Return Measures

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Outline

- Quoted rate = APR
- Compounding and EAR
  - Single period realized return:
    - holding period return
  - Multiple-period realized return:
    - Arithmetic average
    - Geometric average
    - IRR

Quoted Rates and EAR

- Example:
  - Interest rate quoted at 10% compounded semi-annually

- Effective Annual Rate EAR if interest is compounded \( m \) times a year:
  - \( EAR = \left(1 + \frac{\text{quoted rate}}{m}\right)^m - 1 \)

- Example: Which loan is cheapest:
  - 15%, compounded daily
  - 15.5%, compounded quarterly
  - 16%, compounded annually
Continuous Compounding

- Suppose the quoted rate is given.
- Consider increasingly frequent compounding: annually, quarterly, daily, every second,…
- What happens to the EAR?
- When compounding happens “all the time,” it is called continuous compounding
- \[ \text{EAR} = \exp(\text{quoted rate}) - 1. \]
- Challenge: why? (Problem C1.)

APR

- Lenders are required by law to report the Annual Percentage Rate, APR.
- \[ \text{APR} = \text{Quoted Rate} = \text{interest per period} \times \text{number of periods per year} \]
- How do you make a loan seem cheaper?

Single-Period Realized Return

- Holding period return:
  \[ HPR = \frac{\text{ending price} + \text{cash dividend} - \text{beginning price}}{\text{beginning price}} \]
- Annualized holding period return for a holding period of \( t \) years:
  \[ \text{annualized } HPR = (1 + HPR)^{\frac{1}{t}} - 1 \]
  \[ = \left( \frac{\text{ending price} + \text{cash dividend}}{\text{beginning price}} \right)^{\frac{1}{t}} - 1 \]
Multiple-Period Realized Return

- Arithmetic Average:
  \[ \frac{1}{T} (r_1 + r_2 + r_3 + \ldots + r_T) \]
  - Not equivalent per-period return because it neglects compounding
  - Useful for forecasting the return next period

- Geometric Average
  \[ \left[ (1 + r_1)(1 + r_2)(1 + r_3)\ldots(1 + r_T) \right]^{1/T} - 1 \]
  \[ = \left[ \frac{\text{accumulated value}_T}{\text{value}_0} \right]^{1/T} - 1 \]

Multiple-Period Realized Return

- Internal rate of return, IRR
  - Return if one can re-invest cash-flows at this rate
  - “Dollar-weighted average”
  - The IRR in the rate that makes:
    \[ \sum_{t=0}^{\infty} \frac{C(t)}{(1 + \text{IRR})^t} = \text{initial price} = \text{present value of future net profits} \]

\[ P(0) = \sum_{t=0}^{\infty} \frac{C(t)}{(1 + \text{IRR})^t} \]