ACCOUNTING FOR UNOBSERVABLES IN PRODUCTION MODELS:
MANAGEMENT AND INEFFICIENCY

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ABSTRACT

This paper explores the role of unobserved managerial ability in production and its relationship with technical efficiency. Previous analyses of managerial ability have been based on strong assumptions about its role in production or on the use of proxies. We avoid these limitations by introducing managerial ability as an unobserved random variable in a translog production function. The resulting empirical model can be estimated as a stochastic production frontier with random coefficients.

Keywords: Managerial ability, technical efficiency, production frontier, random coefficients model, maximum simulated likelihood.

JEL codes: C5, D2

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This paper explores the role of unobserved managerial ability in production and its relationship with technical efficiency. Previous analyses of managerial ability have been based on strong assumptions about its role in production or on the use of proxies. We avoid these limitations by introducing managerial ability as an unobserved random variable in a translog production function. The resulting empirical model can be estimated as a stochastic production frontier with random coefficients.

1. INTRODUCTION

Management has always been considered an important factor of production. However, the modeling of management is problematic because it is unobservable and for this reason it has been omitted from many production models. This may be a source of important problems because the omission of relevant variables can lead to biased estimates of the remaining parameters of the production function (Griliches, 1957). Economists have coined the term ‘management bias’ to refer to this problem and two remedies have been proposed in the literature. Following Mundlak (1961), some authors have used covariance analysis [e.g. Massell (1987)] or similar tools such as the within transformation to control for the effect of time invariant management by effectively eliminating it from the equation to be estimated. Other studies have used ‘proxies’ for management [e.g. Dawson and Hubbard (1985); Mefford (1986)].

An alternative approach is to consider management as a random effect and model it as part of the stochastic element of the production function. This is the approach implicitly followed by the stochastic production frontier literature (Aigner et al., 1977) where the stochastic structure is composed of two terms: a symmetric term, which accounts for noise, and an asymmetric term that accounts for technical inefficiency. In this literature, it has been common to assume that the inefficiency term picks up, among other things, differences in the level of managerial skills.
The production function\(^1\) and stochastic production frontier literatures have followed parallel but independent paths. On the one hand, the ‘average’ production function literature recognizes the role of management but seldom mentions production inefficiency. On the other, the stochastic frontier literature focuses on estimating technical efficiency (TE) and recognizes that it is related to management ability but it has not provided an analytical linkage between both concepts. For example, Farrell (1957) stated that “technical efficiency indicates the gain that can be achieved by simply ‘gingering-up’ the management,” suggesting that technical inefficiency is the result of a lack of managerial ability. On the other hand, Leibenstein (1966) viewed technical inefficiency as the result of a lack of motivation or effort. In this case, the solution to inefficiency calls for better organization of the work process or greater motivation and supervision of employees, “all of which are commonly considered to be management functions” [Mefford (1986)].

In this paper we explore the analytical linkages between technical efficiency and management within the framework of a translog production model where management is treated as an unobservable fixed input.\(^2\) Starting from an average production function, we construct a production frontier which allows us to derive an explicit relationship between technical efficiency and management. In particular, technical efficiency is shown to be a function of the difference between the firm’s level of managerial input and the amount of managerial input that the firm would need to operate on the frontier.

In the empirical section of the paper we estimate the production function with the latent fixed input described above. It is important to note that in a translog production function the latent input enters the model both additively as well as in interaction with the remaining inputs. This feature is very important for modeling the role of management in production but greatly complicates the estimation of the model. The model that emerges is a type of random coefficients model that has to be estimated by maximum simulated likelihood.

The structure of the paper is as follows. In Section 2 we develop a model in which managerial ability is treated as an unobservable input in a flexible production function. Section 3 discusses estimation issues. Empirical results based on a study of dairy farmers are presented and discussed in Section 4. Finally, some conclusions are drawn in Section 5.

\(^1\) As traditional stochastic production functions have typically been specified with a symmetric error term, in what follows we refer to them as ‘average’ production functions.

\(^2\) This assumes that the production is otherwise fully specified and successfully accounts for other time invariant factors.
2. A PRODUCTION MODEL WITH FIXED MANAGERIAL ABILITY

Our starting point is a translog production function with one time-varying variable input, \( x_{it} \), and managerial ability, \( m_i \), which is considered a fixed input. The flexibility of the production function relaxes ex-ante constraints on the roles of \( m_i \) and \( x_{it} \) in the production process. The translog production function can be written as:

\[
\ln y_{it} = \alpha + \beta_x \ln x_{it} + \frac{1}{2} \beta_{xx} (\ln x_{it})^2 + \beta_m m_i + \frac{1}{2} \beta_{mm} m_i^2 + \beta_x m_i \ln x_{it} m_i + v_{it}
\]

where subscripts \( i \) and \( t \) denote firms and time, respectively, and \( y_{it} \) is the single output. The inputs in the translog model are conventionally expressed in logs, but since managerial ability and its units of measurement are unobservable we express it in levels. We assume that \( v_{it} \) is a symmetric random disturbance with zero mean. Therefore, the model in (1) corresponds to the typical ‘average’ production function. Its key feature for present purposes is the interaction of management with the variable input. Without this interaction, the two management terms collapse into an individual effect and the model does not differ in substance from the standard fixed or random effects production function model.

Greater managerial ability should allow the agent to produce more output from any given amount of input, so we expect production to be monotonically increasing in \( m_i \); that is:

\[
\ln y_{it}^* = \alpha + \beta_x \ln x_{it} + \frac{1}{2} \beta_{xx} (\ln x_{it})^2 + \beta_m m_i^* + \frac{1}{2} \beta_{mm} m_i^{*2} + \beta_x m_i \ln x_{it} m_i^* + v_{it}
\]

Thus, for a given \( x_{it} \), higher values of \( m_i \) imply higher levels of output. The maximal output for given \( x_{it} \) (i.e., the frontier output) is achieved with the maximal level of managerial input \( m_i^* \). The stochastic production frontiers may then be written as:

\[
\ln y_{it}^* = \alpha + \beta_x \ln x_{it} + \frac{1}{2} \beta_{xx} (\ln x_{it})^2 + \beta_m m_i^* + \frac{1}{2} \beta_{mm} m_i^{*2} + \beta_x m_i \ln x_{it} m_i^* + v_{it}
\]

where \( y_{it}^* \) denotes efficient output.

We can now establish a link between technical efficiency and management. This follows from the definition of an output-oriented index of technical efficiency as the ratio of observed to potential output. In log terms this is:

\[
\ln TE_{it} = \ln y_{it} - \ln y_{it}^* = (\beta_m + \beta_x m_i \ln x_{it})(m_i - m_i^*) + \frac{1}{2} \beta_{mm} (m_i^2 - m_i^{*2}) \leq 0
\]
Note that when \( m_i = m_i^* \), that is when the firm is using the amount of management that defines the frontier, \( \ln TE_{it} = 0 \) and the firm is therefore technically efficient. Equation (4) can be rewritten as:

\[
\ln TE_{it} = \theta_i + \theta_{xi} \ln x_{it},
\]

\[
\theta_i = \beta_m (m_i - m_i^*) + \frac{1}{2} \beta_{mn} (m_i^2 - m_i^{**}),
\]

\[
\theta_{xi} = \beta_{xm} (m_i - m_i^*).
\]

Equation (5) shows that \( TE \) has two components. The first can be modeled as a time-invariant individual effect (\( \theta_i \)). The other term, reflecting the interaction of management with input use, is specified as a time-varying component in the production function (\( \theta_{xi} \ln x_{it} \)). Therefore, an interesting feature of expression (5) is that the implied technical efficiency will be time-varying because it depends on \( x_{it} \) even when the observed level of management and the one at the frontier are constant over time. Given the specification in (1), this suggests that it does not seem appropriate to model technical efficiency as a fixed effect since it depends on \( x_{it} \). A consequence of this specification is that the change in managerial input necessary to increase \( TE \) by a given amount differs according to input use.

The special implications for economic analysis of our measure of \( TE \) can be better understood by looking at the effects on \( TE \) of changes in managerial ability and input use. These effects are given by the following derivatives:

\[
\frac{\partial \ln TE_{it}}{\partial m_i} = \beta_m + \beta_{mn} m_i + \frac{1}{2} \beta_{mn} (m_i^2 - m_i^{**}),
\]

\[
\frac{\partial \ln TE_{it}}{\partial \ln x_{it}} = \beta_{xm} (m_i - m_i^*).
\]

The derivative of \( TE \) with respect to managerial input corresponds exactly to the condition for monotonicity of production with respect to managerial ability shown in expression (2). Therefore, an increase in managerial ability increases \( TE \) given conventional inputs if the production function is monotonic in managerial ability. The derivative of \( TE \) with respect to the level of input use is negative if \( \beta_{xm} \) is positive because \( m_i \) is smaller than \( m_i^* \) by definition. Therefore, when \( \beta_{xm} \) is positive the increase in the use of conventional inputs decreases \( TE \) for a given amount of managerial ability.

---

3 This specification represents a departure from previous formulations of panel data frontier models, such as Schmidt and Sickles (1984), which have assumed that inefficiency was time–invariant.
In this section we have shown that the model in (1) raises interesting issues about the relationship between fixed management and technical efficiency. In particular, it shows that TE is not necessarily a fixed effect but instead can vary over time, and that the relationship between TE and managerial ability depends on the amount of managerial ability and conventional inputs. In the next section we discuss the estimation of this model in more general terms, with more than one observed input.

3. ESTIMATION ISSUES

Model (1) cannot be directly estimated because the individual level of management is unobservable. Previous authors have dealt with this problem by introducing a proxy for management in a cost function. [See Dawson and Hubbard (1985), Alvarez and Arias (2003)]. Since the use of a proxy introduces new ambiguities (measurement error) into the model, we will employ a more direct approach to the problem which takes advantage of the panel nature of the data set we will analyze. We will now translate the model in the preceding section into an empirically estimable form.

3.1. Stochastic Frontier Model with Fixed Management

We consider production with $K$ inputs, $x_1, \ldots, x_K$. As before, let $m_i^*$ denote the level of management that defines the frontier and $m_i$ the actual management input for firm $i$. We continue to employ the translog form. The production model will then be

$$\ln y_{it} = \ln y_{it}^* - u_{it}$$
$$= \alpha + \sum_{k=1}^{K} \beta_k \ln x_{itk} + \frac{1}{2} \sum_{k=1}^{K} \sum_{l=1}^{K} \beta_{kl} \ln x_{ilk} \ln x_{lit} + \beta_m m_i^* + \frac{1}{2} \beta_{mm} m_i^2 + \sum_{k=1}^{K} \beta_{km} \ln x_{itk} m_i^* + v_{it} - u_{it}$$

where $u_{it}$ corresponds to the standard definition of technical inefficiency, so that $TE_{it} = \exp(-u_{it})$. For estimation, a critical assumption is the absence of correlation between $u_{it}$ and the input levels in (7). Therefore, it is important to show explicitly the definition of $u_{it}$ in the model:

$$u_{it} = \ln y_{it}^* - \ln y_{it}$$
$$= \left( \beta_m + \sum_{k=1}^{K} \beta_{km} \ln x_{itk} \right) (m_i^* - m_i) + \frac{1}{2} \beta_{mm} (m_i^* - m_i)^2 \geq 0.$$
While $\ln x_{it}$ does appear in $u_{it}$ we assume that it does not influence $(m^*_i - m_i)$. Thus, $u_{it}$ is of the form $a_i + \sum_k (m^*_i - m_i) g(x_{kit})$, and each term $(m^*_i - m_i) g(x_{kit})$ will, by virtue of the presence of the freely varying $(m^*_i - m_i)$ be uncorrelated with $x_{kit}$. We note, this is essentially the argument of Zellner et al. (1966), who argued that in a production function, the input levels would be uncorrelated with the deviation of output from the optimal output, even though they would obviously be correlated with the actual output, itself.

Although (7) involves an unobservable variable $m^*_i$, we can translate it into an empirically estimable form. The result follows from the fact that the unobservable can be seen as a ‘random effect’ in a panel data model. For that purpose, we rewrite (7) as follows:

$$\ln y_{it} = \left(\alpha + \beta_m m^*_i\right) + \frac{1}{2} \beta_{mm} m^*_{i} + \sum_{k=1}^{K} (\beta_k + \beta_{km} m^*_i) \ln x_{it} + \frac{1}{2} \sum_{k=1}^{K} \sum_{l=1}^{K} \beta_{kl} \ln x_{it} \ln x_{it} + v_{it} - u_{it}.$$  

(9)

In this form, the model takes the appearance of a random coefficients stochastic frontier model [See Tsionas (2002), Greene (2003)] although it differs from more familiar random coefficients models in three respects. First, the random component of each random parameter is the same, $m^*_i$. Second, the square of the random component appears in the model. Despite these complications the model is estimable. Third, only the constant and first order terms are randomly distributed.

The estimation of the model has two important requirements that remain to be considered. First, the random coefficients model requires as an identification condition that the random components of the coefficients be uncorrelated with the explanatory variables. The random component of the coefficients in our model is the level of management that defines the frontier $(m^*_i)$, which likely is correlated with at least some of the inputs.\(^5\) In order to avoid this problem, we take the approach suggested for random effect models by Chamberlain (1984) (and borrow from early work by Mundlak) and specify $m^*_i$ as a function of inputs in the following way:

$$m^*_i = \tau \ln x_i + w_i$$  

(10)

\(^5\) Note this is a different issue from the deviation of $m_i$ from $m^*_i$ which we argued above is uncorrelated with the inputs.
where \( \text{lnx} \) is the vector of the means of the logs of inputs, \( \tau \) is a vector of parameters to be estimated (a constant term will not be identified) and \( w_i \) is a random term that follows a standard normal distribution and which we assume is uncorrelated with the inputs.

The second issue concerns the stochastic specification of \( u_{it} \). Maximum likelihood estimation of the model in (9) requires a distributional assumption about \( u_{it} \). In this paper, we model the distribution of \( u_{it} \) as half normal, which produces a random coefficients stochastic frontier model in the spirit of Aigner et al. (1977).

3.2. Estimation of the Random Coefficients Stochastic Frontier Model

This section will describe the method used to estimate the parameters of the random coefficients stochastic frontier model (RCSFM).

From (7), we define

\[
\varepsilon_{it} = v_{it} - u_{it}
\]

In what follows, it is useful to note explicitly that \( \varepsilon_{it} \) will be conditioned on \( m_i^* \). The conditional density for a single observation in the half normal stochastic frontier model is

\[
f(\varepsilon_{it} | m_i^*) = \frac{2}{\sigma} \phi \left( \frac{\varepsilon_{it}}{\sigma} \right) \Phi \left( -\frac{\lambda \varepsilon_{it}}{\sigma} \right)
\]

where \( \phi(z) \) and \( \Phi(z) \) denote the density and CDF of the standard normal variable, respectively. The parameter \( \lambda \) is the ratio of \( \sigma_u/\sigma_v \), while \( \sigma^2=\sigma_v^2+\sigma_u^2 \). [See Aigner et al., (1977)]. The joint density for \( T \) observations on firm \( i \) is

\[
f(\varepsilon_{i1}, \ldots, \varepsilon_{iT} | m_i^*) = \prod_{t=1}^{T} f(\varepsilon_{it} | m_i^*).
\]

This is the contribution to the conditional likelihood for firm \( i \), \( L_i | m_i^* \). The unconditional contribution to the likelihood function is

\[
L_i = \int_{m_i^*} \prod_{t=1}^{T} f(\varepsilon_{it} | m_i^*) g(m_i^*) dm_i^* \quad (14)
\]

where \( g(m_i^*) \) is the marginal density of \( m_i^* \). Consistent with the preceding discussion, there are no new parameters in this density. The log likelihood is

\[
\log L(\delta) = \sum_{i=1}^{N} \log L_i(\delta) \quad (15)
\]
where we use \( \delta \) to denote the full vector of parameters in the model. The maximum likelihood estimates of the parameters are obtained by maximizing (15) with respect to \( \delta \). Since the integral in (14) will not have a closed form, it is not possible to maximize (15) directly. We will use the method of maximum simulated likelihood, instead. [See Train (2003), Greene (2003) and Gourieroux and Monfort (1996) for discussion].

4. EMPIRICAL APPLICATION

Our empirical application uses data from a balanced panel of 247 dairy farms located in Northern Spain. We have data on these farms for a period of six years (1993-1998). Since the farms are specialized in milk production we consider only one output. The variables used in the estimation of the production frontier are described in Table 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milk</td>
<td>131,107</td>
<td>92538</td>
<td>14110</td>
<td>727,281</td>
</tr>
<tr>
<td>Cows</td>
<td>22.12</td>
<td>11.27</td>
<td>4.5</td>
<td>82.3</td>
</tr>
<tr>
<td>Labor</td>
<td>1.67</td>
<td>0.55</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Land</td>
<td>12.99</td>
<td>6.17</td>
<td>2</td>
<td>45.1</td>
</tr>
<tr>
<td>Feed</td>
<td>57941</td>
<td>47981</td>
<td>3924</td>
<td>376,731</td>
</tr>
</tbody>
</table>

We wish to explore the empirical consequences of restrictions on the role of management in the production function. For that purpose, we first estimate a conventional (pooled) stochastic production frontier with four inputs and including time-effects (this is equivalent to estimating equation (7) without considering management). The results of the estimation of the production frontier can be seen in the first column of estimates in Table 2. Since the explanatory variables in the original data were divided by their geometric mean, the first order coefficients can be interpreted as output elasticities evaluated at the mean of the sample. They are positive and significantly different from zero at conventional levels of significance.

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6 We include a summary of the main features of the simulation in the appendix.

7 All models were estimated using LIMDEP 8.0 (Greene, 2002).
Table 2. Estimated Frontier Models

<table>
<thead>
<tr>
<th>Variable</th>
<th>Param.</th>
<th>Stochastic Frontier</th>
<th>Random Coefficients Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Means of Random Parameters, $\beta_k$</td>
<td>Management $\times$ Inputs, $\beta_{km}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stochastic Frontier</td>
<td>Random Coefficients Model</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta_k$</td>
<td></td>
</tr>
<tr>
<td>Cows</td>
<td>$\beta_1$</td>
<td>0.6106 (0.0223)**</td>
<td>0.6597 (0.0113)**</td>
</tr>
<tr>
<td></td>
<td>$\beta_2$</td>
<td>0.0254 (0.0128)**</td>
<td>0.0406 (0.0081)**</td>
</tr>
<tr>
<td>Labor</td>
<td>$\beta_3$</td>
<td>0.0239 (0.0146)**</td>
<td>0.0251 (0.0013)*</td>
</tr>
<tr>
<td>Feed</td>
<td>$\beta_4$</td>
<td>0.4393 (0.0121)**</td>
<td>0.3090 (0.0060)**</td>
</tr>
<tr>
<td>Constant</td>
<td>$\alpha$</td>
<td>11.719 (0.0128)**</td>
<td>11.6761 (0.0041)**</td>
</tr>
<tr>
<td>Management</td>
<td>$\beta_m$</td>
<td></td>
<td>0.1083 (0.0014)**</td>
</tr>
<tr>
<td>Management $\times$</td>
<td>$\beta_{mm}$</td>
<td></td>
<td>-0.0143 (0.0018)**</td>
</tr>
<tr>
<td>Cows $\times$ Cows</td>
<td>$\beta_{11}$</td>
<td>0.5556 (0.1158)**</td>
<td>0.1512 (0.0479)**</td>
</tr>
<tr>
<td>Land $\times$ Land</td>
<td>$\beta_{22}$</td>
<td>-0.1042 (0.0457)**</td>
<td>-0.0425 (0.0173)**</td>
</tr>
<tr>
<td>Labor $\times$ Labor</td>
<td>$\beta_{33}$</td>
<td>0.2116 (0.0371)**</td>
<td>-0.1275 (0.0334)**</td>
</tr>
<tr>
<td>Feed $\times$ Feed</td>
<td>$\beta_{44}$</td>
<td>-0.01150 (0.0964)</td>
<td>0.0962 (0.0151)**</td>
</tr>
<tr>
<td>Cows $\times$ Land</td>
<td>$\beta_{12}$</td>
<td>-0.1078 (0.0507)**</td>
<td>-0.0186 (0.0209)</td>
</tr>
<tr>
<td>Cows $\times$ Labor</td>
<td>$\beta_{13}$</td>
<td>-0.3390 (0.0638)**</td>
<td>0.0936 (0.0241)**</td>
</tr>
<tr>
<td>Cows $\times$ Feed</td>
<td>$\beta_{14}$</td>
<td>0.2230 (0.0753)**</td>
<td>-0.0780 (0.0258)**</td>
</tr>
<tr>
<td>Land $\times$ Labor</td>
<td>$\beta_{33}$</td>
<td>0.0694 (0.0263)**</td>
<td>0.0003 (0.0148)</td>
</tr>
<tr>
<td>Land $\times$ Feed</td>
<td>$\beta_{34}$</td>
<td>-0.0027 (0.0500)</td>
<td>-0.0186 (0.0108)</td>
</tr>
<tr>
<td>Labor $\times$ Feed</td>
<td>$\beta_{34}$</td>
<td>-0.0695 (0.0411)</td>
<td>-0.0074 (0.0143)</td>
</tr>
<tr>
<td>Year 1994</td>
<td>$\delta_{94}$</td>
<td>-0.0363 (0.0120)**</td>
<td>0.0315 (0.0044)**</td>
</tr>
<tr>
<td>Year 1995</td>
<td>$\delta_{95}$</td>
<td>-0.0198 (0.0126)</td>
<td>0.0578 (0.0044)**</td>
</tr>
<tr>
<td>Year 1996</td>
<td>$\delta_{96}$</td>
<td>-0.0013 (0.0117)</td>
<td>0.0675 (0.0044)**</td>
</tr>
<tr>
<td>Year 1997</td>
<td>$\delta_{97}$</td>
<td>-0.0067 (0.0121)</td>
<td>0.0715 (0.0043)**</td>
</tr>
<tr>
<td>Year 1998</td>
<td>$\delta_{98}$</td>
<td>-0.0110 (0.0125)</td>
<td>0.0902 (0.0044)**</td>
</tr>
<tr>
<td>$\lambda$</td>
<td></td>
<td>1.8657 (0.1521)**</td>
<td>1.1994 (0.0490)**</td>
</tr>
<tr>
<td>$\sigma$</td>
<td></td>
<td>0.1932 (0.0061)**</td>
<td>0.0961 (0.0014)**</td>
</tr>
<tr>
<td>Log L</td>
<td></td>
<td>860.649</td>
<td>1401.562</td>
</tr>
</tbody>
</table>

Standard errors in parentheses.
We now estimate the random coefficients model specified in equation (9) following the estimation procedure described previously. The results, which were obtained using 1,000 Halton draws in each replication, can be seen at the right in Table 2. The means for the random parameters remain positive and significant at the geometric mean of the sample. These means differ slightly from the coefficients of the conventional stochastic frontier. The coefficients of management ($\beta_m$, $\beta_{mm}$ and $\beta_{km}$) are significantly different from zero at conventional levels of significance. This can be interpreted as evidence in favor of the random coefficients model with respect to the conventional stochastic frontier approach since the coefficients of the production function change with the level of management of the farm. The coefficients of the interactions of management with inputs ($\beta_{km}$) have a positive sign for cows and labor and a negative sign for feed and land. Finally, the negative sign of management squared can be reasonably interpreted as evidence that management has a positive but decreasing effect on production.

The efficiency levels computed in the model with fixed management can be computed according to Jondrow et al.’s (1982) prescription conditioned on $m_i^*$. 

$$E[u_{it} \mid \varepsilon_{it}, m_i^*] = \frac{\sigma \lambda}{(1 + \lambda^2)} \left[ \frac{\phi(-\varepsilon_{it} | m_i^* \lambda / \sigma)}{\Phi(-\varepsilon_{it} | m_i^* \lambda / \sigma)} \right]$$  (16)

Like other quantities that involve $m_i^*$, this can be computed directly by simulation. We have, instead, computed estimates of $m_i^*$ as described below, then computed the values in (16) conditioned on the estimates of $m_i^*$ and the other data for farm $i$. The value of $m_i^*$ can be computed from the conditional distribution of $m_i^*$ given the data on farm $i$ using Bayes theorem as follows: Let $y_i$ denote the $T \times 1$ vector of logs of the outputs for farm $i$ for the six years. Let the $T \times K$ matrix $X_i$ denote the other data (inputs and year dummy variables, linear and quadratic terms in logs) for farm $i$. The conditional distribution of $m_i^*$ given $y_i$ is

$$f(m_i^* \mid y_i, X_i) = \frac{f(y_i \mid m_i^*, X_i)g(m_i^*)}{f(y_i \mid X_i)}$$

$$= \frac{f(y_i \mid m_i^*, X_i)g(m_i^*)}{\int_{m_i^*} f(y_i \mid m_i^*, X_i)g(m_i^*)dm_i^*}$$  (17)
The denominator is the contribution of farm $i$ to the likelihood function for the sample (not the log likelihood) in equation (14). Thus, we can estimate $m_i^*$ for farm $i$ as the conditional mean from this distribution. This would be

$$
\hat{m}_i^* = \hat{E}(m_i^* \mid y_i, X_i) = \frac{\int_{m_i} m_i^* f(y_i \mid m_i^*, X_i) g(m_i^*) dm_i^*}{\int_{m_i} f(y_i \mid m_i^*, X_i) g(m_i^*) dm_i^*}
$$

(18)

Like the likelihood, itself, this quantity cannot be computed directly, as the integrals will not have a closed form. But, they can be simulated, in the same fashion. Thus, the simulation based estimator of $m_i^*$ is

$$
\hat{E}(m_i^* \mid y_i, X_i) = \frac{(1/R) \sum_{r=1}^{R} m_{i,r}^* \hat{f}(y_i \mid m_{i,r}^*, X_i)}{(1/R) \sum_{r=1}^{R} \hat{f}(y_i \mid m_{i,r}^*, X_i)}.
$$

(19)

where $m_{i,r}^*$ is a draw from the population of $m_i^*$. Note, $\hat{f}$ denotes the portion of the likelihood function for farm $i$, evaluated at the parameter estimates and the current draw of $m_{i,r}^*$. Draws on $m_i^*$ are obtained by drawing $w_i$ in (10) from the standard normal distribution using Halton sequences. With these estimates in hand, estimated inefficiencies for the farms are produced using (16).

Table 3 presents descriptive statistics for the inefficiency estimates produced from the basic stochastic frontier model and from the random coefficients stochastic frontier model. The distribution for the latter set of estimates has a much smaller mean and a much tighter distribution. Figure 1 below suggests the same pattern. This suggests that not accounting for the effect of management on production has somewhat inflated the estimated inefficiency. One might view this as a decomposition of the inefficiency into two parts, one explicitly accounted for by the management effect and the other apparently due to other unexplained factors.
Table 3. Descriptive Statistics for Estimated Inefficiencies and Estimated Management

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated Inefficiency</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic Stochastic Frontier</td>
<td>0.1352</td>
<td>0.0794</td>
<td>0.0107</td>
<td>0.4747</td>
</tr>
<tr>
<td>Random Coefficients SF Model</td>
<td>0.0581</td>
<td>0.0256</td>
<td>0.0116</td>
<td>0.2637</td>
</tr>
<tr>
<td>Estimated Management</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frontier Management, $m_i^*$</td>
<td>0.0634</td>
<td>1.0799</td>
<td>-2.8240</td>
<td>3.4772</td>
</tr>
</tbody>
</table>

Table 3 also presents descriptive statistics for the estimates of $m_i^*$ based on (19). Since the units of measurement of $m_i^*$ are unknown, these values do not have a direct interpretation relative to any observed quantities. However, $m_i^*$ does play a role in the computation of output and estimated elasticities, as seen below.

Our model allows for a complex role of management in the production function. For example, it is possible to gauge the effects of optimal management in the technological characteristics. The output elasticity at the frontier for input $k$ has the following expression:

$$\frac{\partial \ln y_{it}^*}{\partial \ln x_{itk}} = \beta_k + \sum_{i=1}^{K} \beta_{kl} \ln x_{itl} + \beta_{km} m_i^*$$  \hspace{1cm} (20)

We have calculated the set of output elasticities at the frontier for each of the inputs. The results are shown in Table 4 evaluated at the geometric mean of the inputs and estimated mean of $m_i^*$ shown in Table 3.

Table 4. Output elasticities at the frontier

<table>
<thead>
<tr>
<th></th>
<th>Output elasticity at the frontier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cows</td>
<td>0.6344</td>
</tr>
<tr>
<td>Land</td>
<td>0.0408</td>
</tr>
<tr>
<td>Labor</td>
<td>0.0229</td>
</tr>
<tr>
<td>Feed</td>
<td>0.3095</td>
</tr>
<tr>
<td>RTS</td>
<td>1.0076</td>
</tr>
</tbody>
</table>

Expression (20) shows that management affects the output elasticities through the parameters $\beta_{kn}$. In (7), these are the parameters of the interaction between management and conventional inputs.
The estimated values of the parameters $\beta_{km}$ were shown in Table 2. In the present case, management negatively affects the output elasticities of cows and labor and positively affects the output elasticities of land and feed. The estimates of $\beta_{km}$ are significantly different from zero at conventional levels of significance. However, the results in Table 2 show that the estimated values of these parameters is small compared with the value of output elasticities. Consider, however, the direct effect of management, 

$$\frac{\partial \ln y_{it}^{*}}{\partial m_{i}^{*}} = \beta_{m} + \beta_{nm} m_{i}^{*} + \sum_{l=1}^{\kappa} \beta_{lm} \ln x_{i,l}$$

Evaluated at the mean estimated value of $m_{i}^{*}$ of 0.0634 and the geometric means of the inputs, this is roughly equal to 0.1059. While it is difficult to be specific about the margin involved – we could not state specifically what it would mean to increase optimal management by one unit – this result does suggest that in the terms of the unobserved factor, management is clearly important in the determination of output. (We do note that the preceding results are in terms of optimal management, $m_{i}^{*}$, not actual management, $m_{i}$. It seems reasonable to assume that these would at least be highly positively correlated, so at least the qualitative result would persist.)

The sum of output elasticities for the conventional factors shown in Table 4 is a measure of returns to scale. The measure depends on the sum of the parameters $\beta_{km}$, which by our estimates is extremely small. These parameters take on positive and negative values and their sum is a very small number. Therefore, in terms of the observed inputs, management plays a very small role in explaining returns to scale. However, when management is considered directly as an input, it adds 0.1059, or about 11% to the total. This suggests that when management is considered among the factors of production, at least for this application, there is a moderate degree of returns to scale.

In summary, management plays a complex role in production far from the simplistic approach implied by a production frontier with fixed coefficients. We find that the fixed coefficients frontier is not a good instrument to analyze firm behavior when management is unobservable. This result is important in a number of settings. For example, unobserved management is a key factor in explaining firm size and growth (Jovanovic, 1982). Another example is farm policy, where the level of management is important for assessing the effects of increasing the size of farms (Dawson and Hubbard, 1985; Alvarez and Arias, 2003). This is an important issue for policy purposes since in the agricultural sector it is common to implement...
farm policies oriented towards farm growth. These policies usually consist of low interest loans that allow farmers to buy more inputs and therefore disregard the important implications for technology characteristics of holding management constant, as shown in previous sections.

5. CONCLUSIONS

This paper explores the relationship between managerial ability and technical efficiency. For that purpose, fixed managerial ability is introduced as an unobservable input in a translog production function. The interaction between the unobservable input and conventional inputs implies that technical efficiency depends not only on management but also on input use. As a result, fixed management can lead to time variant technical efficiency if input use changes over time. This is an important insight not considered in models with time invariant efficiency based in the assumption of fixed management.

The interaction between the unobservable input and conventional inputs creates a great deal of difficulty in estimating the resulting model. However, the model can be cast as a stochastic frontier with random coefficients. The unobserved variable is integrated out leading to a likelihood function with non trivial definite integrals. This is the reason why the model is estimated by maximum simulated likelihood.

We illustrate the feasibility of the proposed estimation procedure using a sample of dairy farms in Northern Spain. The mean of the random coefficients are of similar magnitude as the coefficients of a standard stochastic frontier, but managerial ability is found to affect the value of a number of random input coefficients. As a result, the random coefficients frontier provides a measure of technical efficiency that can be related with different levels of management depending on the circumstances of the farm (input use and output production). The feasibility of estimating both the farm current level management and the level of management that defines the production frontier is a clear advantage of our model.

The empirical model developed in the paper can be useful in analyzing firm policy issues since management is considered a key variable in assessing the effects of these policies. In fact, the model avoids both the simplistic assumptions about management implied by fixed coefficients frontiers and the econometric problems associated with the use of proxies.
References


Figure 1. Kernel Density Estimates for Distribution of Inefficiency Across Firms
APPENDIX.

Let $m_{ir}^*$ denote the $r$-th random draw from the standard normal population of $m_i^*$ in a sample of $R$ such draws. Then, the contribution to the simulated likelihood function for firm $i$ is

$$L_i^S = \frac{1}{R} \sum_{r=1}^{R} \left( \prod_{t=1}^{T} f(e_{it} | m_{ir}^*) \right)$$  \hspace{1cm} (A.1)

The simulated log likelihood that is maximized is

$$\log L_i^S (\delta) = \sum_{i=1}^{N} \log \left( \frac{1}{R} \sum_{r=1}^{R} \prod_{t=1}^{T} f(e_{it} | m_{ir}^*) \right)$$ \hspace{1cm} (A.2)

where $f(e_{it}|m_{ir}^*)$ appears in (12). This function is smooth and twice continuously differentiable in the parameters. (For conditions under which maximization of the simulated log likelihood produces an estimator with the same asymptotic properties as the true MLE, see Gourieroux and Monfort (1996), Train (2003) and Greene (2003)). The derivatives of the simulated log likelihood are obtained as follows:

$$\frac{\partial \log L_i^S}{\partial \delta} = \frac{\partial L_i^S}{L_i^S}$$

$$= \frac{\sum_{i=1}^{N} \sum_{r=1}^{R} \left( \prod_{t=1}^{T} f(e_{it} | m_{ir}^*) \right) \prod_{t=1}^{T} \left[ \partial \log f(e_{it} | m_{ir}^*) / \partial \delta \right]}{\sum_{i=1}^{N} \sum_{r=1}^{R} \prod_{t=1}^{T} f(e_{it} | m_{ir}^*)}$$ \hspace{1cm} (A.3)

where $\omega_{ir}$ is a set of nonnegative weights that sum over $r$ to one for each $i$ by construction and $g_{it}(m_{ir}^*)$ is the vector of derivatives, $\partial \log f(e_{it} | m_{ir}^*) / \partial \delta$. Let

$$x_{itr}^* = [1, \ldots, \ln x_{it1}, \ldots, \ldots, \frac{1}{2} \ln x_{itk}, \ln x_{itl}, \ldots, m_{ir}^*, \frac{1}{2} m_{ir}^*, \frac{1}{2}, \ldots, \frac{1}{2} \ln x_{itk} m_{ir}^*]$$

so that $e_{it}|m_{ir}^* = y_{it} - \beta x_{it}(m_{ir}^*) = e_{itr}^*$. For convenience, let $h_{itr}^* = \phi(-\lambda e_{itr}^*/\sigma) \Phi(-\lambda e_{itr}^*/\sigma)$. The required first derivatives are

$$\frac{\partial \log f(e_{itr}^*)}{\partial \sigma} = \left( \begin{array}{c} \beta \\ \sigma \\ \lambda \end{array} \right) = \left[ \begin{array}{c} \left( e_{itr}^* / \sigma \right) + h_{itr}^* \lambda \\ e_{itr}^* / \sigma - 1 + e_{itr}^* / \sigma \lambda h_{itr}^* \\ -e_{itr}^* h_{itr}^* \end{array} \right]$$ \hspace{1cm} (A.4)
The integrals in (14) and its derivatives are approximated by obtaining a sufficient number of draws from the population generating \( m_i^* \). The law of large numbers is invoked to infer that the sample averages will converge to the underlying integral. Random draws from the population are sufficient for this process, but not necessary. What is essential is coverage of the range of variation of \( m_i^* \), not randomness of the draws. The method of Halton sequences [see Bhat (1999), for example] is used to provide much more efficient coverage of the range, and in turn, much faster estimation than the method of random simulation. [See, as well, Train (2003, pp. 224-238) for a discussion of Halton sequences]. Thus, \( m_{ir}^* \) is the \( r \)th element of the Halton sequence for individual \( i \). The elements of the Halton sequence, \( H_{ir} \) are spread over the unit interval, \((0,1)\). The draw of \( m_{ir}^* \) is obtained by the inverse probability transform. Thus, \( m_{ir}^* = \Phi^{-1}(H_{ir}) \). The estimated standard errors of the parameter estimators are computed by using the BHHH estimator, as before, with simulation used for the derivative vectors. Since we are only integrating over a single dimension, the gain in efficiency, if this application is like others, is on the order of ten fold - that is, the same results are obtained with only about one tenth the number of draws needed. We have used 1,000 draws in our estimation, which would correspond to several thousand draws were they produced with a random number generator instead.