The Capital Asset Pricing Model (CAPM)

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Outline

- Key questions:
  - What is the equilibrium required return, $E(R)$, of a stock?
  - What is the equilibrium price of a stock?
  - Which portfolios should investors hold in equilibrium?

- Answer: CAPM
  - Assumptions
  - Results:
    - Identify the tangency portfolio in equilibrium
    - Hence, identify investors' portfolios
    - Derive equilibrium returns (and hence prices)

CAPM: Introduction

- Equilibrium model that
  - predicts optimal portfolio choices
  - predicts the relationship between risk and expected return
  - underlies much of modern finance theory
  - underlies most of real-world financial decision making
- Derived using Markowitz's principles of portfolio theory, with additional simplifying assumptions.
- Sharpe, Lintner and Mossin are researchers credited with its development.
- William Sharpe won the Nobel Prize in 1990.
1: The market is in a competitive equilibrium

- Equilibrium:
  - Supply = Demand
  - Supply of securities is fixed (in the short-run).
  - If Demand > Supply for a particular security, the excess demand drives up price and reduces expected return.
  - (Reverse if Demand < Supply)

- Competitive market:
  - Investors take prices as given
  - No investor can manipulate the market.
  - No monopolist

2: Single-period horizon

- All investors agree on a horizon.
- Ensures that all investors are facing the same investment problem.
3: All assets are tradable

- This includes in principle:
  - All financial assets (including international stocks)
  - Real estate
  - Human capital
- This ensures that every investor has the same assets to invest in:
  - all the assets in the world, the “market portfolio”

4: No frictions

- No taxes
- No transaction costs (no bid-ask spread)
- Same interest rate for lending and borrowing
- All investors can borrow or lend unlimited amounts. (No margin requirements.)

5-6: Investors are rational mean-variance optimizers with homogeneous expectations

- Investors choose efficient portfolios consistent with their risk-return preferences
- Investors have the same views about expected returns, variances, and covariances (and hence correlations).
What is the Equilibrium Tangency Portfolio?

- Recall from portfolio theory:
  - All investors should have a (positive or negative) fraction of their wealth invested in the risk-free security, and
  - The rest of their wealth is invested in the tangency portfolio.
  - The tangency portfolio is the same for all investors (homogeneous expectations).
- In equilibrium, supply=demand so:
  - the tangency portfolio must be the portfolio of all existing risky assets, the “market portfolio”!!

The Market Portfolio

- \( p_i \) = price of one share of risky security \( i \)
- \( n_i \) = number of shares outstanding for risky security \( i \)
- \( M \) = Market Portfolio. The portfolio in which each risky security \( i \) has the following weight:

\[
\omega_{Mi} = \frac{p_i \times n_i}{\sum p_i \times n_i}
= \frac{\text{market capitalization of security } i}{\text{total market capitalization}}
\]

In words, the market portfolio is the portfolio consisting of all assets (everything!). However, you also invest in the market portfolio if you buy a few shares of every security, weighted by their market cap.

The Capital Market Line (CML)

- Recall: The CAL with the highest Sharpe ratio is the CAL with respect to the tangency portfolio.
- In equilibrium, the market portfolio is the tangency portfolio.
- The market portfolio’s CAL is called the Capital Market Line (CML).
- The CML gives the risk-return combinations achieved by forming portfolios from the risk-free security and the market portfolio:

\[
E(R_p) = R_f + \left[ \frac{E(R_M) - R_f}{\sigma_M} \right] \sigma_p
\]
The E(R)-SD Frontier and The Capital Market Line

The Required Return on Individual Stocks

- CAPM is most famous for its prediction concerning the relationship between risk and return for individual securities:
  \[ E[R_i] = R_f + \beta_i \left[ E[R_M] - R_f \right] \]
  where \( \beta_i = \frac{\text{cov}(R_i, R_M)}{\sigma_M^2} \)

- The model predicts that expected return of an asset is linear its 'beta'.

- This linear relation is called the Security Market Line (SML).

- The beta measures the security’s sensitivity to market movements.

Deriving CAPM Equation using FOC

- The market portfolio is the tangency portfolio and therefore it solves:
  \[ \max_{w_i, w_M} \frac{E(R_p) - R_f}{\sigma_p} \]
  where \( E(R_p) = \sum w_i E(R_i) + (1 - \sum w_i) R_f \)
  \( \sigma_p = \sqrt{\sum w_i w_j \text{cov}(R_i, R_j)} \)

- The first-order condition (FOC) is:
  \[ 0 = \frac{\partial}{\partial w_j} \frac{E(R_p) - R_f}{\sigma_p} \bigg|_{w_{max}} \]
  that is,
  \[ 0 = \frac{(E(R_i) - R_f)\sigma_M - (E(R_M) - R_f) \frac{\text{cov}(R_i, R_M)}{\sigma_M}}{\sigma_M^2} \]
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Security Market Line (SML)

The graph relates the location of the individual securities with respect to the M-SD frontier to their betas.

E(R)-SD Frontier and the Betas

The Capital Market Line and the Security Market Line
Systematic and Non-Systematic Risk

- The CAPM equation can be written as:
  \[ R_i = R_f + \beta_i (R_M - R_f) + \text{error}_i \]
  where \( \beta_i = \frac{\text{cov}(R_i, R_M)}{\sigma^2_{M}} \)

- This implies the total risk of a security can be partitioned into two components:
  \[ \sigma^2_i = \beta^2_i \sigma^2_M + \sigma^2_{\text{error}} \]

Systematic and Non-Systematic Risk: Example

- ABC Internet stock has a volatility of 90% and a beta of 3. The market portfolio has an expected return of 14% and a volatility of 15%. The risk-free rate is 7%.

- What is the equilibrium expected return on ABC stock?

- What is the proportion of ABC Internet's variance which is diversified away in the market portfolio?

  \[ \sigma^2_i = \beta^2_i \sigma^2_M + \sigma^2_{\text{error}} \]
  \[ (0.9)^2 = 3^2 \times 0.15^2 + \sigma^2_{\text{error}} \]
  \[ \sigma^2_{\text{error}} = 0.06075 \]

- Hence, 75% of variance is diversified away.

Systematic and Non-Systematic Risk

- \( \beta_i \) measures security i's contribution of to the total risk of a well-diversified portfolio, namely the market portfolio.

- Hence, \( \beta_i \) measures the non-diversifiable risk of the stock.

- Investors must be compensated for holding non-diversifiable risk. This explains the CAPM equation:
  \[ E(R_i) = R_f + \beta_i (E(R_M) - R_f), \quad i = 1, \ldots, N \]
Risk Premium

- Recall the SML: $E(R_i) = R_f + \beta_i [E(R_m) - R_f]$
- Stock i's systematic or market risk is: $\beta_i$
- Investors are compensated for holding systematic risk in form of higher returns.
- The size of the compensation depends on the equilibrium risk premium, $E(R_m) - R_f$.
- The equilibrium risk premium is increasing in:
  1. the variance of the market portfolio
  2. the degree of risk aversion of average investor

Estimating Beta

An Example:

Many institutions estimate beta’s, e.g.:
- Bloomberg
- Merrill Lynch
- Value Line
- Yahoo

Estimating Beta by Linear Regressions (OLS)

- CAPM equation: $E[R_i] - R_f = \beta_i [E[R_m] - R_f]$
- Get data on "excess returns": $R'_i(t) = R_i(t) - R_f$, $R'_m(t) = R_m(t) - R_f$
- where $R_f$ is the risk-free rate from time t-1 to time t.
- Estimate $\beta_i$ by running the regression: $R'_i(t) = \alpha_i + \beta_i R'_m(t) + error(t)$
- Typically, 60 months of data are used.
Security Characteristic Line (SCL)

The SCL is the “regression line”:

\[ R_i(t) - R_f = \alpha + \beta (R_m(t) - R_f) + error_i(t) \]

Note:
CAPM implies \( \alpha = 0 \)

Estimating Beta:
Real Life Example, AT&T

- Construct excess returns
- Run the regression, for instance using Excel:
  - apply Tools, Add-ins, Analysis ToolPak
  - use Tools, Data Analysis, Regression
- The result is in the spreadsheet “betareg.xls”
- Excel Regression output:

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<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Stat</th>
<th>P-value</th>
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Estimating Beta:
Real-Life SCL for AT&T
Applications of the CAPM

- Portfolio choice
- Shows what a “fair” security return is
- Gives benchmark for security analysis
- Required return used in capital budgeting to
  - compute NPV of risky project
  - or “hurdle rate” for IRR
- Evaluation of fund manager performance.

Stock Selection and Active Management

- One possible benchmark for stock selection is to find assets that are cheap relative to CAPM (or more advanced models).
- A security’s alpha is defined as:
  \[ \alpha_i = E[R_i] - R_f - \beta_i [E[R_M] - R_f] \]
  where \( \beta_i = \frac{\text{cov}[R_i, R_M]}{\sigma_i^2} \)
- Some fund managers try to buy positive-alpha stocks and sell negative-alpha stocks.
- CAPM predicts that all alpha’s are zero.

Stock Selection

\[ \hat{\alpha}_i > 0 \quad \Rightarrow \quad \text{Under-priced} \]
\[ \hat{\alpha}_i < 0 \quad \Rightarrow \quad \text{Over-priced} \]
Active and Passive Strategies

- An "active" strategy tries to beat the market buy stock picking, by timing, or other methods
- But, CAPM implies that
  - security analysis is not necessary
  - every investor should just buy a mix of the risk-free security and the market portfolio, a "passive" strategy.

Summary

- The CAPM follows from equilibrium conditions in a frictionless mean-variance economy with rational investors.
- Prediction 1: Everyone should hold a mix of the market portfolio and the risk-free asset. (That is, everyone should hold a portfolio on the CML.)
- Prediction 2: The expected return on a stock is a linear function of its beta. (That is, stocks should be on SML.)
- The beta is given by:
  \[ \beta_i = \frac{\text{cov} [R_i, R_M]}{\sigma_M^2} \]
- A stock's beta can be estimated using historical data by linear regression. (That is, by estimating the SCL.)