ECONOMETRICS I

Assignment 1

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Part I. Paradigm.


Your choice
Part II. Distribution Theory

The random variables $y, x, z$ have a multivariate normal distribution with mean vector $\mathbf{\mu} = [1, 2, 4]$ and covariance matrix $\mathbf{\Sigma} = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 5 & 2 \\ 1 & 2 & 6 \end{bmatrix}$. Compute the slope and intercept in the conditional mean function $E[y|x]$. Compute the two slopes and the intercept in the conditional mean function $E[y|x, z]$. Is the slope on $x$ the same in the two functions? Explain. Compute the conditional variance $\text{Var}[y|x]$. Compute the squared correlation between $y$ and $x$. Compute the squared correlation between $y$ and $E[y|x]$. Compute the squared correlation between $y$ and $E[y|x, z]$.

The slope of $E[y|x]$ is $\frac{\sigma_{yx}}{\sigma_x^2} = \beta = \frac{3}{5}$. The intercept is $\mu_y - \beta \mu_x = 1 - \left(\frac{3}{5}\right)2 = -1/5$.

The slopes in $E[y|x, z]$ are $\left(\begin{bmatrix} \beta_x \\ \beta_z \end{bmatrix} = \begin{bmatrix} \frac{\sigma_{zx}^2}{\sigma_{xx}} & \frac{\sigma_{xy}}{\sigma_{xx}} \\ \frac{\sigma_{zx}^2}{\sigma_{zz}} & \frac{\sigma_{zy}}{\sigma_{zz}} \end{bmatrix}^{-1} = \begin{bmatrix} 5 & 2 \\ 2 & 6 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} .615385 \\ -.0384615 \end{bmatrix} = \left(\begin{bmatrix} 16/26 \\ 1/26 \end{bmatrix}\right)$

The intercept is $\mu_y - \beta_x \mu_x - \beta_z \mu_z = 1 - \left(\frac{16}{26}\right)2 - \left(-\frac{1}{26}\right)4 = -1/13$.

The slope on $x$ differs because in the second case, the coefficient is partial, after the effect of $z$. $\text{Var}[y|x] = \text{Var}[y] - \beta^2 \text{Var}[x] = 2 - \left(\frac{3}{5}\right)^2 5 = 1/5$.

The squared correlation between $y$ and $x$ is $\text{Cov}(x, y)^2/\text{Var}(x)\text{Var}(y) = 9/(2*5) = 9/10$.

Since $E[y|x]$ is a linear function of $x$, the squared correlation between $y$ and $E[y|x]$ is the same as that between $y$ and $x$, $9/10$. The squared correlation between $y$ and $E[y|x, z]$ is $\text{cov}(y, E[y|x, z]) = \text{Var}(y)\text{Var}(E[y|x, z])$. $\text{Var}(y) = 2$.

$\text{Var}[E[y|x, z]] = (16/26)^2 \text{Var}(x) + (-1/26)^2 \text{Var}(z) + 2(16/26)(-1/26)\text{Cov}(x,z)$

$= (16/26)^2*5 + (-1/26)^2*6 + 2(16/26)(-1/26)*2 = 1.8076923$.

$\text{Cov}(y, E[y|x, z]) = (16/26)\text{Cov}(y, x) + (-1/26)\text{Cov}(y, z) = (16/26)*3 + (-1/26)*1 = 1.8076923$.

The squared correlation is $1.8076923^2 / (2*1.8076923) = .9038$. 
Part III. Regression

(You will need software to do this exercise. You may use any computer program that you wish. The computations are straightforward.) The data file (which you should download)

http://people.stern.nyu.edu/wgreene/Econometrics/fuelbills.csv

is an Excel CSV file that contains data on fuelbills and number of rooms for 144 homes. Produce a simple scatter (X-Y) plot with ROOMS on the horizontal axis and FUELBILL on the vertical axis. What conclusion do you draw about the relationship between number of rooms and fuelbill?

2. Note that ROOMS only takes a few values, 3,4,5,...,11. Compute the mean value of FUELBILL for the different values of ROOMS. What do you conclude about the conditional mean? Plot the means against the number of rooms. What do you find?

TIP: NLOGIT Users: You can import a csv data set that you have downloaded to your computer simply by using Project → Import → Variables... Then, use the Windows miniexplorer to make your way to the file and select it in the window.

\[
\begin{align*}
\text{plot}: & \text{lhs}=\text{rooms}; \text{rhs}=\text{fuelbill} \\
\text{dstat}: & \text{lhs}=\text{fuelbill}; \text{str}=\text{rooms} \\
\text{sort}: & \text{lhs}=\text{rooms}; \text{rhs}=* \\
\text{create}: & \text{meanbill}=\text{groupmean}(\text{fuelbill}, \text{str}=\text{rooms}) \\
\text{plot}: & \text{lhs}=\text{rooms}; \text{rhs}=\text{fuelbill}; \text{rh2}=\text{meanbill}
\end{align*}
\]

---

Descriptive Statistics for FUELBILL
Stratification is based on ROOMS

<table>
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<tr>
<th>Subsample</th>
<th>Mean</th>
<th>Std.Dev.</th>
<th>Cases</th>
<th>Sum of wts</th>
<th>Missing</th>
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<tr>
<td>ROOMS</td>
<td>3</td>
<td>236.666667</td>
<td>37.859389</td>
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<td>79.936880</td>
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<tr>
<td>ROOMS</td>
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<td>422.027778</td>
<td>66.352085</td>
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<td>70.774278</td>
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<td>26.00</td>
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<tr>
<td>ROOMS</td>
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<td>175.391219</td>
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</table>

---
Part IV. Distribution Theory

Consider the joint distribution of two random variables, \( y \), which is the number of failures of some component (disk drive) in a brand of computer per unit of time and \( x \), the average lifetime of some different but related component (a chip). Note that \( y \) is a discrete random variable and \( x \) is a continuous random variable. Suppose that the conditional distribution of \( y \) is

\[
f(y|x) = e^{-\beta x}(\beta x)^y / y!, \quad y = 0, 1, ..., x \geq 0, \beta > 0,
\]

while the marginal distribution of \( x \) is

\[
f(x) = \theta e^{-\theta x}, \quad x \geq 0, \theta > 0.
\]

Thus, conditioned on \( x \), \( y \) has a Poisson distribution with parameter \( \beta x \), while \( x \), unconditionally, has an exponential distribution.

a. What is the joint distribution of these two random variables, \( f(y,x) \)?

\[
f(y,x) = f(y|x)f(x) = e^{-\beta x}(\beta x)^y / y! \cdot \theta e^{-\theta x} = \theta e^{-\beta x}(\beta x)^y / y! \cdot \theta = \theta e^{-\beta x}(\beta x)^y / y! = \theta e^{-\beta x}(\beta x)^y / y! = \theta e^{-\beta x}(\beta x)^y / y! = \theta e^{-\beta x}(\beta x)^y / y! = \theta e^{-\beta x}(\beta x)^y / y!
\]

b. Show that the unconditional density of \( y \) is \( f(y) = \delta(1-\delta)^y \) where \( \delta = \theta / (\beta + \theta) \).

Integrate \( x \) out of \( f(y,x) \). This is

\[
\int e^{-(\beta+\theta)x} x^y dx = \frac{\theta \beta^y}{\beta + \theta} \Gamma(y+1) = \left( \frac{\theta}{\beta + \theta} \right)^y \Gamma(y+1) = \left( \frac{\theta}{\beta + \theta} \right)^y \Gamma(y+1)
\]

Using the gamma integral for the integration, and note \( \Gamma(y+1) = y! \) so this part cancels. The rest is now obvious.

c. Show that \( E[x] = 1/\theta \) and \( Var[x] = 1/\theta^2 \). \( f(x) \) is the exponential density. This is a standard form. You can prove the results for \( E[x] \) and \( Var[x] \) using gamma integrals. \( E[x] = \int_0^\infty x \theta e^{-\theta x} dx = \frac{\Gamma(2)}{\theta^2} = \frac{1}{\theta^2} \).

For the variance, you need \( E[x^2] = \theta \Gamma(3)/\theta^3 = 2/\theta^2 \). Subtract \( E[x^2] \) to get \( Var[x] = 2/\theta^2 - (1/\theta)^2 = 1/\theta^2 \).

For a discrete random variable \( z \) that has a Poisson distribution with parameter \( \alpha \)

\[
f(z) = e^{-\alpha} \alpha^z / z!, \quad E[z] = Var[z] = \alpha.
\]

It follows then, that in our conditional distribution, \( E[y|x] = Var[y|x] = \beta x \). Note that this “regression” model has a linear conditional mean function. You could obtain \( E[y] \), \( Var[y] \), and \( Cov(y,x) \) from the marginal distribution \( f(y) \) and the joint distribution \( f(x,y) \) by summing and integrating using the definitions. But, there is a much easier way.

d. Using the fundamental results:

\[
E[y] = E_x[E[y|x]]
\]

\[
Var[y] = E_x[Var[y|x]] + Var_x[E[y|x]],
\]

\[
Cov(x,y) = Cov_x[E[y|x]],
\]

show that \( E[y] = \beta/\theta = \gamma \), \( Var[y] = \beta/\theta + (\beta/\theta)^2 = \gamma(1+\gamma) \), and \( Cov[x,y] = \beta/\theta^2 = \gamma/\theta \).

\[
E[y] = E_x[E[y|x]] = E[Bx] = \beta E[x] = \beta/\theta = \gamma.
\]

\[
Var[y] = E_x[Var[y|x]] + Var_x[E[y|x]] = \beta/\theta + \beta^2 (1/\theta^2) = \beta/\theta[1 + \beta/\theta].
\]

\[
Cov(x,y) = Cov_x[E[y|x]] = Cov(x,Bx) = \beta Var[x] = \beta/\theta^2.
\]
Part V. Least Squares Algebra

1. In the December, 1969, American Economic Review (pp. 886-896), Nathaniel Leff reports the following least squares regression results for a cross section study of the effect of age composition on savings in 74 countries in 1964:

\[
\log \frac{S}{Y} = 7.3439 + 0.1596 \log \frac{Y}{N} + 0.0254 \log G - 1.3520 \log D_1 - 0.3990 \log D_2 \quad (R^2 = 0.57)
\]

\[
\log \frac{S}{N} = 8.7851 + 1.1486 \log \frac{Y}{N} + 0.0265 \log G - 1.3438 \log D_1 - 0.3966 \log D_2 \quad (R^2 = 0.96)
\]

where \(\frac{S}{Y}\) = domestic savings ratio, \(\frac{S}{N}\) = per capita savings, \(\frac{Y}{N}\) = per capita income, \(D_1\) = percentage of the population under 15, \(D_2\) = percentage of the population over 64, and \(G\) = growth rate of per capita income. Are these results correct? Explain. Arthur Goldberger raised this question in a comment on Leff’s paper in a comment in the 1973 American Economic Review. The (2 page) paper is on the course website – you can download http://people.stern.nyu.edu/wgreene/Econometrics/Goldberger-on-Leff.pdf. Leff’s (1 page) reply is at http://people.stern.nyu.edu/wgreene/Econometrics/Leff-on-Goldberger.pdf. Read these two papers. What’s your opinion? Specifically, what about Leff’s reaction to the comment?

Since \(\log(S/N) = \log(S/Y) + \log(Y/N)\), the second equation should be exactly equal to the first equation plus \(\log(Y/N)\). All the coefficients should be identical except the coefficient on \(\log Y/N\) in the second equation, which should be 1.1596. Leff’s results are quite far off. In his reply to Goldberger, Leff was very glib about these, but the differences are too large to be rounding error. Goldberger had it right.

2. Regression without a constant term. What is the effect on \(R^2\) of computing a linear regression without a constant term? (Note, of course, it depends on how \(R^2\) is computed.)

\(R^2\) can be nonsense depending on how it is computed. It can be larger than 1 if computed as \(b'X'M^0y/y'M^0y\) or negative if \(1 - e'e/y'M^0y\).

3. Partitioned regression. Suppose a data set consists of \(n\) observations on \(y\), \(K_1\) variables in \(X_1\) and \(K_2\) variables \(X_2\). Do the following three procedures produce the same value for the least squares coefficients on \(X_2\)?

a. Regress \(y\) on both \(X_1\) and \(X_2\). This is the multiple regression.

b. Regress the residuals from a regression of \(y\) on \(X_1\) on the residuals (column by column) of regressions of \(X_2\) on \(X_1\). This is regression of \(M_1y\) on \(M_1X_2\) which by Frisch and Waugh is the coefficient on \(X_2\) in the multiple regression.

c. Same as \(b\), but do not transform \(y\). This is the regression of \(y\) on \(M_1X_2\). This is the same as \(b\) because \(M_1\) is idempotent.

d. Same as \(b\), but do not transform \(X_2\). This is not the same. This is the regression of \(M_1y\) on \(X_2\) which produces a different result.

4. Residual makers. What is the result of the matrix product \(M_1M\) where \(M\) is defined in (3-14) and \(M_1\) is defined in (3-19) in your text? Multiply it out. \(M_1M = [I - X_1(X_1'X_1)^{-1}X_1']M = M\) because \(X_1'M = 0\).

5. Change in the sum of squares. Suppose that \(b\) is the least squares coefficient vector in the regression of \(y\) on \(X\) and that \(e\) is any other \(K\times1\) vector. Prove that the difference in the two sums of squared residuals is

\[
(y - Xc)'(y - Xc) - (y - Xb)'(y - Xb) = (c - b)'X'X(c - b).
\]

Prove that this difference is positive.

\((y-Xc)'(y-Xc) = (y-Xb+Xb-Xc)'(y-Xb+Xb-Xc) = (e+X(b-c))'(e+X(b-c)).\)

When you multiply this out, the middle term is zero because \(e'X = 0\), so what remains is \(e'e + (b-c)'X'X(b-c)\). So \((y-Xc)'(y-Xc) = e'e + (b-c)'X'X(b-c)\). Since \(X'X\) is positive definite, the second term is positive.
6. **The budget model.** Consider a plan to fit least squares regressions using three dependent variables $y_1$, $y_2$, $y_3$ where $y_j$ is the share of total expenditure on durables, nondurables, and services, respectively. Note that the three budget shares sum to 1. All three regressions will use the same $X$ matrix which has 5 columns (variables)

$$X = [\text{a constant term, income, } P_D, P_N, P_S]$$

where $P_m$ is a price index for the $m$th expenditure group.

Denote the the $m$th least squares coefficient vector by $b_m$, $m = D, N, S$. Prove that the sum of the three least squares coefficient vectors is $b_D + b_N + b_S = [1,0,0,0,0]^\prime$. That is, the constant terms sum to 1 and the other coefficients sum to zero. Now, suppose instead of budget shares, we have expenditure data. Moreover, though we would like to use income as the second independent variable, we have only total expenditure, the sum of the three expenditures. Now, what do you get when you add the three least squares coefficient vectors? Prove your answer.

$$b_D + b_N + b_S = (X'X)^{-1}X'y_1 + y_2 + y_3.$$  But, the $ys$ sum to 1, which is the first column in $X$, so

$b_D + b_N + b_S = (X'X)^{-1}X'x_1$. Since $X'x_1$ is the first column of $X'X$, the product is the first column of an identity matrix. That proves the result. If the budget shares are expenditures instead, and they sum to the second column in $X$, then the same result applies, though now the three coefficient vectors sum to $(0,1,0,0,0)$.

7. **(Multicollinearity)** The regression model of interest is

$$y = X_1 \beta_1 + X_2 \beta_2 + \epsilon$$

where $X_1$ is $K_1$ variables, including a constant and $X_2$ is $K_2$ variables not including a constant. It is believed that multicollinearity between the columns of $X_1$ and $X_2$ is adversely affecting the regression. Consider the following ‘cure.’ We will first regress each variable in $X_1$ on all of the variables in $X_1$. By construction, the residuals in these regressions, call them $Z_2 = (z_1,\ldots,z_{k_2})$, are orthogonal to every variable in $X_1$. So, instead of regressing $y$ on $X_1$ and $X_2$, we linearly regress $y$ on $X_1$ and $Z_2$. Denote by $b = (b_1, b_2)$ the least squares coefficients in the original regression, and by $c = (c_1, c_2)$ the least squares coefficients in the regression of $y$ on $X_1$ and $Z_2$. Show the algebraic relation between $b$ and $c$. Is $c$ unbiased?

$$Z_2 = M_1X_2.$$  Note that $X_1'Z_2 = 0$ because $X_1'M_1 = 0$. So in the regression of $y$ on $(X_1, Z_2)$, we can to the two sub regressions separately. That is, $c_1 = (X_1'X_1)^{-1}X_1'y$, which is not equal to $b_1$ if we regress $y$ on $X_1$ and $X_2$. But, $c_2 = (Z_2'Z_2)^{-1}Z_2'y = (X_2'M_1X_2)^{-1}(X_2'M_1y)$ which is equal to $b_2$ in the original regression. So, $c_1$ is biased, but $c_2$ is unbiased.

Using the gasoline data

http://people.stern.nyu.edu/wgreene/Econometrics/gasoline.csv

let $y$ be the variable $G$ in the data set, let $X_2$ denote the three macroeconomic price indexes, $P_d$, $P_n$, and $P_s$, and let $X_1$ denote the other independent variables, constant, $GasP$, and $PCIncome$. Carry out the computations listed above and verify that the algebraic results you obtained do appear in the empirical results.

```r
sample:1-52$
name:x1=one,gasp,pcincome$
nname:x2=pd,pn,ps$
regress;lhs=ps;rhs=x1;res=ed$
regress;lhs=pn;rhs=x1;res=en$
regress;lhs=ps;rhs=x1;res=es$
nname:z2=ed,en,es$
regress;lhs=g;rhs=x1,x2;table=x1_x2$
regress;lhs=g;rhs=x1,z2;table=x1_z2$
maketable;x1_x2,x1_z2$
```
Intermediate results omitted.
|-> maketable;x1_x2,x1_z2

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<th>X1_Z2</th>
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<td>s.d.e(i)</td>
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</tr>
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Part VI. Replication and Extensions

1. The Cornwell and Rupert data used in several examples in class, including the example at the end of slides Econometrics-I-2, are provided on the course home page in

   [http://people.stern.nyu.edu/wgreene/Econometrics/cornwell&rupert.csv](http://people.stern.nyu.edu/wgreene/Econometrics/cornwell&rupert.csv)

This is a generic csv file that can be imported into any modern software. For this part of the assignment, you are to replicate the regression that appears in slides 43-44. Note that in modern research, replicating an author’s results usually starts by obtaining the same coefficients and standard errors. If that is possible (which is often not the case), other differences in reported results can usually be explained. As part of your submission for this assignment, include the specific estimation results that you obtained for this regression.

2. Now that you have replicated the regression, we’ll consider a couple of minor extensions.

   a. Functional Form. The example thus far computes a single, generic effect of education on LWAGE. We’re interested in determining if there is a different effect for men (FEM=0) and women (FEM=1). One compact way to do this is to add an interaction term, FEM*ED to the model. The different effects are the coefficient on ED which is for men and the sum of the two effects, ED and FEM*Ed, for women. Reestimate your model with this additional effect, and report your result.

   b. Standard Errors. Just as an experiment, I’d be interested to compute the standard errors for the model in 1 using bootstrapping. Do this, using 100 replications, and report/discuss the difference you observe.

   NLOGIT tip: You can use

   ```
   procedure$
   regress;lhs=lwage;rhs=one,ed,exp,exp*exp,wks, occ,south,smrsa,ms,fem,union;quietly$
   endproc$
   execute;n=100;bootstrap=b$
   ```

   to compute bootstrapped standard errors. There is also a built-in procedure in Stata for bootstrapping standard errors for a linear regression model. Note, there is no right answer to this exercise because it involves pseudorandom numbers which will differ from one run to the next and definitely across software platforms.
**Ordinary least squares regression**

**LHS=LWAGE**  
Mean = 6.67635

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<th>Error</th>
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<th>Prob.</th>
<th>95% Confidence Interval</th>
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Model was estimated on Oct 01, 2014 at 09:28:05 AM.
Results of bootstrap estimation of model.
Model has been reestimated 100 times.
Coefficients shown below are the original model estimates based on the full sample.
Bootstrap samples have 4165 observations.
Estimated parameter vector is B.
Estimated variance matrix saved as VARB.

<table>
<thead>
<tr>
<th>BootStrp</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>z</th>
<th>Prob.</th>
<th>95% Confidence Interval</th>
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<td>.01485</td>
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***, **, * ==> Significance at 1%, 5%, 10% level.
Model was estimated on Oct 01, 2014 at 09:31:27 AM

Ordinary least squares regression .........
LHS=LWAGE
Mean = 6.67635
Standard deviation = .46151
No. of observations = 4165
DegFreedom = 4153
Mean square = 33.87472
Regression Sum of Squares = 372.622
Residual Sum of Squares = 514.283
Total Sum of Squares = 886.905
Standard error of e = .35190
Root MSE = .35139
Fit R-squared = .42014
R-bar squared = .41860
Model test F[ 11, 4153] = 273.54917 Prob F > F* = .00000

<table>
<thead>
<tr>
<th>LWAGE</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>z</th>
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***, **, * ==> Significance at 1%, 5%, 10% level.
Model was estimated on Oct 01, 2014 at 09:29:38 AM