1. Introduction

The cost of capital for a project (or an investment) is the rate of return required by the investors who put money into it. This rate of return, in turn, will depend on the rate of return that investors can get from risk-equivalent investments elsewhere in the capital markets. If investors cannot get at least the risk-equivalent rate of return from the project, they will have little incentive to put their money into it, and a firm will have a difficult time raising capital. Thus, the cost of capital for a firm’s investment is also the opportunity cost (to investors) of putting their capital into the firm’s investment project. The cost of capital is the appropriate discount rate at which expected free cash flows from a project should be discounted, in order to get its present value (PV).

It would be nice if the cost of capital for a firm or a project could be observed directly, but this is unfortunately not the case: The cost of capital has to be estimated. For the two broad types of investors—equityholders and debtholders—we will see below that required rates of return can be determined for debt, and can be inferred using observed capital market prices for equity. By weighting the market-determined cost of equity and the after-tax cost of debt with the respective proportions of each in an investment’s market value, and adding the two parts together, the opportunity cost of capital for the project can be estimated. This opportunity cost is referred to as the “weighted average cost of capital” or WACC, for short.

2. The Two Typical Sources of Financing: Equity (Stocks) and Debt (Bonds)

The two typical—and most common—sources of finance for firms’ investments are equity (also called stocks or shares) and debt (also called bonds). Both types of securities are traded in financial markets worldwide.

Stocks are non-contractual ownership claims to the residual cash flows of a corporation, and are the basic units of ownership. Owners (stockholders) receive stock certificates for the shares they own of a corporation. There is usually a stated value of the stock certificate that is called a “par value.” The amount of equity contributed to the firm’s over above the par value (e.g., when new stock is issued by the firm) is called “capital surplus.” The amount that is retained in the firm from net income after payment of dividends to shareholders is called “retained earnings.” The book value of equity in a firm is the sum of its par value, capital surplus, and all the retained earnings. The market value of the equity in a firm is the observed stock price in the stock market, multiplied by the number of shares that the firm has outstanding. (In corporate finance, we shall almost always be concerned with the market value of equity, and not book value.)

Bonds represent contractual obligations to repay corporate borrowing—i.e., they must be repaid. It is not an ownership claim on the firm (except in the event of bankruptcy). When corporations borrow, they promise to make regularly scheduled interest payments (called coupons) and to repay the original amount borrowed (called the principal or the face value). There is usually a promise to repay by a certain date, called the maturity date. In all countries with bankruptcy laws, the corporation can legally default at any time on its bonds and declare bankruptcy (for example, by not paying interest), but it must hand over the assets of the firm to its debtors. Unlike dividend
payments on equity, interest payment on debt is considered a cost of doing business and is fully tax deductible in most countries. Therefore, the true ‘cost’ of debt financing to a firm is the after-tax interest payments. Debt can be secured or unsecured: security provides that the secured property can be sold in the event of default to satisfy the debt for which the security is given. Debt can sometimes be called bills, notes, bonds, or debentures. Bills are usually short term debt, with maturity less than a year. Notes are typically medium term debt, with maturity greater than a year but less than seven years. Bonds usually refer to longer term debt (but “notes” and “bonds” are often used interchangeably). Debentures usually refer to long term unsecured debt. Long term debt is typically paid in regular amounts over the life of the debt. The payment of long term debt by installments is called amortization, which is usually arranged through a ‘sinking fund.’ Each year, the firm places money into the sinking fund and that money is used to buy back the bonds.

3. The Weighted Average Cost of Capital (WACC)

Assuming that a project (or firm) is financed entirely with debt and/or equity, if we call the market value of equity in a project E, and the market value of debt in the project D, then the total value of the project or firm—let us call it V—would be equal to E+D.

If we knew (or could estimate) that the required rate of return on equity is \( r_E \), the required rate of return on debt is \( r_D \), and the marginal corporate tax rate is \( t_c \), then the weighted average cost of capital is defined as:

\[
WACC = \left( \frac{E}{V} \right) \cdot r_E + \left( \frac{D}{V} \right) \cdot [1 - t_c] \cdot r_D
\]

where we have weighted the market-determined cost of equity (\( r_E \)) and market-determined after-tax cost of debt (\( (1 - t_c) \cdot r_D \)) in the respective proportions of value in the project claimed by equity (\( E/V \)) and by debt (\( D/V \)). Note that we are interested in the after-tax cost of debt (and pre-tax cost of equity) in situations in which the country’s tax codes allow corporations to deduct interest payments as an expense for tax purposes, but not dividend payments (this is, as mentioned before, the situation in most countries of the world).

To recap, in order to compute the WACC, we need to know the following:

1) The cost of debt for the project or the investment (\( r_D \));
2) The marginal corporate tax rate applicable to the project or the investment (\( t_c \));
3) The cost of equity for the project or the investment (\( r_E \));
4) The ‘optimal’ ratio of the market value of debt to firm value (\( D/V \)), and the market value of equity to firm value (\( E/V \)).

Items (1) – (3) above deal with the problem of assessing the cost of each type of capital. Item (4) deals with the problem of finding the ‘right’ mix of debt and equity for an investment project, and is referred to generically as the problem of finding the ‘optimal capital structure.’ In this note, we shall deal mostly with (1) to (3), on the assumption that (4) had already been determined. However, we will have something brief to say on the considerations that go into deciding whether a project should be financed with equity or with debt, and if so, in what proportions.

We will see below that the cost of debt (\( r_D \)) is relatively easy to calculate (or infer) from current market data. However, calculating the cost of equity (\( r_E \)) requires us to understand the crucial relationship between risk and return, and how investors make portfolio decisions. This will lead us to the concept of something called “beta.” Once we get there, we will see that the actual formula used to calculate \( r_E \) is relatively simple.
In order to concretize the definition of WACC above, let us consider an example. Suppose we are told (or knew) that a Company (Co) has a cost of equity ($r_E$) of 16%, a pretax cost of debt ($r_D$) of 9.5%, faces a corporate tax rate ($t_c$) of 32%, and its debt-to-equity ratio ($D/E$) is 25% (therefore, its debt-to-total value ratio must be 20%). Then, the Company’s overall cost of capital, or its weighted average cost of capital is:

$$WACC_{Co} = (0.8)*(.16) + (0.2)*(1 – 0.32)*(0.095) = 14.09\%.$$  

Let us now turn to the issue of computing $r_D$ and $r_E$.

4. The Cost of Debt Financing ($r_D$)

The (pre-tax) cost of debt is relatively easy to calculate: it is simply the yield-to-maturity (i.e., the internal rate of return) on the company’s bonds. It is not the historical coupon rates on the company’s bonds, but rather, the current yield-to-maturity that would prevail if the firm were to go out and issue bonds given the prevailing market conditions and its credit rating. Typically, the cost of debt can be assessed by examining the current yields-to-maturity on corporate bonds of equivalent credit risk or credit rating. A number of agencies (such as Moody’s or Standard & Poors in the US) regularly put out bond ratings for most of the large, publicly traded companies around the world.

5. The Cost of Equity Financing ($r_E$)

The estimation of the cost of equity financing ($r_E$) is one of the central and most exciting topics in finance. The development of ideas on this issue led to the concept of the “beta.” The beta measures the degree of ‘systematic’ or ‘undiversifiable’ risk associated with an asset, and it will provide us with a simple formula for determining $r_E$. Before we can understand beta, however, it is necessary to understand how expected return is related to risk, how investors use this relation to form diversified portfolios, and how such diversified portfolios result in the elimination of a certain type of risk known as ‘diversifiable’ risk.

5.1. Investor Behavior and the Relation between Risk and Return

Finance theory makes three behavioral assumptions about investors. All else equal: (1) Investors prefer a dollar today to a dollar tomorrow (the assumption of “time preference” or “time value of money”); (2) Investors prefer more wealth to less (the “greed” or “wealth preference” assumption); and (3) Investors prefer less risk to more (the “fear” or “risk aversion” assumption). These three assumptions will lead to a simple prediction: higher expected returns are likely to be associated with higher risks.

The time preference assumption implies that, even for a riskless investment, investors will expect a minimum rate of return. That is, given that money has time value, investors will expect to get some riskfree rate of return—let us call it $r_f$—from investing in a riskless asset (such as, say, US treasury bills). The wealth preference assumption suggests that investors would prefer something more than simply the riskless rate of return from the financial marketplace; however, the

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1 It is pertinent to point out that the idea of “beta” not only resulted in a Nobel prize in Economics a few years ago, but it is also one of the most widely used concepts in the practice of real-world finance.
risk preference assumption implies that risk averse investors prefer less risky investments for a
given level of return. The problem is, however, that if an asset has an expected return that is higher
than the riskfree rate of return, it must also be a riskier asset. (If it were not, nobody would hold the
riskfree asset that gave a lower return.)

Thus, given investor risk aversion, how can we get them to invest in risky assets? Simple:
pay them a “risk premium” over and above the riskfree rate of return. Another way of stating this
is as follows: Given the risk characteristics of a particular asset, there is an appropriate risk
premium at which:

\[
\text{Expected return on} \quad \text{Expected return on} \quad \text{Risk premium for} \\
\text{a risky asset } i \quad \text{the riskless asset} \quad \text{the risky asset } i
\]

or,

\[
r_i = r_f + \text{RP}_i
\]

Two questions then arise: (i) What do we mean by “risk?” and (ii) Given this risk, what is
the appropriate “risk premium” (RP) for the asset? If we can answer these two questions, we are
done, in terms of pricing risk.

5.2. Step 1: Diversification and Risk

There is a rather simple way for investors to reduce the risk of their investments: diversify.
By holding portfolios that consist of different kinds of risky assets (e.g., different types of stock)
which comove less than perfectly with each other, investors can get rid of a major part of the risk
from their investments. The risk that can be thus eliminated is called “diversifiable risk.”

Consider an example. Suppose there are two types of stocks, those belonging to an
emerging equity market (EM) and those belonging to a developed country equity market (DM). Let
us suppose that, when DC stocks do well, EM stocks do poorly, and vice versa. Under three
equiprobable states of the world (State 1, State 2, State 3), suppose that the expected returns on EM
and DM stocks are as follows:

<table>
<thead>
<tr>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>EM</td>
<td>–10%</td>
<td>+10%</td>
</tr>
<tr>
<td>DM</td>
<td>+30%</td>
<td>+10%</td>
</tr>
</tbody>
</table>

If we own either stock by itself, we would do well in the good states of nature, but badly in
the poor states—i.e., there is considerable variability in our expected returns. But if we create a
portfolio by putting 50% of our wealth in EM and 50% in DM, our expected returns are:

EM/DM Portfolio (50% EM, 50% DM)

+10% +10% +10%

By diversifying, we guarantee ourselves a 10% return regardless of the state of nature. (Of
course, this example relied on two stocks with perfect negative correlation. But the general point of
this example holds whenever stocks have less than perfect positive correlation with each other (a
situation that is almost always true in reality). When any group of risky assets have less than
perfect positive correlation among each other, diversification will reduce risk.)
In the real world, however, we cannot get rid of all the risk, because there are common factors (such as interest rate movements, oil price hikes, political turbulence) that affect the returns on all stocks and hence, are a source of underlying systemic risk. As we increase the number of stocks in our portfolio, the overall risk of our portfolio will continue to fall, but at a slower and slower rate. We will reach a point when we cannot reduce any more risk. The risk that can diversified away is called **unique**, or **diversifiable**, or **unsystematic**, or **specific** risk. The risk that remains despite diversification (and that results from common underlying factors affecting all stocks) is called **systematic**, or **market**, or **undiversifiable** risk. The following figure shows how it works in the general case:

---

**Portfolio variance (risk)**

↑

Unsystematic risk

↓

Systematic risk

↑

↓

Number of stocks in portfolio

---

5.3. Step 2: Systematic Risk

In summary, investors in well-functioning markets will want to hold diversified portfolios (or they can also invest in mutual funds which can diversify on their behalf). Such diversification will get rid of a certain component of the risk. This is where the folk wisdom “don’t put all your eggs in one basket” comes from. There are two additional implications of this insight: (1) If an investor chooses not to diversify, financial markets will not compensate him or her for bearing risk that can be diversified away; (2) Since there is systematic or market risk left behind in the portfolio regardless of how much an investor diversifies, the market will compensate them with additional risk premium for bearing only for this latter type of risk.
Thus, since investors will hold well-diversified portfolios, the only risk that is important is the additional contribution to risk made by a particular risky asset to this diversified portfolio. The most diversified portfolio will have, in theory, every risky security in the market—this is called the market portfolio. Thus, in measuring the risk of any asset and in assessing its risk premium, we need a measure of how much risk a single risky asset contributes to the market portfolio. This risk, the systematic risk of a risky asset, is measured by the beta.

5.4. Step 3: The Beta and the Capital Asset Pricing Model (CAPM)

The metric that is used to measure the systematic risk of an asset is called the “beta” ($\beta$). It measures the sensitivity of the expected returns on a risky asset to movements in the market portfolio (which, in turn, is presumably driven by the effects of economy-wide factors). More precisely, it measures the percentage change in the expected return on a risky asset for every unit percentage change in the expected return on a well-diversified market portfolio. For example, a stock with $\beta = 1.5$ means that if the “market” moves up by 1%, the expected return on the stock moves up by 1.5% (and likewise, the expected return falls by 1.5% if the market moves down by 1%); a stock with $\beta = 0.5$ means that if the market moves up by 1%, the expected return on the stock moves up by 0.5%; etc.

The market portfolio has beta of one, by definition (It is tautological: when the market moves up or down by 1%, the market moves up or down by 1%!). We can also similarly guess that the riskfree asset must have a beta of zero (Why?). Now, recall our notion that expected returns should be increasing as the relevant risk of the security increases. We have established that the relevant risk is beta risk. For now, assume that the relation between expected return ($E[r]$) and beta ($\beta$) is a straight line (we will shortly argue why this should be). If that is the case, we know two points on that straight line: the riskfree asset with an expected return of $r_f$ and a beta of zero, and the market portfolio with an expected return of $r_m$ and a beta of one. That is, graphically:

---

2 The exact definition of the beta or the systematic risk, for security i, is $\beta_i = \frac{\text{Cov}(r_m, r_i)}{\sigma_m^2}$, where Cov(,) is the covariance and $\sigma$ is the standard deviation]. Thus, it measures the covariance of security i’s expected return with the expected return on the market portfolio, standardized by the variance of the market portfolio’s returns.
The line that is upward sloping is called the “security market line” (or SML for short), and it is the graphical representation of the famous capital asset pricing model (CAPM). The algebraic representation of the graph above for any risky security \( i \), on the vertical axis, is:

\[
E[r_i] = r_f + \beta_i (r_m - r_f)
\]

This is the CAPM. Its interpretation is as follows: The expected return on a risky asset \( E[r_i] \) is equal to the expected return on a riskfree asset \( r_f \) plus the risk premium for that asset; the risk premium for that asset is the beta \( \beta_i \) of that asset multiplied by the risk premium of the market portfolio \((r_m - r_f)\), also called the ’market risk premium’ or MRP. Going back to where we started:

\[
\begin{align*}
\text{Expected return on} & \quad \text{Expected return on} & \quad \text{Risk premium for} \\
\text{a risky asset } i & \quad \text{the riskless asset} & \quad \text{the risky asset } i \\
\uparrow & \quad \uparrow & \quad \uparrow \\
E[r_i] & = & r_f + \beta_i (r_m - r_f)
\end{align*}
\]

An equivalent (and more intuitive way) is to state the CAPM as follows:

\[
E[r_i] = r_f + \beta_i \times \text{MRP}
\]

where the beta can be interpreted as the “quantity of risk” in the asset, and the MRP can be interpreted as the “market price of risk” for the typical investor in the financial marketplace.

(We implicitly assumed that the SML is a straight line. Let us examine that assumption. Suppose it were not true. Let us say that there was a security with \( \beta = 1.5 \) that was below the SML line. Then according the CAPM formula, that security is expected to generate an expected return that is lower than a security in its beta class—i.e., it is overvalued at its current price. If investors knew this, they would start to sell the security, upon which its price would fall and its returns would rise, toward SML. As a result, in equilibrium, the “fairly priced” security will lie on the SML. Another was of saying this is as follows: in a no-arbitrage equilibrium, the SML will be a straight line, and CAPM will produce a ‘linear’ relation between risk and expected return.)

From the formula above, note the following: (i) The required rate of return on the riskless asset \( \beta = 0 \) is, as it should be, the risk free rate of return, \( r_f \). The required rate of return on the market portfolio \( \beta = 1 \) is, as it should be, \( r_m \).

A few other important implications of CAPM:

- This general model quantifies the risk-return relation for any risky asset in the world: stocks, bonds, real estate, investment projects, art,.....

- We should invest more in assets with expected returns above the SML; We should not invest in assets that lie below the SML.

- Stocks whose returns move with the market have positive betas. They can be aggressive (high beta, \( \beta > 1 \)) or defensive (low beta, \( \beta < 1 \)) stocks.
Examples of aggressive stocks are those in cyclical industries (e.g., construction, housing), those with high operating leverage (e.g., semiconductor, commercial airframes), those whose value is embedded in future growth opportunities and intangibles (e.g., software, R&D- or knowledge-based assets, biotechnology), or those with considerable cash flow volatility (e.g., internet, information technology. Examples of defensive stocks are those with relatively stable and mature, non-cyclical cash flows, or those with low operating leverage (e.g., food, utilities, beverages, tobacco, over-the-counter drugs).

Betas can also be zero, or even negative. A zero beta asset is one all of whose variance is unsystematic—i.e., in a well diversified portfolio, such an asset adds no risk. Thus we would be happy with just the riskfree rate of return for such an asset. A negative beta asset is one that pays off when the market portfolio does poorly (and vice versa) Such assets have value since they do well in the poor states of nature. We would accept less than the riskfree rate of return (perhaps even a negative return) for such an asset.

There is no such thing as a “good” or “bad” beta; the only issue is, given an asset’s beta, does it give its investors a return consistent with its beta risk (as specified by the CAPM).

5.5. And Finally...... The Cost of Equity!!

The cost of equity for a project (or a firm) is simply the required rate of return on a risky asset whose risk equals the risk of the project (or the firm). Therefore, the cost of equity can be calculated form the CAPM formula:

\[ r_E = r_f + \beta_E \times MRP \]

To compute the cost of equity, we need to know (or estimate) three numbers: the currently prevailing riskfree rate of return \(r_f\), the systematic risk (or ‘equity beta’) for the project or the firm \(\beta_E\), and the market risk premium \((MRP)\).

Estimating \(\beta\): \(\beta\) is estimated using a statistical procedure known as linear regression. In such regression analysis, historical data (typically, three to five years of monthly data, or one to two years of weekly data) on the asset returns and the market returns are used to estimate \(\beta\). However, we don’t have to do this—many commercial companies (e.g. Ibbotson Associates for US firms; Disclosure/Worldscope database for companies worldwide) estimate and update betas regularly. Betas are also available (for free) on the Yahoo!Finance website (www.finance.yahoo.com), and for NASDAQ-listed companies, on www.nasdaq.com

Estimating the Riskfree Rate: A commonly used proxy for the riskfree rate is the current yield-to-maturity on US (or similar) government bonds and bills. If the cost of equity is being calculated for capital budgeting or corporate valuation purposes, we should use the yield on long term bonds (ten- to thirty-year bonds). If the cost of equity is being calculated for short term investment purposes, we should use the yield on short term bills (six months to a year).³

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³ Of course, the CAPM is a ‘one-period’ model. For multiple periods, the theoretically correct way is to use a different riskfree rate for each period (which can be calculated as periodic forward rates from the yield curve). However, if the yield curve is relatively flat, the use of long term bond yields is fine in practice.
Estimating the Market Risk Premium: The market risk premium, MRP, represents a ‘behavioral’ construct in that it attempts to capture what the typical investor in the marketplace will require, going forward, as the risk premium over the riskfree rate (e.g., the yield on a relatively safe asset such T-bonds or T-bills) in order to hold a well-diversified portfolio that is similar to the market portfolio. The issue of ‘what is the right MRP’ has been one of much debate in the literature during the past few years.

The traditional (and simple) approach has been to calculate MRP as the average excess long run historical return to a ‘market portfolio’ (typically proxied by the S&P 500 stock index in the US), relative to T-bonds or T-bills. In computing such an average, we can calculate an ‘arithmetic mean’ (AM) which is just a simple average of the annual excess return, or a ‘geometric mean’ (GM) which measures a compounded annual growth rate in returns during the period considered.\(^4\)

In finance theory, the conceptually correct measure of returns is AM, since it measures the ‘expectation’ of annual returns that an investor hopes to get when (s)he invests their money in the market at any random point in time. In the US, the average MRP as measured by the AM and using long-term T-bonds as the benchmark for the historical period 1900-2000 is approximately 7% (and if measured by GM, 5%). If T-bills are used as the benchmark, it is 7.7% and 5.8%, respectively. The MRP of the ‘world’ market relative to T-bonds is 5.6% and 4.9%, respectively. Table 1 (see end of this note) summarizes MRP’s using AM and GM (and T-bills versus T-bonds) for some of the major industrialized countries.

Thus, the US MRP that is consistent with the traditional view (and the one most frequently used in the past) is the AM of 7%. However, there is now substantial evidence that this MRP has been declining over time, and that 7% might be an overestimate of the MRP going forward. Many scholars and practitioners in finance are gravitating toward an MRP of around 5% (and some believe that it could be as low as 2% - 3%).

Going back to our example of the Company (see top of Page 3 of this note), we said that the Company’s cost of equity is currently 16%. If the current yield on a long term US government bond is 5.5%, the Company’s equity beta is 1.5, and if we consider the historical US MRP of 7% as being apt, the Company’s cost of equity can be derived from CAPM as:

\[
\begin{align*}
\text{r}_{E(Co)} & = 5.5\% + 1.5 \times 7.0\% = 16\%, \\
\uparrow & \quad \uparrow \quad \uparrow \\
\text{r}_f & \quad \beta_{Co} \quad \text{MRP}
\end{align*}
\]

6. Tax Rates ($t_c$), Debt-to-Firm Value ($D/V$), and Equity-to-Firm Value ($E/V$)

The three things remaining to be figured out are the appropriate corporate tax rate ($t_c$), and the appropriate ratios of debt and equity to the market value of the firm ($D/V$ and $E/V$).

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\(^4\) In general, AM is greater than GM; the more volatile the stream of historical returns, the greater th difference.
The Tax Rate: The appropriate tax rate, \( t_c \), to use in calculating the WACC is a firm’s marginal tax rate, and not its average tax rate. In other words, it is the tax rate associated with an incremental dollar that comes into the company, relative to where it is today. In many instances it may be the same as the firm’s current average (effective) tax rate.

Equity-to-Firm Value: The appropriate value of equity, \( E \), to use is the market value and not the book value. The total value of equity is given by the number of shares outstanding multiplied by the current market price per share (and not the average of some past prices) — i.e., the price at the time the WACC calculation is being undertaken. This number would then be divided by the market value of the firm, \( V \) (see below).

There is the issue of whether we should consider the number of shares on a ‘fully-diluted’ basis. A more ‘honest’ WACC calculation will, in fact, consider the fully-diluted number of shares at the time of the calculation (however, treasury stock should be excluded from the calculation).

Debt-to-Firm Value: Here too, the appropriate number for \( D \) is the market value. But, unlike the case of equity, market value data for bonds may be more difficult to come by. If the firm is planning to issue fairly good quality bonds to finance the project (i.e., not ‘junk bonds’) and has an investment-grade credit rating in the marketplace, then it is OK to compromise a bit, and to use the book value of debt in place of market value. This is not an ideal situation, but a practical approach.

The more important question is, which ‘debt’ should be included in calculating a firm’s WACC. In general, the only debt that is relevant is the debt that is considered to be a relatively ‘permanent’ part of the capital structure of a project, division, or firm. This would argue for considering only ‘long-term’ debt (i.e., debt that has maturity greater than one year). But there are many who would argue that we should consider all interest-bearing debt and therefore, we should consider both long-term and short-term debt. My view on this is somewhat eclectic (although I have a personal preference for considering only long-term debt). The one thing that we can say for sure is that, if short-term debt is considered as part of the WACC calculation, then it should not be considered in the derivation of free cash flows (i.e., short-term debt should be taken out of the current liabilities prior to deriving the change in net working capital). If only long-term debt is considered in the WACC calculation, then the effects of short-term debt should be included in the derivation of free cash flows. The important point is, be consistent. (One final point: The capitalized value of long term leases should be considered as the equivalent of long-term debt, in WACC calculations).

Firm Value: The value of the firm, \( V \), for the purposes of WACC, is simply the sum of the market value of equity and the value of debt—i.e., \( V = E + D \).

7. Some Observations on Capital Structure Choice (i.e., the ratio \( D/E \))

The question of ‘what is the appropriate debt-equity ratio for a project or a firm’ (or the related question, ‘how much debt should a project or a firm take on’) is a huge topic of research in finance, and one that perennially confronts CEOs. It is beyond the scope of this note to address all the nuances and issues involved. Moreover, unlike other parts of finance theory and practice, there is no simple formula that helps us to determine the optimal capital structure—the issues involved are somewhat qualitative and even judgmental. However, there are some generalized insights we can take away on factors that affect the capital structure decision.
All else equal, a firm or a project can bear more, rather than less, debt if:

- There are tax shield benefits;
- The bankruptcy risk is low (e.g., there is sufficient interest coverage from earnings before interest and taxes);
- The firm or project has low beta (generally, ‘defensive’ assets);
- The firm or project has low operating leverage (i.e., fixed costs are low) and low financial leverage (i.e., it is not already loaded up with debt);
- The firm or project has sufficient tangible assets (i.e., collateral).

There may be some additional considerations as well:

- **Control:**
  - Will the debt bring with it new covenants on management behavior?
  - If equity is repurchased in the process of the debt issue, is there a (potentially troublesome) large shareholder whose share ownership might increase too much?

- **Flexibility:**
  - Is the decision reversible?
  - Will it constrain future access to capital markets?
  - Is there an option value associated with maintaining some financial ‘slack’?

- **Signalling:**
  - What kind of signals will the financial markets read?
  - Is the firm moving toward, or away from, industry D/E norms?

- **Transaction Costs:**
  - How much will it cost to do the issue? What are the direct and indirect costs?
  - Will it be a one-time decision, or an incremental decision?

- **And Finally.....**
  - Does the firm need debt?
  - Does the firm want debt?
  - How will it affect the corporate culture?
  - Could there be a potential takeover threat if the capital structure change is not implemented?
  - If “it ain’t broke,” maybe you shouldn’t “fix it?”

As you can see, it is not an easy decision!
8. Impact of Capital Structure on the Cost of Equity

There is one other important issue we have to deal with, in relation to the cost of equity: It turns out that the cost of equity is sensitive to the debt-equity ratio. When this ratio changes, the cost of equity also changes. More specifically, when the debt-equity ratio goes up, the cost of equity goes up too. Since debt has priority over equity on the firm’s risky cash flows, most of the risk of the business (the asset risk) is imposed on the equityholders, since after all, they only get what is left after the debtholders are paid off first. This is referred to as additional ‘financial risk’ for equityholders. This implies that, as the firm uses more debt, equityholders require a higher rate of return in order to compensate them for the financial risk (on top of the asset risk). There are two general approaches used to quantify the effects of leverage changes on cost of equity: the ‘M-M 2 approach,’ and the ‘unlevering/relevering betas approach.’

M-M 2

One way to quantify the effect of leverage changes on cost of equity is by using another important (and famous) result, called the “Miller-Modigliani Proposition 2” (M-M 2):

\[
    r_E = r_A + (r_A - r_D)(1 - t_c)[D/E],
\]

where \( r_E, r_D, D, E, \) and \( t_c \) are as before, and \( r_A \) is the underlying business risk of the asset (Note: \( r_A \) is also the required rate of return on equity for an all-equity firm). The required rate of return for levered equity is equal to the expected rate of return on the firm’s assets, \( r_A \), plus a financial risk premium, \( (r_A - r_D)(1 - t_c)[D/E] \) which in turn is a function of the firm’s capital structure. Thus:

\[
    r_E = r_A + (r_A - r_D)(1 - t_c)[D/E]
\]

\[
    \uparrow \quad \uparrow \quad \uparrow
\]

Cost of equity \quad Return to compensate \quad Additional return to compensate
for a levered firm \quad for business risk \quad for financial risk from leverage

Going back to our example of the Company, given its levered cost of equity, its tax rate, and its current \( D/E \) ratio, we can now calculate the required rate of return on its assets. From M-M 2 above:

\[
    16\% = r_{A(Co)} + (r_{A(Co)} - 9.5\%)(1 - 0.32)(25\%)
\]

Therefore, \( r_{A(Co)} = 15.06\% \). This number represents the rate of return that reflects purely the Company’s business risk as a company. This is also the rate of return that would be required by its shareholders if it was financed as an all-equity firm. By having a 25% \( D/E \) ratio, the Company has imposed additional risk on its shareholders, who then demand an extra 0.94% as the financial risk premium.

Using M-M 2, we can also answer a question such as: What would the Company’s cost of equity be if it decided to increase its \( D/E \) ratio to, say 50%? Since we now know its “pure” asset rate of return (15.06%), we can answer this question. At a \( D/E \) of 50%, the Company’s cost of equity would be:

\[
    r_{E(Co)} = 15.06\% + (15.06\% - 9.5\%)(1 - 0.32)(50\%)
    = 16.94\%.
\]
As the Company increases leverage from 25% to 50%, its cost of equity goes up from 16% to 17%.

**Unlevering/Relevering Betas**

The adjustment to reflect the effects of leverage can also be done with the betas of the firm. In there are no corporate taxes, given the value-additivity of CAPM, the beta of a firm’s assets (call it $\beta_A$) must be equal to the weighted average of the betas of its equity and debt financing. That is:

$$\beta_A = \beta_E(\frac{E}{E+D}) + \beta_D(\frac{D}{D+E})$$

where $\beta_A$ is the firm’s equity beta and $\beta_D$ its debt beta. If $\beta_D = 0$ (i.e., the firm has very high-quality debt and its yield is almost risk-free), then this can be simplified to:

$$\beta_A = \beta_E(\frac{E}{E+D})$$

or,

$$\beta_E = \beta_A(\frac{1}{1 + (D/E)})$$

In other words, given a firm’s unlevered or ‘all-equity’ beta ($\beta_A$; also called the ‘pure’ beta) and its debt-equity ratio, we can ‘relever’ to calculate its equity beta ($\beta_E$). In reverse, if we know the firm’s equity beta and its debt-equity ratio, we can ‘unlever’ to calculate its asset beta. If we incorporate corporate taxes, the formula above is modified slightly: it becomes:

$$\beta_E = \beta_A[1 + (1 - t_c)(D/E)]$$

where $t_c$ is the (marginal) corporate tax rate. Another modification, which takes into account the fact that debt can be risky (i.e., $\beta_D$ is not equal to zero) is the following, which says that the levered beta is given by:

$$\beta_E = \beta_A + (\beta_A - \beta_D)(1 - t_c)(D/E)$$

where $r_D$ is calculated using the CAPM as $r_D = r_f + \beta_D \times MRP$.

Yet another modification is based on the ‘Miles-Ezzell’ formula, which says that the levered beta is:

$$\beta_E = \beta_A + (\beta_A - \beta_D)(1 - t_c \times Q)(D/E)$$

where $Q = r_D/(1+r_D)$, and $\beta_D$ is calculated as in Footnote 6.

**Which Approach To Use?**

Each of these approaches can result in slightly different cost of equity $r_E$, therefore, slightly different WACC. Therefore, which is the ‘correct’ approach? Under the most general conditions, the most ‘precise’ estimate is provided by the Miles-Ezzell approach, since that is the method that will ensure equivalence between two most commonly used methods of corporate and project valuation, namely the ‘weighted average cost of capital’ (WACC) method and the Adjusted Present Value (APV) method (we will not go into them here).

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5 In the event that the $r_D$ is known or given, the debt beta, $\beta_D$, can be derived from the CAPM formula for cost of debt. In other words, $\beta_D = (r_D - r_f)/MRP$. 

13
But, keeping in mind the dictum that ‘good financial analysis is about being approximately right rather than précisely wrong,’ and given the fact our estimates of parameters such betas, market risk premium, risk free rates and so forth will have errors in estimation, it is OK, in practice, to use the approach that says that $\beta_C = \beta_A \left[ 1 + (1 - t)(D/E) \right]$. It turns out that this formula produces estimates that are reasonably close to those we would obtain with the Miles-Ezzell approach.

9. Cost of Capital in the Cross-border Setting

Capital budgeting and valuation of projects in the cross-border setting can be approached one of two ways:

1. Forecast the foreign currency project cash flows, discount to the present at the foreign currency discount rate, and convert it back into the home currency present value using the currently prevailing spot exchange rate between the home and foreign currency; or,

2. Forecast the foreign currency project cash flows, convert into home currency cash flows using forecasted exchange rates, and discount to the home currency present value using the home currency discount rate.

If both uncovered interest parity (UIP) and purchasing power parity (PPP) held at all times, both approaches will result in the same valuations.

However, a few basic questions arise, the most important of which are the following: (1) In which currency should we undertake the valuation? (2) How do we convert a home currency WACC into a foreign currency WACC (or vice versa)? (3) How do we find the ‘right’ measure of systematic risk and the ‘right’ cost of equity in the cross-border setting? (4) How do we forecast exchange rates for cross-border valuation purposes?

- Which Currency: We can choose either the home currency or the foreign currency in determining cash flows. Foreign currency cash flows should be discounted at the foreign currency discount rate, and home currency cash flows at the home currency discount rate. If the value of a project is likely to be particularly sensitive to exchange rate fluctuations, it might be helpful to explicitly forecast foreign currency cash flows and then convert them into home currency cash flows using a specific set of forecasted exchange rates.

- WACC: It may be expedient (and probably accurate) to assume that the investors are indigenous to the home country in which the parent firm is headquartered. This means that we can use a home-country-benchmarked cost of capital (adjusted, of course, for the risk characteristics of the project vis-a-vis the home country). If we need to convert the WACC from one currency to another, use the following formula for uncovered interest parity:

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6 This is an advanced section, and can be skipped if cross-border issues are not being covered in the course. Prior to reading this section, it is presumed that participants have read Sundaram and Black, Chapter 3, “Foreign Exchange: The Basics,” in The International Business Environment: Text and Cases, Prentice-Hall, 1995. Specifically, participants should be familiar with the theories of Uncovered Interest Parity and (Relative) Purchasing Power Parity.

\[ \text{WACC}_{\text{Home}} = \text{WACC}_{\text{Foreign}} \times (1 + \Delta e) + \Delta e \]

where \( \Delta e \) is the “expected percentage annual change in the value of the home currency against the foreign currency.” Using the direct quote (i.e., the number of units of the home currency it takes to buy each unit of the foreign currency), if \( \Delta e \) is greater than zero, then it would be an expected depreciation of the home currency, and if \( \Delta e \) is less than zero, it would be an expected appreciation of the home currency.

In the WACC calculation, the cost of equity capital will depend on whether equity markets are integrated or segmented. If markets are segmented, we should use the home country market index; if integrated, a world index. But in the case of countries such as the US, the correlation coefficient between the two indexes is quite high, and so it doesn’t matter too much.

- **Country Risk and \( \beta \):** Whether or not a risk premium for country risk (CRP) should be included in the WACC calculation depends on two judgments (and therefore, assumptions) that we need to make: (i) Whether or not the country risk can be considered as systematic risk; (ii) Whether or not it is already captured in the \( \beta \) for the project. If the judgment is that it is unsystematic, we should ignore it.

For bonds (debt) it is always necessary to add a CRP; for equity, it may not be relevant. However, there is increasing consensus that, at least with respect to projects in emerging markets, it may be necessary to add a CRP (although it may not be necessary to do so in industrialized country markets). The most commonly used approach to calculate the CRP is called the ‘yield-spread method’. First, calculate the average yield differential between equivalent maturity (and equivalent currency) Eurobond issues of investment-grade issuers of the home country and foreign country. Next, add this to the expected return from CAPM (the modified CAPM would now be written as \( r_E = r_f + \beta_E \times MRP + CRP \)). For instance, if the average yield differential between US$ Eurobond issues of Brazilian and US firms is 300 basis points (3%), and if \( r_f \) (in US$) = 5.5%, \( \beta_E = 1.5 \), and the MRP of US investors is 7%, then the US$ cost of equity for the Brazilian project would be \( r_E = 5.5\% + 1.5 \times 7\% + 3\% = 19\% \). If, from the US perspective, \( \Delta e = -5\% \) (i.e., the US$ is expected to appreciate by 5% annually against the Brazilian Real, B$), then the B$ cost of equity is:

\[
r_{E(B$)} = \left[ r_{E(US$)} - \Delta e \right]/\left[1 + \Delta e\right] = \left[19\% +5\%\right]/0.95 = 25.26\%
\]

Note that, reflecting the higher expected inflation in Brazil relative to the US, investors will expect a higher return (relative to the US) to their cash flows denominated in B$.

- **Forecasting Exchange Rates:** The two most commonly used approaches to forecast exchange rates (i.e., to derive \( \Delta e \)) are relative purchasing power parity (RPPP) and uncovered interest parity (UIP). According to RPPP, the expected percentage change in the value of the domestic currency against the foreign currency, \( \Delta e \), is given by:

\[
\Delta e = \left[ (1 + \Delta P_{\text{Dom}})/(1 + \Delta P_{\text{Foreign}}) \right] - 1
\]

where \( \Delta P_{\text{Dom}} \) is the expected inflation rate in the home country and \( \Delta P_{\text{Foreign}} \) is the expected inflation rate in the foreign country.
Thus, for example, if the expected inflation rate in the US is 5% and that in Japan is 3%, then the US$ would be expected to change by \((1.05/1.03) - 1 = 1.94\%\). Since this is a positive number, using direct quotes (from the US$ standpoint), it would represent an expected depreciation of the US$ by 1.94% against the Japanese ¥.

According to UIP, the expected percentage change in the value of the domestic currency against the foreign currency, \(\Delta e\), is given by:

\[
\Delta e = \left[\frac{1 + r_{\text{Dom}}}{1 + r_{\text{Foreign}}}\right] - 1
\]

where \(r_{\text{Dom}}\) is the domestic nominal interest rate, and \(r_{\text{Foreign}}\) is the foreign nominal interest rate. Thus if the nominal interest rate in the US is 8% and the risk-equivalent nominal interest rate in Japan is 5.94%, then, according to uncovered interest parity, the US$ would be expected to depreciate against the Japanese ¥ by \((1.08/1.0594) - 1 = 1.94\%\) (consistent with the RPPP prediction).
### Table 1a: Market Risk Premia in Major Economies (based on data for 1900-2000, relative to short-term risk-free bills)

<table>
<thead>
<tr>
<th>Country</th>
<th>Arithmetic Mean Risk Premium</th>
<th>Geometric Mean Risk Premium</th>
<th>Adjusted Ex-post Risk Premium*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>8.5%</td>
<td>7.1%</td>
<td>-</td>
</tr>
<tr>
<td>Belgium</td>
<td>5.1%</td>
<td>2.9%</td>
<td>-</td>
</tr>
<tr>
<td>Canada</td>
<td>5.9%</td>
<td>4.6%</td>
<td>-</td>
</tr>
<tr>
<td>Denmark</td>
<td>3.4%</td>
<td>1.8%</td>
<td>-</td>
</tr>
<tr>
<td>France</td>
<td>9.8%</td>
<td>7.4%</td>
<td>-</td>
</tr>
<tr>
<td>Germany</td>
<td>10.3%</td>
<td>4.9%</td>
<td>-</td>
</tr>
<tr>
<td>Ireland</td>
<td>5.4%</td>
<td>3.5%</td>
<td>-</td>
</tr>
<tr>
<td>Italy</td>
<td>11.0%</td>
<td>7.0%</td>
<td>-</td>
</tr>
<tr>
<td>Japan</td>
<td>9.9%</td>
<td>6.7%</td>
<td>-</td>
</tr>
<tr>
<td>Netherlands</td>
<td>7.1%</td>
<td>5.1%</td>
<td>-</td>
</tr>
<tr>
<td>Spain</td>
<td>5.3%</td>
<td>3.2%</td>
<td>-</td>
</tr>
<tr>
<td>Sweden</td>
<td>7.7%</td>
<td>5.5%</td>
<td>-</td>
</tr>
<tr>
<td>Switzerland</td>
<td>6.1%</td>
<td>4.3%</td>
<td>-</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>6.5%</td>
<td>4.8%</td>
<td>2.4%</td>
</tr>
<tr>
<td>United States</td>
<td>7.7%</td>
<td>5.8%</td>
<td>4.1%</td>
</tr>
<tr>
<td><strong>World</strong></td>
<td><strong>6.2%</strong></td>
<td><strong>4.9%</strong></td>
<td><strong>3.1%</strong></td>
</tr>
</tbody>
</table>

* Not available for all countries. Premia are geometric means, ‘adjusted’ for the impact of unanticipated cash flows and impact of fall in the ‘required’ risk premium as calculated by Dimson, Marsh and Staunton (2002).