Options

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New York University

- Puts and Calls
- Put-Call Parity
- Combinations and Trading Strategies
- Valuation
- Hedging
*Options*

**Payoffs and Profits on Options at Expiration - Calls**

**Notation**
- Stock Price = $S_T$
- Exercise Price = $X$

**Payoff to Call Holder**
- $(S_T - X)$ if $S_T > X$
- $0$ if $S_T \leq X$

**Profit to Call Holder**
- Payoff - Purchase Price
Payoff Profiles for Calls

Payoff Profiles for Puts
**Put-Call Parity Relationship**

<table>
<thead>
<tr>
<th>Condition</th>
<th>Payoff for Call Owned</th>
<th>Payoff for Put Written</th>
<th>Total Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_T \leq X )</td>
<td>0</td>
<td>(- (X - ST))</td>
<td>( S_T - X )</td>
</tr>
<tr>
<td>( S_T &gt; X )</td>
<td>( S_T - X )</td>
<td>0</td>
<td>( S_T - X )</td>
</tr>
</tbody>
</table>

**Payoff of Long Call & Short Put**

Payoff: Combined = Leveraged Equity

Long Call: \( \Rightarrow \)

Short Put: \( \Leftarrow \)

Stock Price: \( \nabla \)
**Put Call Parity - Disequilibrium Example**

Stock Price = 110  Call Price = 17  
Put Price = 5  Risk Free rate = 10.25%  
Maturity = .5 yr  Strike price X = 105  

\[ C - P > S_0 - X / (1 + r_f)^T \]

17 - 5 > 110 - (105/1.05)  
12 > 10  

Since the leveraged equity is less expensive, acquire the low cost alternative and sell the high cost alternative.
### Put-Call Parity Arbitrage

<table>
<thead>
<tr>
<th>Position</th>
<th>Immediate Cashflow</th>
<th>Cashflow in Six Months</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$S_T &lt; 105$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$S_T \geq 105$</td>
</tr>
<tr>
<td>Buy Stock</td>
<td>-110</td>
<td>$S_T$</td>
</tr>
<tr>
<td>Borrow</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X/(1+r)^T = 100$</td>
<td>+100</td>
<td>-105</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-105</td>
</tr>
<tr>
<td>Sell Call</td>
<td>+17</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-(S_T-105)</td>
</tr>
<tr>
<td>Buy Put</td>
<td>-5</td>
<td>105-S_T</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

### Option Strategies

- **Protective Put**
  - Long Stock
  - Long Put
- **Covered Call**
  - Long Stock
  - Short Call
- **Straddle (Same Exercise Price)**
  - Long Call
  - Long Put

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Option Combinations

- Four classic ways of combining an option with a futures to create the opposite option

When Use Options to Take a View?

- View on direction
- View on volatility

<table>
<thead>
<tr>
<th>Direction: Volatility</th>
<th>Security rising</th>
<th>Security falling</th>
<th>No trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility rising</td>
<td>Buy call</td>
<td>Buy put</td>
<td>Buy straddle</td>
</tr>
<tr>
<td>Volatility falling</td>
<td>Sell put</td>
<td>Sell call</td>
<td>Sell straddle</td>
</tr>
<tr>
<td>No trend in volatility</td>
<td>Buy forward</td>
<td>Sell forward</td>
<td>Arbitrage</td>
</tr>
</tbody>
</table>
View on Direction, Volatility or Both?

Option Valuation
Option Values

+ Intrinsic value - profit that could be made if the option was immediately exercised
  ◆ Call: stock price - exercise price
  ◆ Put: exercise price - stock price
+ Time value - the difference between the option price and the intrinsic value
**Option Pricing Model**

**Factors Influencing Option Values:**

**Calls**

<table>
<thead>
<tr>
<th>Factor</th>
<th>Effect on value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock price</td>
<td>increases</td>
</tr>
<tr>
<td>Exercise price</td>
<td>decreases</td>
</tr>
<tr>
<td>Volatility of stock price</td>
<td>increases</td>
</tr>
<tr>
<td>Time to expiration</td>
<td>increases</td>
</tr>
<tr>
<td>Interest rate</td>
<td>increases</td>
</tr>
<tr>
<td>Dividend Rate</td>
<td>decreases</td>
</tr>
</tbody>
</table>
Option Pricing

Option Price

= Intrinsic value + Time value

Time value depends on

- Time
- Volatility
- Distance from the strike price

Option Pricing Model

ENTER THESE DATA:

-------------------

-> FUTURES PRICE 94.75
-> STRIKE PRICE 94.5
-> TIME IN DAYS 300
-> INTEREST RATE 7
-> STD DEVIATION 15

CALL PRICE IS....... 0.40
PUT PRICE IS....... 0.17
**Value of Call Option**

Intrinsic Value: Expected value of profit given exercise.

Futures Price: Probability distribution of the log of the futures price on the expiration date for values above the strike.

Time Value: Expected value of profit given exercise.

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**Black-Scholes Option Valuation**

Call value = \( S_0 N(d_1) - X e^{-rT} N(d_2) \)

\[ d_1 = \frac{\ln \left( \frac{S_0}{X} \right) + (r + \frac{\sigma^2}{2})T}{\sigma \sqrt{T}} \]

\[ d_2 = d_1 - \sigma \sqrt{T} \]

where

- \( S_0 \) = Current stock price
- \( X \) = Strike price, \( T \) = time, \( r \) = interest rate
- \( N(d) \) = probability that a random draw from a normal distribution will be less than \( d \).
**Black-Scholes Option Valuation**

- **X** = Exercise price.
- **e** = 2.71828, the base of the natural log.
- **r** = Risk-free interest rate (annualizes continuously compounded with the same maturity as the option).
- **T** = time to maturity of the option in years.
- **ln** = Natural log function
- **σ** = Standard deviation of annualized cont. compounded rate of return on the stock

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**How a Change in the Futures Price Changes the Option’s Price**

- **F** = Futures price
- **K** = Strike price
- **F-K** = Time Value
- **E(FJ+J>FJ>K)** = Call option price line
- **Time Value**
- **Profit (gain or loss)**

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Options28
**Call Option Example**

\[ S_0 = 100 \quad X = 95 \]
\[ r = .10 \quad T = .25 \text{ (quarter)} \]
\[ \sigma = .50 \]
\[ d_1 = \left[ \ln \left( \frac{100}{95} \right) + (0.10 + (0.5^2/2)) \right] / (0.5 \times 0.25^{1/2}) \]
\[ = 0.43 \]
\[ d_2 = 0.43 - ((0.5)(0.25^{1/2})) \]
\[ = 0.18 \]

**Probabilities from Normal Dist.**

\[ N(0.43) = 0.6664 \]

<table>
<thead>
<tr>
<th>(d)</th>
<th>(N(d))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.42</td>
<td>0.6628</td>
</tr>
<tr>
<td>0.43</td>
<td>0.6664</td>
</tr>
<tr>
<td>0.44</td>
<td>0.6700</td>
</tr>
</tbody>
</table>
**Probabilities from Normal Dist.**

\[ N(.18) = .5714 \]

<table>
<thead>
<tr>
<th>d</th>
<th>N(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.16</td>
<td>.5636</td>
</tr>
<tr>
<td>.18</td>
<td>.5714</td>
</tr>
<tr>
<td>.20</td>
<td>.5793</td>
</tr>
</tbody>
</table>

**Call Option Value**

\[ C_o = S_o N(d_1) - X e^{-rT} N(d_2) \]
\[ C_o = 100 \times .6664 - 95 \times e^{-0.10 \times 0.25} \times .5714 \]
\[ C_o = 13.70 \]

**Implied Volatility**

Using Black-Scholes and the actual price of the option, solve for volatility. Is the implied volatility consistent with the stock?
**Put Option Valuation: Using Put-Call Parity**

\[
P = C + PV(X) - S_0 = C + Xe^{-rT} - S_0
\]

**Using the example data**

- \( C = 13.70 \)
- \( X = 95 \)
- \( S = 100 \)
- \( r = .10 \)
- \( T = .25 \)

\[
P = 13.70 + 95 e^{-0.10 \times 0.25} - 100 = 8.35
\]

**Using the Black-Scholes Formula**

**Hedging: Hedge ratio or delta**

The number of stocks required to hedge against the price risk of holding one option

- Call = \( N(d_1) \)
- Put = \( N(d_1) - 1 \)

**Option Elasticity**

Percentage change in the option’s value given a 1% change in the value of the underlying stock
**Portfolio Insurance - Protecting Against Declines in Stock Value**

- Buying Puts - results in downside protection with unlimited upside potential
- Limitations
  - Tracking errors if indexes are used for the puts
  - Maturity of puts may be too short
  - Hedge ratios or deltas change as stock values change

**Trading Options: Risk Management Concepts**

- Hedger’s view: “An option is an insurance premium—you can gain, but you cannot lose more than the premium you pay.”
- Trader’s view: “An option is just another security—I can buy it or sell it, take a long or short position in puts or calls. Its price fluctuates with market variables. For any position, the gain or loss is the change in price of the position.”
We’ve written an option

Delta Hedging

<table>
<thead>
<tr>
<th>HEDGE RATIO</th>
<th>Option Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>15%</td>
<td></td>
</tr>
<tr>
<td>50%</td>
<td></td>
</tr>
<tr>
<td>80%</td>
<td></td>
</tr>
</tbody>
</table>

Forward or Futures Price
Trading Options: Delta Hedging with Futures

- Delta or hedge ratio is the change in the option price for a given change in the price of the underlying.
- Delta is also the probability of exercise.
- Delta is also the hedge ratio that tells one how many futures you need to hedge a given options position.

Trading Options: Delta Hedging

We’ve written a put option

Hedged with 40% short futures

Delta
Options Trading: Delta Hedging and the Gamma

- Delta is not constant: it’s low for out of the money options, high for in the money options.
- Change in delta makes it tough to know exactly how many futures to use.
- Change in delta is the gamma. Positive gamma is nice. Writing options produces negative gamma, which is nasty.

Goal: Understand Options’ Sensitivity

An option trader has a portfolio of options with different deltas, gammas, etc. The goal is to discover the sensitivities of the portfolio to changes in rates, time, volatility, etc, and to neutralize them.

<table>
<thead>
<tr>
<th>Greek</th>
<th>Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta$</td>
<td>Delta</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Gamma</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Theta</td>
</tr>
<tr>
<td>$\Lambda$ (Vega)</td>
<td>Lambda</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Rho</td>
</tr>
</tbody>
</table>
Options: Summary

- Puts and Calls
- Put-Call Parity
- Combinations and Trading Strategies
- Valuation
- Hedging

Appendix:
Option Pricing: Trees

The Idea:
A risk-free portfolio must earn the risk-free rate of return.
**Binomial Tree**

Consider a call option on a Treasury Bill
- The strike price is set at $21.
- The underlying bill could go to $22 or $18 in 1 month.

**UP**
- Bill=$20
- Call=?
- Bill=$22
- Call=$1

**DOWN**
- Bill=$18
- Call=$0
- Bill=$22
- Call=$1

A risk-free portfolio must earn the risk-free rate of return.

---

**Binomial Tree**

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- Call=$1

**DOWN**
- Bill=$18
- Call=$0
- Bill=$22
- Call=$1

PORTFOLIO:
- Long $\Delta$ bills
- Short 1 Call

Portfolio value:
- $22 \Delta - 1$
- $18 \Delta$

What’s the delta that will make the portfolio risk free?

A risk-free portfolio must earn the risk-free rate of return.
**Binomial Tree**

Consider a call option on a Treasury Bill
- The strike price is set at $21.
- The underlying bill could go to $22 or $18 in 1 month.

**PORTFOLIO:**
- Long $\Delta$ bills
- Short 1 Call

**Bill=$22**
- Call=$1
**Portfolio value** = 22 $\Delta - 1$

**Bill=$18**
- Call=$0
**Portfolio value** = 18 $\Delta$

**UP**
- Bill=$20
- Call=?

**DOWN**
- Bill=$18
- Call=$0

If $\Delta = 0.25$, Portfolio = $4.5$

*A risk-free portfolio must earn the risk-free rate of return.*

Discounting the *certain outcome* of $4.5 by the risk-free rate (10%) and subtracting cost of $\Delta$ bills leads to the call price: $PV(4.5) - 5$, ie $0.91$.

---

**Plain Vanilla**

<table>
<thead>
<tr>
<th>Option</th>
<th>Strike</th>
<th>Type</th>
<th>Maturity</th>
<th>Coup. Rate</th>
<th>Expired</th>
<th>FWd rate</th>
<th>On the bond</th>
<th>Fwd Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$99.00$</td>
<td>$99.00$</td>
<td>European</td>
<td>$&lt;5$</td>
<td>$10.00%$</td>
<td>$78.95$</td>
<td>$2.00%$</td>
<td>$39.33%$</td>
<td>$15.94%$</td>
</tr>
<tr>
<td>$99.00$</td>
<td>$99.00$</td>
<td>European</td>
<td>$&lt;5$</td>
<td>$10.00%$</td>
<td>$78.95$</td>
<td>$2.00%$</td>
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<td>$15.94%$</td>
</tr>
<tr>
<td>$99.00$</td>
<td>$99.00$</td>
<td>European</td>
<td>$&lt;6$</td>
<td>$10.00%$</td>
<td>$78.95$</td>
<td>$2.00%$</td>
<td>$39.33%$</td>
<td>$15.94%$</td>
</tr>
<tr>
<td>$99.00$</td>
<td>$99.00$</td>
<td>European</td>
<td>$&lt;6$</td>
<td>$10.00%$</td>
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<td>$39.33%$</td>
<td>$15.94%$</td>
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</table>

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Options 48
Barrier Options

This Down-and-In option is an example of path-dependent options, where how you get there matters.

Options

- Puts and Calls
- Put-Call Parity
- Combinations and Trading Strategies
- Valuation
- Hedging
- Valuation of Exotics
Option Valuation: Applications

- Warrant bonds
- Convertibles
- Callable bonds
- Corporate valuation

What's a Company Worth?

- Required returns
- Types of Models
  - Balance sheet models
  - Dividend discount models
  - Corporate cash flow models
  - Price/Earnings ratios
- Estimating Growth Rates
- Application
**What’s a Company Worth?**

**Alternative Models**

+ The options approach
  - Option to expand
  - Option to abandon
+ Creation of key resources that another company would pay for
  - Patents or trademarks
  - Teams of employees
  - Customers
+ Examples?

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**What’s a Company Worth?**

**The Options Approach**

Value of the Firm or project

Present Value of Expected Cash Flows if Option Exercised
The Option to Expand

An Example of a Corporate Option

- J&J is considering investing $110 million to purchase an internet distribution company to serve the growing on-line market.

- A financial analysis of the cash flows from this investment suggests that the present value of the cash flows from this investment to J&J will be only $80 million. Thus, by itself, the corporate venture has a negative NPV of $30 million.

- If the on-line market turns out to be more lucrative than currently anticipated, J&J could expand its reach a global on-line market with an additional investment of $150 million any time over the next 2 years. While the current expectation is that the cash flows from having a worldwide on-line distribution channel is only $100 million, there is considerable uncertainty about both the potential for such an channel and the shape of the market itself, leading to significant variance in this estimate.

- This uncertainty is what makes the corporate venture valuable!
Valuing the Corporate Venture Option

- Value of the underlying asset \((S)\) = PV of cash flows from purchase of on-line selling venture, if done now = $100 Million
- Strike Price \((K)\) = cost of expansion into global on-line selling = $150 Million
- We estimate the variance in the estimate of the project value by using the annualized variance in firm value of publicly traded on-line marketing firms in the global markets, which is approximately 20%.
  - Variance in Underlying Asset’s Value = 0.20
- Time to expiration = Period for which “venture option” applies = 2 years
- 2-year interest rate: 6.5%

Black-Scholes Option Valuation

\[
\text{Call value} = S_o N(d_1) - X e^{-rT} N(d_2)
\]
\[
d_1 = \frac{\ln(S_o/X) + (r + \sigma^2/2)T}{\sigma \sqrt{T}}
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d_2 = d_1 - \sigma \sqrt{T}
\]

where
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- We estimate the variance in the estimate of the project value by using the annualized variance in firm value of publicly traded on-line marketing firms in the global markets, which is approximately 20%.
  - Variance in Underlying Asset’s Value = 0.20
- Time to expiration = Period for which “venture option” applies = 2 years
- 2-year interest rate: 6.5%
  
  Call Value = 100 (0.7915) - 150 (exp(-0.065)(2) (0.3400)
  = $ 52.5 Million

Summary

+ Puts and Calls
+ Put-Call Parity
+ Combinations and Trading Strategies
+ Valuation
+ Hedging
+ Valuation of Exotics
+ Applications to corporate securities and corporate valuation