1 Overview

This class discusses how market liquidity affect security prices. Liquidity is, as we shall see, a complex concept. Said simply, liquidity is the ease of trading a security. A security can be illiquid because it has high exogenous transaction costs such as brokerage fees or the fees paid to a real estate agent. Hence, every time such a security is traded, the buyer and/or seller incurs a transaction cost, and the buyer anticipates further costs upon a future sale, and so on.

Another source of illiquidity is demand pressure and inventory risk. Demand pressure arises because not all agents are present in the market at all times, which means that if an agent needs to sell a large block of securities quickly, then the natural buyers may not be immediately available. In this case, the seller may trade with a market maker who will buy in anticipation of being able to later lay off the position. The market maker is exposed to the risk of price changes in the meantime, however, and the market maker must be compensated for this risk.

Also, a security can be illiquid because the traders of the security have private information. E.g. the buyer of a stock may be afraid that a potential seller has private information that the company is loosing money, and the seller may be afraid the buyer has private information that the company will do well because of a drug is about to be approved or because the company is about to be taken over. In addition to such private information about the fundamentals of the security, agents can also have private information about order flow. For instance, if a trading desk knows that a hedge fund needs to liquidate a large position and that this liquidation will depress prices, then the trading desk can sell early at relatively high prices and buy back later at lower prices.

Another source of illiquidity is that it is simply difficult to locate a counterparty who is willing to trade a particular security, and, once a counterparty is located, the agents must negotiate the price in a less than perfectly competitive environment since alternative trading partners are not immediately available. This search friction is particularly relevant in over-the-counter (OTC) markets in which there is no central market place.

Finally, a security market can be affected by short sale constraints. Certain assets cannot be shorted at all (e.g. real estate) while most financial securities can in principle be shorted, but institutional arrangements sometimes make this difficult.
Illiquidity gives rise to a bid-ask spread, that is, to a difference between the price at which one can buy or sell. In addition, illiquidity affects the level of prices and the required return. Cross-sectional differences in securities’ liquidity have implications for relative prices. Needless to say, understanding how liquidity affects pricing is important, for instance, because asset pricing affect corporations’ cost of capital and, hence, the allocation of the economy’s real resources.

1.1 Background: Standard Asset Pricing

Standard asset pricing is based on the assumption of frictionless (or, perfectly liquid) markets, that is, every security can be traded at no cost all of the time, and agents take prices as given. This assumption is combined with one of the following three assumptions: no arbitrage, agent optimality, and equilibrium.

No arbitrage means that one cannot make money in one state of nature without paying money in at least one state of nature. In a frictionless market, the assumption of no arbitrage is essentially equivalent to the existence of a stochastic discount factor $m_t$, and a martingale measure (or risk-neutral measure) $Q$ such that the price process $p_t$ of any security with dividend process $d_t$ satisfies

$$
p_t = E_t \left( \left( p_{t+1} + d_{t+1} \right) \frac{m_{t+1}}{m_t} \right) \quad (1)
$$

$$
= E_t^Q \left( \left( p_{t+1} + d_{t+1} \right) \frac{1}{1 + r_f^t} \right) \quad (2)
$$

where $r_f^t$ is the riskfree rate of interest.

Equation (1) is the main building block of standard asset pricing theory. It can also be derived by agent optimality. In particular, if an insatiable investor trades in a frictionless market, his optimal portfolio choice problem only has a solution in the absence of arbitrage — otherwise he will make an arbitrarily large profit and consume an arbitrarily large amount. Further, the first-order condition to the investor’s problem has the form (1). In particular, if the investor’s preferences are represented by an additively separable utility function $E_t \sum_s u(c_s)$ for a consumption process $c$, then $m_t = u'(c_t)$ is the marginal utility of consumption.

Finally, in a competitive equilibrium with complete markets and agents $i = 1, \ldots, I$ with separable utility functions $u^i$, (1) is satisfied with $m_t =$
$u'_\lambda(c_t)$, where $u_\lambda = \sum_i \lambda^i u^i$ is the utility function of the representative investor and $\lambda_i$ are the Pareto weights that depend on the agents’ endowments.

1.2 On the Impossibility of Frictionless Markets

One could argue that, if there were a friction that lead to large costs for agents, then there would be an institutional response to profit by alleviating this friction. According to this view, there cannot be any (important) frictions left in equilibrium.

Alleviating frictions is costly, however, and the institutions who alleviate frictions may be able to earn rents. For instance, setting up a market requires computers, trading systems, clearing operations, risk and operational controls, legal documentation, marketing, information systems, and so on. Hence, if frictions did not affect prices then the institutions that alleviated the frictions would not be compensated for doing so. Therefore, no one would have an incentive to alleviate frictions, and, hence, markets cannot be frictionless. 1

1.3 Liquidity and Asset Pricing: The Point of Departure

If markets are not frictionless, that is, if market are plagued by some form of illiquidity, then the main building blocks of standard asset pricing are shaken. First, the equilibrium aggregation of individuals’ utility function to a representative investor may not apply. Second, individual investor optimality may not imply that (1) holds with $m_t = u'(c_t)$ at all times and for all securities. This is because an investor need not be “marginal” on any security if trading frictions make it suboptimal to trade it.

Hence, illiquidity implies that we cannot easily derive the stochastic discount factor from consumption, much less from aggregate consumption. Then, what determines asset pricing?

Some people might argue that the key cornerstone of standard asset pricing is the mere existence of a stochastic discount factor, not necessarily its relation to consumption. Indeed, powerful results — such as the theory of derivative pricing — follow from the simple and almost self-evident premise

\[^1\text{Grossman and Stiglitz (1980) use this argument to rule out informationally efficient markets.}\]
of no arbitrage. It is, however, important to recognize that the standard
no-arbitrage pricing theory relies not only on the absence of arbitrage, but
also on the assumption of frictionless market.

To see why the assumption of frictionless market is crucial, consider the
basic principle of standard asset pricing: securities, portfolios, or trading
strategies with the same cash flows must have the same price. This simple
principle is based on the insight that, if securities with identical cash flows
have different prices, then an investor could buy — with no trading costs —
the cheaper security and sell — with no trading costs — the more expensive
security, and, hence, realize an immediate arbitrage profit at no risk. Another
way to see this is to iterate (1) to get

\[ p_t = E_t \left( \sum_{s=t+1}^{\infty} d_s \frac{m_s}{m_t} \right) \]  

(3)

which shows that the price \( p_t \) only depends on the pricing kernel and the cash
flows \( d_s \). With trading costs, however, this principle need not apply. Indeed,
with transaction costs, securities with the same cash flows can have different
prices without introducing arbitrage opportunities.

Do real-world securities with the same cash flows have the same price?
Perhaps surprisingly, the answer is “no,” certainly not always. For instance,
on-the-run (i.e. newly issued) Treasuries often trade at lower yields than
almost identical off-the-run Treasuries. Shares that are restricted from trade
for a two years trade at an average discount of about 30% relative to shares of
the same company with identical dividends that can be traded freely (Silber
(1991)). Chinese “restricted institutional shares,” which can be traded only
privately, trade at a discount of about 80% relative corresponding exchange-
traded shares in the same company (Chen and Xiong (2001)). Options than
cannot be traded over their life trade at large discounts relative to identical
tradable options (M. Brenner and Hauser (2001)). The put-call parity is
sometimes violated when it is difficult to sell short, implying that a stock
trades at a higher price than a synthetic stock created in the option market
(Ofek, Richardson, and Whitelaw (2004)). Further, in so-called “negative
stub value” situations, a security can trade at a lower price than another
security, which has strictly lower cash flows (e.g. Lamont and Thaler (2003)).

The existence of securities with identical cash flows and different prices
implies that there does not exist a stochastic discount factor \( m \) that prices
all securities, that is, there does not exist an \( m \) such that (3) holds for all
Another important difference between standard asset pricing and liquidity asset pricing is that the latter sometimes relaxes the assumption of price taking behavior. Indeed, if prices are not always at fundamentals and are affected by the nature of the trading activity, then agents may take this into account. For instance, if an agent is so large that his trades significantly affect prices, he will take this into account, or if agents trade in a bilateral over-the-counter market then prices are privately negotiated.

1.4 Liquidity and Asset Pricing: Where it will Take Us (in This Class)

The prices of securities are determined by the general equilibrium of the economy. Hence, the price \( p_j^t \) of a security \( j \) is some function, say \( f \), of the security’s cash flow \( d_j^t \), the cash flows of other securities \( (d_k^t)_{k \neq j} \), the utility functions of all agents \( (u^i)_{i \in I} \), and the agents’ endowments \( (e^i) \),

\[
p_j^t = f_t (d_j^t, (d_k^t)_{k \neq j}, (u^i), (e^i))
\]  

(4)

The strength of a frictionless economy is that a security’s cash flows \( d_j^t \) and the pricing kernel \( m \) are sufficient statistics for this pricing operation as described by Equation (1). This means that the pricing kernel summarizes all the needed information contained in utility functions, endowments, correlations with other securities, etc.

In an economy with frictions, prices depend additionally on the liquidity \( (l_k^t) \) of all securities,

\[
p_j^t = f_t (d_j^t, l_j^t, (d_k^t)_{k \neq j}, (l_k^t)_{k \neq j}, (u^i), (e^i))
\]  

(5)

In some models of illiquidity, prices would be arbitrage free even without frictions, and, hence, there exists a pricing kernel \( m \) such that (1) holds. In this case, the illiquidity affects \( m \), but the pricing of securities can still be summarized using a pricing kernel. This is the case in models in which certain agents can trade all securities without costs. For instance, in the models of demand pressure and inventory risk that follow \(^\text{?}\), competitive market makers can trade all securities at no cost which ensures the absence of arbitrage. Garleanu, Poteshman, and Pedersen (2004) show explicitly how \( m \) depends on demand pressure in a multi-asset model.
In other models of liquidity, however, there is no pricing kernel such that (1) applies. For instance, in the transaction cost models, securities with the same dividend streams have different prices if they have different transaction costs. Hence, a security’s transaction costs not only affects the nature of the equilibrium, it is a fundamental attribute of the security.

When there does not exist a pricing kernel, then the computation of equilibrium asset prices becomes more difficult. Indeed, the general equilibrium prices with illiquidity may depend on the fundamental parameters in a complicated way that does not have a closed-form expression. Nevertheless, we can derive explicit prices under certain special assumptions such as risk neutrality, specially rigged trading horizons, partial equilibrium, normally distributed dividends, and so on. While the difficulty of general equilibrium with frictions often forces us to use such special assumptions to get closed-form results, we can still gain important insights into the main principles of how liquidity affects asset prices.

This abstract way of thinking about asset pricing will be much clearer in the context of specific frictions and specific economies. In this class, we will explicitly derive asset prices in the context of fixed transaction costs, inventory risk, asymmetric information, search frictions, and short-sale constraints, and show how these frictions affect optimal trading decisions. Further, we will consider a variety of related topics such as derivative pricing in illiquid markets, and other topics that the participants are interested in.