CHAPTER II

ELEMENTS FOR A POSITIVE THEORY OF EQUITY VALUATION:
SOME BACKGROUND ISSUES

2.1 Participant Objectives

The causal mechanisms which translate observable phenomena into
investor preferences for common stocks are probably a complex synthesis
of deductive reasoning, inference, and personal behavioral idiosyncracies.
In order to begin to understand this decision process, it is necessary to
abstract from the individualistic elements of the process and concentrate
on those variables that seem to be a part of the consideration of large
groups of individuals. It is possible, of course, that severe simplifica-
tion will obscure the real-world patterns one is trying to explain. But
there is a great deal of faith—and some evidence—that the relative
valuation of common stock equities can be at least partially explained
by relating certain economic and financial variables of the firm directly
to equity share prices.

It is probable that at least some of the firm financial figures one
would like to use as independent variables to determine equity share prices
are partially determined themselves by equity share prices. As Figure II-1
suggests, a dynamic equilibrium situation in the equity markets contains
feedback loops to both investor and firm participant groups. The goals
of these different groups are not necessarily equivalent, and this has
contributed to the bifurcation of the question, "What are the determinants
of equity share prices?" to the constrained forms: (a) "What are the
determinants of equity share prices given firm financial variables?" and (b) "What are the determinants of firm financial variables given equity share prices?".

2.1.1 Investor Objectives

There seems to be general agreement in the literature that investor objectives derive from two behavioral propositions: (1) an investor's utility of money function is positively sloped and probably concave downward; (2) an investor's investment strategy is the maximization of expected utility.\(^1\) Although the concept of utility has undergone considerable modification over the years,\(^2\) some sort of indifference mapping has always been used to explain away certain paradoxes and derive general equilibrium conditions.

Paradoxes arise, for example, when attempts are made to substitute the single purpose goal "expected wealth maximization" for expected utility maximization. This substitute goal would lead to certain pathological examples\(^3\) and to behavior modes that are simply not observed in the real world.\(^4\) The result has been the development of a two dimensional goal,


\(^3\)See, for example, David Durand, "Growth Stock and the Petersburg Paradox," Journal of Finance, 12 (Sept., 1957), pp. 348-363.

"expected risk-return," to describe investor objectives.  

There is surprisingly little discussion in recent literature about the advisability of using a two-dimensional risk-return vector as a surrogate for utility goals in the investments area. Indeed, the issue seems so settled that efforts are now being directed toward incorporating this mode of analysis into a general equilibrium theory of financial goods prices.  

This is surprising for two reasons. First, the definitional concepts of return and risk remain uncomfortably vague for having such widespread acceptance. Second, the process of defining goals for the firm and translating these goals into desired operating magnitudes is developing along somewhat broader lines than the process of defining investor goals. It is not clear that investor allocation decisions are any less complex in theory than firm allocation decisions.

2.1.2 Firm Objectives

Traditional theory of the firm models ascribe to the firm operating in a perfectly competitive market with given prices and technology the single-purpose objective of maximizing net profits. As was true of


investor expected wealth maximization, "expected profit maximization"
has hardly ever been taken literally but has been constrained in various
ways. Originally the constraints may have been informal conditions, but
more recently they have become a formal part of the analysis in theory of
the firm models. The constraints are of two types. The first type might
be characterized as organizational-institutional constraints. These
operate directly on the firm goal function, splintering the single-purpose
profit-maximization objective into a vector of firm goals. These goals,
imposed by various groups of firm participants, are not necessarily compat-
able, so ultimate fulfillment depends on group power, resource slack, and
behavioral dynamics.

The second type of constraints could be characterized as behavioral-
technological constraints. These are usually management-imposed subsidiary
goals or technological constraints that operate on the profit objective
function but do not splinter it into multiple goals. Firm models of this

7 Examples might include such statements as "profit maximization in
the long run" or "survival is the first firm goal."

8 For a review of the theory of the firm, see Richard M. Cyert and

9 Attributes of the behavioral theory of the firm have their fullest
development in:

Kalman J. Cohen and Richard M. Cyert, Theory of the Firm, (Englewood

Cyert and March, op. cit.

type could be positive descriptions of firm behavior, but they tend to be normative programming models.

Thus, the developing models of the theory of the firm tend to describe the allocation of the firm's resources in a framework that postulates multiple goals and a matrix of environmental constraints. At the same time, theory of the investor models describe the allocation process in terms of two-dimensional goals and only limited attention to subsidiary constraints.

It is probable that at the most general systems level, all these complementary approaches will eventually be synthesized into a general theory of the allocation of financial resources. In the meantime, however, the type of objectives prescribed and the environmental constraints considered have a very real impact on the type of variables that are considered for incorporation into models of equity valuation.

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11 For examples of the developing work in this area that attempts to incorporate corporate goals and financial constraints, see:


Bruce P. Fitch, "Elements in Corporate Equity Valuation," working papers, University of California (Berkeley), 1965.

12 The possibilities of more complex analysis have not been completely ignored, but it cannot be said such possibilities have been developed to any great extent. For some ideas, see:

Markowitz, Portfolio Selection: Efficient Diversification of Investments, pp. 129-132.

2.2 Specification of Return Measures

In this section, we shall consider several problems related to the return dimension of the investor's goal set. These problems include a review of the concept of return, a discussion of the problems of translating expectational variables into observable data measures, a discussion of the impact of growth, of taxes, of economic externalities on equity returns. To begin we consider the rights of equity shareholders.

2.2.1 The Rights of Equity Shareholders

The common-stock shareholder is the legal owner of a company. This does not mean that control of the resources of a company or power to effectively make decisions concerning the use of resources or other corporate policy resides with the stockholder. Quite the contrary, for the responsibility and power of the average American shareholder to affect the ongoing existence of the corporation are negligible.\(^{13}\) Legal ownership entitles the stockholder to four basic rights:

(1) The right to possess a piece of paper declaring Mr. Stockholder to be the owner of (x) shares of the XYZ Company.

(2) The right to vote for company directors and certain other matters of policy, such as mergers and large acquisitions requiring additional equity capital.

(3) The right to buy or sell a stock at a time of one's choosing or at a price of one's choosing (but not necessarily at the time and price of one's choosing) if an opposite party to the transaction can be found.

(4) The right to such periodic payments called dividends as may be declared by company management. Dividend payments are not a guaranteed certain flow, but are uncertain cash streams periodically determined by company management.

Note that legal ownership does not give the stockholder a right to any of the following: (1) a direct claim on cash, securities, or other assets which the corporation may own; (2) a voice in determining the management of a company (although many companies with "inside board of directors" have a high degree of overlap between directors and high corporate officials); (3) a voice in setting contracts determining product pricing policies, product mix strategies or virtually any other decision that is crucial to the competitive position of the company; (4) a direct claim on the earnings from corporate activities. The stockholder simply does not have control over the internal processes or flows of the firm.

The relation between valuation and those "rights" a stockholder does have is a tenuous one at best. There is little doubt that at various times all four rights do have economic value, but except in transient situations it is difficult to disentangle the effects of the first three rights from other aspects of equity valuation. The fourth right--the

14One might consider it ridiculous to pay something for a piece of paper per se. Yet many of the purchasers of Communications Satellite Corporation stock were reported to have bought the stock just so they could tell their grandchildren they were one of the original stockholders--even though they fully expected the stock to perform poorly in every respect compared to alternative issues.

The right to vote becomes noticeably valuable when there are groups competing for control of the company, so that demand for stock to vote results in a premium price over the valuation that might otherwise be placed on the company by the interactions of the market.

The right to buy or sell at a time or price of one's choosing is seldom abrogated, but there have been times when trading in the organized markets has been suspended for single issues or for the entire market.
right to dividends—will be examined in the next section since it is an integral part of the concept of return.

2.2.2 The Concept of Return

The concept of return is dependent upon at least three definitional variables: (a) change in wealth measures; (b) a base wealth measure for comparison; (c) a time interval or point of reference. Given the absence of transactions costs and taxes, investor return for the period \( t \) is almost universally defined as:

\[
(2.1) \quad k(t) = \frac{dv(t)}{P(t-1)} + \frac{P(t) - P(t-1)}{P(t-1)}
\]

\[dv = \text{dividends per share for the period}\]
\[P = \text{market price of the stock}\]
\[t = \text{end of period subscript}\]

Such a definition obscures certain issues at the micro (individual investor) level and is rather misleading when aggregated to indicate total return for the firm's stockholders.

At any point in time, historical return for an individual small shareholder is a result of two types of changes in the shareholder's wealth. First, the stockholder has accumulated dividends. Second, the stockholder may have a change in the asset value of the security held. The simplest return measure would relate these gross flows and stock value changes to original investment.

\[
(2.2) \quad k_1 = \frac{\Sigma dv(t) + P(t) - P(o)}{P(o)}
\]

This index is suitable for comparing returns when one is indifferent to the time horizon \( T \) and to the distribution of dividend flows within the interval \((o,T)\).
As an historical indication of return, \( k_1 \) may be a suitable index. As a measure of expected return, however, this index is probably not adequate. In an uncertain world where new investment opportunities are continuously created and where each investment opportunity has its own informational code, the ability to forecast expected return will be a function of the information code of the particular project. This is true not only for current projects, but also for future projects in which eventual investment will be possible. These uncertainties, arising from differential informational codes associated with known projects and from unknown possibilities of future projects, make it desirable to incorporate the time horizon and distributional differences in flows into a return index.

If we assume for mathematical convenience that dividends are distributed continuously and that dividends and price can be described as current measures times some time path function, the measure of return \( k_1 \) can be restated as follows:

\[
(2.3) \quad k_1 = \frac{\int_0^T dv(o).G(t)dt + P(o).H(T) - P(o)}{P(o)}
\]

where \( G(t) \) is some function describing the time path of dividends and \( H(t) \) is some function describing the time path of price movements.

A measure of return incorporating the time horizon \((o, T)\) into it can now be defined as:

\[
(2.4) \quad k_2 = \frac{\int_0^T dv(o).G(t)dt / T + P(o).H(T) / T - P(o) / T}{P(o)}
\]

This index defines an average rate of return per time unit for the project. If the time interval is suitably short, the measure can be
reduced to a marginal rate of return:

\[ k_3 = \frac{dv(o).G(t) + P(o).H'(t)}{P(o)} \]

where \( H'(t) \) is the first derivative of \( H(t) \)

Expected yield over the interval \((o,T)\) for this instantaneous rate of return is just the average rate of return \( k_2 \) defined in equation (2.4).

Note that expected yield under this definition results from two sets of projected adjustment processes. One adjustment process, the dividends change \([G(t)]\) is largely a discretionary management variable. The other adjustment process, the price level change \([H'(t)]\), is determined by market interactions arising from investor decisions. These investor decisions are a result of multiple variable changes, including changes in equity price levels and changes in dividends.

While expected yield as defined by equation (2.4) will discriminate between two securities with the same gross returns but different time horizons, \( k_2 \) will not discriminate between two securities with the same gross returns and time horizons but different dividend distributions within the horizon. To make this discrimination, each point within the horizon \((o,T)\) must be given a different weighting. The standard weighting scheme is to discount each point by the expected rate of return raised to a power equal to the interval length between \((o)\) and the point. This is just the familiar discounted rate of return.

\[ o = \sum dv(t).w^{t-1} + P(T).w^{T-1} - P(o) \]

where \( w = \frac{1}{1 + k_4} \)
In terms of continuous variables, the value for $k_4$ results from the solution to equation (2.7):

$$\text{(2.7) } P(o) = \int_0^T dv(o).G(t)e^{-k_4\cdot t} \; dt + P(o).H(T)e^{-k_4\cdot T}$$

Obviously, alternative weighting schemes could have been used in equation (2.6) to discriminate between two different dividend flow patterns. The use of $(w)$ is convenient since it permits a ready interpretation of $k_4$ as the discounted rate of return.

There are serious problems if $k_4$ is to be used by individuals as the return decision index. In an expression such as equation (2.6), it is not possible to separate $k_4$ into independent dividends and capital gains returns as was true for equation (2.4). But it seems clear from much of the investment advisory literature that the process of forming an overall expected return is more nearly described by a model where dividend return and capital gains return can be separately considered. Thus, if $k_4$ is a better surrogate for the return index used by investors than the index $k_2$, it must be supposed investors somehow implicitly translate dividend and capital gains returns into a single index that is not a simple addition of the two types of returns.

A problem that must be considered in all these indexes is the definition of capital gains. One must distinguish rather sharply between realized and unrealized gains when calculating a capital gains index. Even ignoring taxes and transactions costs, realized and unrealized capital gains will not be equivalent variables unless one can exchange the unrealized gains for desired goods. This is rarely
possible;¹⁵ indeed, it is seldom possible to even borrow the full value of an unrealized equity gain. Another consideration is that the decision when an unrealized gain becomes a realized gain is an investor decision over which the firm has no control. Even though realized and unrealized gains are different variables, it might be that from a behavioral viewpoint they could be treated as equivalent variables. Such an assumption permits great simplification in equity valuation theories. Since there seems to be little evidence on the proposition one way or the other, the assumption of behavioral indifference between realized and unrealized capital gains remains a convenient conjecture almost universally adopted.

In this section return indexes that are a function of the expected dividend stream, capital gains, and time horizon have been considered. For such indexes to be meaningful, the time horizon [o,T] must be the same for determining expected dividend returns and expected capital gains returns. More important, under ceteris paribus conditions,¹⁶ the investor must be strictly indifferent between different combinations of expected dividend-capital gains returns that produce the same expected aggregate return, if a single valued return index is to be a meaningful decision variable.

¹⁵In recent years, there have developed a few "exchange funds," which provide opportunities for individuals with large single holdings to exchange these holdings for the fund's own securities. This permits diversification without incurring immediate capital gains taxes.

¹⁶It is particularly important that combinations from different securities be picked so that overall risk is held constant.
2.2.3 The Impact of Expectational Surrogates

Most of the developed economic models of equity valuation are formulated in terms of expectational variables. Although there is a considerable body of research\(^{17}\) that examines aggregate forecasts, businessmen's anticipations, and expected price movements with regard to business and economic statistics, there is almost no developed research in the area of surveying or describing investor expectations in the equity markets. The result has been a strong tendency to rely on historical firm financial statistics as surrogates for expectational measures.

Part of one of the older issues in the area of equity valuation seems to revolve around the question of the correct formulation of expectational measures. The issue is whether—in the absence of taxes and under *ceteris paribus* conditions—stockholders value a dollar of dividends or earnings more.\(^{18}\) A closely related issue is whether stock values under *ceteris paribus* conditions are a function of corporate earnings independent of dividends.\(^{19}\) A major factor contributing to the difficulty in resolving these issues is the inability of researchers to satisfactorily specify


values for expected dividends and expected earnings, particularly earnings.

If future dividends are known with certainty and if people behave in an economically rational manner so that market equilibrating forces are dominant most of the time, pure dividends models may be an adequate explanation of equity prices. If, however, dividends are not known with certainty, the problem becomes considerably more complex. The explanation of consumer stock prices can be considered a two-stage process if the dividend stream be uncertain:

1. the development of a stock valuation model;

2. the development of a dividends prediction model to assist in the specification of the dividends variables in the stock valuation model.

Fortunately, for the vast majority of American corporations there is evidence that dividends are a function of certain economic variables, rather than some arbitrary capricious decision by the Board of Directors of a firm. In particular, dividends are a function of the earnings generated by the corporation. The decision processes that determine dividend payments are quite varied, so that for the individual firm almost nothing can be said about the functional form of the generating process. Investors are not privy to the decision processes of the Board of Directors of a firm. This suggests that if the majority of transactions in a stock are by "outsiders", that is, buyers and sellers not aware of the detailed decision process for determining dividends, it is only necessary to worry about investor perceptions of the functional form of the generating process and not the true dividends-earnings relation for each firm.
But what sort of assumptions do investors usually make about dividends-earnings relations? Three conditions are usually specified:

(a) dividend payments are less than current net income;
(b) dividends tend to rise as earnings rise;
(c) dividends are relatively more stable than earnings.

Condition (a) is a statement of the normal policy guideline for dividend-paying corporations. Ordinary stockholder returns come from the returns to the corporation. Only under depressed economic conditions or under conditions where part of the corporate assets are being liquidated and the funds returned to stockholders, will (a) be violated. Condition (b) suggests that corporations are more likely to raise their dividends as earnings rise than they are to lower dividends. The relation is not necessarily a proportional one, however, but may be more nearly a step function. Condition (c) must be interpreted with some caution. It does not necessarily mean that $\sigma_{dv} < \sigma_{ni}$. What people may mean when talking about dividend stability is that over a given time interval dividends are more likely to remain stable or to change fewer times than are earnings, and in addition, if dividends do change, it is likely to be an upward shift. That is, an upward dividends shift consistent with rising earnings is not taken to be a sign of dividends instability.

It should be noted at this point that the discussion has centered on two different types of dividends models. One type may be called the actual dividends model. Actual dividends are a function of several variables:

$$\text{(2.8) Actual dividends} = f(\text{expected net income values, internal investment opportunities, operating cash constraints, stockholder pressures, external financing activity, directors' goals and values}).$$
The dependent variable for this equation can be measured exactly, but only approximate measures exist for some of the independent variables, particularly for those observers who are outside the firm. The other type of dividends specification model could be called the investors' dividends expectation model. It may be reasonable to suppose that investor expected dividends are a function of at least the following variables:

\[
(2.9) \quad \text{Investor expected dividends} = f(\text{present dividends, indicated management desired payout ratio, expected net income values})
\]

Data are readily available to determine some of the independent variables of equation (2.9), but for this equation no data exist for the dependent variable unless each individual investor is interviewed.

Several things are possible given these two types of dividends expectations models. First, it is possible that both equations predict the same values for expected dividends. If, for example, expected net income and net income trends are the dominant variables in determining actual dividends, it seems likely that this information can be quickly diffused to the general investor, particularly if expectations about income are closely related to recent historical income patterns. Second, it is possible that both models (2.8) and (2.9) exist in the market place simultaneously. If one believes that there are at least two distinct investor classes--one a large class of general investors with only limited information, and the other a smaller investment group with large dollar resources to invest and to use in the search for information--it seems reasonable to assert that there may be some investors who actually talk to managements and study them in an effort to ascertain actual dividend policy. To the extent this information is sold or passed on to the
larger general investor group, it may also partially modify the expectations of these investors about dividends. The third possibility is that the model suggested by equation (2.9) is indeed a first approximation to the formation of investor expectations about dividends and hence attempts to approximate (2.9) by variations of equation (2.8) may befuddle the real relationships. Finally, it should be noted that the current major models of equity valuation do assume that the correct mechanism for describing investor expectations about dividends is described by the model of equation (2.8), not equation (2.9). In particular, the current models seem to attach some importance to investment decisions and capital structure changes in the firm. These investment flow variables are sometimes linked directly to the dividends variable, but more often the link is indirect with the emphasis on predicting retained earnings.

There are some who would ascribe to earnings a more important role than that of predictor for an uncertain dividends stream.\(^\text{20}\) Indeed, this school inverts the mechanism previously described and suggests that equity prices are a function of an uncertain earnings stream and dividends are merely an information surrogate for management expectations about future earnings. In a now famous article,\(^\text{21}\) Professors Miller and Modigliani

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argue that with perfect markets, rational behavior, and perfect certainty, it is easy to demonstrate that the value of the firm must be independent of the dividends payout decision. In order to justify this proposition, it is necessary to strip away not only all the encumbrances of real world markets, but also any real world interdependencies in corporate policies. For example, the current investment budget $I(t)$ is assumed to be completely independent of the current dividends decision. As John Lintner pointed out in a subsequent article, even a slight relaxation of these economic wonderworld assumptions to admit the possibility of conditions such as (a) issue or transfer costs, (b) differential tax rates on capital gains and dividends, (c) the nonuniformity of investor expectations, means that current prices will be a function of the dividends stream of the firm.

The need to create a highly artificial static economic environment devoid of behavioral differentiations in order to demonstrate the equivalence of earnings and dividends theories has not deterred some from attempting to empirically "prove" the superiority of earnings to dividends as an explanatory variable. If one ignores the logical basis for such models, there might even be a crude justification for such tests. For example, suppose that over a period of years earnings had steadily increased for a company while dividends had remained constant; suppose further that the price of the company's stock had increased during the period.

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Clearly a regression of price on earnings will give a superior result to a regression of price on measured dividends. It is rather difficult to develop a theory based on measured earnings or dividends however. For one thing, there exist corporations which have large positive stock prices and which have never reported any earnings or dividends. There also exist corporations which have either long periods of negative earnings or a few years of very large losses, so that almost any weighting system that considers all earnings for the corporation would imply that for some year the price of the stock should be negative for an earnings model. There are at least two strategies that have been employed to deal with negative earnings and nonexistent earnings and dividends: (1) ignore these firms and hope they will go away; (2) shift from an empirical "theory" to a model that is supposedly based on expectations, and then throw out firms when the statistics don't agree with \textit{a priori} notions about expectations.

Some of the studies that have been made seem to strongly indicate that a dollar of dividends is worth significantly more to shareholders than a dollar of earnings, other things being equal.\footnote{See, for example, Myron J. Gordon, "Dividends, Earnings, and Stock Prices," \textit{Review of Economics and Statistics}, 41 (May, 1959), pp, 99-105.} Unfortunately, such
studies may be subject to serious bias since measured dividends and earnings are used as surrogates for expected values. In the last two decades, large American corporations have tended to follow a dividend stabilization policy. This means that current dividends are often a good surrogate for near-term future dividends. Earnings, however, tend to have a large random component that makes current earnings a rather gross predictor for future earnings. If current earnings are used as a surrogate for expected earnings, the error could substantially bias downward the estimate for the earnings coefficient.\textsuperscript{24} Most attempts to alleviate this problem use some sort of smoothed historical series to derive an estimate of "normal" current earnings. The smoothed series usually gives a parameter estimate for the earnings coefficient that is significantly higher and closer to the dividends coefficient than the unsmoothed earnings series.\textsuperscript{25}

There might be an alternative way to try to distinguish dividends and earnings effects. That procedure would depend on being able to find a suitable sample where investor expectations were clearly incorrect.

Potential situations such as the following might be investigated:

(a) a corporation with slowly declining earnings suddenly announces a dividends increase;

(b) a corporation with a long history of paying the same dividends announces an unexpected dividends shift;

\textsuperscript{24} For some attempts to correct this problem, see M. Miller and F. Modigliani, "Financial Policy and the Cost of Capital: Some Empirical Results," multilithed notes from remarks delivered at Pittsburgh, Pa. meeting of the Econometric Society (December 28, 1962).

(c) a corporation with rising earnings announces a dividends cut;

(d) a corporation with rising earnings announces a dividends increase (anticipated) and then a short time later announces another dividends increase (unanticipated).

Given such exceptions, the impact of the unanticipated change on price might be clearly discerned. Of course, there must be some mechanism, independent of any price changes, for ascertaining that these dividends changes were in fact unanticipated. All this is not to suggest that we absolutely cannot distinguish between earnings and dividends hypotheses. It does seem to suggest that, given present limitations in statistical techniques, and given the very weak theoretical foundations for earnings hypotheses thus far developed, the task will not be easy.

Another type of problem the researcher using historical financial statistics must face is the fact that if the capital markets are assumed perfect, new information will be disseminated much more quickly than the annual financial statistics themselves. For example, most models use yearly income and balance sheet data, but quarterly figures and preliminary yearly figures are available long before annual reports are published.

There is some evidence, from both the random walk theorists' research on stock price movements\(^{26}\) and from direct examination of the impact of changes in earnings and dividends,\(^{27}\) which suggests that equity values


adjust rather quickly to new information. Current expectational measures are generally not sensitive to announcement changes of any type.

2.2.4 The Impact of Growth

In a previous section (2.2.2), it was suggested that expected return is a function of investor horizon, the time distribution of dividend payments, and the time distribution of capital gains. Since it is not likely that most such time distributions would be represented by simple mathematical expressions, it would be convenient if the time vectors could be represented by alternative expressions. Also, it is not likely that investors would make point estimates of the necessary yields for every period in the investor's time horizon. The simplest way around these problems is to redefine the yield streams in terms of a specified yield times a compound growth rate.

For example, this might be done in the following manner:

\[
\frac{dv^*}{P(o)} = \text{expected dividend yield}
\]

\[
= \int_0^T \frac{dv(o) \cdot G(t) dt}{P(o) \cdot T} \quad \text{let } G(t) = e^{gt}
\]

then

\[
\frac{dv^*}{P(o)} = \int_0^T \frac{dv(o) \cdot e^{gt} dt}{P(o) \cdot T}
\]

\[
= \frac{dv(o)}{P(o)} \cdot \frac{[e^{gT} - 1]}{gT}
\]

\[
= \frac{dv(o)}{P(o)} \cdot \frac{[(gT) + (gT)^2/2 + (gT)^3/6 + (gT)^4/24 + ---]}{gT}
\]

\[
= \frac{dv(o)}{P(o)} \cdot [1 + (gT)/2 + (gT)^2/6 + ---]
\]

Here the dividend yield has been summarized in terms of the current yield, an expected growth rate parameter, and the time horizon. There are many ways the time-path functions for variables could be approximated
by growth parameters, so the equation form selected in each instance will probably depend on simplicity considerations and the closeness of fit to historical data.

The choice of functions for specifying growth rates has sometimes created paradoxes that became separate issues in the development of models of equity valuation.\textsuperscript{28} For example, if exponential functions are assumed for the discounted rate of return index we will have:

\[
(2.9) \quad P(o) = \int_0^T dv(o).G(t)e^{-kt}dt + P(o).H(T)e^{-kT} \\
= \int_0^T dv(o).e^{gt}e^{-kt}dt + P(o).e^{hT}e^{-kT} \\
= \frac{dv(o)}{k-g} \left[ \frac{1-e^{(g-k)T}}{1-e^{(h-k)T}} \right]
\]

If an infinite time horizon is assumed and \((k > g, k > h)\) equation (2.9) reduces to the standard price model:\textsuperscript{29}

\[
(2.10) \quad P(o) = \frac{dv(o)}{k-g}
\]

The paradox, of course, is the implication for \(P(o)\) if \((g)\) or \((h)\) is not less than \((k)\). If choice is based on a return index, then \((k)\) is the dependent variable and the possibility of \((k)\) being less than \((g\) or \(h)\) would never occur in an equilibrium situation. That is, \(k = f(g,h,T)\), just as it is in equation (2.4). If \((k)\) is an independent variable—perhaps a minimum acceptable rate of return—then at the micro (individual


\textsuperscript{29} For a discussion of the development of this formulation, see M. J. Gordon and Eli Shapiro, "Capital Equipment Analysis: The Required Rate of Profit," Management Science, 3 (October, 1956), pp. 102-110, and Myron J. Gordon, "Dividends, Earnings, and Stock Prices."
investor) level, a situation where \( g \geq k \) is a situation where the security would be acceptable to the investor at any price if (a) the time horizon were infinite, or (b) the time horizon were finite but \( h > k \). For a finite time horizon and \( h < k \), the equilibrium price would be finite and equal to:

\[
(2.11) \quad P(o) = \frac{dv(o)}{(g-k)} \left[ \frac{e^{(g-k)T-1}}{1-e^{(h-k)T}} \right]
\]

for situations where \( g > k \). It must be remembered that return is only one of the decision criteria. Equations (2.9-2.11) say nothing about the risk associated with assuming a situation where \( g > k \) for an infinite time horizon. Furthermore, even if all individuals held the same expectations that \( g > k \) for equation (2.10), this does not mean the price of the stock is unboundedly large. It simply means that the stock will not be traded in the market so long as current expectations do not change.

Another type of issue that has been raised is whether or not increases in magnitude per se are indications of growth. Some researchers have argued that it is not expansion which is important in the valuation process, but expansion which provides a rate of return greater than that currently available on the firm's capital. Thus, neither plant expansions nor mergers with other firms having the same investment opportunities is true growth. The shares of a firm will be capitalized at a premium only if the marginal investment opportunities indicate a higher rate of return than that currently implied by the firm's cost of capital. It should be noted that this type of growth theory assumes a complete market equilibrium so that the firm's cost of capital is identical to investors' \( k \).

2.2.5 The Impact of Taxes, Interest Rates, and Other Economic Externalities

In the vast majority of equity valuation models developed in recent years, taxes, transaction costs, interest rate levels, and other economic externalities that are not reflected in firm financial statistics are ignored. The absence of such institutional factors in models is reasonable if these factors do not differentially affect participants in the stock market or if the effects are so small as to be statistically insignificant.

Such assumptions are probably not warranted in the case of taxes. Taxes influence both firm financial variables and the return expectations of different investor groups. It is not only net income that is affected by taxes in the firm. Tax policy also plays an important role in determining firm capital structure, dividend payments, and future investment decisions. Since interest is a deductible expense for tax purposes, while common and preferred dividends are not deductible, there is some bias in favor of debt financing relative to equity financing. This is true whether one holds with the traditional view on the impact of debt on corporate financial structure, or one believes in the relevancy of the Modigliani-Miller abstractions for the real world.

As for dividend payments, it would seem tax policy exercises a dual role. There is reason to believe that two of the determinants of dividends are cash flows and foreseen investment opportunities.  

31 See David Durand, "Costs of Debt and Equity Funds for Business."


investment profitabilities are a function of tax policy concerning depreciation and depletion allowances. The differences in allowable depreciation rates have become so great from industry to industry that many analysts now use cash flow instead of net income after taxes as a measure of short-term corporate performance. The other way tax policy may influence dividends is through the codes taxing dividends to individuals at the income tax rates but taxing capital appreciation at a lower capital gains rate. This policy has implications for the willingness of individuals to adjust their portfolios where capital gains have occurred.\textsuperscript{34} It also makes it more attractive for many investors to search for those securities that are likely to offer returns as capital gains rather than dividend payments. Since corporate officials are not unaware of this differential taxation on their stockholders, they may retain more for internal investment than would be the case if the double taxation did not exist. This may also mean that dividend yields by themselves (without considering capital gains yields) may seem relatively lower than one would predict from an overall assessment of the risk characteristics of the equity markets.

Transaction costs are probably not a significant factor in the determination of equity share prices. Although the rates are moderately scaled to provide some marginal reduction in rates as the dollar volume increases, these reductions are only applicable to the unit of trading--usually 100 shares. This means that buyers of several thousand shares enjoy no advantage over buyers of one hundred shares. Although there

have been some recent attempts to bypass or reduce transaction fees in
large block sales, it is not clear that such sales affect relative equity
share prices. Transactions costs do affect overall portfolio return.
Since such costs are a function of the frequency of trading, portfolio
strategies that require active in and out of the market trading are likely
to find a rather significant transactions bill.

Interest rate levels should play a critical role in the determination
of equity share prices. Most researchers assume that the capital markets
are closely interrelated with rapid adjustment among the various rates.
The pure rate of interest plays a critical role in numerous theories as
a key upon which other rates are built.\footnote{See, for example, the article by Tobin, \emph{op. cit.}, pp. 67-75.} Virtually all researchers make
time comparisons of cross-section sample performance. That is, parameter
estimates for the year (t) for a sample are often compared to parameter
estimates for the same cross-section sample for the year (t + x). Despite
these factors, not one of the models under investigation makes any adjust-
ment for temporal differences in interest rate levels. Indeed, in the
entire literature surveyed, there is only passing reference to this
problem.\footnote{For explicit attempts to incorporate interest rates into models,
L. Fisher, "Determinants of Risk Premiums on Corporate Bonds," \emph{Journal
of Political Economy}, 67 (June, 1959), pp. 217-237.}

There are several reasons why money rate indexes may not have been
utilized. First is the fact that most researchers are dealing with cross-
section samples. So long as one restricts oneself to statements about
parameter significance, it is not necessary to make such adjustments. Unfortunately, researchers do not so restrict themselves, but also talk about the comparative magnitudes of parameter estimates. For the types of models specified in equity valuation theory, it seems clear that interest rate levels would affect variable coefficients as well as constant terms. Second, it may be that the impact of money price-level changes is small relative to other effects. This is a rather strong assumption to make in the absence of supporting evidence. Third, there are very great theoretical and empirical difficulties in deciding what type of index should be used and how it should be incorporated into particular models. It is fairly clear that yields in the capital markets do not always move concomitantly. This is particularly true when comparing equity yields to rates in the other capital markets. Indeed, we have the curious situation of stock prices being advocated by some as a leading economic indicator and yet lagging important interest rate adjustments. With such uncertainties about the relationship between money and stock prices, it is not surprising that an interest rate level index has not been successfully incorporated into current equity models.

Finally, there are other variables that might properly be considered for inclusion in an equity valuation model if parameter estimates are to be compared across years. Investor expectations may be influenced by factors other than those presently or ultimately revealed in firm financial statistics. Such factors could include political stability, labor-relations

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stability, or general economic exuberance—perhaps measured by the degree of participation in the stock market itself. There are also changing environmental constraints on the financial markets that could result in a major impact on the stock market. Such changes are not captured in existing models and probably cannot be until a complete flow of funds model of the capital markets is developed.

2.3 Specification of Risk Measures

In this section, we shall consider several problems related to the risk dimension of the investor's goal set. The problems include the definition of risk, the impact of financial structure on investor returns, possible measures of the riskiness of firm financial flows, and risk elements associated with security price changes. A proper discussion of risk would require a monograph at least as long as the present work. Fortunately, such a task is not necessary for a comparative study of existing equity valuation models. Although risk is a crucial dimension in most portfolio selection models, and is incorporated in some form in many equity valuation models, the facts are that risk variables as specified in equity valuation models are generally neither statistically significant nor of an important magnitude. This does not imply that risk is not an important dimension, for there is serious doubt that existing equity valuation models have properly incorporated this variable into their theoretical framework.

38 Thus, the rapid growth of mutual funds in the last decade has had little, if any, impact on the financial variables of the firm. These funds may have had a very major impact, however, on changes in the temporal returns available on equity securities.
2.3.1 The Concept of Risk

Section (2.1.1) suggested that an investor's utility function with regard to future wealth could be related to a two-dimensional return-risk space. The risk dimension is concerned with measures of the variability of return over the relevant investor horizon. The concept of risk is broadly construed to include measures of what might be called (a) financial structure stability; (b) financial flows stability; and (c) investor returns stability. But it is the variability in expected returns or future wealth that is the dominant consideration in defining risk.

If we assume that an investor commits an amount \( W_0 \) of his present wealth to investment and that over the desired horizon this wealth increases to \( W_T \), a gross measure of return on investment is given by:

\[
R = \frac{W_T - W_0}{W_0}
\]

Assume this is not a certain return. Then the investor can be supposed to associate some type of subjective probability distribution with the expected value of \( R \). We shall designate \( \sigma_R \) as the index of dispersion of this subjective probability distribution and call this index the "risk" measure associated with the expected value of \( R \).

The question arises as to the form \( \sigma_R \) should assume. It turns out that computational procedures and statistical testing problems are vastly simplified if \( \sigma_R \) is in fact assumed to be the standard deviation associated with the expected return \( E_R \). This is not the only form the risk

\[39\] See Markowitz, Portfolio Selection: Efficient Diversification of Investments, pp. 257-286.
index could take. Indeed, as Markowitz points out in his pioneering study, if a single variable must be used to measure the risk dimension, variance may not be the most desirable choice. Other measures reviewed by Markowitz include (a) the semivariance, (b) expected loss, (c) expected absolute deviation, (d) probability of loss, and (e) the maximum loss.\footnote{\textit{Ibid.}, pp. 288-297.} All these measures are intended to decrease the weight of positive deviations about \(E_R\). That is, the possibility of exceeding the expected return is not regarded as being bad. The desired risk decision variable would be an index under these alternatives mostly of the possibility of falling below the desired return. Unfortunately, with each of these alternatives there are formidable computational algebraic and statistical problems at present. Therefore, risk indexes are predominantly variance measures.

If the capital markets are assumed to be reasonably perfect and participants make investment decisions according to return-risk criteria, efficient portfolios will fall along the capital market line:

\begin{equation}
(2.13) \quad k_i = a + b \sigma_i
\end{equation}

Here \(k_i\) is the expected return for the \(i\)'th efficient portfolio and \(\sigma_i\) is the standard deviation of this return. The parameter \(a\) should be the pure rate of interest and \(b\) is the risk premium.\footnote{See Tobin, \textit{op. cit.}, pp. 71-76 and William F. Sharpe, "Risk-Aversion in the Capital Markets: Some Empirical Evidence," \textit{Journal of Finance} 20 (September, 1965), pp. 416-422.} Equation (2.13), usually rewritten to the form of equation (2.14), is a standard procedure for bringing risk into an investments model.
(2.14) \( k_i = k^* (1 + c \sigma_i) \)

Unfortunately, it does not follow that because a linear capital market line exists for efficient portfolios, an equivalent situation exists for individual securities. Portfolio selection theory and the empirical evidence on the wide dispersion of objective risk-return statistics for individual firms suggest such a simple relation does not in fact exist. In very recent attempts to extend risk-return analysis to the determination of equilibrium prices of individual securities, Sharpe and Lintner indicate the type of relationship that may exist.42 Instead of the traditional market line, Sharpe suggests that the relation for individual securities is approximately given by:

\[
(2.15) \quad k_i = a + b(\psi_i)
\]

\[
(\psi_i) = \frac{(r_{ig})(\sigma_i)}{(\sigma_g^2)}
\]

Again, \((a)\) is the pure rate of interest and \((b)\) is a slope parameter. \((\psi_i)\) is a measure of the relation between the return on security \((i)\) and the return on some efficient portfolio \((g)\) containing this security. The variable \((r_{ig})\) is the correlation between returns on \((i)\) and \((g)\). Thus, the index \((\sigma_i)\) from equation (2.13) is now adjusted by a factor of \([(r_{ig})/(\sigma_g)]\) for individual securities. Since there is still an unsystematic component in \((\sigma_i)\) not accounted for by \((\psi_i)\), the relation in (2.15) will not be exact.43

42See Sharpe, "Capital Asset Prices" and Lintner, "Security Prices, Risk, and Maximal Gains from Diversification."

43For a derivation of these results, see Sharpe, "Capital Asset Prices," pp. 436-442.
An alternative procedure for specifying risk is to concentrate on the independent variables in the expected returns equations rather than the dependent variable \( k \). Under this alternative, risk measures are associated with expected dividends, capital gains, and any other surrogate variables (such as growth, leverage) used to describe expected return. This procedure seeks to translate expected values of the independent variables into certainty equivalent values by deflating by the appropriate risk index. Using the return index suggested by equation (2.4), the two alternative ways of handling risk would be:

\[
(2.16) \quad k^* (1 + \psi) = \frac{E(dv)}{P(o)} + \frac{E(cf)}{P(o)}
\]

\[
(2.17) \quad k^* = \frac{E(dv)(\phi_1)}{P(o)} + \frac{E(cf)(\phi_2)}{P(o)}
\]

where \( k^* \) is the pure rate of interest, \( E \) is the expectation operator, \( (cg) \) is capital gains, and \( (\phi) \) is a risk index.

The same sort of relations would carry through in any of the return indexes described in section (2.2.2). The risk indexes \( (\phi) \) may in fact be standard deviations for the distributions of the associated variables. Even so, the certainty equivalent approach implied by adjusting the discount rate (2.16) is not likely to give the same results as the certainty equivalent approach suggested by (2.17) where the expectational variables are individually adjusted.\(^4\) Unfortunately, the developed theory of portfolio selection (and hence equity valuation theory) uses the adjusted discount approach, but almost all empirical models of equity valuation use risk specifications analogous to (2.17). This may be another factor

contributing to the inability of current models to measure risk.

2.3.2 The Impact of Financial Structure

Not infrequently, equity valuation models are concerned with the problem of financial risk. Financial risk is not properly a risk dimension if the measures in the previous section are accepted. Financial risk is concerned with measures of the degree of exposure of the flows to stockholders to prior claims of corporate lenders. We shall be concerned here primarily with the contractual claims of the holders of senior corporate securities--bonds and preferred stocks. The measure of risk that is of interest is the financial leverage ratio [debt/equity]. The question that is of concern to equity valuation theorists is, "What is the impact of financial leverage on the value of the firm's leverage?"

The traditional position on this question is that over a reasonable range, the debt/equity ratio will not influence the expected yield on the common stock of the corporation.\(^45\) This implies that the equity value and total value of the corporation can be increased by debt financing. This position can be maintained despite the increased risk to stockholders (measured by earnings variability) because imperfect capital markets prevent complete arbitrage. In particular, institutional constraints on large financial intermediaries mean that yields on high-grade corporate debt are lowered, as these instruments are in great demand. Therefore, the yield differential between debt and equity will be greater than that required for pure risk differentials. Thus, the average cost of capital for the firm could be decreased by the introduction of moderate amounts of debt.

\(^45\)For the classical exposition of this position, see Durand, "Cost of Debt and Equity Funds for Business."
Professors Modigliani and Miller have argued at some length that the traditional theory contradicts standard economic theory. Such theory suggests that, in any perfect market, under ceteris paribus conditions the "price of a commodity representing a bundle of two other commodities cannot be consistently different from the weighted average of the two components." Suppose firms are divided into homogeneous risk classes, where risk is associated with the variability of the income flows generated by firm assets. Then for a given risk class, the price per dollar of a levered stream of earnings should be a weighted average of the price per dollar of an unlevered stream in that risk class and the price per dollar of a debt stream. That is, expected yield on the common stock should be an increasing function of the debt/equity ratio. Except for tax benefits, the total value of the corporation would not be changed by the particular type of financing undertaken. Under these circumstances, the average cost of capital would not change as moderate amounts of debt are introduced. The cost of capital would in fact be the cost associated with an unlevered structure for a given risk class.

The theory postulated by M-M is not much in dispute. In the constrained framework they postulate, the derived propositions are quite reasonable. What is in dispute is the applicability of such a theory to the real world. Are the capital markets perfect enough for their propositions to hold? This is essentially an empirical question. Unfortunately, as we shall see in the following chapters, tests on the impact of leverage

46 The basic article is Modigliani and Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment."

47 Ibid., p. 279.
are not easy to make. Statistical problems are created by the equation forms and by the fact that mis-specification of more important variables (such as expected dividends, earnings, growth rates, risk measures) is likely to create errors that swamp any attempt to measure the impact of leverage.

2.3.3 Some Measures of Risk

The concept of risk as described in the previous sections is thus broadly construed to include measures of what might be called (a) investor returns stability, (b) firm financial structure stability, and (c) financial flows stability. The majority of risk measures is some type of variance statistic—usually the standard deviation or coefficient of variation of the distribution under consideration. Some of the types of risk measures that have been used include:

(1) historical rate of return variance
(2) price variance or price change variance
(3) dividend variance
(4) earnings variance using different earnings measures
(5) earnings variance about a trend or specified growth function
(6) dividends/earnings ratios
(7) debt/equity ratios
(8) fixed charges relative to cash flows or liquid asset stocks
(9) fluctuations in working capital levels
(10) firm size indexes.

Firm size indexes are used because there is some possibility that the increasing diversification within American firms makes business risk more a function of relative firm size than traditional industrial classifications. Firm size measures, such as sales, total assets, total market value, may be surrogates for diversification of product groups, production processes, management personnel, or financial holdings. To the extent that increased size implies increased public awareness and hence increased
stockholder interest, there may also be risk reduction due to a broader financial market for a firm's stock.

Estimated variances for capital gains returns or estimated variances of stock price change variables may turn out to be rather unstable statistics. There is at least suggestive evidence that stock price changes do not fit theoretical distributions with a finite variance. Instead, relative price changes seem to follow a stable Paretian distribution which does not have a finite variance.\(^{48}\) This means that sample variances for relative price changes will show erratic behavior.\(^{49}\) It also has implications for the entire concept of risk-return decision making which can only be vaguely perceived at this time.

Finally, there is the question of how accurately any risk measures computed from historical data match current subjective risk estimates made by investors. It is possible that subjective risk is partly a function of investor pleasure, where pleasure is related to the current level of return and the duration of that level. For example, in a cyclic upswing, investors might view each period's additional capital gains as evidence that fundamental business conditions had changed (or some such rationalization) and that security returns were no longer as risky as they once had been. This reduction in subjective risk at the same time investors are probably raising their expectations about future return levels is a type of adaptive behavior that undoubtedly leads to speculative excesses.


\(^{49}\)See Fama, op. cit., pp. 94-98.
at market tops and bottoms. Most current measures of risk, based on actual
firm financial statistics, cannot completely capture these changes in
subjective evaluations. This is one more reason it has been so difficult
to incorporate measures of risk into models of equity valuation.