Management of Interest-Rate Risk

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Outline
- Interest rate sensitivity
- Duration
- Cash-flow matching
- Duration matching: immunization
- Convexity

Interest-Rate Sensitivity
- First order effect: Bond prices and yields are negatively related
- Maturity matters: Prices of long-term bonds are more sensitive to interest-rate changes than short-term bonds
- Convexity: An increase in a bond’s YTM results in a smaller price decline than the price gain associated with a decrease of equal magnitude in the YTM.
Duration

- The duration \( D \) of a bond with cashflows \( c(t) \) is defined as minus the elasticity of its price \( (P) \) with respect to 1 plus its yield \( (y) \):
  \[
  D = - \frac{dP}{dy} \frac{1+y}{P} = \sum t f(t) c(t) \frac{1+y}{1+y}\frac{1+y-P}{P} \]
- We see that the duration is equal to the average of the cash-flow times \( t \) weighted by \( f(t) \), the fraction of the present value of the bond that comes from \( c(t) \).
- The relative price-response to a yield change is therefore:
  \[
  \frac{\Delta P}{P} = D \frac{\Delta y}{1+y} \approx D \frac{\Delta y}{1+y} \frac{1+y}{1+y} \Delta y = -D \Delta y = -D \Delta y \]

Example: Duration of a Coupon Bond

- What is the duration of a 3-year coupon bond with a coupon rate of 8% and a YTM of 10%?
- If the YTM changes to 10.1%, what would be the (relative) change in price?
- If the YTM changes to 11%, what would be the (relative) change in price?

Duration Facts

- The duration of a portfolio is the weighted average of the durations: \( D_p = \sum w_i D_i \)
- What is the duration of a zero-coupon bond?
- What must be true for the duration of a coupon bond?
- What happens to the duration of a coupon bond if (all else equal) the coupon rate increases?
- The duration of a perpetuity is: \((1+y)/y\)
Interest-Rate Management

- Investors and financial institutions are subject to interest-rate risk, for instance,
  - homeowner: mortgage payments
  - bank: short-term deposits and long-term loans
  - pension fund: owns bonds and must pay retirees

- A change in the interest rate results in:
  1. price risk
  2. re-investment risk

- Want to construct a portfolio which is insensitive to interest-rate changes.

Example: GM’s Pension Fund

- General Motor’s pension fund had
  - liabilities (payments to retirees) with duration of about 15 years
  - assets (bonds) with duration of about 5 years

- When the interest rate fell:
  - the value of the bonds increased, but
  - the present value of the liabilities increased more!

- Reinvestment risk:
  - At the new interest rate, the assets could not be reinvested to make the future payments.

Cash-Flow Matching

- Match exactly the cash-flows of assets and liabilities:
  - Then the portfolio is risk-free, in particular;
  - it has no interest-rate risk.
  - Can in principle be done using, for instance, zero-coupon bonds

- Problems with cash-flow matching:
  - it can be difficult to find securities that enables a perfect match
  - the needed securities may be illiquid and have high transactions costs
Duration Matching: Immunization

- If cash-flow matching is impossible or very costly, one can instead use duration matching.
- Duration matching means to make the duration of assets and liabilities equal.
- Then, the sensitivity to interest-rate changes is:

\[
\Delta P = \Delta y \left( \frac{D_{\text{assets}}}{1 + y} \right) - \left( \frac{D_{\text{liabilities}}}{1 + y} \right) = 0
\]

- Interest rate changes makes the values of assets and liabilities change by (approximately) the same amount.

Example: Immunization

- Suppose:
  - a pension fund must pay $100M in 15 years
  - the current market interest rate is 6% at all maturities
  - the fund’s current assets equals $100M / 1.06^{15} = $41.73M

- The pension fund wants to invest in 1-year and 30-year zero-coupon bonds
  - How much money should they invest at each of those maturities?
  - What are the prices of these bonds?
  - How many securities should the fund buy (assume a face of $100)?

- Right after the fund buys the bonds, the interest rate rises to 7%
  - what is the new value of the fund’s bond position?
  - what is the new present value of the fund’s liabilities?

Problems with Immunization

- Requires rebalancing
- It is an approximation that assumes:
  - flat term structure of interest
  - only risk of changes in the level of interest; not in the slope of the term-structure or other types of shape changes
Convexity

- The sensitivity of price with respect to yield is approximated by a linear function when using duration.
- The relation is really non-linear, in particular, it is convex.
- The convexity of a bond is the curvature of its price-yield relationship:

\[
\text{convexity} = \frac{d^2 P}{dy^2} \sum \frac{w_i \cdot (t+1)}{(1+y)^i} \quad \text{where} \quad w_i = \frac{\text{Cashflow}_i}{(1+y)^i}
\]

- The relative price response to a yield change can be better approximated using convexity:

\[
\frac{d^2 P}{dy^2} = \frac{\Delta y \cdot \text{convexity} \cdot (\Delta y)}{2}
\]