Calculating the Annual Return (Realized Compound Yield) on a Coupon Bond

William L. Silber

Objective:

To show that the annual return actually earned on a coupon-bearing bond will equal its yield to maturity only if you can and do reinvest the coupons at the yield to maturity.

PROOF FOR ANNUAL PAY BONDS

1. Assume:  \( F = 1000 \quad C = $80 \quad t=4 \) years

2. If \( P=100 \) we know that YTM = 8%

3. Definition of Annual Return

\[
r_{ann} = \left( \frac{V_t}{V_0} \right)^{1/t} - 1,
\]

where \( V_t = \$ \) amount at the end and \( V_0 \) is the \$ amount at the beginning. In our case

\[
V_0 = $1000 \quad \text{and} \quad t = 4
\]

therefore

\[
r_{ann} = \left( \frac{V_t}{\$1000} \right)^{1/4} - 1
\]

4. To calculate \( r_{ann} \) we must calculate \( V_t \). To calculate \( V_t \) we must account for the reinvestment of the annual 8% coupon (=\$80 per annum). Assuming we reinvest these coupons at 8%, we have the following cash flows on the bond:

<table>
<thead>
<tr>
<th>Cash Flows</th>
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<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>1st coupon</td>
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<tr>
<td>2nd coupon</td>
</tr>
<tr>
<td>3rd coupon</td>
</tr>
<tr>
<td>4th coupon + principal</td>
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<tr>
<td>FINAL TOTAL ( (V_t) ) =</td>
</tr>
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5. In this case,

\[ r_{ann} = \left( \frac{1360.49}{1000} \right)^{1/4} - 1 = 0.08 \]

Thus

\[ r_{ann} = YTM \] if you reinvest the coupons at the YTM

6. If you reinvest the coupons at more than 8\% you accumulate more than $1360.49 and earn an annual return > 0.08 and if you reinvest the coupon at less than 8\% you accumulate less than $1360.49 and earn an annual return < 0.08.

FOR SEMI-ANNUAL PAY BONDS: AN EXERCISE

1. Assume: \( F = $1000 \quad C/2 = $40 \quad t = 4 \) years

2. If \( P = 100 \) we know YTM = 8\%

3. Calculate the annual return assuming you reinvest the coupons at the YTM/2 or at 0.08/2 = 0.04

4. What is the relationship between YTM and \( r_{ann} \) in this case?