SOLUTIONS AND EXPLANATION FOR MINI-QUIZ 1

The original question is in bold below. I only graded parts (1) and (2) of the question as many of you seemed confused on the third section. Additionally, I realize I wasn’t clear that when I ask for the value of debt, this means the “market-value” of debt. Some of you reported the face value. I understand the confusion so I gave credit here as well if you got the value of equity correct.

A firm is financed with debt and equity. The debt has a face value of $8. Today a judge will determine if the firm is guilty of fraud. If the judge finds the firm guilty, it will be worth $3; otherwise, the firm will be worth $9. The market believes there is a 1/4 chance the firm will be found guilty.

Define the following notation:
- \( V \) = The value of the firm
- \( F \) = The face value of debt
- \( D \) = The value of debt (or the market value of debt)
- \( E \) = The value of equity

Here is what we know from the text:
- \( \Pr(\text{firm is guilty}) = \frac{1}{4} \)
- \( \Pr(\text{firm is not guilty}) = \frac{3}{4} \)
- \( V = \begin{cases} 3 & \text{if verdict is guilty} \\ 9 & \text{if verdict is not guilty} \end{cases} \)
- \( F = 8 \)

You need three facts to do the questions:
- Firm value = value of debt plus value of equity: \( V = D + E \) (Always!)
- Debt holders are paid first and thus have a claim on the firm assets up to the face value of debt: \( D = \min(V, F) \)
- Before an outcome is realized, the value of a security is equal to it’s expected payoff.

(1) Suppose the judge finds the firm guilty.
- What is the value of debt?
  \[ = \min(V, F) = \min(3, 8) = 3 \]
- What’s the value of equity?
  \[ = \max(0, V - F) = \max(0, -5) = 0 \]

(2) Suppose the judge finds the firm not guilty.
- What is the value of debt?
  \[ = \min(V, F) = \min(9, 8) = 8 \]
- What’s the value of equity?
  \[ = \max(0, V - F) = \max(0, 1) = 1 \]
(3) Before the verdict is cast,

• What is the value of debt?
  
  \[ \text{Value of debt} = \Pr(\text{guilty}) \cdot D_{\text{guilty}} + \Pr(\text{not guilty}) \cdot D_{\text{not guilty}} \]
  
  \[ = \frac{1}{4} \cdot 3 + \frac{3}{4} \cdot 8 \]
  
  \[ = \frac{27}{4} \]

• What’s the value of equity?
  
  \[ \text{Value of equity} = \Pr(\text{guilty}) \cdot E_{\text{guilty}} + \Pr(\text{not guilty}) \cdot E_{\text{not guilty}} \]
  
  \[ = \frac{1}{4} \cdot 0 + \frac{3}{4} \cdot 1 \]
  
  \[ = \frac{3}{4} \]

If you haven’t been getting emails from me, what is your email address?