<td>$33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11:01</td>
<td></td>
<td>D</td>
<td>Buy</td>
<td>Limit Order</td>
<td>2,000</td>
<td>$29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11:04</td>
<td></td>
<td>E</td>
<td>Sell</td>
<td>Limit Order</td>
<td>1,000</td>
<td>$32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11:07</td>
<td></td>
<td>F</td>
<td>Buy</td>
<td>Limit Order</td>
<td>5,000</td>
<td>$28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11:12</td>
<td></td>
<td>G</td>
<td>Sell</td>
<td>Market Order</td>
<td>500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11:16</td>
<td></td>
<td>H</td>
<td>Buy</td>
<td>Limit Order</td>
<td>500</td>
<td>$32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11:19</td>
<td></td>
<td>I</td>
<td>Sell</td>
<td>Market Order</td>
<td>1,000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a): Bid-ask spread is 2 at 11:08, 3 at 11:13, 3 at 11:17, and 3 at 11:20.

(b): The most recent transaction price is 30 at 11:13, 32 at 11:17, and 29 at 11:20.


(d): The bid ask spread can vary over-time, the size of which depends on market liquidity. The transaction price can also vary greatly over-time depending on the current difference between the bid-ask spread. The concept of bid-ask bounce refers to the bounce in the transaction price between the bid price and the ask price. The bid-ask bounce will be larger if the bid-ask spread is larger.
7. (Intended to make you think - give this question your best shot!) When a company, say IBM, sells shares in the primary market, it receives money directly, and therefore it has reason to care deeply about the price it receives. In contrast, when a share of IBM is sold by one investor to another in the secondary market, IBM itself is not directly involved. Why does, or should, IBM care about the price of its shares in the secondary market?

Solution. There are many different reasons why a company may care about its share price after it has already conducted its primary market transactions. At a most basic level, the share price reflects the current value of the company. Since the management of a company is appointed by shareholders to maximize the share price, the failure of the management to realize a favorable share price could result in the shareholders voting to replace management. To incentivize management to maximize share price, the compensation packages of managers is closely tied to the secondary market share price. Therefore managers care about maximizing the share price because it maximizes the value of their own personal compensation packages. If the stock performance is very poor, the company could become a takeover target. Poor performance also brings the company closer to default if the company has issued any debt. Once a default action is triggered, debt-holders can seize various control rights to maximize the value of debt, potentially at the expense of the value of the company’s share price. This is a brief list but if you are interested in these issues, you should consider taking a corporate finance class or a corporate governance class.