**Fixed-Income Securities**

Professor Lasse H. Pedersen

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**Main Features of Bonds**

1. **Issuer:**
   - US Treasury/Government
   - States, municipalities, and agencies
   - Foreign governments (sovereign bonds)
   - Corporations

2. **Term (number of years to maturity):**
   - Short (less than 1 yr)
     - T-bills, CD’s, Commercial papers
   - Long (more than 1yr)
     - T-bonds, corporate bonds
     - Consols

3. **Price vs. par value = face value**
   - par bond
   - discount bond
   - premium bond

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**Outline**

- Main features of bonds
- Yield to maturity
- Realized return
- Forward rates
- Yield curve or term structure of interest
Main Features of Bonds

4. **Coupon**
   - Coupon rate: total annual interest payment per dollar face value
   - Period (usually semi-annual)
   - Fixed or variable (floaters and inverse floaters)
   - Nominal or inflation-indexed
   - Possibly no coupons (zero-coupon bond)

5. **Currency**
   - Yankee bonds, Eurobonds
   - Gold-denominated

6. **Credit risk**
   - Risk free
   - Default able

7. **Seniority and security**
   - Senior, subordinated senior, junior...
   - Secured by properties and equipment, other assets of the issuer, income-stream, etc
   - Sinking fund provisions (sinkers)

8. **Covenants**
   - Restrictions on additional issues, dividends, and other corporate actions.

9. **Option provisions**
   - Callability: After a certain period, issuer has the right to pay back the loan before it matures.
   - Putability: After a certain period, bondholder has the right to demand payment of the loan before maturity.
   - Convertibility: After a certain period, bondholder had the right to exchange the bond for stocks of the issuer.

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Yield to Maturity (YTM) on Annual-Pay Coupon Bonds

- For an annual-pay coupon bond, the YTM is the same as the IRR.
- Hence, YTM is the rate that solves:

\[
price = \sum_{t=1}^{T} \frac{coupon}{(1+YTM)^t} + \frac{face\ value}{(1+YTM)^T}
\]
Example: Price and YTM for an Annual-Pay Coupon Bond

- Suppose a 3-year bond has a face value of 100 and annual coupon payments of 8.
- If the YTM is 8% then what is the price?
- If the YTM is 10% then what is the price?
- If the price is 106 then what is the YTM?
- If the YTM is higher (or lower) than the coupon rate, then what must be true about the price?
- What must be true about the YTM:
  - for par bonds?
  - for discount bonds?
  - for premium bonds?

Example: YTM for Zero-Coupon Bonds

- Suppose a zero-coupon bond pays 100 after 3 years and has a price of 90.
  - What is its YTM?
- In general, what is the YTM of a zero with price $P$ and maturity $T$?

Yield to Maturity on Semi-Annual-Pay Coupon Bonds

- For a semi-annual-pay coupon bond, the YTM is computed in 2 steps:
  1. Find the semi-annual IRR, that is, the rate $r$ that solves:
     \[
     \text{price} = \sum_{n=1}^{N} \frac{\text{coupon}}{(1+r)^n} + \frac{\text{face value}}{(1+r)^N}
     \]
  2. The YTM is the corresponding annual percentage rate: $YTM = 2r$
- The corresponding effective annual yield is $(1+r)^2 - 1 = (1+YTM/2)^2 - 1$
Example: YTM on a Semi-Annual-Pay Coupon Bond

- Suppose that a 2-year bond has a face value of 100 and pays semi-annual coupons of 4.
- If the YTM is 10%:
  - what is the price?
  - what is the effective annual rate?
- If the price is 100 then what is the YTM?
- If the price is 105 then the YTM must be higher or lower than what number?
- How are the price and YTM related?

Realized Return vs. YTM

- Suppose that you buy a bond.
- Will the return on your investment be equal to the YTM?
- The return on your investment is equal to the YTM of the bond if:
  - you can re-invest the coupons at the same rate, and
  - you hold the bond until maturity
- The return on your investment is different from the YTM if:
  - you must re-invest the coupons at a different rate, or
  - you sell the bond before maturity at a price that corresponds to a different yield-to-maturity. (Market yields can change.)

Realized Holding Period Return

- Suppose that:
  - at time 0, you buy a bond for \( P(0) \)
  - you re-invest all dividends until date \( t \)
  - at time \( t \), you sell the bond and the re-invested dividends for a total price of \( V(t) \)
- The annual holding period return (HPR) is the solution to:
  \[
  P(0) (1 + HPR)^t = V(t)
  \]
- Hence, the annual HPR is:
  \[
  HPR = \left( \frac{V(t)}{P(0)} \right)^{\frac{1}{t}} - 1
  \]
Example: Realized Return on Zero-Coupon Bond

- Suppose that a 3-year zero-coupon bond has a YTM of 5%
  - What is the bond’s current price?
- Next year, the YTM changes to 7%
  - What is the price in that year?
  - What is the realized (holding period) return over the one year period?
- What if the YTM in year 1 had remained 5%
  - What would be the price that year?
  - What would be the realized return over the one year period?

Example: Realized Return on an Annual-Pay Coupon Bond

- Suppose that a 2-year coupon bond has face value 100, coupons of 10, and a YTM of 10%
  - What is the bond’s current price?
- Suppose that, next year, the coupon payment is re-invested at a yield of 10%
  - What is total cash flow after 2 years when the bond matures?
  - What is the annual realized return over the 2 year period?
- Suppose instead that, next year, the coupon payment is re-invested at a yield of 4%
  - What is total cash flow after 2 years when the bond matures?
  - What is the annual realized return over the 2 year period?

Forward Rates

- A firm foresees the need for short-term funds one year from now but is worried about interest rate rises. Can they “lock in” a rate for a one-year loan, starting one year from now?
- A company will receive a payment next year and must make a payment two years from now. The company is worried about the re-investment risk related to the incoming payment. Can the company lock in a lending rate, starting one year from now?
- A forward rate is an interest rate on a future loan that is fixed today.
- The forward rate for 1-year lending starting $t$ years from now is denoted $r(t)$. 


Example: Forward Rates

- Suppose that
  - a 2-year zero has a YTM of 6%
  - a 3-year zero has a YTM of 7%
- What are the prices of these bonds?
- How can you trade these bonds to replicate a loan between year 2 and year 3?
- (You create a "synthetic" loan.)
- What is the interest rate on that loan?
- This interest rate is the 2-year forward rate, \( f(2) \).

Forward Rates

- The forward rate is determined by arbitrage to be:
  \[
  f(t) = \frac{P(t)}{P(t+1)} - 1
  \]
  where \( P(t) \) is the price of a 1-year zero-coupon bond
- Forward rates are also traded directly:
  - FRA’s: Forward Rate Agreements
  - Eurocurrency Interest Rate Futures.
  - Bond Futures

The Yield Curve or the Term-Structure of Interest

- The collection of YTM of zero-coupon bonds has many names:
  - the term structure of zero-coupon bond yields
  - the term structure of interest
  - the yield curve
- Typical shapes of the term structure of interest:
  - flat
  - upward sloping (most typical)
  - downward sloping
  - hump shaped
What determines the Shape of the Term-Structure?

Theory 1: The Expectations Hypothesis

- The YTM on a long-term bond is determined by the expected future short-term interest rates
- The expected holding period return on all bonds is the same
- The expected future 1-year interest rate is equal to the forward rate: \( E(r(t)) = f(t) \)
- What does an upward-sloping term structure imply about the expected future short-term interest rate?

Example: The Expectations Hypothesis

- Suppose that
  - a 2-year zero has a YTM of 6%
  - a 3-year zero has a YTM of 7%
- Recall the 2-year forward rate, \( f(2) = 9.028 \).
- Suppose that you invest $1M in 3-year zeros
  - how much money do you have after 3 years?
- Suppose, instead, that you invest $1M in 2-year zeros and, after 2 years, you re-invest the money for 1 year at the prevailing interest rate.
  - how much money do you have after 3 years
    - if, in year 2, the 1-year yield is 5%
    - if, in year 2, the 1-year yield is 9.028%
    - if, in year 2, the 1-year yield is 15%

What determines the Shape of the Term-Structure?

Theory 2: Liquidity Preference Theory

- Buyers of long-term bonds want to be compensated
  - for "tying up" money for a long time
  - for having a price risk if they need to sell before maturity
- Issuers of bonds are willing to pay a higher interest rate on long-term bonds because
  - they can lock in an interest rate for many years
- The associated risk premium is denoted the liquidity premium
- Based on this theory,
  - what is the typical shape of the term-structure?
  - can you identify a statistical arbitrage in the bond market?
What determines the Shape of the Term-Structure?
Theory 3: Market Segmentation Theory (Preferred Habitat Theory)

- Some investors trade short-term bonds:
  - short-term interest rates are determined by supply and demand among these investors

- Other investors trade long-term bonds:
  - long-term interest rates are determined by supply and demand among these investors