The Mathematics of Credit Derivatives: Instruments and Structures

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Any questions?

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- If your question is of general interest:
  Post it on www.wilmott.com and send me an email. I’ll post my answer.
Default Risk

Default risk is the risk that an obligor does not honour his payment obligations.

- linked to (every) individual payment obligation
- linked to legal rules governing obligations
- via legal rules: default risk of one payment obligation connected to other payment obligations of the same obligor

Properties:

- Default events are rare (high legal penalties).
- They may occur unexpectedly.
- Default events involve significant losses.
- The size of these losses is unknown before default.
Components of Credit Risk

- arrival risk
- timing risk
- recovery risk
- market risk*, price correlation risk*
- default correlation risk

* must be consistent with other risks
Credit Derivatives

Credit Derivatives are derivative securities that are used to trade and hedge default risks.

Usually, their payoff is made contingent on the occurrence of a Credit Event (e.g. a payment default or a bankruptcy).

Credit Derivatives enable the user to trade the credit risk of an obligor in isolation, i.e. independently from the obligor’s bonds or loans.
Credit Default Swaps
Credit Default Swap (CDS)

**CDS Fee / Rate:** paid by A to B in compensation of default risk

**Default Payment:** w.r.t. a reference asset issued by C. Alternatives are:

- Notional minus market value of the reference asset (*cash settlement*)
- Notional for delivery of the reference asset (*physical settlement*)

**Effect of the CDS:** Transfer of the C-default risk from B to A.
Documentation

Key terms: (negotiable, but standardised by ISDA)

- notional, maturity, currency
- CDS fee
- reference entity (reference credit)
  - reference assets
  - deliverable obligations
- credit event definitions
- settlement method
Notional, Maturity

Typical values

- notional amount: 5-100m USD/EUR, median 10m
- maturity: up to 10 years, most liquidity at 5 years
- currency: usually USD or EUR
- also note:
  - trade date
  - effective date: the date from which the transaction is valid (usually trade date + 3 business days)
  - scheduled termination date: fancy word for: maturity

Example:
10m USD notional, 5 years to maturity.
CDS: Fees

- The fee is the *price* of the default protection. Initially, it is chosen such that the CDS has initial value zero to both sides.
- Fee payments are *terminated at a credit event*, but the accrued fee up to the credit event is paid.
- Technical payment details
  - quoted as basis points of the notional of the CDS
  - payment in regular intervals (quarterly or semi-annually)
  - payment is made at the end of each interval
  - usual swap daycount conventions apply (e.g. act/360)

Example:
168 bp, quarterly
Reference Credit / Reference Entity / Reference Obligor

The entity whose default risk is transferred.

Potential problem scenarios:

- Mergers, demergers
- Takeovers
- Spin-offs (“successor”-problem)
- Conglomerates: Mother / daughter
  i.e.: If you buy protection on Ford Motor Co., are you protected against a default of Ford Motor Credit Co. (a 100% daughter) and vice versa?

Example:
Ford Motor Co.
Reference Assets

• a set of bonds or other assets issued by the reference credit (RC).
• Purpose: Determination of the credit event.
• Usually a very broad definition: (up to “owed money”)

Deliverable Obligations

• the set of deliverable assets in default (for physical delivery)
• the basis for the price and recovery determination mechanism (for cash settlement).
• definition usually narrower, or a concrete list of bonds.
  Selection criteria: seniority, tradeability, liquidity
• Open dispute: delivery of convertible bonds
Credit Events

ISDA:

- Bankruptcy (RC)
- Failure to pay (RA)
- Obligation default (RA)
- Obligation acceleration (RA)
- Repudiation / moratorium (RC)
- Restructuring (RC) (controversial / must be negotiated)

But also other definitions are possible:

- industrial disruption (=strikes)
- government action (=nationalisation)
- convertibility events (sovereigns)
- armed hostilities (sovereigns)

Notification of a credit event must be given (usually by the protection buyer).
Default Payment

Physical delivery:

- protection buyer delivers deliverable obligations (notional = CDS notional)
- protection seller pays CDS notional

Protection buyer has a delivery option.

Cash settlement:

determination of recovery / loss in default:

- calculation agent
- dealer poll, repeated, averaging (elimination of highest and lowest)
- protection seller pays [CDS notional] minus [recovery value].
Default Payment II

Effect:

- speed: payment or delivery within 2-6 weeks after default event
- independence of legal considerations and bankruptcy law
- problems with market manipulation and liquidity/short squeezes
- physical delivery most common whenever possible
- cash settlement for: credit-linked notes, synthetic CDOs etc.

*There is a lot of money in the default and the payment specifications!*
Motivation, Advantages and Applications

- credit risk can be traded in isolation
- no economic transfer of ownership
- flexibility: specified to the needs of the counterparties
- confidentiality (customer relations)
- possibility to short credit risk
- portfolio management or regulatory capital
- Cost of funding advantages
- Off-balance sheet
Market rates: 5-year CDS on Daimler-Chrysler

Basis points p.a., Bid / Offer prices of different brokers, Source: ABN Amro
Market rates: 5-year CDS on Ford

Basis points p.a., Bid / Offer prices of different brokers, Source: ABN Amro
Market rates: 5-year CDS on Fiat

Basis points p.a., Bid / Offer prices of different brokers, Source: ABN Amro
Trading CDSs: Technicalities

- Trading takes place: OTC, intra-bank, via brokers (e.g. GFInet), some online exchanges (creditex, credittrade)

- Large investment banks as market makers
  (JPMorgan, Deutsche Bank, Lehman Brothers, Goldman etc.)

- Bid/Ask Convention: The traded asset is protection
  (this is different from corporate bond prices)

  ★ Bid: what the broker bids if you offer him protection
  ★ Ask: what the broker asks for if you want to buy protection
  ★ Ask-Bid = 12% of price (approximately).

- Collateral/margin required from lower rated counterparties if they sell protection.
• Documentation (partially) standardised by ISDA. Price differences for documentation variations (credit event definition (reorganisation), delivery of convertible bonds)

• Most liquid products: Single-Name Credit Default Swap
  ⋆ documentation according to ISDA standard
  ⋆ 5 years maturity
  ⋆ notional 10-100m
  ⋆ quarterly fee payments, USD or EUR
  ⋆ reference entity: large corporation, bond issuer
Other Single-Name Credit Derivatives
Credit-Linked Notes

Basic Note:

- **Coupon**: LIBOR + Spread

- **Principal redeemed at maturity.**

- **Issuer high credit quality**
  (AAA-rated Special Purpose Vehicle)

- **Reference Credit (RC) and Reference Asset (RA) are defined.**
Total Return Credit-Linked Note:

- Coupon as usual

- Principal redemption:
  Notional + Payoff of Asset Swap on RA

  \[ \text{Notional} + \text{Change in Value RA} \times \text{Notional} \]

- Early redemption at default of RC

- Investor: Synthetic exposure to RC

- Leverage can be introduced (with cap at 0 redemption)

- Can be linked to index
Credit Default Linked Notes

- Coupon as usual

- Principal redemption:
  Notional + Payoff of CDS on RA

- Note: Default payment is fully collateralised.

- designed to replicate RA,

- link between CDS market and corporate bond market
Example: Wal-Mart Credit Linked Note

Issuer: JP Morgan, September 1996
(via a AAA trust)

Buyers receive:

- coupon (fixed or floating)

- principal if no default of reference (Wal-Mart) until $T$

- recovery rate if there was a default.

Default Risk: buyers
equivalent to debt issued by Wal-Mart.
Common Features:

- Combination of note with CRD
- Issuer buys protection
- May ease access to desired exposure
- Protection fully collateralised
- Investor faces counterparty risk
- RC may have objections (demand for his credit risk will decrease)
- Investor has information disadvantage.
Variations: Callability, coupons at risk (instead of principal), baskets as underlying
Default Digital Swaps

Like CDS, but:

- Default Payment = 1
- *no* recovery / Loss In the Event of Default (LIED) determination

Motivations

- avoid complicated settlement
- exposure not related to LIED
- bet on recovery rate
Recovery-Rate Bets

1. A long position in a CDS with notional of 1:
   Fee: \( \bar{s} \)
   Payoff in default: \( 1 - \text{Recovery} \).

2. A long position in a DDS with notional of \( \bar{s}/\bar{s}^{DDS} \):
   Fee: \( \bar{s}/\bar{s}^{DDS} \times \bar{s}^{DDS} = \bar{s} \)
   Payoff in default: \( \bar{s}/\bar{s}^{DDS} \).

Same fee payments before default. Therefore, the payoffs in default must be equally valuable, too.

This means that the implied recovery rate is

\[
\text{Recovery}^{\text{impl}} = 1 - \frac{\bar{s}}{\bar{s}^{DDS}}.
\]
CDS with Exotic Payoffs

- define default event, fees, etc. like CDS
- default payment: mark-to-market value of an (exotic) derivatives transaction
- Example: FX swaps, interest-rate swaps
- Note: The delivery option becomes very important
- Motivation: Management of counterparty risk in derivatives transactions.
- Can also (partially) manage counterparty risk with default digital swaps.
Credit Spread Options

Possible variations:

- Options on CDS:
  Option to enter a CDS at a given time and for a given CDS spread.
  Enter as protection buyer: Call option.
  Enter as protection seller: Put option.

- Option to extend / cancel an existing CDS. (As above.)

- Options on Asset swap packages:
  Option to buy/sell a defaultable bond in form of an asset swap package for a
given asset swap spread.

- Yield spread options:
Option to buy/sell a bond at a given yield spread over a pre-specified benchmark bond (unusual).

- Plain bond option:
  Option to buy/sell a defaultable bond for a given price.
  (Also carries interest-rate risk.)

- Related:
  Coupon step-up upon rating downgrade of a defaultable bond.

- Motivation
  - hedging of callable bonds, options in loans;
  - regulatory capital (maturity matching)
  - insure against adverse credit spread moves while keeping the upside.
Asset Swap Packages

Eliminate interest-rate exposure from defaultable bonds (and isolate credit exposure).

An *asset swap package* is a combination of

- a defaultable bond (the asset)
- with an interest-rate swap contract

The interest-rate swap swaps the coupon of the bond into a payoff stream of Libor plus a spread $s^A$.

This spread is chosen such that the value of the whole package is the par value of the defaultable bond.

Usually, the bond is a fixed-coupon bond and the interest-rate swap a fixed-for-floating interest-rate swap.
Payoffs Asset Swaps

A sells to B for 1 (the notional value of the C-bond):

- a fixed coupon bond issued by C with coupon \( c \) payable at coupon dates \( T_i, \ i = 1, \ldots, N \),
- a fixed for floating swap (as below).

The payments of the swap: At each coupon date \( T_i, \ i \leq N \) of the bond

- B pays to A: \( c \), the amount of the fixed coupon of the bond,
- A pays to B: Libor + \( s^A \).

Problem: What to do with the interest-rate swap at a default event?
Asset

Investor

Funding

Swap

Coupon 100

Funding Rate 100

Libor + S

Coupon
Asset Swap Convertible Options Trades ASCOTs

this structure is also known as an asset swap, but is significantly different.

- the bank buys the convertible bond from the original investor
  enters an interest-rate swap on the coupon (asset swap part)

- and simultaneously sells a call option on it.
  Strike price: [conversion price of the bond]-[value of interest-rate swap].
  Exercise dates: Conversion dates of the convertible.

Effect Investor:

- Default risk is hedged.
- If the conversion is optimal, the investor can exercise the option and convert the bond.
• The position is funded.
• Additional uncertainty because strike depends on interest-rates.
**Total Rate of Return Swaps TRS**

Motivation:

Completely exchange payoff streams / returns from two investments without legal transfer of ownership.

One investment is the defaultable bond, the other is default-free Libor.

To limit counterparty risk: Mark-to-Market at regular intervals.

Similar to exchange-traded Futures contracts.
Total Rate of Return Swap

TROR Receiver

Libor + Spread

Coupon

Price depreciation

Price appreciation

TROR Payer
Payoffs

Counterparty A pays to counterparty B at regular intervals:

- the coupon $\bar{c}$ of the bond issued by C (if there was one)
- the price appreciation $(C(T_{i+1}) - C(T_i))^+$ of bond C since the last payment
- the principal repayment of bond C (at the final payment date)
- the recovery value of the bond (if there was a default)

B pays at the same intervals

- a regular fee of Libor $+s^{TRS}$
- the price depreciation $(C(T_i) - C(T_{i+1}))^+$ of the bond C since the last payment  (if there was any)
- the par value of the bond (if there was a default in the meantime)

These payments are netted.
Effect of a TRS

- The investor (TR receiver) assumes all risks and cash flow of the reference asset.

- The bank (TR payer) passes through all payments of the RA.

- The investor makes regular payments (akin to funding cost).

- Mark-to-market: Regular exchange of payments reflecting price changes of the RA.
Problems with TRS:

dependence on a single reference asset:

- default risk only of this one asset
- mark-to-market risk if maturity is not matched
- generally not enough flexibility
- but a frequent and useful tool in more complex structures

Potential Problem: Representation in case of default and information disadvantage of TR receiver vs. loan originator.
Hedge-Based Pricing
... in this unit

• some robust, but inaccurate pricing methods for credit derivative
  ★ asset swap packages
  ★ total return swaps
  ★ credit default swaps

• the problems in the implementation of these hedge strategies

• why market prices may deviate from these prices

• why we need better pricing models
Short Positions in Defaultable Bonds

needed to implement positions which \textit{gain} at a default of the reference credit.

\textbf{Repo Transactions}

Repurchase (repo) transactions were first used in government bond markets.

Recently, a repo market for corporate bonds has developed which can be used to implement short positions in corporate bonds.

A repurchase (repo) transaction consists of a sale- and a repurchase part:

• Before: \textbf{A} owns the defaultable bond $\overline{C}$.
• \textbf{B} buys $\overline{C}$ from \textbf{A} for the price $\overline{C}(0)$
• At the same time, \textbf{A} and \textbf{B} enter a \textit{repurchase} agreement:
  \textbf{B} agrees to sell the bond back to \textbf{A} at time $t = T$ for the forward price $K$. \textbf{A}
agrees to buy the bond.
This agreement is binding to both sides.

The forward price $K$ is the spot price $\bar{C}(0)$, adjusted for intermediate coupon payments, and increased by the repo rate $r^{repo}$:

$$K = (1 + T r^{repo}) \bar{C}(0).$$

(For a term of less than one year.)
Short Positions with Repo

To implement a short position, B:

- At time $t = 0$, B sells the bond in the market for $C(0)$.
- At time $t = T$ (in order to deliver the bond to A), B has to buy the bond back in the market for the then current market price $C(T)$.

B’s profit/loss:

$$C(0)(1 + r^{\text{repo}}T) - C(T)$$

From A’s point of view, the transaction can be viewed as a collateralised lending transaction:

A has borrowed from B the amount of $C(0)$ at the rate $r^{\text{repo}}$, and as collateral he has delivered the bond to B.
Risks

• Counterparty risk: generally small
  \( B \) is exposed to the risk of \( A \)'s default on the payment of \( K - \overline{C}(T) \) at time \( T \)

• Liquidity risk:
  There may be no bonds in the market at \( T \) that \( B \) can buy back.

• Short squeezes:
  \( A \) could buy up the supply of bonds and then not extend the repo but force \( B \) to buy at his prices.
## Asset Swaps

<table>
<thead>
<tr>
<th>Time</th>
<th>Defaultable Bond</th>
<th>Swap</th>
<th>Net</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 0$</td>
<td>$-\overline{C}(0)$</td>
<td>$-1 + \overline{C}(0)$</td>
<td>$1$</td>
</tr>
<tr>
<td>$t = T_i$</td>
<td>$\overline{c}^*$</td>
<td>$-\overline{c} + L_{i-1} + s^A$</td>
<td>$L_{i-1} + s^A + (\overline{c}^* - \overline{c})$</td>
</tr>
<tr>
<td>$t = T_N$</td>
<td>$(1 + \overline{c})^*$</td>
<td>$-\overline{c} + L_{i-1} + s^A$</td>
<td>$1^* + L_{i-1} + s^A + (\overline{c}^* - \overline{c})$</td>
</tr>
</tbody>
</table>

Default Recovery unaffected | Recovery

* denotes: payoff only in survival.
Replication of the Asset Swap for A

At each $T_i$, A receives $c_i$ for sure, but must pay $L_{i-1} + s^A$. This can be replicated as follows:

Buy a default-free bond with coupon $c_i - s^A$ and borrow 1 at $L$

<table>
<thead>
<tr>
<th>Time</th>
<th>Default-free Bond</th>
<th>Funding</th>
<th>Net</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 0$</td>
<td>$-C'(0)$</td>
<td>+1</td>
<td>$1 - C'(0)$</td>
</tr>
<tr>
<td>$t = T_i$</td>
<td>$\bar{c}_i - s^A$</td>
<td>$-L_{i-1}$</td>
<td>$\bar{c}<em>i - L</em>{i-1} - s^A$</td>
</tr>
<tr>
<td>$t = T_N$</td>
<td>$1 + \bar{c}_N - s^A$</td>
<td>$-L_{i-1} - 1$</td>
<td>$\bar{c}<em>i - L</em>{i-1} - s^A$</td>
</tr>
</tbody>
</table>

Default unaffected unaffected unaffected

where $C' =$ value of a default-free bond with coupon $\bar{c} - s^A$.

all cash-flows after $t = 0$ are identical to the asset-swap transaction.
Initial Cash-Flows of A

- $1 - C'(0)$ from replication
- $1 - \overline{C}(0)$ from the asset swap package
  - buy the defaultable bond for $\overline{C}(0)$
  - combine it with the interest-rate swap (no initial payment)
  - sell it to $B$ for par ($= 1$)
- Result:
  Consider the defaultable coupon bond. We do two things:
  - we remove the default risk (make it default-free)
  - the bond becomes more valuable. New value:
    $C'(0) =$ value of a default-free bond with coupon $\overline{c}$.
  - we reduce its coupon by $s^A$
  - the bond becomes less valuable again. New value:
    $C''(0) = C'(0) - s^A A(0)$
  $s^A$ is chosen such that both effects cancel exactly.
Result

The asset swap rate $s^A$ of a defaultable coupon bond $\overline{C}$ with coupon $\bar{c}$ expresses the price difference between

- the defaultable coupon bond $\overline{C}$ and
- a default-free coupon bond with the same coupon $\bar{c}$, the same payment dates and the same maturity.

This price difference is expressed in terms of units of the annuity $A$, i.e. the cash price-difference is $s^A(t)A(t)$.

$$s^A(0) = \frac{1}{A(0)} \left( C(0) - \overline{C}(0) \right).$$
Analysis

• At a default of $C$:
  
  \[(C(t) - \overline{C}(t))\] becomes $C(t) - \text{recovery}$, very large, but not infinitely large.

• over time, $C(t) - \overline{C}(t)$ will become smaller, but also $A(t)$.
Cash-and-Carry Hedging of CDS:

**Portfolio I**

- 1 defaultable Bond: Coupon $c$, maturity $T$
- 1 protection in form of a CDS, rate $-\bar{s}$

**Portfolio II**

- 1 default-free Bond: Coupon $c - \bar{s}$, maturity $T$

Identical payoffs in survival.

BUT: Same payoffs in default?

- Portfolio I: Value = 1 (quite precisely)
- Portfolio II: Value = ??? (depends on the default-free bond price, i.e. default-free interest-rates)
Alternative: Asset-Swap Hedging of CDS

Bond Side

- 1 defaultable Bond as *asset swap package*:
  - coupon $\text{Libor} + s^A$, maturity $T$
- 1 protection in form of a CDS, rate $-\bar{s}$

Funding Side

- borrow 1 (initial price of asset swap package) at Libor until $T$.

In Survival: Regular positive payoff (*carry*) of

$$s^A - \bar{s}.$$ 

BUT: What about default?
<table>
<thead>
<tr>
<th>Time</th>
<th>Portfolio I</th>
<th>Portfolio II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Asset Swap Pckg.</td>
<td>CDS</td>
</tr>
<tr>
<td>$t = 0$</td>
<td>$-1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$t = T_i$</td>
<td>$L_{i-1} + s^A$</td>
<td>$-\overline{s}$</td>
</tr>
<tr>
<td>$t = T_N$</td>
<td>$1 + L_{i-1} + s^A$</td>
<td>$-\overline{s}$</td>
</tr>
<tr>
<td>Default $t = \tau$</td>
<td>Recovery + Value of Swap</td>
<td>$1 - \text{Recovery}$</td>
</tr>
</tbody>
</table>
Asset-Swap Position at Default

- **Asset** → **Investor** through **Recovery**
- **Investor** to **Funding** through **Value of Swap**
- **Funding** to **Investor** through 0
- **Funding** to **Asset** through 100

Diagram:

- Asset
- Investor
- Funding
- Swap
Result

provided the value difference in default is not too large:

\[ \bar{s} \approx s^A \]

The asset swap rate is a good indicator for a fair credit default swap spread. The accuracy of this relationship depends on the degree to which the following assumptions are fulfilled

(i) The initial value of the underlying bond is at par.
(ii) Interest-rate movements and defaults occur independently.
(iii) Short positions in the asset swap market are possible.
(iv) At default, the default-free floater trades at par.
If Bond Short Sales are Impossible:

The following strategy must not generate a risk-free profit:

- Buy a defaultable bond as asset swap package:
  - receive $s^A_{offer}$
- Buy default protection as CDS:
  - pay $-s^{offer}$

$CDS_{offer} \geq s^A_{offer}$ \hspace{1cm} Asset Swap offer.

If on the other hand short sales of the asset swap package are possible, then we have in addition the following inequality:

$s^{bid} \leq s^A_{bid}$, \hspace{1cm} if short sales are possible.

Note: In a CDS we trade protection, in an asset swap we trade the bond.
If the Bond Trades Below Par:

e.g. “fallen angels”. Similar analysis yields:

\[ \bar{s} > s^A \quad \text{if } C'(0) < 1. \]

- Plausibility: Assume very high default risk: \( \bar{s} \to +\infty \), but the asset swap rate \( s^A \) remains bounded.

The asset-swap / CDS basis is defined as:

\[ \text{basis} = \text{CDS bid} - \text{Asset Swap offer} = \bar{s}^{\text{bid}} - s^A^{\text{offer}}. \]
## Trading the Basis: Examples

<table>
<thead>
<tr>
<th></th>
<th>Default Swap (bp)</th>
<th>Asset Swap (bp)</th>
<th>Basis (bp)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bid / Offer</td>
<td>Bid / Offer</td>
<td>(CDS Bid – AS Offer)</td>
</tr>
<tr>
<td>Bank of America</td>
<td>48/55</td>
<td>46/43</td>
<td>5</td>
</tr>
<tr>
<td>Bank One</td>
<td>60/75</td>
<td>65/60</td>
<td>0</td>
</tr>
<tr>
<td>Chase Corp.</td>
<td>40/48</td>
<td>35/30</td>
<td>10</td>
</tr>
<tr>
<td>Citigroup</td>
<td>38/45</td>
<td>30/27</td>
<td>11</td>
</tr>
<tr>
<td>First Union</td>
<td>68/85</td>
<td>66/63</td>
<td>5</td>
</tr>
<tr>
<td>Goldman Sachs</td>
<td>45/55</td>
<td>46/41</td>
<td>4</td>
</tr>
<tr>
<td>Lehman Brothers</td>
<td>70/80</td>
<td>68/63</td>
<td>7</td>
</tr>
<tr>
<td>Merrill Lynch</td>
<td>40/50</td>
<td>28/23</td>
<td>17</td>
</tr>
<tr>
<td>Morgan Stanley</td>
<td>45/55</td>
<td>28/23</td>
<td>22</td>
</tr>
</tbody>
</table>

(Source: Deutsche Bank)
Discussion

• Cash-and-carry is very robust and simple to implement. But:

• It is inexact:
  ★ default-free bond must be close to par
    (or interest-rate swap close to zero)
  ★ ideally: bond is floater
  ★ Delivery Option of the CDS must not have large value

• often hard to implement:
  ★ Replication of CDS requires a short position in a defaultable bond
  ★ a defaultable bond with matching maturity of the CDS must exist

• does not work for more complicated credit derivatives

Therefore: We need a pricing model.