SOLUTIONS TO FINALS 00.01

IE USEFUL FORMULAS

(The section number indicates the principal reference in the text.)

Perpetuity (3-2)
The value of a perpetuity of $1 per year is:

\[ PV = \frac{1}{r} \]

Annuity (3-2)
The value of annuity of $1 per period for \( t \) years (\( t \)-year annuity factor) is:

\[ PV = \frac{1}{r} - \frac{1}{r(1 + r)^t} \]

A Growing Perpetuity (the "Gordon" model) (3-2)
If the initial cash flow is $1 at year 1 and if cash flows thereafter grow at a constant rate of \( g \) in perpetuity,

\[ PV = \frac{1}{r - g} \]

Continuous Compounding (3-3)
If \( r \) is the continuously compounded rate of interest, the present value of $1 received in year \( t \) is:

\[ PV = \frac{1}{e^{rt}} \]

Equivalent Annual Cost (6-3)
If an asset has a life of \( t \) years, the equivalent annual cost is:

\[ \frac{PV(\text{costs})}{t\text{-year annuity factor}} \]

Measures of Risk (7-2 to 7-4)
Variance of returns = \( \sigma^2 \)
- expected value of \((\bar{r} - \eta)^2\)

Standard deviation of returns = \( \sqrt{\text{variance}} = \sigma \)

Covariance between returns of stocks 1 and 2
\( \sigma_{12} = \text{expected value of } [(r_1 - \bar{r}_1)(r_2 - \bar{r}_2)] \)

Correlation between returns of stocks 1 and 2
\( \rho_{12} = \frac{\sigma_{12}}{\sigma_1\sigma_2} \)

Beta of stock \( i = \beta_i = \frac{\sigma_{im}}{\sigma_m^2} \)

\[ P_0 = \frac{\text{DIV}_1}{r - g}, \quad P_0 = \sum_{t=1}^{\infty} \frac{\text{DIV}_t}{(1 + r)^t} \]

\[ r = \frac{\text{DIV}_1}{P_0} + g \]

\[ P_0 = \frac{\text{DIV}_1}{1 + r} + \frac{\text{DIV}_2}{(1 + r)^2} + \ldots + \frac{\text{DIV}_H + P_H}{(1 + r)^H} \]

Company cost of capital = \( r_{\text{assets}} = r_{\text{portfolio}} \)

\[ = \frac{\text{debt}}{\text{debt + equity}} r_{\text{debt}} + \frac{\text{equity}}{\text{debt + equity}} r_{\text{equity}} \]

\[ r_A = \left( \frac{D}{D + E} \times r_D \right) + \left( \frac{E}{D + E} \times r_E \right) \]

\[ r_E = r_A + \frac{D}{E} (r_A - r_D) \]

\[ \beta_A = \left( \frac{D}{D + E} \times \beta_D \right) + \left( \frac{E}{D + E} \times \beta_E \right) \]

\[ \beta_E = \beta_A + \frac{D}{E} (\beta_A - \beta_D) \]

Capital Asset Pricing Model
The expected risk premium on a risky investment is:

\[ r - r_f = \beta(r_m - r_f) \]

\[ r = r_f + \beta(r_m - r_f) \]

\[ \sigma_j^2 = \beta_j^2 \sigma_m^2 + \sigma_e^2 \]

\[ \sigma_p = \beta_p \sigma_m \]

\[ \beta_p = \sum_{i=1}^{n} w_i \beta_i \]

\[ \text{PV(tax shield)} = \frac{T_c(P_D)}{r_D} = T_c P_D \]

4
1. 

\[
\begin{array}{cccccc}
 & 30 & 30 & 30 & 30 & 30 \\
& .5 & & & & \\
-150 & .5 & & & & \\
20 & 20 & 20 & 20 & 20 & \\
\end{array}
\]

(a) NPV without information

\[\text{Expected cash flow} = -150 + 0.5 (184.3) + 0.5 (122.9)\]
\[= -150 + 92.15 + 61.45\]
\[= 153.6\]
\[= 3.6\]

(b) NPV with information

\[0.5 (34.3) = 17.15\]

Value of information \[17.15 - 3.60 = 13.55\]

[Avoid the negative NPV of investing in the bad state \[-13.55\]]

14 m > 18.55 not buy into
2. \[ \text{PV of all cash savings} \] \[ \frac{620}{0.14 - 0.04} = 6200 \]

\[ 6200 > 6000 \]

would pay.

3. (a) True. Net present rule valuation rule.

(b) True.

(c) False.

(d) \[ \gamma_p = \gamma_f + 1.5 (\gamma_m - \gamma_f) \]

\[ \gamma = 12\% \]

(4) \begin{align*}
& t = 1 & t = 2 & t = 3 & t = 4 \\
& 20 & 20(1.2) & 20(1.2)^2 & 20(1.2)^3 \\
& 20(1.2)^4 & 20(1.2)^5 \\
\end{align*}

\[ \text{PV} = \frac{F_1}{1 + \gamma} + \frac{F_2}{(1 + \gamma)^2} + \cdots + \frac{F_6}{(1 + \gamma)^5} + \frac{H_6}{(1 + \gamma)^6} \]

\[ = \frac{20(1.2)^5(1.05)}{0.12 - 0.05} \]

\[ = 506.4 \]

\[ \gamma = 0.12 \]
5. (a) \( \sigma_p = \beta \sigma_m \)
\[ 50\% = \beta (25\%) \]
\[ \beta = 2 \]

(b) \( \beta = 1.2 \quad \text{syst. risk} = 1.2 (25\%) = 30\% \)
\[ \frac{30\%}{40\%} = 75\% \]
\[ \gamma = \gamma_f + 1.2 (20\% - \gamma_f) \]

(c) \( \beta_a = \beta_e = 4.0 \quad \gamma_e = \gamma_a = 26\% \)
\[ \gamma_f = 6\% \\
\begin{align*}
4.0 &= 0.25 \beta_a + 0.75 \beta_e \\
4.0 &= 0.75 \beta_e
\end{align*} \]
\[ 5\frac{1}{3} = \frac{16}{3} = \beta_e \]

(b) \( \beta_a = 4.0 \)

(c) \( \gamma_e = 26\% \)

(d) \( \delta_m - \delta_f = 5\% \quad \gamma_e = 6\% + 5\frac{1}{3}(5\%) \)
(d) \[ 6\% + 25\frac{2}{3}\% = 32\frac{2}{3}\% \]

(7) \[ \alpha_f = 8\% \quad \alpha_m = 12\% \]
\[ \beta_a = 2.0 \quad t = 0.40 \]
\[ \beta_p \leftrightarrow \alpha_p \]
\[ \alpha_p^* = \alpha_p (1 - t \frac{D}{V}) \quad \text{tax adjusted discounted rate} \]

(a) \[ \beta_p = \beta_a = 2.0 \]
\[ \alpha_p = 8\% + 2.0(4\%) = 16\% \]
\[ \alpha_p^* = 0.16 \left(1 - 0.4 \left( \frac{4}{100} \right) \right) = 0.16 \left(1 - 0.16 \right) = 0.1344 \]
\[ \text{APV} = -200 + \frac{30}{0.1344} = 23.21 > 0. \text{ Accept.} \]

(b) \[ \alpha_p = 8\% + 4(4\%) = 24\% \]
\[ \alpha_p^* = 0.24 \left(1 - 0.4 \left( \frac{4}{100} \right) \right) = 0.2016 \]
\[ \alpha_p^* > \text{IRR} = 20\% \quad \text{Do not accept.} \]
(c) \[ A \quad \gamma_a = 8\% + 2.5(4\%) = 18\% \]
\[ \gamma_a^* = 0.18(84) = 0.1512 \]
\[ \gamma_b = 8\% + 4.5(4\%) = 26\% \]
\[ \gamma_b^* = 0.26(84) = 0.2184 \]
\[ APV_a = -130 + \frac{31.68}{0.1512} = 79.5238 \]
\[ APV_b = -100 + \frac{34.32}{0.2184} = 57.1429 \]

8. (a) beta of Xerox can be estimated by regressing $X_{X}$ on $X_{M}$.

\[ \beta_a = 0.50 \beta_d + 0.50 \beta_e \]

(b) 1. independent projects (not mutually exclusive).
2. Normal cash flows

(c) [Diagram]

- Reject low-risk good project
- Accept high-risk bad project
Exp: \[ \frac{19/2}{1^1} = 95K \]

(a) \[ \text{NPV} = -100K + 95K = -5K \]

\[ -\frac{100K + \frac{15}{1^1}}{.5} = 50K \]

\[ -\frac{0}{.5} = 0 \]

(b) \[ \text{NPV (with info)} = C + 25K \]

\[ \text{NPV (without info)} = 0 \]

\[ \text{NPV (with info)} = 25K - C \]

\[ \text{Max } C = 25K \]
\[ \sigma_m = 25\% \quad \mu_m = 0.20 \quad \mu_F = 0.08 \]

(a) Risk of a well-div = \( \frac{\beta_p \sigma_m}{\sigma_F} \)

\[ \beta_p = \frac{\sigma_F}{\sigma_m} = \frac{.15}{.25} = .6 \]

(b) \( \sigma_F = 40\% \)

sys. risk = \( .8 \times (25\%) = 0.20 \)

\[ \rho = \frac{.20}{.40} = .5 = 50\% \]

\[ \tau = R_F + \beta \left( \mu_m - \mu_F \right) \]
\[ = .08 + .8 \times (.2 - .08) \]
\[ = 0.176 \]

3. \( \beta_E = 2.0 = \beta_A \quad \mu_A = \mu_E = 0.16 \)

\[ \frac{D}{E} = \frac{\beta_A}{2} = \frac{1}{2} \quad \beta_D = 0 \]

(a) \( \beta_E = \beta_A + \frac{1}{2} (\beta_A - \beta_D) \)
\[ = 2 + \frac{1}{2} (2) = 3 \]

(b) \( \beta_A = 2 \) (same).

(c) \( \tau = 16\% \) reqd. return

(d) \( \beta_E = 3 \quad \tau = .06 + 3 \times (.05) = 0.21 \)
4. **Cost** $24m

\[ \Delta D = $6m \]
\[ \Delta C = $18m \]

**Annual cash flow (after tax)**

\[ = \Delta C(1-t) + t \Delta D \]
\[ = 18(0.6) + 0.4(6) \]
\[ = 10.8 + 2.4 \]
\[ = 13.2 \]

\[ t=0 \quad t=1 \quad t=2 \quad t=3 \quad t=4 \]
\[ -24 \quad 13.2 \quad 13.2 \quad 13.2 \quad 13.2 \]

\[ 24 + 13.2(\text{PVIFA})_{4\%,r} = 0 \]

\[ (\text{PVIFA})_{4\%,r} = \frac{24}{13.2} \]

\[ = 1.8182 \]

(a) **IRR** \[\approx 41\%\]

(b) **Cost of cap = required rate of return**

\[ \beta = 2.5 \]
\[ r = 0.08 + 2.5(0.08) \]
\[ = 0.28 \]

(c) \[\text{PVIFA}_{4\%, 28} = 2.241 \]

\[ \text{NPV} = -24 + 29.581 = 5.581 \]
(d) Change in \[ NPV \] \[ \frac{5}{(1 + 28)^4} \]

\[ -5 + \frac{5}{1 + 28} \approx 1.863 \]

\[ = -0.943 - 3.132 \]

5. \[ P_0 = \frac{D_1}{\alpha - 9} \]

\[ 100 = \frac{10}{\alpha - 0.05} \]

\[ \alpha = \frac{10}{100} + 0.05 \]

\[ = 0.15 \]

6. \[ t = 0 \quad t = 1 \quad t = 2 \quad t = 3 \quad t = 4 \quad t = 5 \quad t = 6 \quad t = 7 \]

\[ P_0 = 10(1.1)^0 \]

\[ V = \frac{10m}{1.1} + \frac{10m}{1.1} + \frac{10}{1.1} + \frac{10}{1.1} + \frac{10}{1.1} + \frac{10}{1.1} + \frac{10(1.1)^5}{1.1^6} \]

\[ = \frac{60}{1.1} + \frac{210}{1.1} = 245.45 \]
Consider as investment

**Second way**

Free cash flow

\[ P_0 = \frac{210 \times 0.6}{1.1} = 114.545 \]

\[ r = r_F + \beta_P (r_M - r_F) \]

\[ \tilde{r}_A = r \left( 1 - \frac{D}{V} \right) \]

\[ D = 0.50 \quad r_F = 0.08 \quad \beta_D = 0 \]

\[ \beta_A = 2.0 \quad \beta_B = 4.0 \]

\[ \alpha = 0.08 + 3(0.08) \quad \alpha = 0.08 + 4(0.08) \]

\[ = 0.32 \quad = 0.40 \]

\[ \tilde{r}_A = 0.32 \left( 1 - 0.4(0.5) \right) \]

\[ = 0.256 \]

\[ NPV_A = -100 + \frac{26}{1256} \quad NPV_B = -100 + \frac{32}{32} = 0 \]

\[ = 1.563 \]

**NPV_A > NPV_B accept**
8 (a) **Both**

1. Normal cash flow pattern
   
   \[ /0 \quad /0 \quad +/0 \quad +/0 \quad +/0 \quad \]

2. Independent projects

(b)

1. Mistake by rejecting low \( \beta \), low return

2. Accept high \( \beta \) high return projects

(c) First estimate the \( \beta \) of GM by regressing \( \hat{\gamma}_{GM} \) on \( \hat{\delta}_M \):

\[ \beta_A = \]