First-Purchase Rights: Rights of First Refusal and Rights of First Offer

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This paper analyzes rights of first refusal and rights of first offer in a multiple-buyer, sequential bargaining setting. A right of first refusal entitles the right-holder to purchase a subject asset on the same terms as those accepted by a third party. A right of first offer requires a seller to first offer the right-holder to buy a subject asset and prohibits the seller from subsequently selling the asset to a third party on better terms than those offered to the right-holder. We examine when and how such rights yield benefits to, or impose costs, on the right-holder and the seller. We show that a right of first refusal transfers value from other buyers to the right-holder, but may also force the seller to make suboptimal offers. A right of first offer induces the seller to lower his first-period offer, which will tend to increase the net surplus to the seller and right-holder, but also forces the seller to make suboptimal subsequent offers. We find conditions under which it is in the \textit{ex ante} interest of the seller and the right-holder to contract for a right of first refusal or a right of first offer, respectively. (JEL K12, C78)

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1. Introduction

This paper presents an analysis of rights of first refusal and rights of first offer. A right of first refusal is triggered when a seller of an asset subject to such right has agreed to sell the asset to a third-party buyer. The holder of a right of first refusal then has the option to purchase the asset on the same terms as those accepted by the third-party buyer. A right of first offer requires a seller who wishes to sell an asset, subject to such right, to offer the right-holder to buy that asset before it is offered to other potential buyers. If the right-holder declines the offer, the seller can sell the asset to a third-party buyer, but only on terms no better (for the third-party) than those offered to the right-holder. We will refer to rights of first refusal and rights of first offer collectively as ‘first-purchase rights.’

First-purchase rights are employed in a variety of contractual settings. They are found, for example, in real estate sales and lease contracts, in agreements among shareholders of closely held corporations, in joint venture and franchise agreements, and in professional sports collective bargaining agreements. See, e.g., Mueller (1989), Bartok (1991), Daskal (1995), Johnson and Stanford (1997), Smith (1997) and Platt (1996). In addition, state law creates miscellaneous rights of first refusal (for example, for franchisees, with respect to the establishment of new franchises). See, e.g. Keenan (1987), Lawless (1988) and Hess (1995).

Consider, for example, the agreement between the World Wrestling Federation Entertainment (WWFE), Inc. and USA Cable, a cable-network operator. The agreement, initially signed in 1983, provided that USA Cable will have the right to teletcast wrestling events organized by the WWFE. The agreement further provided that, upon its expiration, USA cable will have a right of first refusal whereby, if WWFE wished to enter an agreement with a third-party cable network, it would have to notify USA Cable of the terms negotiated with that third-party network and give USA Cable 10 days to match the negotiated terms. If USA Cable chose to not match the terms of the negotiated agreement, WWFE would be free to enter a telecasting agreement with the third-party network at the previously-negotiated terms. A right of first offer would have required WWFE to offer USA Cable a set

of contract terms, before negotiating with any other network. USA Cable would then have the right to accept WWFE’s offer. If USA Cable rejected WWFE’s offer, WWFE could enter an agreement with a third-party network at terms no more favorable than those proposed to USA Cable.

To examine the circumstances in which parties will employ first-purchase rights, we consider a sequential bargaining game between one seller and many buyers. Buyers’ valuations are independently drawn from a common distribution function. In each period, the seller makes a take-it-or-leave-it offer to one buyer. The buyer must decide whether or not to incur investigation costs to observe his valuation of the asset. If the buyer incurs investigation costs and his valuation is greater than the seller’s offer, the buyer accepts the offer. Otherwise, the game proceeds to the next period. In the right-of-first-refusal case, one buyer—the right-holder—has the right to buy the asset at the price offered by the seller to another buyer if that buyer accepted the seller’s offer. In the right-of-first-offer case, the seller must first offer the asset to the right-holder. If the right-holder declined the seller’s offer, the seller may offer the asset to other buyers but only at price no lower than the price offered to the right-holder.

The paper’s analysis proceeds by comparing the seller’s optimal sequence of offers, and the seller’s and right-holder’s joint profits, in the presence and absence of a first-purchase right. In the absence of a first-purchase right, the seller’s optimal offer in each period depends on the number of remaining buyers. The fewer buyers remain, the more eager the seller is to sell his asset and, consequently, the lower is the seller’s optimal offer. The seller’s optimal sequence of offers is thus decreasing. In addition, to induce a potential buyer to incur investigation costs, the seller’s offer cannot exceed the buyer’s investigation constraint.

A right of first refusal may affect the seller’s optimal sequence of offers by tightening the investigation constraint of any buyer other than the right-holder. Especially, a right of first refusal may lower the upper bound of offers that induce buyers to investigate. The intuition behind this is as follows. A buyer will incur investigation costs if the expected profit from acquiring the assets exceeds his investigation costs. The buyer’s expected profit, in turn, depends on his expected value of the asset conditional on acquiring it. In the no-rights case, the seller commits to selling the asset at the price offered to the buyer. Thus, the buyer’s probability of acquiring
the asset is equal to the probability that, following the buyer’s investigation, the buyer’s value is greater than the price offered by the seller. In the right-of-first-refusal case, in contrast, the buyer will only acquire the asset if he accepts the offer and the right-holder does not exercise his right. As a result, the probability of the buyer acquiring the asset for any offer is lower than in the no-rights case. To induce the buyer to incur investigation costs, the seller’s offer may have to be lower than in the no-rights case.

If the investigation constraint is not binding, the seller’s optimal sequence of offer in the right-of-first-refusal case is identical to that in the no-rights case. If the investigation constraint is binding, in contrast, the seller makes different offers in the right-of-first-refusal case. Finally, if buyers’ investigation costs are sufficiently high, no offer could induce buyers to incur investigation costs and the asset will become non-marketable: The seller will only be able to sell the asset to the right-holder.

A right of first offer affects the seller’s optimal sequence of offers by imposing the additional constraint that offers made subsequent to the first-period offer may not be lower than the first-period offer. Thus, as opposed to the no-rights case and the right-of-first-refusal case, the seller’s optimal sequence of offers is either flat or has an inverted U-shape which has its maximum at the second-period offer and where the first-period offer is equal to the last-period one.

The intuition for this result is as follows. By the terms of the right of first offer, no offer may be lower than the first-period offer made to the right-holder. Because the first-period offer imposes a lower bound on all subsequent offers, the seller’s optimal first-period offer is lower than the first-period offer in the corresponding no-rights case. Moreover, the last-period offer in the right-of-first-offer case is equal to the first-period offer because (i) the last-period offer may not (by the terms of the right) be lower than the first-period offer; and (ii) it is never optimal for the seller, as we show, to make a last-period offer which is higher than the first-period offer. The optimal sequence of offers subsequent to the first-period offer is either decreasing (for the reason given in the no-rights case) or flat (because the optimal last-period offer is sufficiently high that it is no longer optimal for the intermediate offers to be even higher).

Because both types of first-purchase rights constrain the seller’s optimal sequence of offers (relative to the no-rights case), the seller’s expected
profit is greatest in the no-rights case. The seller, however, may profit from selling a right of first refusal or a right of first offer to the right-holder. The question then is whether the joint profits of the seller and right-holder in the presence of a first-purchase right exceed their joint profits in the no-rights case. If they do, then the seller and right-holder are better off agreeing to a first-purchase right. Note that, to compare the seller and right-holders’ joint profits, we must designate one buyer in the no-rights case as the (purported) right-holder, even though, obviously, that buyer has no first-purchase right.

The paper goes on to show that no one arrangement (no-rights, right of first refusal, right of first offer) dominates any other. Rather, whether a first-purchase right generates surplus depends on the type of right, buyers’ valuation distribution, buyers’ investigation costs, and the number of buyers, and the cost of delay.

Consider first the effects of a right of first refusal on the seller’s and right-holder’s expected profits. If buyers’ investigation costs and the costs of delay are sufficiently low and the discount factor is sufficiently high, a right of first refusal transfers value from other buyers to the right-holder. When buyers’ investigation costs are sufficiently low, the seller need not alter his optimal sequence of offers to induce buyers to investigate. As for the right holder, because the seller’s optimal sequence of offers is decreasing, the right-holder under a right of first refusal can buy the asset at a price equal to, or lower than, the one that he is offered in the no rights case. Moreover, because the right-holder can exercise his right in every period and is offered to buy the asset in the last period, the right-holder’s probability of buying the asset is higher when compared with the no-rights case. If the costs of delay are sufficiently low, the right-holder would rather pay less for the asset in a later period than pay more earlier on. Because the seller’s expected profit is identical to the no-rights case and the right-holder’s expected profit is higher, the seller’s and right-holder’s joint profit is higher under a right of first refusal.

Consider next the effects of a right of first offer. As explained, the optimal first-period offer in the-right-of-first-offer case is lower than the first-period offer in the corresponding no-rights case. As we show, however, the seller’s optimal offer in the no-rights case is higher than the offer which maximizes the seller’s and the respective buyer’s joint profits (because the seller makes an offer that maximizes his expected profit, but does not take
into account the benefit of a lower offer to the right-holder). As a result, the lower first-period offer induced by a right of first offer tends to increase the seller and right-holder’s joint profits. But a right of first offer also requires the seller to modify his subsequent offers, which in turn reduces the seller’s expected profit (when compared with the no-rights case) without generating any benefits for the right-holder. Depending on which effect dominates, a right of first offer may generate positive or negative surplus.

This paper further shows that first-purchase rights tend to generate a greater surplus when the right-holder’s investigation costs are lower than (rather than equal to) those of other buyers and the investigation constraint of other buyers is binding. The intuition behind this result is as follows. In the no-rights case, the seller exploits the right-holder’s low investigation costs by making him a higher offer. But as explained in the preceding paragraph, the seller’s optimal offer to the right-holder in the no-rights case is higher than the offer which maximizes the seller and right-holder’s joint profits. Both a right of first refusal and a right of first offer impede the seller’s ability to exploit the right-holder’s low investigation costs through a high offer, and thus tend to make such rights more attractive. This result comports with the fact that first-purchase rights are often employed when the potential right-holder had a previous relationship with the seller with respect to the subject asset.

Despite their prevalent use, there has been little formal analysis of first-purchase rights. The previous literature has focused primarily on rights of first refusal. Walker (1999) argues that prospective buyers’ search and negotiation costs coupled with the right-holder’s idiosyncratic valuation will make potential buyers reluctant to bid for an asset encumbered with a right of first refusal. He accordingly concludes that rights of first refusal are intended to exclude third-party buyers and thereby increase the probability that the seller successfully negotiates with the right-holder (who is likely to value the seller’s asset more highly than other buyers).

Choi (2009) considers the effects of rights of first refusal on the outcome of a two-bidder auction. A right of first refusal, Choi shows, confers on the right-holder the advantage of observing the other bidder’s bid before making his own bid. Choi shows that, when the unprivileged bidder wins the auction, a right of first refusal increases the seller’s and right-holder’s joint profits when compared with an English auction (while not affecting the seller’s and
right-holder’s joint profits otherwise). The reason is that the outside bidder’s bid must be higher than the right-holder’s valuation to preempt the latter’s right, whereas in an English auction that bidder would win so long as his bid is greater than the right-holder’s bid.

Hua’s (2012) is the only other paper to provide an analysis of rights of first offer. His paper evaluates the social welfare effects of a right of first offer, rather than the private incentives to employ this right, as this paper does. Hua considers a model in which, if the right-holder rejects the seller’s offer, the seller and right-holder can renegotiate the right. Following the renegotiation, the seller can sell the asset in a subsequent market with either multiple buyers, who share a common value for the seller’s asset, or a single buyer, who has an idiosyncratic value for the seller’s asset. Hua shows, as do we, that the right forces the seller to lower his offer to the right-holder and thereby curbs the seller’s incentive to extract rent from the right-holder. The effect on social welfare, Hua shows, depends on the relative bargaining power of the seller and right-holder and the nature of the subsequent market.

Other papers have considered the effects of rights of first refusal in more specialized circumstances. Harris (1985) examines how rights of first refusal may limit opportunism in multiple-owner production coalitions. Bikhchandani et al. (2005) consider the effect of a right of first refusal in a sealed-bid second-price auction in which bidders privately observe signals about their valuations. They show that a right of first refusal exacerbates the winner’s curse when bidders’ valuations are correlated and conclude that a right of first refusal may result in inefficiency. Grosskopf and Roth (2009) analyze a specific combination of a right of first offer and a right of first refusal, whereby the right of first refusal is activated if the right of first offer is violated (the right of first offer thus precedes the right of first refusal chronologically). They show that in a two-buyer, sequential bargaining framework, this hybrid right strengthens the seller’s bargaining position vis-a-vis the right-holder, thereby disadvantaging the right-holder.

The analysis here differs from previous works in several respects: First, this paper is the first\(^2\) (together with Hua, 2012) to formally model and

\(^2\) See Kahan et al. (2007).
analyze rights of first offer and the only one to compare rights of first offer with rights of first refusal. We thereby offer a more comprehensive explanation for the use of rights of first refusal and rights of first offer, consistent with the circumstances under which these rights are commonly observed.

Second, the paper analyzes these rights in a sequential bargaining, rather than an auction, framework. We motivate this framework on several grounds. As a practical matter, the seller may not be able to assemble all potential buyers at the same time (or it may be costly to do so) in order to conduct an auction. Indeed, most assets are not sold in auctions but rather through sequential bargaining. Moreover, an implicit assumption underlying the design of rights of first offers is that the seller approaches buyers sequentially. Last, a sequential bargaining setting brings to the fore the significance of costs of delay in evaluating first-purchase rights.

Third, a key aspect of our model is that potential buyers may have to incur investigation costs to learn their valuation of the asset subject to a right of first-purchase. If a potential buyer has to incur investigation costs, he will only consider an offer by the seller if his expected profit from investigation is at least equal to his investigation costs. A potential buyer’s investigation constraint—i.e., the maximum offer which ensures investigation by potential buyers—and whether that constraint is binding on the seller depends on the buyer’s distribution of valuation, his investigation costs, and, as we will show, the presence of a right of first refusal or a right of first offer. This, in turn, has a significant effect on whether such rights generate costs or benefits to the seller and the right-holder.

The paper proceeds as follows. Part 2 presents the benchmark model where no potential buyer has a first-purchase right. Part 3 examines rights of first refusal. Part 4 examines rights of first offer. Part 5 compares rights of first refusal and rights of first offer in the specific case in which buyers’ valuations are drawn from a uniform distribution. Proofs are relegated to the appendix.

3. Walker (1999) (in the only other paper that examines the effects of buyers’ costs) also notes that a right of first refusal might exclude potential buyers, but does not provide a formal analysis.
2. The Benchmark Model: No-Rights Case

Consider a single risk-neutral seller, $S$, with one indivisible asset for sale. $S$’s valuation of the asset is normalized to zero. There is a finite number $n \geq 2$ of potential buyers for the asset. For simplicity, we refer to any potential buyer as ‘buyer.’ Buyers arrive sequentially and are risk-neutral. Each buyer’s valuation of the asset, $v$, is a random draw from the same differentiable distribution $F$ on support $[\underline{v}, \bar{v}]$, where $\underline{v} < 0 < \bar{v}$, with a strictly positive density $f$. We assume the distribution $F$ is common knowledge and that the hazard rate, $f(\cdot)/(1 - F(\cdot))$, is strictly increasing for $v \in [0, \bar{v})$.

We also assume that buyers can learn their valuation of the asset at a cost $c \in [0, E(v|v \geq 0))$, where $E(v|v \geq 0) = \int_{\underline{v}}^{\bar{v}} vf(v) \, dv$. Finally, we assume that $\int_{\underline{v}}^{\bar{v}} vf(v) \, dv < 0$ so that, for any offer by the seller, a buyer will not buy the asset without first learning his valuation.

We designate one buyer, a potential ‘right-holder,’ by $R$. Holders of first-purchase rights often have a special relationship to $S$ with respect to the asset subject to a first-purchase right. $R$, therefore, may learn his valuation of the asset at lower costs than other buyers’. We accordingly assume that $R$’s investigation costs, $c_R$, may be lower than other buyers’; that is, $c_R \leq c$.

A relationship between $S$ and $R$ also implies that $R$ may be first in the line of buyers approached by $S$. Moreover, we later show that $S$ never benefits from not approaching $R$ first in the no-rights case. We thus assume that $R$ is approached first in the no-rights case. For completeness, however, throughout the analysis we discuss how our results would change if the assumption that $R$ is approached first were relaxed.

As is common in models of sequential bargaining, we assume that bargaining takes place without recall (e.g., Riley and Zeckhauser, 1983). In each period, $S$ makes an offer to a buyer. The buyer then chooses whether to consider the offer. If the buyer does not consider the offer, the game proceeds to the next period. If the buyer chooses to consider the offer, he incurs investigation costs $c$ and observes his valuation of the asset. If the buyer’s valuation is higher than, or equal to, $S$’s offer, the buyer accepts the offer and the game ends. If the buyer’s valuation is lower than $S$’s offer, the buyer rejects the offer and the game proceeds to the next period. If the asset has not been sold after all buyers have been approached, the game terminates and $S$ receives a payoff of zero.
2.1. Offers in the No-rights Case

We begin by describing $S$’s set of feasible offers to outside buyers, i.e.,
offers that would induce an outside buyer to consider $S$’s offer. A buyer
whose valuation of the asset is $v$ will accept an offer $k$ if and only if $v \geq k$.
If the buyer accepts the offer, his profit is $v - k$. The buyer’s expected profit
from investigation is thus $\int_{k}^{\bar{v}} (v - k) f(v) \, dv$. A buyer will incur investigation costs if and only if his expected profit from investigation is greater than or equal to his investigation costs. $S$’s offer must therefore satisfy the following condition:

$$E[(v - k)^+] \geq c,$$

where $(\cdot)^+ = \max\{\cdot, 0\}$.

Let $\bar{k}$ be the value of $k$ that satisfies (1) as an equality. Note that $\bar{k}$ is a
decreasing function of $c$. We will refer to $\bar{k}$ as the investigation constraint
(‘ICN’) of outside buyers. The set of $S$’s feasible offers to outside buyers is
thus a set of the form $K^N = [0, \bar{k}]$ (the superscript stands for ’no-rights’).
The assumption $c \in [0, E(v))$ ensures that ex ante there are gains to trade
for $S$ and each buyer (because for $k = 0$ buyers’ expected profit from inves-
tigation is strictly positive).

To solve for $S$’s optimal sequence of offers, we proceed by backward
induction. Let $k_n$ denote $S$’s offer in the last period. The probability that an
offer $k_n$ will be accepted is $1 - F(k_n)$. $S$’s profit if the offer is accepted is
$k_n$. $S$’s optimal offer in the last period maximizes the product of the offer,
$k_n$, and the probability that the offer is accepted:

$$\max_{k_n \in K^N} k_n (1 - F(k_n)).$$

Let $k^*_n$ denote $S$’s optimal last-period offer and $V_n$ the maximized objective
function as a function of $k^*_n$.

Next, consider $S$’s problem in period $n - 1$, the second-to-last period
(when this is not the first period). $S$’s solves:

$$\max_{k_{n-1} \in K^N} k_{n-1} (1 - F(k_{n-1})) + \delta V_n F(k_{n-1}),$$

where $0 < \delta \leq 1$ is the discount factor per period. Note that the lower the
discount factor is, the higher is $S$’s cost of delaying the sale of the asset.
The first expression is $S$’s expected profit given that his offer is accepted.
The second expression is $S$’s discounted continuation value ($\delta V_n$) multiplied by the probability that $S$’s offer is rejected. Let $k_{n-1}^*$ and $V_{n-1}$ denote $S$’s optimal offer and maximized objective function, respectively, in the second-to-last period.

The procedure is completed by induction. $S$’s problem in period $i = 2, \ldots, n - 1$, when there are $n - i$ buyers remaining, is

$$\max_{k_i \in K^N} k_i (1 - F(k_i)) + \delta V_{i+1} F(k_i),$$  \hspace{1cm} (4)

where $V_{i+1}$ is $S$’s continuation value in period $i$. Thus, $S$’s optimal period-$i$ offer is

$$k_i^* = \min\{\bar{k}_i, \tilde{k}\},$$  \hspace{1cm} (5)

where $\bar{k}_i$ is the value of $k$ such that $k = \delta V_{i+1} + (1 - F(k))/f(k)$.

The assumption that $S$ approaches $R$ first implies that $S$’s optimal first-period offer depends on $R$’s investigation costs. Let $\tilde{k}(c_R)$ be the value of $k$ such that $E[(v - k)^+] = c_R$; that is, for $k = \tilde{k}(c_R)$ the right-holder’s expected profit from investigation is zero. $\tilde{k}(c_R)$ is thus the right-holder’s investigation constraint. $S$’s optimal first-period offer, $k_1^*$, is thus $\min\{\bar{k}_1, \tilde{k}(c_R)\}$, where $k_1$ is the value of $k$ such that $k = \delta V_2 + (1 - F(k))/f(k)$ and $V_2$ is $S$’s continuation value in the second period.\(^4\)

It follows from (3) that $S$’s optimal unconstrained offer in each period is increasing in the number of remaining buyers.\(^5\) $S$’s optimal sequence of unconstrained offers is thus strictly decreasing. The intuition is that as the number of remaining buyers decreases, $S$ becomes more eager to sell the asset to avoid a payoff of zero if the asset is never sold, and thus $S$ keeps lowering her offers. That $S$’s optimal sequence of unconstrained offers is decreasing also implies that, if the ICN is binding on any of $S$’s optimal offers, it is also binding on all prior offers (if all the previous buyers have the same investigation costs).\(^6\)

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4. It is straightforward to show that $k_i^*$, $i = 1, \ldots, n$, satisfies the second-order condition for maximum. In addition, the assumption that $F$ has a strictly increasing hazard rate ensures that $S$’s optimal offers are unique.

5. Note that the cross derivative of $S$’s objective function (in (3)) in period $i < n$ with respect to $V_{i+1}$ and $k_i$ is $\delta f(k_i) > 0$.

6. It follows from (4) that $S$’s optimal unconstrained offers (except the last-period offer) are increasing in the discount factor. This is because the higher the discount factor is, the less costly it is for $S$ to make a higher offer, which is more likely to be rejected.
We conclude this section by considering the relation between $S$'s optimal first-period offer and the offer that maximizes $S$'s and $R$'s joint profits.

**Lemma 1 (Seller and Right-holder’s joint profits)**

(a) Let $\hat{k}_1$ be the first-period offer that maximizes $S$'s and $R$'s joint profits. Then $\hat{k}_1$ is equal to the discounted value of $S$'s expected profit from subsequent periods (‘continuation value’) and is lower than the offer that maximizes $S$'s expected profit; that is, $k_1^* > \hat{k}_1 = \delta V_2$.

(b) $S$ and $R$’s joint profits are strictly decreasing for $k_1 \in (\hat{k}_1, k_1^*)$.\(^7\)

*Proof.* See the appendix. \(\square\)

To gain intuition for part (a), observe that $S$’s and $R$’s joint profits are equal to

$$\int_{k_1}^{\bar{v}}vf(v)\,dv + F(k_1)\delta V_2.$$ \(^6\)

The first expression represents $S$’s and $R$’s joint profits in the first period; the second expression represents $S$’s discounted continuation value multiplied by the probability of no sale in the first period. Note that $S$’s and $R$’s joint profits in the first period decrease in $S$’s first-period offer (with a maximum at zero). Because $S$ ignores the benefit to $R$ when choosing her optimal first-period offer, that offer is higher than the offer that maximizes the parties’ joint profits. When the first-period offer is equal to $S$’s discounted continuation value, the marginal joint profits to $S$ and $R$ from increasing the first-period offer is equal to the marginal cost to $S$ from doing so, a cost which results from the lower probability that $S$ reaches subsequent periods.\(^8\)

Part (b) states that, for offers higher than $\hat{k}_1$, $S$’s and $R$’s jointly optimal offer, $S$’s and $R$’s joint profit is decreasing with $S$’s offer. This in turn implies that $S$’s and $R$’s joint surplus from lowering $S$’s first-period offer is increasing in $S$’s (privately) optimal first-period offer.

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7. The proof of Lemma 1 (in the appendix) shows that $\hat{k}_1 \leq \hat{k}(c_R)$.

8. If $R$ had idiosyncratic (high) valuation of $S$’s asset (e.g., $R$’s distribution of valuation first-order stochastically dominated other buyers’), $S$’s privately-optimal first offer would be even more exploitative to $R$. We will occasionally comment on this case.
2.2. Example: The Uniform Distribution Case

To illustrate the results in Section 2.1, consider the case in which buyers’ valuations are distributed uniformly on [0, 1] and \( c \in [0, 0.5] \). Recall that a buyer whose valuation of the asset is \( v \) will accept an offer \( k \) if and only if \( v \geq k \). The probability that a buyer accepts an offer \( k \in [0, 1] \) is thus \( 1 - k \). If the buyer accepts the offer, his profit is \( v - k \). It follows that, conditional on accepting the seller’s offer, the buyer’s expected profit is \( (1 - k)/2 \). The buyer’s expected profit from investigation is accordingly \( (1 - k)((1 - k)/2) \), which simplifies to \( (1 - k)^2/2 \).

Now, a buyer will incur investigation costs if and only if his expected profit from investigation is greater than or equal to his investigation costs. Thus, to induce a buyer to incur investigation costs of \( c \), S’s offer must satisfy the following condition:

\[
\frac{(1 - k)^2}{2} \geq c. \tag{7}
\]

The investigation constraint is the value of \( k \) which satisfies (7) as an equality: \( \tilde{k} = 1 - \sqrt{2c} \). S’s set of feasible offers to outside buyers is thus \( K^N \in [0, 1 - \sqrt{2c}] \). Likewise, the right-holder’s investigation constraint is \( \tilde{k}(c_R) = 1 - \sqrt{2c_R} \).

Next, consider S’s optimal offers. S’s problem in the last period is

\[
\max_{k_n \in K^N} k_n(1 - k_n), \tag{8}
\]

where \( k_n \) is S’s profit if the last buyer accepts an offer of \( k_n \), and \( 1 - k_n \) is the probability that the last buyer’s valuation is higher than \( k_n \). It follows

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9. For the sake of simplicity, we assume that buyers always ascribe a positive value to S’s asset, but must know the value of S’s asset to use it properly. This implies that buyers would not buy the asset if they did not incur investigation costs. The example would not change substantially if there were a positive probability that buyers ascribe a negative value to S’s asset (in this case, buyers’ investigation constraint would be lower).

10. To see why, note that the buyer accepts S’s offer if his valuation is higher than \( k \). If the buyer’s valuation is exactly \( k \), his expected profit is zero; if his valuation is 1, his expected profit is \( 1 - k \). Thus, on average, the buyer’s valuation, given that he accepts the seller’s offer, is \( (1 - k)/2 \).
that $S$’s optimal last-period offer is $k_n^* = \min\{\frac{1}{2}, 1 - \sqrt{2c}\}$.\footnote{More specifically, $S$’s optimal last-period offer is}

$S$’s expected profit from the last stage is accordingly $V_n = \min\{\frac{1}{4}, \sqrt{2c}(1 - \sqrt{2c})\}$.

$S$’s problem in period $i = 2, \ldots, n - 1$, when there are $n - i$ buyers remaining, is

$$\max_{k_i \in K_n} k_i (1 - k_i) + \delta \cdot k_i V_{i+1}, \quad (9)$$

where the first expression in the maximand is $S$’s expected profit given that the buyer accepts his offer and the second expression is $S$’s discounted continuation value.

$S$’s optimal period-$i$ offer is accordingly $k_i^* = \min\{\frac{1}{2}(1 + \delta V_{i+1}), 1 - \sqrt{2c}\}$. By a similar analysis, $S$’s optimal period-$1$ offer (to the potential right-holder) is $\min\{\frac{1}{2}(1 + \delta V_2), 1 - \sqrt{2cR}\}$.

To find the first-period offer that maximizes $S$’s and $R$’s joint profit, note that $S$’s and $R$’s joint profit from transferring the asset to $R$ is equal to $R$’s expected valuation conditional on accepting $S$’s offer ($(1 + k)/2$) and that, if $R$ rejects $S$’s offer, $S$’s continuation value is $V_2$. The first-period (unconstrained) offer that maximizes $S$’s and $R$’s joint profit must therefore maximize the following expression:

$$\frac{(1 - k^2)}{2} + k\delta V_2. \quad (10)$$

The first expression is $S$’s and $R$’s joint profit from transferring the asset to $R$, multiplied by the probability that $R$ accepts $S$’s offer. The second expression is $S$’s continuation value multiplied by the probability that $R$ rejects $S$’s offer. Differentiating (10) with respect to $k$ and equating to zero gives $\hat{k}_1 = \delta V_2$, which is lower than $S$’s optimal first-period offer.

Table 1, which concludes this example, presents $S$’s optimal sequences of offers and $S$’s and $R$’s jointly-optimal offers for different numbers of buyers given that $\delta = 1$:

\footnote{More specifically, $S$’s optimal last-period offer is}

$$k_n^* = \begin{cases} \frac{1}{2} & \text{if } c \leq \frac{1}{8}, \\
1 - \sqrt{2c} & \text{if } c > \frac{1}{8}. \end{cases}$$
Table 1. Seller’s Optimal Unconstrained Offers and Offers that Maximize Joint Profits

<table>
<thead>
<tr>
<th>$\delta = 1$</th>
<th>$k_{n-9}$</th>
<th>$k_{n-8}$</th>
<th>$k_{n-7}$</th>
<th>$k_{n-6}$</th>
<th>$k_{n-5}$</th>
<th>$k_{n-4}$</th>
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<td>$n = 3$</td>
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</table>

$\delta = 1$ : $k_{n-9} = 0.7751$, $k_{n-8} = 0.7417$, $k_{n-7} = 0.6953$, $k_{n-6} = 0.6250$, $k_{n-5} = 0.5000$, $k_{n-4} = 0.3906$, $k_{n-3} = 0.2500$, $k_{n-2} = 0.0000$

$\delta = 0.6953$, $k_{n-9} = 0.6250$, $k_{n-8} = 0.5000$, $k_{n-7} = 0.3906$, $k_{n-6} = 0.2500$, $k_{n-5} = 0.0000$

$\delta = 0.7751$, $k_{n-9} = 0.7417$, $k_{n-8} = 0.6953$, $k_{n-7} = 0.6250$, $k_{n-6} = 0.5000$, $k_{n-5} = 0.3906$, $k_{n-4} = 0.2500$, $k_{n-3} = 0.0000$
3. Right of First Refusal

A right of first refusal (‘RFR’) provides $R$ with the right to buy $S$’s asset at a price offered by $S$ to another buyer conditional on the offer being accepted by that buyer. The RFR game proceeds as follows. Each period $S$ makes an offer to a buyer other than $R$. If the buyer fails to consider the offer or rejects the offer outright, the game proceeds to the next period. If the buyer accepts the offer, $R$ then incurs investigation costs and observes his valuation of the asset. If $R$’s realized valuation of the asset is higher than or equal to $S$’s offer, $R$ exercises his right; if not, the asset is sold to the other buyer. This procedure tracks the legal terms of an RFR. We further assume that if $S$ has made offers to all buyers other than $R$ and none of these offers were accepted, $S$ makes an offer directly to $R$ in the last period. This assumption reflects the notion that the presence of an RFR should not (and by its terms does not) prevent $S$ from approaching $R$ directly when $R$ is the only buyer remaining.

We first examine the effect of an RFR on $S$’s optimal offers. Next, we consider when contracting for an RFR increases $S$’s and $R$’s joint profits when compared with the no-rights case.

3.1. Offers in the RFR Case

We begin by characterizing $S$’s set of feasible offers under an RFR. $R$ will incur investigation costs (either when the RFR is triggered or when $R$ is made an offer directly) for any $k \leq \bar{k}(c_R)$. A buyer other than $R$ will incur investigation costs if and only if his expected profit from investigation is positive. Any offer to a buyer other than $R$ must therefore satisfy the following condition:

$$F(k)E[(v - k)^+] \geq c.$$  \hspace{1cm} (11)

$F(k)$ is the probability that $R$’s valuation is lower than $k$ so that $R$ does not exercise his right. The expectation expression is the buyer’s expected profit conditional on the buyer’s valuation exceeding $S$’s offer. Let $c^m = \max_k F(k)E[(v - k)^+]$ denote outside buyers’ maximum profit from investigation under an RFR. If buyers’ investigation costs are higher than $c^m$, then $S$’s set of feasible offers is empty. For $c > c^m$, therefore, $S$ makes a single offer to $R$, whose value may depend on $R$’s investigation
costs. We will assume for the remainder of this section that \(c < c^m\) so that \(S\) does not skip buyers.

Because buyers are subject to \(R\)'s right of first refusal, their expected profit from investigation for any offer \(k\) is lower than in the no-rights case. Moreover, for strictly positive investigation costs, there exists a set of sufficiently small offers such that buyers’ expected profit from investigation is negative (because for offers in this set, the probability that the right-holder exercise his right is high). Thus, the set of offers that induce buyers to investigate is smaller in the RFR case than in the no-rights case. Let \(K^R\) denote \(S\)'s set of feasible offers in the RFR case. Then \(K^R \subseteq K^N\), where \(K^N\) is the set of feasible offers to outside buyers in the no-rights case.

Observe that the smaller set of feasible offers in the RFR case cannot make \(S\) better off when compared with the no-rights case. As a consequence, \(S\)'s expected profit under an RFR is equal to, or lower than, his expected profit in the no-rights case. The solution to \(S\)'s optimal sequence of offers under an RFR is analogous to the no-rights case, except that \(S\)'s set of feasible offers in all periods other than the last period is now \(K^R\).
Before proceeding to compare $S$’s optimal sequence of offers under an RFR and in the no-rights case, note that (i) as in the no-rights case, $S$’s optimal sequence of offers in the RFR case is monotone decreasing except for the second-to-last and the last period (the intuition here is identical to that in the no-rights case); and (ii) in contrast to the no-rights case, the last-period offer in the RFR case may be higher than the second-to-last period offer (because the offer in the penultimate period is subject to the RFR ICN, whereas the last-period offer is subject to the laxer no-rights ICN). We denote such state of affairs as a ‘spike.’ A spike may exist because the last-period offer in the RFR case is made to $R$ and thus is not subject to the RFR ICN as are previous offers.

**Lemma 2** (Seller’s optimal offers in the RFR case when compared with the no-rights case (if seller does not skip buyers)). For any offer other than the last-period offer:

(a) If $S$’s optimal offer under an RFR is unconstrained, then that offer is identical to $S$’s equivalent-period optimal offer in the no-rights case.

(b) If the ICN is binding on any of $S$’s optimal offers in the no-rights case, then that offer is higher than $S$’s equivalent-period optimal offer in the RFR case.

(c) If the ICN is not binding on any of $S$’s optimal offers in the no-rights case, then $S$’s equivalent-period optimal offer in the RFR case may either be equal to, higher than, or lower than that in the no-rights case.

The last-period offer:

(d) If the ICN in the no-rights case is not binding on the optimal last-period offer or if $c_R = c$, then the last-period offer is identical in the no-rights case and the RFR case. If the ICN in the no-rights case is binding on the optimal last-period offer and $c_R < c$, then the last-period offer is higher in the RFR case than in the no-rights case.

**Proof.** See the appendix.

Part (a) stems from the fact that $S$’s set of feasible offers under an RFR is smaller than that in the no-rights case. Consequently, when $S$’s optimal offer
under an RFR is unconstrained, so is the equivalent-period optimal offer in the no-rights case.

The intuition for part (b) is as follows. When the ICN is binding on any of S’s optimal offers in the no-rights case, outside buyers’ expected profit from investigation is zero. Because outside buyers in the RFR case are subject to R’s preemptive right, their expected profit in the RFR case is strictly lower than in the no-rights case for any offer made by S. To induce investigation in the RFR case, S must therefore make a lower offer than in the no-rights case.

Part (c) results from the fact that, unlike in the no-rights case, outside buyers’ expected profit in the RFR case does not necessarily increase as S’s offer decreases. A lower offer increases the probability that R will exercise his RFR and thus reduces the offeree’s expected profit. (For example, when S’s offer is zero, outside buyers’ expected profit is zero because R is certain to exercise his right.) This effect can outweigh the benefit to a potential buyer from a lower offer if the buyer ends up buying the asset. In order to induce investigation under an RFR, S may therefore have to increase her offers.

The intuition for part (d) is that the last-period offer in the RFR case (which is extended to R) is subject to the same constraint as the last-period offer in the no-rights case and will only differ if R’s investigation costs are lower than those of other buyers and the investigation constraint is binding in the no-rights case (when the last-period offer is not extended to R).

3.2. Example: The Uniform Distribution Case

To illustrate the results in Section 3.1, consider the case in which buyers’ valuations are distributed uniformly on [0, 1]. Recall that a buyer whose valuation of the asset is v will accept an offer k if and only if v ≥ k. The joint probability that a buyer accepts an offer k ∈ [0, 1] and that the right-holder does not exercise his right of first refusal is thus k(1 − k). If the buyer accepts the offer, his profit is v − k. It follows that, conditional on accepting the seller’s offer and on the right-holder not exercising his right, the buyer’s expected profit is (1 − k)/2. The buyer’s expected profit from investigation is therefore k(1 − k)((1 − k)/2), which simplifies to k(1 − k)^2/2. Thus, to induce a buyer to incur investigation costs of c, S’s offer must satisfy the
following condition:

\[
\frac{k(1-k)^2}{2} \geq c. \tag{12}
\]

\[S\]'s set of feasible offers under an RFR consists of the values of \(k\) that satisfy (12). This set is a closed interval. We denote the right endpoint of this interval as the ‘Upper RFR ICN’ and the left endpoint the ‘Lower RFR ICN.’ Note that, because the left-hand side of (12) reaches a maximum on \([0, 1]\) at \(k = \frac{1}{3}\), the highest investigation costs at which an outside buyer will investigate for any offer made by \(S\) is \(c_m = \frac{1}{3}(1-\frac{1}{3})^2 = 0.07407\).

Figure 1 presents the upper and lower RFR investigation constraints as well the investigation constraints in the no-rights case. As Figure 1 shows, the no-rights ICN is higher than the upper RFR ICN, which in turn is higher than the lower RFR ICN. Note that, for \(c = c_m = 0.07407\), the upper and lower RFR ICNs obtain the same value.

In all periods other than the last period, \(S\)'s optimal offer is equal to the minimum of the Upper RFR ICN as a function of outside buyers’ investigation costs and \(S\)'s unconstrained optimal offer. Note that the Lower RFR ICN is not binding on \(S\)'s optimal offers in this example. \(S\)'s optimal last-period offer is equal to the minimum of the no-rights ICN as a function of \(R\)'s investigation costs and \(S\)'s unconstrained optimal offer. Because the Upper RFR ICN is lower than the no-rights ICN, offers in the RFR case are equal to or lower than those in the no-rights case.
3.3. Joint Profits in the RFR Case

An RFR will be contracted for \textit{ex ante} if it increases S’s and R’s joint profits. Let $V_1$ and $V'_1$ denote S’s expected profits in the no-rights case and the RFR case, respectively; let $B$ and $B'$ denote R’s expected profits in the no-rights case and the RFR case, respectively. Thus, an RFR will be contracted for if and only if:

$$B' + V'_1 > B + V_1. \quad (13)$$

We call the difference between the joint profits in the RFR case and the no-rights case ‘surplus,’ whether or not such difference is positive.

**Proposition 1** (Transfer of value from other buyers) Let $C = \{c : k^* \in K^r \}$ be the set of buyers’ investigation costs such that S’s optimal sequence of offers in the no-rights case is identical to that in the RFR case ($C$ is not empty because it contains $c = 0$). If $c \in C$ and $\delta = 1$, then an RFR generates positive surplus.

\textit{Proof.} See the appendix. \hfill \Box

According to Proposition 1, if S’s optimal sequence of offers under an RFR is unconstrained and the discount factor is 1, an RFR generates positive surplus. The intuition is as follows. When the RFR ICN is not binding, the optimal sequence of offers in the no-rights case is identical to that in the RFR case ($C$ is not empty because it contains $c = 0$). If $c \in C$ and $\delta = 1$, then an RFR generates positive surplus.

$R$ is assured to be offered to buy the asset both in the no-rights case (as he is approached first) and in the RFR case. But the offer price in the RFR case is lower than (or equal to) the offer price in the no-rights case because under an RFR, the asset may be offered to $R$ at periods later than the first period and hence at a lower price. As long as the discount factor is sufficiently high (and always when the discount factor is (1), $R$’s expected profit under an RFR is higher than his expected profit in the no-rights case. Proposition 1 illustrates that an RFR is valuable to the seller and right-holder because it transfers value from outside buyers to the right-holder. The surplus generated by an RFR in this case is independent of the right-holder’s investigation costs.
Proposition 1 does not depend on the assumption that $R$ is approached first in the no-rights case. In fact, if $R$ is not approached first in the no-rights case and thus not assured to be offered to buy the asset in this case, an RFR yields the additional benefit of assuring an offer to $R$. As a result, when $R$ is not approached first in the no-rights case, the minimum discount factor under which Proposition 1 holds is lower; and when $R$ is approached last in the no-rights case, Proposition 1 holds for any discount factor.

Rights of first refusal in sports collective bargaining agreements between players (sellers) and teams (buyers) may represent an example of Proposition 1. Investigation costs of teams are likely to be relatively low and the discount factor is likely to be high because it is easy for a player to approach several teams within a short time span. Granting the incumbent team a right of first refusal thus has little cost to the player, but benefits the incumbent team.

Proposition 2 (Prevention of exploitation when $R$ is different from other buyers) Assume that the ICN in the no-rights case is binding on $S$’s optimal first-period offer (to $R$), but is not binding on $S$’s optimal last-period offer. Then the surplus generated by an RFR is decreasing in $R$’s investigation costs.

Proposition 2 holds that an RFR becomes relatively more attractive to $S$ and $R$ when $R$ faces lower investigation costs than other buyers and other buyers face substantial investigation costs. The intuition for Proposition 2 derives from the fact that the first-period offer that maximizes $S$’s and $R$’s joint profits in the no-rights case is lower than $S$’s optimal first-period offer (see Lemma 1). In the no-rights case, if the investigation constraint would be binding on the first-period offer to $R$ if $R$ had the same investigation costs as other buyers, $S$’s optimal first-period offer increases, and thereby reduces the parties’ joint profits, as $R$’s investigation costs decrease. But as long as the ICN is not binding on the last-period offer in the no-rights case, $S$’s optimal offers in the RFR case (and thus the parties’ joint profits) are invariant to $R$’s investigation costs. Because $R$’s lower investigation costs reduce $S$’s and $R$’s joint profits in the no-rights case but not in the RFR case, an RFR becomes more attractive in this case. Proposition 2 does not depend on the assumption that $R$ is approached first in the no-rights case.
Proposition 2 fits the observation that parties often contract for first-purchase rights when $R$ has some prior relationship with $S$ that reduces $R$’s investigation costs and investigation costs for outsiders are high, e.g., in joint ventures or among co-owners of a closely-held corporation. The proposition shows that rights of first refusal tend to generate greater surplus under such conditions.\textsuperscript{17}

\textit{A Comment on the Discount Factor and the Number of Buyers.} A decrease in the discount factor decreases $S$’s expected profit both in the no-rights case and in the RFR case (because it decreases the present value of payments received in later periods). A decrease in the discount factor increases $R$’s expected profit in the no-rights case (because $S$’s optimal first-period offer is lower and $R$ is approached first),\textsuperscript{18} but may increase or decrease $R$’s expected profit in the RFR case.\textsuperscript{19} Thus, the net effect of a decrease in the discount factor on the surplus generated by an RFR is indeterminate.

An increase in the number of buyers increases $S$’s expected profit both in the no-rights and in the RFR case. An increase in the number of buyers decreases $R$’s expected profit in the no-rights case. In the RFR case, an increase in the number of buyers decreases $R$’s expected profit if there is no spike (because the additional offers to earlier buyers are higher), and has an indeterminate effect on $R$’s expected profit if there is a spike (because the additional offers to earlier buyers decrease the probability that the game will proceed to the last period, in which a spiked offer is made to $R$). Thus, the

\textsuperscript{17} If $R$ had idiosyncratic valuation of $S$’s asset (e.g., $R$’s distribution of valuation first-order stochastically dominated other buyers’), then the surplus from an RFR would be either higher or lower. On one hand, $S$’s optimal first offer in the no-rights case would be even more exploitative to $R$. Prevention of exploitation would thus create a greater surplus. On the other hand, the set of $S$’s feasible offers under an RFR would be smaller (because for any offer, $R$ would be more likely to exercise his right). As a result, $S$’s optimal sequence of offers under an RFR would more likely be different than his optimal sequence of offers in the no-rights case.

\textsuperscript{18} If $R$ were approached last in the no-rights case, a decrease in the discount factor would decrease $R$’s expected profit in the no-rights case. For intermediate positions, the effect is indeterminate.

\textsuperscript{19} A lower discount factor decreases $S$’s optimal offers to other buyers under an RFR; this, as explained, has an indeterminate effect on $R$’s expected profit. In addition, a lower discount factor decreases the discounted value of $R$’s expected profit from later periods.
net effect of an increase in the number of buyers on the surplus generated by an RFR is likewise indeterminate.

4. Right of First Offer

This section explores the case of a Right of First Offer (‘RFO’). The game proceeds as in the no-rights case with the following modifications. If $R$ is granted an RFO, any sequence of offers must satisfy two conditions:

- The first offer must be made to $R$;
- If $R$ rejects the offer, $S$ cannot offer the asset to another buyer for a price lower than that offered to $R$.

We denote the case in which $R$ is granted an RFO by attaching the superscript ‘$o$’ to the relevant expressions. We first examine the effect of an RFO on $S$’s optimal offers. Next, we consider when contracting for an RFO increases $S$’s and $R$’s joint profits when compared with the no-rights case.

4.1. Offers in the RFO Case

We begin by characterizing $S$’s set of feasible offers under an RFO as a function of $S$’s first-period offer. $S$’s set of feasible offers under an RFO is a set of the form $K^O = [k^o_1, \bar{k}]$, where $k^o_1$ is the first-period offer in the RFO case and $\bar{k}$ is buyers’ investigation constraint. Note that the investigation constraint in the RFO case is identical to the one in the no-rights case. It follows that $K^O \subseteq K^N$, where $K^N$ is $S$’s set of feasible offers in the no-rights case.

The additional constraint introduced by an RFO on the set of $S$’s feasible offers cannot make $S$ better off. Recall that $S$’s optimal sequence of offers in the no-rights case is non-increasing. Introducing a lower bound on the set of feasible offers may thus affect the feasibility of some of these offers. As a consequence, $S$’s expected profit under an RFO is lower than or equal to his expected profit in the no-rights case.\footnote{Profits are equal only in the special case where $S$ makes identical offers in the no-rights case, i.e., where the ICN is binding on $S$’s optimal last-period offer in the no-rights case and $c = c_R$.}

\footnote{This was an assumption in the no-rights case. In the RFO case, it follows from the legal terms of an RFO.}
To solve for $S$’s optimal sequence of offers, we use the fact that $S$’s optimal first-period offer under an RFO (i.e., the offer that is made to $R$) is equal to $S$’s optimal last-period offer. This is because (i) $S$’s first-period offer may not be higher than the last-period offer due to the RFO constraint; and (ii) $S$’s optimal first-period offer is not lower than $S$’s optimal last-period offer for any choice of last-period offer (this follows because $F(\cdot)$ has a strictly increasing hazard rate, which implies, in turn, that $S$’s expected profit is increasing on $[0, k^*(V_2)]$). \(^{22}\)

As we more fully show in the appendix, $S$’s problem is reduced to finding an optimal first-period offer that maximizes $S$’s expected profit. This formulation of $S$’s problem helps us to characterize $S$’s optimal sequence of offers and is useful in solving for $S$’s optimal sequence of offers within a specific parameterization setting.

Before proceeding to compare $S$’s optimal sequence of offers in the RFO case to that in the no-rights case, note that it may be optimal for $S$ in the RFO case to make a single offer to $R$, which is higher than the investigation constraint of other buyers and thus precludes $S$ from approaching other buyers. We denote such state of affairs as ‘skipping buyers.’ For $S$ to skip buyers, it is necessary that $R$’s investigation costs be lower than other buyers’ and that the ICN would be binding on $S$’s optimal last-period offer in the RFO case if $S$ were to approach all buyers. \(^{23}\)

**Lemma 3** (Seller’s optimal offers in the RFO case [if the seller does not skip buyers])

(a) $S$’s optimal sequence of offers in the RFO case is either constant or forms a reverse U-shape peaking at the second-period offer, where $S$’s optimal first-period offer is equal to $S$’s optimal last-period offer.

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\(^{22}\) More specifically, for any sequence of subsequent offers, $S$’s optimal first-period offer is (weakly) higher than the last-period offer. The fact that the hazard-rate is increasing implies that $S$’s profit is increasing with his first-period offer up to the optimal first-period offer.

\(^{23}\) More specifically, when $V_n > V^o_1$, $S$’s expected profit from making a single offer to $R$ is greater than $S$’s expected profit from approaching all buyers and making offers in compliance with the RFO constraint. $S$, therefore, will skip buyers. If $S$’s single optimal offer to $R$ were lower than or equal to $\bar{k}$, then $S$ could increase her expected profit by making additional offers in compliance with the RFO constraint. It follows that a single offer to $R$ must be higher than $\bar{k}$. 

(b) $S$’s optimal first-period offer in the RFO case is (weakly) lower than $S$’s optimal first-period offer in the no-rights case.

**Proof.** See the appendix. \qed

Part (a) follows from the fact that, as in the no-rights case, the optimal sequence of unconstrained offers—from the second to the last period—is strictly decreasing. However, due to the RFO constraint, $S$’s first-period offer may not be higher than any of the subsequent offers.

The intuition for part (b) is as follows. Because the RFO constraint forces $S$ to make buyers suboptimal offers, $S$’s continuation value in any period is (weakly) higher in the no-rights case than in the equivalent-period RFO case. As a result, $S$’s optimal second-period offer in the no-rights case is (weakly) higher than $S$’s optimal second-period offer in the RFO case. This, in turn, implies that $S$’s optimal first-period offer in the no-rights case is (weakly) higher than his optimal first-period offer in the RFO (because, in the no-rights case, the optimal sequence of offers is decreasing, while in the RFO case the first-period offer is no higher than the second-period offer).\(^24\)

For any first- and last-period offers, $S$’s optimal intermediate offers involve lower continuation value in the RFO case than in the no-rights case, and help to explain the flatter optimal sequence of (unconstrained) offers in the RFO case. More specifically, because $S$’s optimal offer is decreasing with $S$’s continuation value and $S$’s optimal last-period offer must be higher in the RFO case than in the no-rights case (for otherwise $S$ could increase her profit by raising both the first- and last-period offers), $S$’s optimal intermediate (unconstrained) offers under an RFO are flatter than in the no-rights case.

4.2. Example: The Uniform Distribution Case

To illustrate the results in Section 4.1, consider $S$’s maximization problem under an RFO when buyers’ valuations are distributed uniformly on $[0, 1]$. Using computational software, we obtain $S$’s optimal sequence of

\(^{24}\) The offers would be the same only if the investigation constraint were binding on these offers.
unconstrained offers for different values of $n$ and $\delta$. We present the results in Table 2.

For $c_R = c$, $S$’s optimal offer is equal to the minimum of the investigation constraint as a function of $c$ and the unconstrained optimal offer. For $c_R = 0$, $S$ instead skips buyers and makes a single offer of 0.5 to $R$ if $c$ is higher than the following values (Table 3).

4.3. Joint Profits in the RFO Case

An RFO will be contracted for *ex ante* if it increases $S$’s and $R$’s joint profits. Let $V_1$ and $V_1^o$ denote $S$’s expected profits in the no-rights case and the RFO case, respectively; let $B$ and $B^o$ denote $R$’s expected profits in the no-rights case and the RFO case, respectively. Thus, an RFO will be contracted for if and only if:

$$B^o + V_1^o > B + V_1.$$  

(14)

We call the difference between the joint profits in the RFO case and the no-rights case ‘surplus,’ whether or not such difference is positive.

**Proposition 3** (Prevention of exploitation when $R$ is different from other buyers and $S$ does not skip buyers)

(a) If the ICN is binding on $S$’s optimal first-period offer to $R$ in the no-rights case, then the surplus generated by an RFR is decreasing in $R$’s investigation costs.

(b) (high investigation costs of other buyers)

If the ICN is binding on $S$’s optimal last-period offer in the no-rights case and $R$’s investigation costs are lower than other buyers’, then an RFO generates positive surplus.

*Proof.* See the appendix. □

Proposition 3(a) holds that, when $R$ has lower investigation costs than other buyers (and these buyers, in turn, have substantial investigation costs), an RFO becomes relatively more attractive to $S$ and $R$. The intuition for part (a) is similar to that of Proposition 2. The first-period offer that maximizes $S$’s and $R$’s joint profits in the no-rights case is lower than $S$’s optimal
Table 2. Seller’s Optimal Unconstrained Offers under a Right of First Offer

<table>
<thead>
<tr>
<th>δ = 0.99</th>
<th>$k_{n-9}$</th>
<th>$k_{n-8}$</th>
<th>$k_{n-7}$</th>
<th>$k_{n-6}$</th>
<th>$k_{n-5}$</th>
<th>$k_{n-4}$</th>
<th>$k_{n-3}$</th>
<th>$k_{n-2}$</th>
<th>$k_{n-1}$</th>
<th>$k_n$</th>
</tr>
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<tbody>
<tr>
<td>$n = 3$</td>
<td></td>
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<td></td>
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<td>$n = 5$</td>
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</tr>
<tr>
<td>$n = 10$</td>
<td>0.7303</td>
<td>0.7497</td>
<td>0.7449</td>
<td>0.7377</td>
<td>0.7303</td>
<td>0.7303</td>
<td>0.7303</td>
<td>0.7303</td>
<td>0.7303</td>
<td>0.7303</td>
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<tr>
<td>δ = 0.95</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$n = 3$</td>
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<tr>
<td>$n = 5$</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$n = 10$</td>
<td>0.7602</td>
<td>0.8293</td>
<td>0.8156</td>
<td>0.7985</td>
<td>0.7766</td>
<td>0.7602</td>
<td>0.7602</td>
<td>0.7602</td>
<td>0.7602</td>
<td>0.7602</td>
</tr>
</tbody>
</table>

Table continued...
first-period offer (see Lemma 1). In the no-rights case, if the investigation constraint would be binding on the first-period offer to $R$ if $R$ had the same investigation costs as other buyers, $S$’s optimal first-period offer increases, and thereby reduces the parties’ joint profits, as $R$’s investigation costs decrease. But as long as $S$ is not skipping buyers, $S$’s optimal offers in the RFO case (and thus the parties’ joint profits) are invariant to $R$’s investigation costs. Because lower investigation costs to $R$ reduce $S$’s and $R$’s joint profits in the no-rights case but not in the RFO case, an RFO becomes more attractive.

Proposition 3(a) does not depend on the assumption that $R$ is approached first in the no-rights case. Proposition 3(a) also fits the observation that parties often contract for first-purchase rights when $R$ had some prior relationship with $S$ that reduces $R$’s investigation costs as compared to other buyers, and other buyers face substantial investigation costs.

Part (b) presents circumstances in which an RFO is certain to generate positive surplus. This is the case when investigation costs are sufficiently high so that the investigation constraint is binding on $S$’s optimal offers to all buyers other than $R$ in the no-rights case and $R$ faces lower investigation costs than other buyers. The rationale for part (b) is as follows. When the ICN is binding on all of $S$’s optimal offers (other than that made to $R$) in the no-rights case (which is the case whenever it is binding on $S$’s optimal last-period offer in the no-rights case), $S$’s optimal sequence of offers to outside buyers in the no-rights case is constant. The optimal offers in the no-rights case (other than the offer to $R$) are thus identical to those in the RFO case. This implies that $S$’s cost of granting an RFO stems only from the lower first-period offer made to $R$ in the RFO case as compared to the no-rights case. But then, as was shown in Lemma 1(b), $S$’s and $R$’s joint profits from the first-period offer increase as that offer decreases to the level of the second-period offer (which is higher than $S$’s discounted continuation

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\delta = 1$</th>
<th>$\delta = 0.99$</th>
<th>$\delta = 0.95$</th>
</tr>
</thead>
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<tr>
<td>3</td>
<td>0.2781</td>
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<td>0.2751</td>
</tr>
<tr>
<td>5</td>
<td>0.2810</td>
<td>0.2804</td>
<td>0.2779</td>
</tr>
<tr>
<td>10</td>
<td>0.2812</td>
<td>0.2806</td>
<td>0.2780</td>
</tr>
</tbody>
</table>
value). This result as well does not depend on the assumption that \( R \) is approached first in the no-rights case.

Before proceeding to the next proposition, let us make a general comment on the different effects of an RFO on \( S \)'s and \( R \)'s joint profits. First, as was shown in Lemma 3, an RFO may force \( S \) to make a lower first-period offer to \( R \) when compared with the no-rights case. Such a lower first-period offer, as was shown in Lemma 1, may increase \( S \)'s and \( R \)'s joint profits. Second, the RFO constraint may also force \( S \) to make offers in subsequent periods that would not maximize \( S \)'s expected profit. Because \( R \) derives no benefit from these subsequent constrained offers, the decrease in \( S \)'s expected profit will reduce the surplus generated by an RFO.

Whether an RFO generates positive or negative surplus thus depends on which of these effects dominates. Given the generality of the model, the magnitudes of these effects are hard to quantify. Part (a) of Proposition 4 presents a special case where an RFO does not reduce the first-period offer so that the net effect of an RFO is determinate. Part (b) considers instances where both of these effects are at play and the net effect of an RFO is indeterminate.

**Proposition 4 (surplus under an RFO)**

(a) *Moderate investigation costs.* If the ICN is binding on \( S \)'s optimal first-period offer in the RFO case, then an RFO generates negative surplus.

(b) *Low investigation costs.* If the ICN is not binding on \( S \)'s optimal first-period offer in the RFO case, then an RFO may generate positive or negative surplus.

**Proof.** See the appendix.

Consider first part (a). When the ICN is binding on \( S \)'s optimal first-period offer under an RFO, it is also binding on \( S \)'s optimal first-period offer in the no-rights case. \( R \)'s expected profit in both the RFO case and the no-rights case is thus zero. But \( S \)'s expected profit is never higher in the RFO case than in the no-rights case. An RFO therefore generates negative surplus. This result does not depend on the assumption that \( R \) is approached first in the no-rights case.
The intuition for part (b) is as follows. If the ICN is not binding on $S$’s optimal first-period offer in the RFO case, then $S$’s optimal first-period offer in the RFO case is lower than $S$’s optimal first-period offer in the no-rights case (see Lemma 3(b)). As shown in Lemma 1, a lower first-period offer may increase the joint profits to $R$ and $S$ in the first period. However, if the ICN is not binding on $S$’s optimal first-period offer in the RFO case, then the RFO constraint will force $S$ to make offers in subsequent periods that yield $S$ lower expected profit than the equivalent-period offers in the no-rights case. This is costly to $S$, but yields no benefit to $R$. Whether an RFO produces positive or negative surplus depends on which effect dominates. This result as well does not depend on the assumption that $R$ is approached first in the no-rights case.

**A Comment on the Discount Factor and the Number of Buyers.**

A decrease in the discount factor, which flattens $S$’s optimal sequence of offers, decreases $S$’s expected profit and increases $R$’s expected profit in both the no-rights case and the RFO case. An increase in the number of buyers increases $S$’s expected profit and, because it increases $S$’s optimal first offer, it decreases $R$’s expected profit in both the no-rights case and the RFO case. The net effect on surplus in both cases is indeterminate.

### 5. Specific Parameterization

#### 5.1. Positive Surplus under an RFR and an RFO

To illustrate some of the results in the previous sections, we again consider the case in which buyers’ valuations are distributed uniformly on $[0, 1]$. We consider two alternative assumptions on $R$’s investigation cost: (i) $R$’s investigation costs are equal to other buyers’ investigation costs; or (ii) $R$’s investigation costs, as contrasted to other buyers’, are equal to zero. We consider the cases where $n = 3, 5, 10$ and $\delta = 1, 0.99, 0.95$. Table 4 presents

---

25. If $R$ had idiosyncratic valuation of $S$’s asset (e.g., $R$’s distribution of valuation first-order stochastically dominated other buyers’), then the surplus from an RFO would be either higher or lower. On the one hand, $S$’s optimal first offer in the no-rights case would be even more exploitative to $R$. Prevention of exploitation would thus create a greater surplus. On the other hand, because $R$ is more likely to value the asset highly, $S$’s optimal first-period offer under an RFO would also be higher.
Table 4. Ranges of Buyers’ Investigation Costs in which First-Purchase Rights Generate Positive Surplus

<table>
<thead>
<tr>
<th>Panel A: RFR $c_R = c$</th>
<th>Panel B: RFR $c_R = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n = 3$</td>
<td>$n = 3$</td>
</tr>
<tr>
<td>[0, 0.0732]</td>
<td>[0, 0.0734]</td>
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<tr>
<td>[0, 0.0741]</td>
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<tr>
<td>[0, 0.0741]</td>
<td>[0, 0.0740]</td>
</tr>
<tr>
<td>$n = 5$</td>
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<td>[0, 0.0484]</td>
<td>[0, 0.0491]</td>
</tr>
<tr>
<td>[0, 0.0521]</td>
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<tr>
<td>[0, 0.0585]</td>
<td>[0, 0.0612]</td>
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<tr>
<td>$n = 10$</td>
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<td>[0, 0.0235]</td>
<td>[0, 0.0244]</td>
</tr>
<tr>
<td>[0, 0.0294]</td>
<td>[0, 0.0285]</td>
</tr>
<tr>
<td>[0, 0.0298]</td>
<td>[0, 0.0348]</td>
</tr>
<tr>
<td>Panel C: RFO $c_R = c$</td>
<td>Panel D: RFO $c_R = 0$</td>
</tr>
<tr>
<td>$n = 3$</td>
<td>$n = 3$</td>
</tr>
<tr>
<td>[0, 0.0612]</td>
<td>[0, 0.0616]</td>
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<td>[0, 0.0628]</td>
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<td>[0, 0.2775]</td>
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<tr>
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</tr>
<tr>
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<tr>
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<td>[0, 0.2779]</td>
</tr>
<tr>
<td>$n = 10$</td>
<td>$n = 10$</td>
</tr>
<tr>
<td>no surplus</td>
<td>no surplus</td>
</tr>
<tr>
<td>no surplus</td>
<td>[0.156, 0.2812]</td>
</tr>
<tr>
<td>no surplus</td>
<td>[0.2806]</td>
</tr>
<tr>
<td>no surplus</td>
<td>[0.2780]</td>
</tr>
</tbody>
</table>

Table 4 shows:

- An RFR generates positive surplus for low investigation costs (Panel A and Proposition 1).

- The range of buyers’ investigation costs in which an RFR generates positive surplus is decreasing in the number of buyers. The reason is that as the number of buyers increases, the upper RFR ICN is binding on a greater number of $S$’s optimal offers. As a result, as the number of buyers increases, granting an RFR entails greater costs to $S$.

- Under both an RFR and an RFO, surplus is positive for a wider range of other buyers’ investigation costs where the right-holder’s investigation costs are lower than other buyers’ (Panels B and D versus A and C; and Propositions 2 and 3).

- An RFO generates positive surplus when buyers’ investigation costs are high and $R$’s investigation costs are lower than other buyers’ (Panel D and Proposition 4(a)).

- For $c_R = 0$, an RFO generates positive surplus so long as $S$ does not skip buyers.

Note that an RFR does not generate positive surplus where outside buyers’ investigation costs exceed $c^m = 0.0741$, so that they are excluded from
considering the subject asset. Similarly, an RFO does not generate positive surplus where outside buyers’ investigation costs are sufficiently high and the difference between the right-holder’s and other buyers’ investigation costs induces $S$ to skip buyers.

5.2. Comparison of Positive Surplus under an RFR and an RFO

Although we are not able to make any general observations on the relative positive surplus generated by an RFR and an RFO, we can make some observations on the case where buyers’ valuations are drawn from a uniform distribution on $[0, 1]$. We provide results for $n = 3, 5, 10$ and $\delta = 1, 0.95, 0.9$.

Our first observation is that when the right-holder’s investigation costs are identical to other buyers’ ($c_R = c$), an RFR always generates more surplus than an RFO. Whether an RFR produces more or less positive surplus than an RFO when $c_R = 0$ depends on the number of buyers, their investigation costs, and the discount factor. Table 5 presents the ranges of outside buyers’ investigation costs in which one right produces more surplus than the other, given that both rights generate positive surplus:

Table 5 shows:

- An RFR (RFO) produces more surplus for a low (high) range of other buyers’ investigation costs.

- The effect of an increase in the discount factor on the relative surplus generated by an RFO and an RFR depends on the number of buyers: for $n = 5$, an increase in the discount factor increases the range in which an RFR generates more surplus as compared to an RFO; for $n = 10$, in contrast, an increase in the discount factor decreases the range in which an RFR generates more surplus as compared to an RFO.

- The effect of an increase in the number of buyers on the relative surplus generated by an RFO and an RFR depends on the discount factor: for $\delta = 1, 0.99$, an increase in the number of buyers decreases the range in which an RFR generates more surplus as compared to an RFO; for $\delta = 0.95$, in contrast, an increase in the number of buyers
Table 5. Ranges of Buyers’ Investigation Costs for which First-Purchase Rights Maximize Joint Surplus ($c_R = 0$)

<table>
<thead>
<tr>
<th></th>
<th>$\delta = 1$</th>
<th>$\delta = 0.99$</th>
<th>$\delta = 0.95$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RFR</td>
<td>RFO</td>
<td>RFR</td>
</tr>
<tr>
<td>$n = 3$</td>
<td>$[0, 0.0740)$</td>
<td>$(0.0740, 0.2781)$</td>
<td>$[0, 0.0740)$</td>
</tr>
<tr>
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<td>$(0.0409, 0.2810)$</td>
<td>$[0, 0.0388)$</td>
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<tr>
<td>$n = 10$</td>
<td>$[0, 0.0270)$</td>
<td>$(0.0270, 0.2812)$</td>
<td>$[0, 0.0273)$</td>
</tr>
</tbody>
</table>
decreases, and then increases, the range in which an RFR generates more surplus as compared to an RFO.

6. Conclusion

This paper analyzes first-purchase rights within a sequential-bargaining framework. A seller and a potential right-holder have incentives, *ex ante*, to bargain for a first-purchase right if such a right is expected, *ex post*, to increase their joint profits. We showed that the *ex post* effect of first-purchase rights depends, *inter alia*, on the right-holder’s investigation costs, other buyers’ investigation costs, the number of buyers, the discount factor, and buyers’ valuation distribution.

The paper highlights several potential effects of first-purchase rights. A right of first refusal has the potential of transferring value from other buyers to the right-holder. However, if potential buyers have to incur investigation costs, the presence of a right of first refusal reduces their incentives to incur these costs and thus reduces the marketability of the seller’s asset. A right of first offer induces the seller to lower her offer to the right-holder, which will tend to increase the seller’s and right-holder’s joint profits. But it also forces the seller to make suboptimal subsequent offers to other potential buyers (if the asset is not sold to the right-holder). Both rights, moreover, make it harder for the seller to exploit the right-holder—who may attribute a higher value to the seller’s asset or have lower investigation costs—by making him too high an offer.

The paper further showed that no one arrangement (no-rights, right of first refusal, right of first offer) is always superior to any other. However, rights of first refusal always generate positive surplus when buyers’ investigation costs are sufficiently low and the discount factor is sufficiently high; rights of first offers generate positive surplus when buyers’ investigation costs are sufficiently high and the right-holder’s investigation costs are lower than other buyers’.

Moreover, we showed that first-purchase rights tend to be more valuable when the right-holder’s investigation costs are lower than other buyers’. In such instances, these rights constrain the *ex post* ability of the seller to exploit the right-holder by making a sub-optimally high offer to the right-holder. This result comports with the common use of first-purchase rights...
in settings where the seller and the right-holder had previous relationship with respect to the subject asset.

Appendix

This appendix provides proofs for Lemmas 1–3, Propositions 1–4, as well as a derivation of the seller’s maximization problem in the RFO case.

Proof of Lemma 1. Consider the first-period offer that maximizes $S$’s and $R$’s joint profits:

\[
\hat{k}_1 = \arg \max_{k_1} \int_{k_1}^{\bar{v}} v f(v) \, dv + F(k_1) \delta V_2.
\]

(A1)

Differentiating the objective function with respect to $k_1$ gives $f(k_1)(\delta V_2 - k_1)$. Equating to zero yields $k_1 = \delta V_2$. The second derivative of the objective function with respect to $k_1$ at $\hat{k}_1$ is $-f(\hat{k}_1)$. Thus, $S$’s and $R$’s joint profits are maximized at $\hat{k}_1$.

Now, recall that $S$’s optimal unconstrained first-period offer, $\tilde{k}_1$, is that value of $k_1$ which solves $k_1 = \delta V_2 + (1 - F(k_1))/f(k_1)$. Because $(1 - F(\cdot))/f(\cdot) > 0$, it follows that $\tilde{k}_1 > \delta V_2 \equiv \hat{k}_1$.

To show that $S$’s constrained first-period offer, $\tilde{k}(c_R)$, is greater than $\hat{k}_1$, observe that

\[
\tilde{k} = \sum_{i=0}^{\infty} [F(\tilde{k})]^i \cdot \tilde{k} \cdot [1 - F(\tilde{k})],
\]

(A2)

because the right-hand side is an infinite geometric series with a constant ratio of $F(\tilde{k})$ and a first element of $\tilde{k} \cdot [1 - F(\tilde{k})]$.

Because $n$ is finite, it follows from (A2) that

\[
\tilde{k} > \sum_{i=0}^{n} [F(\tilde{k})]^i \cdot \tilde{k} \cdot [1 - F(\tilde{k})] > V_2 \geq \delta V_2 \equiv \hat{k}_1,
\]

(A3)

which implies that $\tilde{k} > \hat{k}_1$. Because $\tilde{k}(c_R) \geq \tilde{k}$, it follows that $\tilde{k}(c_R) > \hat{k}_1$.

Finally, because $f(k_1)(\delta V_2 - k_1) < 0$ for $k_1 > \delta V_2$, $S$’s and $R$’s joint profits are decreasing on $(\hat{k}_1, k_1^*)$, where $k_1^* = \min(\hat{k}_1, \tilde{k}(c_R))$. □

Proof of Lemma 2. Parts (a) and (b) follow directly from the text. Consider part (c). To show that $S$’s optimal offer under an RFR may
be either lower than or higher than S’s equivalent-period optimal offer under the no-rights case, recall that outside buyers’ expected profit in the RFR case is $F(k)\int_{k_1}^{\hat{y}}(v-k)f(v)\,dv - c$. Differentiating with respect to $k$ yields $f(k)\int_{k_1}^{\hat{y}}(v-k)f(v)\,dv - F(k)[1 - F(k)]$. Because both $(1 - F(k))/\int f(k)$ and $F(k)/\int_{k_1}^{\hat{y}}(v-k)f(v)\,dv$ are increasing in $k$, the sign of this expression may be positive or negative. □

Proof of Proposition 1. Let $k_i^*$ and $k_i^{sr}$ denote S’s optimal offer in period $i$ in the no-rights case and under an RFR, respectively. Because $k_i^* = k_i^{sr}$ for any $i = 1, \ldots, n$, it follows that $V_1 = V'_1$. Consider next $R$’s expected profit in the no-rights case and in the RFR case. Recall that $R$’s expected profit in the no-rights case is $B = E[(v - k_1^*) - c_R]$. The proof is completed by showing that $B' > B$.

Because $k_i^* = k_i^{sr}$, we can write $R$’s expected profit in the RFR case as follows:

$$B' = [1 - F(k_1^*)](E[(v - k_1^*)^+] - c_R) + F(k_2^*) \times [1 - F(k_2^*)](E[(v - k_2^*)^+] - c_R) + F(k_3^*) \times [1 - F(k_3^*)](E[(v - k_3^*)^+] - c_R) + \cdots + F(k_{n-1}^*) \times [1 - F(k_{n-1}^*)](E[(v - k_{n-1}^*)^+] - c_R). \tag{A4}$$

Now, consider $R$’s expected profit in the RFR case from the last two periods:

$$B'_{n-1} = [1 - F(k_{n-1}^*)](E[(v - k_{n-1}^*)^+] - c_R) + F(k_n^*)(E[(v - k_n^*)^+] - c_R). \tag{A5}$$

Because $E[(v - k_i^*)^+] < E[(v - k_i^{sr})^+]$ for any $i > 1$ if the RFR ICN is not binding, it follows that $B'_{n-1} > B$ (this completes the proof for $n = 2$).

Consider next $R$’s expected profit in the RFR case from the last three periods:

$$B'_{n-2} = [1 - F(k_{n-2}^*)](E[(v - k_{n-2}^*)^+] - c_R) + F(k_{n-2}^*)B'_{n-1}. \tag{A6}$$

Because $E[(v - k_i^*)^+] < E[(v - k_i^{sr})^+]$ for any $i > 1$ if the RFR ICN is not binding and because $E[(v - k_1^*)^+] < c_R = B'_{n-1}$, it follows that $B'_{n-2} > B$ (this completes the proof for $n = 3$).
Letting $B_i^r > (1 - F(k_i^*)) (E[v - k_i^*]) + F(k_i^*) B_{i+1}^r$ for $n - 1 \geq i \geq 1$ and proceeding by induction we obtain

$$B_1^r > E[v - k_1^*] + c_R = B.$$ (A7)

Finally, because $B_1^r = B^r$, it follows that $B^r > B$. □

**Proof of Proposition 2.** The proof follows directly from the text. □

*S’s maximization problem under an RFO.*

*S’s optimal offer in each period other than the first and last periods can be written as a function of *S*’s first-period offer:

$$k_i^o (k_1^o, \bar{k}) = \arg \max_{k_i \in K^0} k_i (1 - F(k_i)) + F(k_i) \delta V_{i+1},$$ (A8)

where $V_{i+1}$ is *S*’s expected profit from period $i + 1$ to $n$. That is, in each period, other than the first period and the last period, *S*’s optimal offer is equal to the maximum of the optimal unconstrained offer and the first-period offer, subject to buyers’ investigation constraint. Let $k_i^o$ denote *S*’s optimal period-$i$ offer, for $i \neq 1, n$, under an RFO as a function of the first-period offer, $k_1^o$, and the investigation constraint, $\bar{k}$.

Setting $F(k_0^o) = 1$, we can now write *S*’s problem as follows:

$$\max_{k_1 \in [0, \bar{k}]} \sum_{i=1}^{n-1} \prod_{j=0}^{i-1} [F(k_j^o)] \delta^{i-1} k_i^o (1 - F(k_i^o))$$

s.t. $k_1^o = k_n^o$. (A9)

The objective function is *S*’s expected profit from periods 1 to $n$. The constraint ensures that the last-period offer is equal to the first-period offer.

In the uniform distribution case, *S*’s maximization problem is:

$$\max_{k_1 \in [0, \bar{k}]} \sum_{i=1}^{n-1} \prod_{j=0}^{i-1} [k_j^o] \delta^{i-1} k_i^o (1 - k_i^o)$$

s.t. $k_1^o = k_n^o$, (A10)

where $k_0^o = 1$ by definition and $k_i^o, i \neq 1, n$, is defined in (16).

**Proof of Lemma 3.** Part (a) follows directly from the text. Consider part (b). Let $k_i^{o*}$ and $V_i^0$ denote *S*’s optimal period-$i$ offer and maximized
expected profit, respectively, in period $i$ under an RFO. Let $k^*_i(\cdot)$ denote $S$’s optimal first-period offer in the no-rights case as a function of $S$’s continuation value. Then $k^*_1(V_2) \geq k^{o_1}_1(V_{2}^o) \geq k^{*o_2}_1$, where the first inequality follows because $V_2 \geq V_2^o$ (because $K^O \subseteq K^N$) and the second inequality follows because $S$’s optimal first-period offer in the no-rights case is (weakly) higher than $S$’s optimal second-period offer. Now, because $k^*_1 \geq k^{*o}_2$ and $k^{*o}_2 \geq k^{*o}_1$ (by Lemma 3(a)), it follows that $k^*_1 \geq k^{*o}_1$.

Proof of Proposition 3. Part (a) follows directly from the text. Consider part (b). First, note that if the ICN is binding on the last-period offer in the no-rights case, then the ICN is binding on all of $S$’s optimal offers subsequent to the first-period offer in the no-rights case and on all of $S$’s optimal offers in the RFO case. It follows that $S$’s second-period continuation values in the no-rights case and the RFO case are equal: $V_2 = V_2^o$. Next, note that $V_2$’s optimal first-period offer in the no-rights case is higher than $\bar{k}(c)$, because $R$’s investigation costs are lower than other buyers’. It follows that $k^*_1 > k^{*o}_1$. Because $S$’s and $R$’s joint profits are decreasing on $[\bar{k}, \bar{k}(c))$ (see Lemma 1(b)), $S$’s and $R$’s joint profits are higher in the RFO case than in the no-rights case.

Proof of Proposition 4. Part (a) follows directly from the text. Consider part (b). The difference between $S$’s and $R$’s joint profits in the RFO case and the no-rights case is given by

$$\int_{k^*_1}^{k^{*o}_1} v f(v) \, dv + (F(k^*_1)\delta V_2 - F(k^{*o}_1)\delta V_2^o). \quad (A11)$$

The first expression represents the surplus from the lower first-period offer under an RFO as compared to the no-rights case. The second expression represents the negative surplus from the lower discounted continuation value in the RFO case when compared with the no-rights case.

To show that the net surplus may be either negative or positive, consider the case in which buyers’ valuations are distributed uniformly on $[0, 1]$, $n = 3$, and $\delta = 1$. For $c = 0.05$ and $0.06$, the ICN is not binding on $S$’s optimal first-period offer in the RFO case. When $c = 0.05$ an RFO generates positive surplus, but for $c = 0.06$ an RFO generates negative surplus (see Table 4, Panel C).
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References


