Information gathering externalities for a multi-attribute good

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June, 2011

Abstract

Most goods and services vary in numerous dimensions. Customers choose to acquire information to assess some characteristics and not others. Their choices affect firms’ incentives to invest in quality and so lead to indirect externalities in consumers’ choices. We illustrate these ideas by characterizing a model in which a monopolist invests in the quality of a product with two characteristics, and consumers are heterogeneous ex-ante. Indirect externalities in information gathering arise because consumers do not consider the effects on the firm’s investment incentives when choosing which information to acquire. Therefore, a fall in the cost of acquiring information, by changing the pattern of consumers’ information gathering and thereby the firm’s investment, can paradoxically reduce consumer surplus, profits, and welfare.

JEL: D83, L15

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1We thank the Editor, and referees for their useful suggestions and particularly helpful audiences at the 2005 NY Commuters’ IO day, Alicante, Chicago GSB, Kellogg School of Management, Oxford University, Southampton University, University of Edinburgh, Technion, UCL, Université de Cergy-Pontoise, ESSET 2005 and 2006, HOC 2006, CECT 2006, 6th Annual Duke/Northwestern/Texas IO Theory Conference, Mark Armstrong, Mariagiovanna Baccara, Matthew Bidwell, Justin Johnson, Michael Katz, Alessandro Lizzeri, Meg Meyer, David Myatt, Volker Nocke, Laura Veldkamp and Lawrence White. Contact info: Bar-Isaac: heski@nyu.edu; Department of Economics, Stern School of Business, NYU, 44 West 4th street 7-73, NY, NY 10012 USA; Caruana: caruana@cemfi.es; Casado del Alisal 5, 28014 Madrid, Spain; and Cuñat: v.cunat@lse.ac.uk. LSE, Houghton Street, WC2A 2AE, London, UK.
1 Introduction

When shopping for goods with multiple characteristics that are not easily observable, consumers’ decisions on what to observe and whether to buy affect the type of good offered and, indirectly, other consumers’ welfare. Think of a prospective student deciding on a business school. She is likely to be concerned by her expected salary upon graduation and also by long-term career prospects. Business schools can tailor their programs in ways that affect these differently. Historically, students have relied on talking to alumni and on other independent research to gain relevant information. Currently, it is much quicker and easier to consult guides and rankings which have made it cheaper for students to learn some programme aspects, such as average salaries upon graduation or the school’s facilities. At the same time, they provide little information on some other dimensions, such as long term career prospects or opportunities in particular specializations. As a consequence, a greater fraction of potential students may only investigate and make decisions based on average starting salaries or facilities. This has induced business schools to invest in relationships with (particular kinds of) recruiters and otherwise tailor their programs to the rankings on which potential students are rationally basing their decisions. As noted by academic and popular commentators, the overall effect of this process may be detrimental for students.\footnote{See Brickley and Zimmerman (2001) and DeAngelo, DeAngelo and Zimmerman (2005). Schatz (1993) states that: “the rankings resulting from the surveys are too one-dimensional to be taken seriously. Most likely, no single MBA program is best for everyone, and almost every program is best for someone. The match has to be individualized.” (See also Education & Social Science Library (2005) for a wide-ranging survey of critiques of school rankings.) In itself this observation need not imply adverse welfare consequences; however, Schatz argues that: “people foolishly tend to believe that there is significance to the order in which the schools appear”. Our model, without assuming that people are foolish, shows that the rankings are relevant but, through their easy availability, can have adverse consequences.}

This is an example of a more general phenomenon. There are many goods and services that are, by nature, multidimensional. Given that it is costly to verify all the characteristics of a good, consumers often make purchase decisions based only on partial information. In addition, firms have to decide whether and how much to invest in the quality of different dimensions. These decisions take into account consumers’ search behaviour. As a consequence there are, in effect, indirect externalities among consumers: In equilibrium, their individual choices affect firm investments and thereby other consumers.\footnote{Strictly speaking these are quasi-externalities as they arise from anticipated equilibrium behaviour.} Our contribution in this paper is to highlight this mechanism, its implications for investment incentives, consumer surplus, and welfare; and, in particular, to show that a drop in the costs of acquiring information can lead to a welfare loss.

We consider a market in which a monopolist sells a good with two characteristics. The quality of each characteristic is uncertain. Before deciding whether or not to buy the good, a continuum of customers can decide to observe either realized quality at a cost. This cost should be interpreted as incorporating the time, effort, and sometimes explicit costs of obtaining and thinking through the relevant information. Aggregated over all potential consumers, such costs may be substantial. Meanwhile, the firm can invest to improve the likelihood of a high quality realization.

A simple revealed preference argument guarantees that if the firm’s behaviour is fixed, then a fall in the costs of assessing one characteristic unambiguously raises consumers’ welfare. This is not the case when the firm can vary its investments in response to the fall in search costs.
Consumers’ expected choices affect the producer’s incentives to invest in the quality provision of each of the dimensions. Consider a group of consumers whose purchase decision is contingent on the information they gather on a certain dimension. This behaviour provides the firm the incentives to make investments in that characteristic. Instead, when assessment costs change, the same group of consumers might switch to a different assessment that is not as effective in appropriately disciplining firm behaviour. This indirect effect of a fall in assessment costs can have negative consequences that overwhelm the direct effect that assessment is cheaper.

Customers cannot coordinate and commit to their choices and do not take into account their cumulative effect on the firm’s behaviour. There are many “missing” contracts in this market environment: between the firm and individual consumers but also between different consumers. If a consumer could individually, credibly commit to the monopolist on the information that she intended to gather and how she was going to use it, this would have no effect at all on the firm since there are a continuum of consumers. To change the firm’s behaviour requires commitment by a mass of consumers in a coordinated manner. The key contractual failure, therefore, is that consumers have no means to coordinate their assessment and purchase behaviour. It is in this sense that we focus on “externalities.”

1.1 Related Literature

Our main results build on the fact that consumers’ and firm behaviour changes as the costs of information acquisition change. Similarly, Jin and Leslie (2003) provide evidence that as information on restaurant hygiene becomes more easily available, restaurants change their behaviour, presumably in response to changes in consumer behaviour. More broadly, our results are related to a broad literature on the effects of information disclosure. Dranove and Jin (2010) provide a recent review of both theoretical and empirical work.

Dranove and Satterthwaite (1992) presents a theoretical framework, and Dranove et al. (2001) empirically highlights negative welfare consequences of quality “report cards” on physicians. Their modelling approach is somewhat different and, in particular, their environment does not contemplate the active role of consumers when gathering information. While we consider active consumers who choose which information to acquire (i.e. which attribute to focus on), they consider consumers who search among sellers but at each seller visited, consumers receive information exogenously. In another paper on information provision in health markets, Lu (2009) presents empirical findings consistent with our results. Lu shows that, following the introduction of mandatory report cards, quality in nursing homes improved somewhat in some of the reported dimensions, but deteriorated in the unreported ones.

Although, our work, in considering multi-dimensional goods is reminiscent of the multi-tasking literature, as in Holmström and Milgrom (1991,1994), the model and mechanisms are quite different. Holmström and Milgrom consider a contractual, moral hazard, multi-task environment. They

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4 A similar effect is at play in Varian’s (1980) model of informed and uninformed consumers, where the presence of fully informed customers leads to lower prices for all consumers of a good of fixed quality. In our model, however, the good has multiple attributes on which consumers can get informed, so that different agents can become informed on different attributes. This leads, for example, to non-monotonic effects on profits.
show that it is difficult to provide strong and well-balanced incentives: Under the assumption that efforts are substitutes in the cost function, strengthening incentives in one dimension leads to lower efforts on others. Our model differs from theirs because the economic forces we describe result from an environment in which agents interact in a market setting, and all parties lack important commitment abilities. This contrasts with their optimal contracting Principal-Agent approach. As such, firm’s incentives arise endogenously from decentralized individual consumers’ decisions. Furthermore, we do not assume substitution in the firm’s investment costs; instead, in our baseline model, there is effort substitution in consumer’s information-gathering. Moreover, in Section 5.4 we argue that substitutability in search effort on the consumer side can arise even if the costs of gathering information on different attributes are separable.

A recent literature addresses the possibility that some features of a good are more salient than others. These papers tend to assume that salience is exogenous and, often, that consumers are naive in their beliefs on non-salient aspects. In contrast, in our model “saliency” is the result of the endogenous decisions of fully rational consumers who take into account the costs and benefits of information acquisition. Further, we highlight the role of externalities among consumers. Hastings et al. (2007) provide some support for our approach. They conduct a field experiment in which parents are provided with simplified fact-sheets about schools. They show that this simplified information had a significant impact on parental school choice. Most relevant for this paper, they argue that their results are more a consequence of the reduction in the costs of obtaining the information provided on the fact-sheets rather than a consequence of the saliency of this information.

Several papers have highlighted qualitatively different mechanisms through which allowing for more information can be damaging for welfare. Schlee (1996) shows that making more information publicly available might also benefit strategic rivals. Kessler (1998) illustrates that committing to an information structure can change the resulting choice of an explicit contract. Meuer and Stahl (1994) show that, in a competitive market, better information might lead to greater differentiation, thereby softening price competition to the detriment of consumers. Finally, Glazer and McGuire (2005) show that revealing a quality index rather than its constituents can reduce the firm’s feasible choices in such a way as to counter-act monopoly power.

2 Model

Consider a monopoly producer of a single complex good, which is exogenously priced at \( p > \frac{1}{2}. \)

This good has two different characteristics or dimensions \((a \text{ and } b)\). The good can be of high or low quality in each of its two dimensions. The effective quality realization of each of them is independent and stochastic. For one of the characteristics the probability of high quality is fixed, but for the other it depends on the firm’s investment. Consumers incur costs in determining the

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5Similar results obtain in an environment with non-contingent contracts when incentives arise through career concerns, as in Dewatripont et al. (1999). However, the career concerns framework does not address the indirect externalities across employers (the equivalent of our consumers) at the heart of this paper.

6Ellison (2006) provides a review of this literature.

7As we discuss below, similar effects and qualitative results arise when prices are endogenous. The restriction that \( p > \frac{1}{2} \) limits the number of cases to consider and simplifies exposition.

8It is reasonable to think that firms choose how much to invest in each of the dimensions. We consider this case in Section 5.2.
quality of an attribute before making a purchase decision. The quality realization can be viewed either as reflecting the nature of the production technology or, in our preferred interpretation, as reflecting idiosyncratic consumer tastes. In this latter case, higher investments in quality cater to a wider range of such tastes.

Timing is as follows:

1. The firm invests $x$ in the quality of characteristic $b$.

2. Each customer, without observing the firm investment decision, chooses to assess the quality realization on characteristic $a$, characteristic $b$, or none.

3. Each customer chooses whether or not to buy the good.

Specifically we make the following assumptions with respect to the firm and consumers:

The firm invests $x \in [0, 1]$ in the quality of characteristic $b$ at a cost of $c(x)$. We assume $c(x)$ to be differentiable, convex, and with $c'(0) = 0$ and $c'(1) = \infty$.\footnote{More generally it is enough to assume that $c'(1)$ is large enough so that in equilibrium $x < 1$.} This is a sunk cost that is incurred regardless of the number of sales. The marginal cost of production is 0. Production leads to consumer-specific stochastic quality realizations, where $q_i \in \{0, 1\}$ denotes the quality realization of the good in characteristic $i \in \{a, b\}$. While the probability of high quality, $q_a = 1$, in attribute $a$ is exogenous, for characteristic $b$ it depends on the firm’s investment. Specifically, we suppose that $\text{Prob}(q_a = 1) = \frac{1}{2}$ and $\text{Prob}(q_b = 1) = x$. Note that stochastic realizations of quality give consumers some incentives for assessing it.

There is a continuum of ex-ante heterogeneous consumers indexed by $t$, where $t$ is continuously distributed on $[0, 1]$ according to some distribution function $F(\cdot)$. That is, all consumers value both dimensions of the good, but consumers with a high $t$ give more importance to dimension $a$ while consumers with a low $t$ give more importance to dimension $b$.\footnote{The role of consumer heterogeneity in the modelling is partly instrumental, as it smooths out consumer behavior and facilitates the analysis. It also allows us to tackle the issue of consumer diversity on welfare (see Section 5.3).}

Consumers cannot directly observe the quality of a good in each dimension; however, they can incur costs to assess the quality realization of either dimension. In particular, by incurring a cost $A$, a consumer can perfectly determine whether the quality on dimension $a$ is high or low, and by incurring a cost $B$ she can perfectly learn the quality on dimension $b$.\footnote{For simplicity we assume perfect monitoring. An imperfect monitoring technology would change none of the qualitative results of the model.}

We suppose that the consumer can search either in one dimension or the other. She may also choose to buy without assessing quality or choose to neither assess quality nor buy. We preclude the possibility of assessing both dimensions for simplicity; though, as discussed in Section 5.4, allowing for this would not overturn our qualitative results. Note that it is only worthwhile for a consumer to incur costs to assess quality if the information learnt has an impact on her purchase behaviour, so that she buys when she finds high quality, but not otherwise.

If the realization of the quality and the consumer’s assessment behaviour leads the consumer to
purchase a good of quality \((q_a, q_b)\) at the price \(p\), her overall utility can be written as:

\[
U_t = tq_a + (1 - t)q_b - p - AI_{s=a} - BI_{s=b}
\]  

(1)

where \(I_{s=a}\) takes the value 1 if the consumer assessed dimension \(a\), and 0 otherwise. \(I_{s=b}\) is similarly defined. If the consumer does not buy the good, her utility is simply

\[
U_t = -AI_{s=a} - BI_{s=b}.
\]  

(2)

3 Equilibrium analysis

We characterize the optimal behaviour, first of consumers, then of the firm, and bring these together to determine equilibrium.

3.1 Consumers

In general, a consumer of type \(t\) has four choices: assessing characteristic \(a\) or \(b\) (and subsequently purchasing if they find high quality), buying with no assessment, and simply not buying the good. We write the net utilities associated with these possibilities for a type \(t\) as \(U_a(t)\), \(U_b(t)\), \(U_n(t)\) and 0, respectively; where

\[
U_a(t) = \frac{1}{2}(t + (1 - t)x - p) - A,
\]

(3)

\[
U_b(t) = x(1 - t + \frac{1}{2}t - p) - B, \text{ and }
\]

(4)

\[
U_n(t) = \frac{1}{2}t + (1 - t)x - p.
\]

(5)

Among these four alternatives, she chooses whichever gives highest expected utility. The fraction of consumers who assess characteristic \(b\) is crucial for determining the firm’s behaviour. Suppose that there is some consumer \(T\) who is indifferent between assessing \(b\) and the best of her other choices. Assuming that \(T\) is interior it is implicitly defined by the equation:

\[
U_b(T) = \max\{U_a(T), U_n(T), 0\}.
\]

(6)

Given that \(\frac{\partial U_a}{\partial t} > \frac{\partial U_n}{\partial t}, 0 > \frac{\partial U_b}{\partial t}\) it is easy to see that all consumers with \(t < T\) prefer to search on dimension \(b\) (and subsequently purchase if and only if they discover high quality on that dimension).

Note that the threshold \(T\) can take any value between zero and one. A value \(T = 0\) corresponds to no agents assessing dimension \(b\), while \(T = 1\) corresponds to all agents doing so.

3.2 Firm

The firm chooses an investment in quality \(x\), in order to maximize profits given the anticipated equilibrium behaviour of consumers. Note that \(x\) affects the purchasing behavior of consumers that assess dimension \(b\), but has no impact on those assessing on \(a\) and those who do not assess. In particular, the proportion \(F(T)\) of consumers that search on dimension \(b\) only buy when they find
high quality on that dimension (which occurs with probability \( p \)). If we denote by \( D_a \) the mass of purchases from consumers that have assessed characteristic \( a \), and by \( D_n \) the purchases from those that do not assess at all, the firm’s problem can be expressed as

\[
\max_x \Pi = (F(T)x + D_a + D_n)p - c(x).
\]

It follows that the optimal investment \( x \), determined by the first order condition, can be expressed as:

\[
F(T)p = c'(x).
\]

### 3.3 Equilibrium

In equilibrium firm investment and the mass of consumers assessing in \( b \) is characterized by the solution to equations (6) and (8). The behaviour of consumers in the range \([T, 1]\) depends on the alternative to which consumer \( T \) is indifferent. Thus, different sorts of equilibria may arise. In Appendix A we provide a full characterization. Here, we focus on the case in which all consumers decide to assess; some on \( a \) and the rest on \( b \). Thus, here we assume that \( U_a > \max\{U_n, 0\} \) and from (6), we obtain:

\[
T = p - x(2p - 1) + 2(A - B).
\]

Given that \( \frac{\partial U_a}{\partial t} > \frac{\partial U_n}{\partial t} > 0 > \frac{\partial U_b}{\partial t} \), all agents with \( t > T \) prefer to search on \( a \). Thus an equilibrium in which all consumers assess one dimension or the other is characterized by conditions (9) and (8).

We can establish the following lemma:

**Lemma 1** There exists at most one equilibrium in which all consumers either assess one dimension or the other. A necessary condition for this equilibrium to exist is that \( 2(A - B) + p \geq 0 \).

**Proof.** Substituting for \( T \) from (9) into (8), we obtain

\[
pF(p - x(2p - 1) + 2(A - B)) = c'(x).
\]

Note that the left hand side of (10) is decreasing in \( x \) (since \( p > \frac{1}{2} \)) and the right hand side is increasing in \( x \). Thus there is at most one solution for \( x \). Given that \( c'(0) = 0 \) and \( c'(1) = \infty \), a solution exists iff the left hand side is higher than the right hand side when evaluated at \( x = 0 \). This is satisfied iff \( 2(A - B) + p \geq 0 \). 

Note that \( 2(A - B) + p \) can be negative if \( B \) is much larger than \( A \). In this case \( T = 0 \); that is, no consumer would assess characteristic \( b \), and, therefore, the firm investment is 0. In the rest of the paper we focus on equilibria with positive investment provisions by the monopolist.

### 3.4 Multiplicity

In the equilibrium analyzed above, the firm invests at the equilibrium level because of the fraction of consumers assessing dimension \( b \). However, there may be other coexisting equilibria for the same parameter values. For example, if consumers expect no investment by the monopolist, none would assess dimension \( b \), which makes no investment the best response for the firm. Thus, the source of
multiplicity can be understood as a coordination issue: consumer expectations of high investment lead many of them to search on dimension \( b \), which in turn triggers firm incentives to invest. On top of this non-investment equilibrium, there may be other ones of the types studied in Appendix A.

In order to deal with the multiplicity of equilibria we use Pareto-dominance as a selection criterion. The following proposition shows that this criterion selects a unique equilibrium. Given the lack of conflict of interest, its use seems reasonable.

**Proposition 2** There exists one equilibrium that Pareto dominates all others. Moreover, this is the one with highest quality investment by the part of the monopolist.

**Proof.** First, note that there is only one equilibrium for any given investment level \( x \). This is immediate, as every consumer \( t \) (except for the indifferent ones, which have measure zero) has a unique best response strategy. Now, consider any two equilibria \( 1 \) and \( 2 \) with different investment levels \( x_1 > x_2 \).

First, each consumer is at least as well off with firm investment \( x_1 \) as with \( x_2 \). This follows since, in equilibrium 1 a consumer could deviate to behave as in equilibrium 2, and by doing so would achieve at least the same utility level as in equilibrium 2.

Second, note that \( \frac{\partial U_a}{\partial x} = \frac{1}{2}(1 - t); \frac{\partial U_b}{\partial x} = (1 - \frac{1}{2}t - p); \text{ and } \frac{\partial U_n}{\partial x} = (1 - t). \) Thus we have that \( \frac{\partial U_a}{\partial x} > \frac{\partial U_b}{\partial x} > \frac{\partial U_n}{\partial x} > 0. \)

Third, note that \( S_1 \geq S_2 \). The logic here makes use of the marginal effects computed above: (i) If a given type \( t \) assesses \( a \) in equilibrium 2 then in equilibrium 1 she will either assess \( a \) or buy without assessment; (ii) If she assesses \( b \) in 2, she will assess in \( b \) in 1 as well (because condition (8) shows that on equilibrium \( x \) and \( T \) are positively related); (iii) If she buys without assessment in 2 then she would do the same in 1. In all cases, since \( x_1 > x_2 \) sales in equilibrium 1 can be no lower than sales in equilibrium 2.

Finally, we show that \( \Pi_1 \geq \Pi_2 \). Suppose for contradiction that \( \Pi_2 > \Pi_1 \). Then in equilibrium 1, given the assessment behaviour of consumers, the firm has a profitable deviation to invest \( x_2 \). Sales under this deviation can be no lower than the sales in equilibrium 2: the investment is the same and (for the same reasons that \( S_1 > S_2 \)) consumers are more prone to assess and buy. Therefore deviation profits \( \Pi_D = pS_D - \frac{x_2^2}{4} \geq pS_2 - \frac{x_2^2}{4} = \Pi_2 > \Pi_1 \), which provides the contradiction. ■

4 Comparative Statics on Assessment Costs

In this section, we restrict attention to the case in which the Pareto dominant equilibrium is of the type described in Lemma 1. That is, in which all consumers are assessing, some dimension \( a \) and some \( b \). In this case, the indifferent consumer \( T \) is characterized by equation (9).

Our primary interest is in the comparative statics of welfare with respect to \( A \), the cost of assessing attribute \( a \).\(^{12}\) Ultimately we are interested in profits, consumer surplus and welfare;

\(^{12}\)If similar comparative statics are applied to \( B \), consumer surplus can be shown to be monotonically decreasing in \( B \); however, profits and welfare can be non-monotonic in \( B \). Given that quality investment in dimension \( a \) is exogenous, this analysis is not that meaningful.
however, it is useful to begin by considering the comparative statics of the endogenous parameters $T$ and $x$. By applying the implicit function theorem to equations (8) and (9), we obtain

\[
\frac{dx}{dA} = \frac{2f(T)p}{c''(x) + pf(T)(2p - 1)} > 0 \tag{11}
\]
\[
\frac{dT}{dA} = \frac{2c''(x)}{c''(x) + pf(T)(2p - 1)} > 0 \tag{12}
\]

where the inequalities are immediate on noting that $f(\cdot)$ is a density function, $c(\cdot)$ is convex and $p > \frac{1}{2}$. Thus, as it becomes less expensive to gather information on attribute $a$, fewer consumers gather information on attribute $b$ (the marginal consumer $T$ goes up), while average quality $x$ decreases. Now we can examine the effect on firm profits:

**Proposition 3** Firm profits can be non-monotonic in $A$, the cost of assessing attribute $a$. In this case, there exist a value $\tilde{A}$ such that profits are decreasing below it and increasing above.

**Proof.** We can write

\[
\frac{d\Pi}{dA} = \frac{\partial\Pi}{\partial T} \frac{dT}{dA} + \frac{\partial\Pi}{\partial x} \frac{dx}{dA} = \frac{\partial\Pi}{\partial T} \frac{dT}{dA}, \tag{13}
\]

since $\frac{\partial \Pi}{\partial x} = 0$, as $x$ is chosen to maximize the firm’s profits. Thus,

\[
\frac{d\Pi}{dA} = p(x - \frac{1}{2}) \frac{dT}{dA} \tag{14}
\]

Given that $\frac{dT}{dA} > 0$, it follows that $\frac{d\Pi}{dA} > 0$ if and only if $x > \frac{1}{2}$. If profits are non-monotonic, given that we have established that $\frac{dx}{dA} > 0$, we can conclude that there exist a value $\tilde{A}$ for which the investment on equilibrium is $x = \frac{1}{2}$, and that profits are decreasing for $A < \tilde{A}$ and increasing for $A > \tilde{A}$. $\blacksquare$

A fall in the cost of assessing $a$ drives more consumers to assess characteristic $a$, and fewer to assess $b$. This increases firm profits whenever high quality is more likely to be realized on characteristic $a$ (that is where $x < \frac{1}{2}$), and decreases profits in the opposite case. Thus $\frac{d\Pi}{dA}$ may be either negative (when $x$ is sufficiently low, which is associated with low $A$ and many consumers searching on dimension $a$) or negative (in opposite circumstances).

Next we turn our attention to consumer surplus:

\[
CS = \int_0^T (x(1 - t + \frac{t}{2} - p) - B)f(t)dt + \int_T^1 (\frac{1}{2}(t + (1-t)x - p) - A)f(t)dt, \tag{15}
\]

where the first integral corresponds to the fraction $T$ of consumers who search on dimension $b$ and the second corresponds to the fraction of consumers who search on dimension $a$.

**Proposition 4** Consumer surplus is in general non-monotonic in $A$, the cost of assessing attribute $a$. A sufficient condition that ensures at most one turning point, whereby consumer surplus is first decreasing and then increasing, is $c''(.) \leq 0$. 

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Proof. Taking the total derivative with respect to $A$ yields:

$$\frac{dCS}{dA} = \frac{\partial CS}{\partial A} + \frac{\partial CS}{\partial T} \frac{dT}{dA} + \frac{\partial CS}{\partial x} \frac{dx}{dA}. \quad (16)$$

We consider each term of this expression in turn. First, the direct effect $\frac{\partial CS}{\partial A} = -(1 - F(T))$ is negative. For those assessing characteristic $a$ an increase in the cost of doing so reduces their utility.

Next, $\frac{\partial CS}{\partial T} = 0$, that is, there is no direct effect on consumer surplus of a marginal increase in $T$. All it causes is the shift of marginal $T$ consumer. But this consumer was indifferent between searching on one or the other dimension, thus her welfare is marginally not affected.

Finally, the indirect effect of raising $A$ through the firm’s investment $x$ on consumer surplus is unambiguously positive. The effect of raising $A$ on the equilibrium investment $x$ is positive, as shown above in equation (11). Given this, for those assessing $b$, there is a greater likelihood of finding quality, and for those assessing $a$, there is a higher likelihood of enjoying quality on the dimension that they do not assess. More formally,

$$\frac{\partial CS}{\partial x} = \int_0^T (1 - \frac{t}{2} - p)f(t)dt + \int_T^1 \frac{1}{2}(1-t)f(t)dt > 0. \quad (17)$$

The second integral is positive trivially. Consider the first: $(1 - \frac{t}{2} - p)$ must be positive in the range $[0, T]$ for those consumers to have an incentive to search.\(^{13}\)

Putting these elements together, overall we can write:

$$\frac{dCS}{dA} = -(1 - F(T)) + \frac{\partial CS}{\partial x} \frac{dx}{dA}. \quad (18)$$

Since the first term is negative and the second term is positive, in general, we cannot sign $\frac{dCS}{dA}$, and, indeed, it may be non-monotonic.

However, note that consumer surplus is convex if

$$\frac{d^2CS}{dA^2} = f(T) \frac{dT}{dA} + \frac{d}{dA} \left( \frac{\partial CS}{\partial x} \frac{dx}{dA} \right) + \frac{\partial CS}{\partial x} \frac{d^2x}{dA^2} \quad (19)$$

First note that doing some algebra and using equation (11) one can show that: $f(T) \frac{dT}{dA} + \frac{d}{dA} (\frac{\partial CS}{\partial x} \frac{dx}{dA}) = f(T) \frac{dT}{dA} (1 - \frac{2p - 1}{2} \frac{dx}{dA}) = f(T) \frac{dT}{dA} (1 - \frac{p(2p - 1)}{c''(x) + p(2p - 1)}) > 0$ since $c''(x)$. Thus, since $\frac{\partial CS}{\partial x} > 0$, a sufficient condition that ensures that $\frac{d^2CS}{dA^2} > 0$ is that $\frac{d^2x}{dA^2} = -\left(\frac{2p}{c''(x) + p(2p - 1)}\right)^2c'''(x) \frac{dx}{dA} > 0$, which is satisfied when $c''' \leq 0$. The convexity of consumer surplus in $A$ is sufficient to guarantee at most one turning point, and that if $CS$ is non-monotonic it must be first decreasing and then increasing in $A$. ■

Note that when $T$ is close to 1 (that is, when $A$ is high with respect to $B$ and most consumers are searching on attribute $b$) the direct effect of a change in $A$ is negligible. The indirect effect of decreasing $A$ is to decrease $x$, and consequently to reduce consumer surplus. Conversely, as can

\(^{13}\)From Equation (6) we know that $x(1 - \frac{t}{2} - p) \geq B > 0$ at $t = T$. Further, as this condition is linear in $t$, it also holds in the range $[0, T)$. 

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readily be verified, if most consumers are searching on dimension \( a \), the direct effect dominates and consumer surplus increases if \( A \) falls.

We can finally turn to welfare. In the discussion above we have argued that both profits and consumer surplus are likely to be decreasing as \( A \) falls when the cost of assessing quality on dimension \( a \) is relatively high and the cost of assessing quality on dimension \( b \) is relatively low; and increasing when \( A \) falls if \( A \) is relatively low and \( B \) is relatively high. Thus, as an immediate consequence of the two previous propositions we get:

**Proposition 5** Welfare can be non-monotonic in the cost of assessing attribute \( a \).

**Proof.** \( W = CS + \Pi \) and so \( \frac{dW}{dA} = \frac{dCS}{dA} + \frac{d\Pi}{dA} \). Since both \( \frac{dCS}{dA} \) and \( \frac{d\Pi}{dA} \) can be non-monotonic and first increasing then decreasing in \( A \), then \( W \) can be non-monotonic and increasing for small values of \( A \) and decreasing for large values. ■

Intuitively, welfare increases as \( A \) falls when the indirect effects of a lower assessment costs are significant as compared to the direct effects of lower costs of assessing characteristic \( a \) for those who do so. These indirect effects result from a lack of commitment by all parties: the firm and every single consumer. If \( A \) falls, all parties expect others to pay “less attention” to dimension \( b \). As a consequence, in equilibrium there is both less investment by the firm and less assessment by consumers devoted to characteristic \( b \). This reduces the value of the resulting good and can end up hurting everyone. As argued below, it is easy to find cases for which welfare, consumer surplus and profits are non-monotonic in \( A \).

The non-monotonicity of consumer surplus and welfare in \( A \) can be interpreted as arising as a consequence of “externalities” in the following sense: Suppose that all agents could commit and coordinate their behaviour.\(^\text{14}\) Then at \( A' < A \), agents could simply behave as if the cost of assessment were \( A \). In this case, the firm’s investment would be at the same level as if the cost were \( A \), and the only difference in consumer surplus (and welfare) at \( A' \) as compared to at \( A \) would be the direct effect of the lower cost of assessment: \( (A - A')(1 - T) \). It is the failure of agents to coordinate on this scheme, or their inability to commit to it, that leads the firm to invest less in equilibrium. This change in investment can overcome the direct effect and lead to lower consumer surplus and welfare, despite the fact that assessment costs are lower.

The previous comparative static results (Propositions 3, 4 and 5) prove that there is in principle scope for non-monotonicities. It is nevertheless very easy to constrict examples that prove that these are prevalent. For instance, consider the particular case where \( c(x) = \frac{x^2}{4} \) and consumer types are uniformly distributed on \([0, 1]\) so that \( F(t) = t \). One can then obtain closed-form solutions for the equilibrium in which all consumers are assessing, and illustrate how profits, consumer surplus and welfare vary non-monotonically with the assessment cost \( A \).\(^\text{15}\) In this equilibrium, profits

\(^{14}\) Note that both aspects, commitment and coordination, are key. Since all consumers are atomistic, individual commitment by consumers would have no effect: no consumer would gain from committing to anything other than her preferred equilibrium behaviour.

\(^{15}\) Concretely, the strategies \( T = \frac{2(A-B)+p}{1-2p+4p^2} \) and \( x = 2p\frac{2(A-B)+p}{1-2p+4p^2} \) define the equilibrium.
decrease when the cost of assessing attribute $a$ falls if and only if

$$A > B + \frac{1 - 2p}{8p}.$$  \hspace{1cm} (20)

That is, firm profits decrease when the cost of assessing attribute $a$ falls as long as the cost of assessing attribute $a$ is sufficiently high, the cost of assessing attribute $b$ is sufficiently low, and the price $p$ is high enough. Consumer surplus falls when the cost of assessing attribute $a$ falls if and only if

$$A > B + \frac{2p - 1}{2}(2p(3p - 1) + (2p - 1)^3).$$ \hspace{1cm} (21)

Thus consumer surplus decreases when the cost of assessing attribute $a$ falls, as long as this cost is sufficiently high, the cost of assessing attribute $b$ is sufficiently low and the price $p$ is not too high. Finally, welfare decreases when the cost of assessing attribute $a$ falls if and only if

$$A > B + \frac{(2p - 1)^2}{2}(3p + (2p - 1)^2)} \times \frac{2}{(4p^2 + 1)}.$$ \hspace{1cm} (22)

In particular, this requires that the cost of assessing attribute $a$ is sufficiently high, the cost of assessing attribute $b$ is sufficiently low and the price $p$ is not too high. It is easy to find parameters for $B$ and $p$ where such non-monotonicities arise.\(^{16}\)

5 Discussion

5.1 Endogenous prices

There are many interesting markets, for example in the area of education and healthcare, where prices are either literally exogenously determined or where its determination is severely constrained. In these cases the assumption of exogenous prices may be a reasonable approximation. However, it is also natural to consider markets where prices are endogenously determined.

The model of Section 2 can be extended to encompass this feature: Suppose that the monopolist first chooses the price $p \in \mathbb{R}$, which becomes public knowledge. Then we repeat the timing of the previous model: the firm decides on investment, which is not verifiable, and consumers take their assessment and purchase decisions.\(^{17}\) Thus, Section 2 can be seen as the equilibrium for the subgame once a particular $p$ has been chosen.

The drawback of this extended model is that a full analytical analysis such as the one we did previously is not feasible. One needs to make particular assumptions on the cost technology $c(\cdot)$ and the distribution of consumers $F(\cdot)$ in order to numerically compute equilibria. As an illustrative exercise we consider the case of $c(x) = \frac{x^2}{2}$ and $F(t) = t$. Note that the only remaining parameters are the verification costs $(A, B)$. As above, we apply the Pareto-criterion to select a unique equilibrium

\(^{16}\)For example, at $p = \frac{1}{3}$ and $B = \frac{1}{41}$, profits are non-monotonic around $A = \frac{1}{127}$, consumer surplus around $A = \frac{397}{2026}$ and welfare around $A = \frac{397}{6100}$.

\(^{17}\)Note, that if the timing is reversed, so that the firm invests first and then chooses price, the outcomes identified here will continue to be equilibria.
for each such subgame, and, working backwards, an overall unique equilibrium.\footnote{Note that Proposition 3 applies whenever the monopolist chooses $p > \frac{1}{2}$, for prices below $\frac{1}{2}$, we solve for all equilibria and verify that the Pareto criterion selects a unique equilibrium.} Thus, we can compute equilibrium investments, prices, and consumer assessment and purchasing behaviour for any values of these costs $(A, B)$. This exercise allows us conclude that similar qualitative results to those of Section 2 still arise. In particular, welfare, consumer surplus and profits can be non-monotonic in the costs of assessment.

As above (see Appendix A for details), there are three potential types of equilibria that can emerge on the basis of consumer behaviour. First, in a “no investment” equilibrium no consumer assesses dimension $b$ (and so there is no firm investment). Second, we call an equilibrium an “all assess” equilibrium when all agents assess either one dimension or another (this is the one analyzed in detail in Section 2). Finally, there are equilibria with investment where some consumers do not assess (some or all of these may always buy, or never buy), we term such an equilibrium a “some do not assess” equilibrium. For any values of $(A, B)$ we calculate which kind of equilibrium arises. The results are summarized in the left panel of Figure 1, where the horizontal axis represents different values of $A$ and the vertical axis different values of $B$.

Figure 1: Equilibrium types and welfare levels that arise depending on the assessment costs $A$ and $B$.

The bottom left region corresponds to all-assess equilibria, where all agents assess either the $a$ or the $b$ dimension. When we increase the assessment cost for dimension $a$ we move to some-buy-without-assessment equilibria: some intermediate consumers buy without assessing either dimension, while those consumers very interested in only one dimension continue assessing their preferred dimension before purchase. When $B$ is sufficiently high, there is no firm investment. Higher values of $A$ and $B$ than those depicted in Figure 1 lead to the same outcomes as the equilibria in the boundaries of the figure. At the upper boundary of the figure, no one is verifying $b$ and so further
increases in $B$ have no effect. A similar argument applies for $A$. That is why the left panel of Figure 1 represents a full characterization of equilibrium behaviour.

The right panel of Figure 1 represents the welfare levels for all the equilibria characterized in the left panel. In the figure darker regions correspond to lower welfare. Note that non-monotonicities arises within an equilibrium regime (and in particular within all-assess equilibria), though changes in equilibrium regimes can also generate non-monotonic effects. Summarizing, this analysis allows us to conclude the following result:

**Proposition 6** When prices are chosen by the monopolist, welfare, profits and consumer surplus can be non-monotonic in the cost of assessing attribute $a$.

Interested readers are referred to Bar-Isaac, Caruana and Cuñat (2008) for further details on profits, consumer surplus and the characterization more generally. There we show that, as in Section 2, not only welfare, but also consumer surplus and profits, are first falling and then rising in $A$.

### 5.2 Investment in both dimensions

So far we assumed that investment in one of the two characteristics was fixed. We did so for the sake of clear exposition, so that we could perform a parsimonious analysis of the model.\footnote{For instance, with investment in two dimensions there is more scope for multiplicity of equilibria, and also equilibria need not be Pareto ranked. For example, there may be an equilibrium where there is only investment in characteristic $a$, and another equilibrium where there is only investment in characteristic $b$.} Allowing for investment in both dimensions need not overturn any of the qualitative results of the paper.

Consider an investment $y$ in characteristic $a$ analogous to that in $b$ in the model of Section 2 with an exogenous price and a general cost function $c(x,y)$. Following a similar analysis, one can see that an additional effect in the comparative statics arises through the investment in the quality of $a$. For example, analogous to (16), one obtains

$$
\frac{dCS}{dA} = \frac{\partial CS}{\partial A} + \frac{\partial CS}{\partial x} \frac{dx}{dA} + \frac{\partial CS}{\partial y} \frac{dy}{dA},
$$

(23)

where again $\frac{\partial CS}{\partial A} < 0$ and $\frac{\partial CS}{\partial x}, \frac{\partial CS}{\partial y} > 0$. Consider now an equilibrium in which all consumers are either assessing one characteristic or the other. Then, analogous to (8), the first order conditions for the firm are

$$
\frac{dc(x,y)}{dx} = pF(T)
$$

(24)

$$
\frac{dc(x,y)}{dy} = p(1 - F(T)).
$$

(25)

In the case where the cost of investing in characteristic $a$ is independent of the cost of investing in $b$ (so that $c(x,y)$ can be written as $c(x,y) = g(x) + h(y)$) then $\frac{dx}{dA}$ and $\frac{dy}{dA}$ have opposite signs. This is intuitive: an increase in $A$ would cause some consumers who were previously assessing $a$ to switch and assess dimension $b$ instead. This increases the incentives to invest in $b$, but at the same
time it decreases the incentives to invest in \( a \).\(^{20}\)

As a result of the discussion above, Equation (23) shows that, in addition to the direct effect on consumer surplus caused by a change in the assessment cost \( A \), there are two indirect and opposing effects through an increase in the investment in one dimension, and a decrease in the other. The relative sizes of these indirect effects will depend on the current level of investment and the convexity of the cost of investment. Thus, one can see that the overall effect on consumer surplus and welfare can be either positive or negative.\(^{21}\) In terms of our opening example on business schools, the emergence of rankings has had a direct effect in reducing information-gathering costs for prospective students. But this reduction occurs only for those attributes captured in the ranking, such as facilities. Consequently schools invest in new buildings and dorms at the expense of neglecting other aspects which are not so easily captured by the rankings (like teaching quality).

### 5.3 Benefits to diversity

The indirect externalities among consumers presented in this paper can explain a benefit of diversity in preferences. When an agent assesses one particular dimension, this benefits all other agents by raising the firm’s level of investment in that dimension. In particular, an agent assessing dimension \( a \) might prefer others in the population having preferences that induce them to assess \( b \). Moreover, all agents might prefer a heterogeneous population to a homogeneous one. We illustrate this possibility through Example 1 in Appendix B.

This discussion relates our work to wider discussions on the benefits of diversity. Waldfogel (2003) treats this issue empirically and focuses on product proliferation with heterogeneous consumers. In his framework consumers unambiguously benefit from the presence of more consumers with similar preferences. Our example shows a contrasting result. In our setting verifying quality is costly and thus there can be a benefit from diversity in consumer search, which arises from underlying diversity in consumer preferences. Further, many formal models in economic geography directly incorporate a benefit of diversity through complementarity in the production function or returns to specialization. Rather than highlighting the role of diversity in production, our arguments stress that there may be benefits of diversity on the demand side. That such benefits play an important role in large multi-cultural cities has been eloquently if informally suggested in Diamond et al. (2004).

### 5.4 Assessing both dimensions

So far we have not allowed consumers to assess both dimensions before deciding whether or not to purchase. It is important to realize that the expected consumer benefit from assessing one dimension can decrease if the other dimension has already been assessed. This induces some degree of substitutability between assessing one dimension or the other, even if the costs of assessing each of them are independent. Information gathering externalities are therefore also present when

\(^{20}\)Other types of equilibria do not have such a direct relationship between \( \frac{dy}{dA} \) and \( \frac{dx}{dA} \). For example, if marginal investors are indifferent between assessing and not buying, then lower investment in either dimension suggests that fewer consumers assess both dimensions thereby reducing investment incentives. That is, \( \frac{dy}{dA} < 0 \) and \( \frac{dx}{dA} < 0 \).

\(^{21}\)For the case \( g(x) = h(x) = \frac{x^2}{4} \) and uniformly distributed consumers, numerical analysis confirms that welfare can be non monotonic in \( A \) for a given \( B \) and viceversa.
consumers are able to assess both dimensions.

There are two straightforward ways to allow consumers to assess both dimensions: First, to let consumers assess both characteristics simultaneously, and, second, sequentially. In the latter case, their decision to assess the second characteristic depends on what they have learnt about the first. One can construct examples in both cases in which a fall in assessment costs might harm welfare. In Example 2 in the Appendix B, we follow the second approach to illustrate that allowing consumers to assess both dimensions does not invalidate our results.

As noted above, once an agent has assessed characteristic $b$, there may be less to be gained from assessing $a$. As a result, even if agents had the opportunity to search on both dimensions, there would still be consumers who switch from searching on $a$ to searching on $b$; and this group of consumers is the key driving force in our analysis. While gathering information on one characteristic does not increase the cost of gathering information on the other, it can reduce the benefit of doing so: there may be less value in learning the information on the second characteristic once the realization on the first is known.

This observation contrasts with the multi-tasking literature. At the heart of those results lies a substitutability of effort in costs—an increase in productive effort in one dimension makes it more costly to exert effort in another. Meanwhile, our results hinge on some consumers switching their behaviour. This example clearly shows that this change in behaviour does not rely crucially on the assumption made in the main model that if a consumer is assessing characteristic $A$, the cost of assessing characteristic $B$ is infinite. Even when assessing one characteristic does not affect the cost of assessing the other, the benefits are endogenously altered.

### 5.5 Commitment to investment

Up to this point, we have assumed that consumers do not observe the firm’s investment. For some applications it is more reasonable to suppose that consumers already know the firm’s investment when they take their decisions. Since investment leads to a stochastic realization on characteristic $b$, consumers still have a reason to assess before purchase. As a consequence, and given that consumers decisions are decentralized, our earlier results and considerations still apply. In particular, a rise in the cost of assessing $a$, can increase consumer surplus.

Again, just as in Section 2, we focus on the case where all agents assess. It is worth noting that in this model with commitment there is never multiplicity.

Given the firm investment $x$, the marginal consumer $T(x)$ indifferent between assessing $a$ and $b$ is characterized by:

$$ T(x) = p(1 - 2x) + x - 2(B - A). \quad (26) $$

The firm’s problem differs from before because now the firm’s investment decision directly affects consumer behaviour:

$$\max_x \Pi = \{(1 - F(T(x)) \frac{1}{2} + F(T(x))x)p - c(x), \quad (27)$$

and so the first order condition yields
\[ \frac{c'(x)}{p} = F(T(x)) - f(T(x))(x - \frac{1}{2})(2p - 1). \] (28)

Equations (28) and (26) determine equilibrium investment and consumer behaviour.\(^{22}\) One can then calculate consumer surplus, welfare and profits just as in the no-commitment case.

In the case where types are uniformly distributed and \( c(x) = \frac{x^2}{2} \), equation (28) can be written as \( x = \frac{p}{4(4+A+4p-1)} \). Note that increasing \( A \) leads to a rise in \( x \) and, so, just as in the analysis following Equation (16), there is an indirect effect which can lead to a rise in consumer surplus and welfare. As a result consumer surplus, profits and welfare can be non-monotonic in \( A \).\(^{23}\)

6 Final Remarks

The model highlights a number of effects at work in consumer markets for multi-dimensional goods where consumers can actively gather information. We have stressed, in particular, that individual’s information gathering influences firm investment and thereby other consumers’ welfare. Since consumers have no way of internalizing the effect of their aggregate choices on the firm’s investment in quality, a change in these choices induced by a fall in the costs of information can reduce welfare.

Our results seem to accord with the evidence that more easily available information has lead to adverse welfare effects in the context of health markets and with the related popular perception for the case of business schools. One rationale for an exogenous change in the cost of acquiring information is the appearance of intermediaries, as in the example of school rankings. While a comprehensive analysis of the incentives and industrial organization of information intermediaries is of considerable interest, it is not the goal of this paper. Nevertheless, our framework can be used to analyze the introduction of an intermediary.

In particular, even an intermediary who works on behalf of consumers may be constrained. By their nature some characteristics of goods have idiosyncratic appeal and are hard to describe. It is easy to describe average salary on graduation from a business school, but harder to describe the extent to which the teaching style and culture will appeal.\(^{24}\) In the context of our model, it may be impossible for the intermediary to enable individuals to cheaply determine whether they would enjoy a particular characteristic when tastes are idiosyncratic. Thus, the intermediary might act to reduce assessment costs on some characteristics but not others, with the consequences implied by the model. It is still conceivable that an intermediary could convey the average quality of a characteristic (by conducting surveys for example). Then consumers, before further investigation, would know their expected satisfaction from that characteristic. This is analogous to Section 5.5 in which the monopolist could commit to quality investment. Thus, welfare can still be non-monotonic in the costs of assessment.

Interesting dynamic implications can arise if it takes time for agents to change their behaviour. In particular, a fall in the cost of acquiring information could initially raise consumer surplus

\(^{22}\)One must verify that this is indeed an equilibrium. That is, that all consumers prefer to assess both to not buy and buy without assessment. In fact, it is sufficient to check it for \( T(x) \).

\(^{23}\)This is the case, for example, at \( B = 0.05 \) and \( p = 0.5 \).

\(^{24}\)Scitovsky’s (1976) enlightening analysis of consumer behaviour and the nature of goods provides another example: It is easy to convey the calorific value but not the taste of food.
and overall welfare. However, the subsequent reaction of the firm to the consumers’ new search behavior may lead to a reduction in welfare in the long run. This will be the case even when all agents are fully aware of these mechanisms due to the inability of consumers to coordinate and commit to an aggregate assessment strategy. This might explain why policies or innovations such as the introduction of school league tables in the UK, while initially welcomed are eventually reviled.

The presence of externalities in consumers’ assessment behavior suggests scope for policy intervention. It also highlights that determining appropriate regulation might be very challenging. Market interventions, such as information provision, price caps, and quality guarantees can appear to be good measures to boost consumer surplus and welfare. However, our results suggest that such interventions can change the pattern of consumer assessment behavior and, thereby, firm investment, with the possibility of adverse consequences for welfare. Thus, this paper highlights the importance of indirect effects through the firm investments. If an intervention has little effect on consumer assessment behavior or if firms have limited ability to respond through their investment choices, then interventions play their intuitive roles: more information, price caps and quality guarantees (for some characteristics) benefit consumers. Otherwise, confounding effects suggest no easy conclusions.

A natural extension of this paper is to consider competition. The effects highlighted in this paper would still be present, as consumer assessment behavior would interact with firm investment. Of course, whether competitive pressures overwhelm our qualitative results is beyond the scope of the paper. Further, we have treated the cost of acquiring information as exogenous. In related work, Bar-Isaac, Caruana and Cuñat (2010) investigate the firm’s choice of marketing and advertising to highlight, or to the contrary, obfuscate product characteristics.

References

We organize our characterization around the marginal consumer who assesses dimension $b$, as described in Equation (6).

First consider the corner solution when $T = 0$. Because of equation (8) this implies that investment $x = 0$. Then, trivially $U_b(t) < 0$, $U_n(t) < 0$ and $U_a(t) = \frac{t}{2} - A$. Thus, the “no investment” equilibrium involves consumers in the range $[0, p + 2A]$ not buying, and those (if any exist) within $[p + 2A, 1]$ assessing $A$.
Next, suppose that the equilibrium involves positive investment. In Section 3.3, we analyzed the case in which the marginal $T$ was indifferent between assessing dimension $b$ and dimension $a$. This is the “all assess” equilibrium. It remains to consider “some do not assess” equilibria. There are two possibilities to consider in terms of Equation (6): (i) where $T$ is indifferent between assessing dimension $b$ and not buying; and (ii) where $T$ is indifferent between assessing dimension $b$ and buying without assessment. We consider each case in turn:

(i) $T$ is determined by

$$x(1 - T + \frac{1}{2}T - p) - B = 0,$$  \hspace{1cm} (29)

and $x$ is determined by Equation (8). In equilibrium, both (29) and (8) must hold and so, as long as a solution exists there are two possibilities

$$T = \frac{p - p^2 - \sqrt{-Bp + p^2 - 2p^3 + p^4}}{p},$$  \hspace{1cm} (30)

$$T = \frac{p - p^2 + \sqrt{-Bp + p^2 - 2p^3 + p^4}}{p}.  \hspace{1cm} (31)$$

In principle both these values, and the associated levels of investment may be interior, and so there may be two equilibria of this form.

Having characterized $T$, one can substitute back into Equation (8) to determine $x$. All that remains is to determine whether consumers in $[T, 1]$ prefer not to buy, buy without assessing, or assess $a$. First note that buying without assessing is never optimal: given that $U_a(T) < 0$, it would be necessary that $\frac{\partial U_a}{\partial T} = \frac{1}{2} - x > 0$, but when $x < \frac{1}{2}$, trivially $U_a < 0$ for all $t$. Finally, and given that $\frac{\partial U_a}{\partial T} > 0$ we can compute $T_{a0}$, the consumer indifferent between assessing $a$ and not buying

$$T_{a0} = \frac{2A + p - x}{1 - x},$$  \hspace{1cm} (32)

and conclude that consumers in the range $[T, T_{a0}]$ prefer not to buy (or assess), while those within $[T_{a0}, 1]$ assess $a$, though this last range may have no mass.

(ii) $T$ is determined by

$$x(1 - T + \frac{1}{2}T - p) - B = \frac{T}{2} + x(1 - T) - p.$$  \hspace{1cm} (33)

and, again, $x$ is determined by Equation (8). There are two possibilities:

$$T = \frac{1 + 4p^2 + \sqrt{16Bp - 8p^2 + 16p^4 + 1}}{4p},$$  \hspace{1cm} (34)

$$T = \frac{1 + 4p^2 - \sqrt{16Bp - 8p^2 + 16p^4 + 1}}{4p}.  \hspace{1cm} (35)$$

As in case (i), all that remains is to determine consumer behavior in the range $(T, 1]$. We can define the consumer indifferent between buying without assessment and not buying, $T_{a0} = \frac{2p - 2A}{1 - 2A}$, and the consumer indifferent between buying without assessment and assessing $a$, $T_{na} = \frac{2A - p + x}{x}$. Depending on the relationship between these two values there are two possible equilibrium configurations: If $T_{na} > T_{a0}$ then all consumers in the range $[T, T_{a0}]$ buy without assessment, those in $[T_{a0}, T_{na}]$ do not buy, and those in $[T_{a0}, 1]$ assess $a$. Otherwise, all consumers in the range $[T, T_{na}]$ buy without assessment, while those on $[T_{na}, 1]$ assess $a$.

B Examples

**Example 1** Suppose the firm chooses investments $x$ and $y$ in dimensions $b$ and $a$ at a cost of $\frac{1}{2}(x^2 + y^2)$. These investments result in high quality with probabilities $x$ and $y$ respectively. The price of the good is 0.5. There is a mass 1 of consumers of two types: $\alpha$ consumers with utility $0.55q_a + 0.45q_b$ from consuming a good of quality $(q_a, q_b)$ and $\beta$ consumers with utility $0.45q_a + 0.55q_b$. Finally, a consumer can pay a cost 0.04 to learn the quality realization on either one of the two dimensions, that is $A = B = 0.04$. We consider
two cases:

I. **(benchmark)** Suppose that there are equal numbers of type $\alpha$ consumers and type $\beta$ consumers. In this case there is an equilibrium in which $x = y = 0.25$, the type $\alpha$ consumer assesses dimension $a$, and the type $\beta$ consumer assesses dimension $b$. In this equilibrium both types of consumer enjoy positive expected surplus (and the firm makes positive profits).

II. **(homogeneity can harm)** Consider varying the distribution of consumer types. If all consumers are $\alpha$ type consumers (or all are $\beta$ type) the unique equilibrium is one with no investment, sales or assessment.

Example 2  Consider Example 1 with an equal number of each type of consumer. Consumers can verify dimension $a$, dimension $b$, both or neither. The firm’s quality in dimension $a$ is fixed: it yields high quality with probability $\frac{1}{2}$. Investment in dimension $b$ is either 0, which ensures low quality, or, else, comes at a cost 0.1 and yields high quality with probability $\frac{1}{2}$.

I. **(benchmark)** If $A = B = 0.04$ there is an equilibrium in which the firm invests, $\alpha$ consumers assess characteristic $A$, and $\beta$ consumers assess characteristic $B$. Here, consumer surplus, profits and welfare are positive.

II. **(similar qualitative results)** If $B = 0.04$ but $A = 0.01$, the unique equilibrium is one in which there is no investment, $\beta$ consumers do not buy and $\alpha$ consumers assess $a$.

Profits and consumer surplus are higher in the benchmark equilibrium.