Portfolio Selection with Two Risky Securities.

Professor Lasse H. Pedersen

Outline

- Portfolio: expected return and SD
- Diversification
- Investment opportunity set
- Investor preference: risk-return tradeoff
- Optimal portfolio choice with 2 risky assets

Portfolio Expected Return and Standard Deviation

- The expected return on the portfolio is:
  \[ E(R_p) = \sum_{i} \omega_i E(R_i) \]
- With 2 securities, the portfolio variance is:
  \[ \sigma_p^2 = \omega_1^2 \sigma_1^2 + \omega_2^2 \sigma_2^2 + 2 \omega_1 \omega_2 \rho_{12} \sigma_1 \sigma_2 \]
- The standard deviation is:
  \[ \sigma_p = \sqrt{\sigma_p^2} \]
Diversification with 2 assets: Example
- Suppose we have two assets, US and JP, with:
  - mean volatility
  - US: 13.6% 15.4%
  - JP: 15.0% 23.0%
  - and with correlation 27%.
- If an investor holds 60% in the US and 40% in JP, what is the mean and volatility of the portfolio?
- ‘volatility’ is another word for ‘standard deviation’

Portfolio mean:
\[ E(R_p) = 0.6 \times 0.136 + 0.4 \times 0.150 = 14.2\% \]

Portfolio variance:
\[ \text{var}(R_p) = (0.6)^2 (0.154)^2 + (0.4)^2 (0.230)^2 \\
+ 2 \times 0.6 \times 0.4 \times 0.27 \times 0.154 \times 0.230 \]
\[ = 0.022 \]
\[ \sigma_p = 14.7\% \]
- This portfolio has higher expected return and lower risk than the US market alone!

Diversification with 2 assets: Example
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- Portfolio variance:
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+ 2 \times 0.6 \times 0.4 \times 0.27 \times 0.154 \times 0.230 \]
\[ = 0.022 \]
\[ \sigma_p = 14.7\% \]
- Risk and Return with Varying Weights
- Let \( \omega \) be the weight in the US, and \( 1-\omega \) the weight in JP.
- The expected return of the portfolio is:
\[ E(r_p) = \omega \times 0.136 + (1-\omega) \times 0.150 \]
- The variance of the portfolio return is:
\[ \text{var}(r_p) = \omega^2 (0.154)^2 + (1-\omega)^2 (0.230)^2 \\
+ 2 \times \omega \times (1-\omega) \times 0.27 \times 0.154 \times 0.230 \]
- What happens when we vary \( \omega \)?
Portfolio Terminology

- The investment opportunity set consists of all available risk-return combinations.
- An efficient portfolio is a portfolio that has the highest possible expected return for a given standard deviation.
- The efficient frontier is the set of efficient portfolios. It is the upper portion of the minimum variance frontier starting at the minimum variance portfolio.
- The minimum variance portfolio (mvp) is the portfolio that provides the lowest variance (standard deviation) among all possible portfolios of risky assets.
Investment Opportunity Set with Varying Correlations

Optimal Portfolio Choice with 2 Risky Assets

- Any (mean-variance) investor should choose an efficient portfolio to benefit from diversification.
- The specific choice depends on the investor’s risk aversion.
- A more risk-averse investor should choose a portfolio with
  - lower risk
  - lower expected return