Yield Calculations for Treasury Bills

William L. Silber

Question

Suppose you could buy a 91-day T-bill at an asked price of $98 per $100 face value and you could sell to the dealer at a bid price of $97.95 per $100 face value. What are the quotation conventions on this bill and how is the yield calculated? What is the best measure of the yield on a T-bill?

Answer

1) This T-bill would be listed in a table as follows:

<table>
<thead>
<tr>
<th>Days to Maturity</th>
<th>Bid</th>
<th>Ask</th>
<th>Ask Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>91</td>
<td>8.11</td>
<td>7.91</td>
<td>8.186</td>
</tr>
</tbody>
</table>

2) The ask yield in the last column is the bond yield equivalent (b.y.e) of this T-bill. This is the yield (assuming simple interest) if you bought the bill at the ask price of 98 per 100 face value and held it for the full 91 days.

\[
B.Y.E. = \left( \frac{F - P}{P} \right) / t
\]

F = Face Value (=$100)
P = Price Paid
\( t \) = Fraction of a year

In our case,

\[
B.Y.E. = \frac{100 - 98}{98} \left( \frac{91}{365} \right)
\]

\[= .08186 = 8.186\%
\]

NB: This formula is simple interest because it comes from:

\[ P(1 + rt) = F \]
which can be solved for \( r \) as

\[
r = \left( \frac{F - P}{P} \right) / t
\]

3. The 7.91 under the word Ask in the table comes from the discount rate calculated on the bill. The discount rate is defined as:

\[
discount\ rate = \left( \frac{F - P}{F} \right) / \left( \frac{X}{360} \right)
\]

where \( X = \text{days to maturity} \)

In our case (using the ask price):

\[
(\text{ask})\ discount\ rate = \left( \frac{100 - 98}{100} \right) / \left( \frac{91}{360} \right)
\]

\[= .0791 = 7.91\%
\]

4) The 8.11 in the table under the word bid uses the same discount rate calculation as above except it uses the bid price (=97.95) in the formula

\[
(bid)\ discount\ rate = \left( \frac{100 - 97.95}{100} \right) / \left( \frac{91}{360} \right)
\]

\[= .0811 = 8.11\%
\]

5) Note that the (ask) discount rate will always be lower than the ask yield based on the b.y.e. formula because \( F \) appears in the denominator of the discount rate formula while \( P \) is in the denominator of the b.y.e. formula (and \( F>P \) as long as yields are positive). In addition, \( 360<365 \) in the year part of the formulas and those numbers wind up in the numerator.

6) Of these calculations, the best measure of yield earned when buying a T-bill is the b.y.e. since it uses \( P \) as the base rather than \( F \) (and because 365 is correct). Why does the discount rate calculation exist? Because it is a shorthand calculation that was easier before hand calculators existed. It allowed people to translate price into yield quickly. In fact, this tradition is perpetuated by dealers who quote T-bills in discount
rates rather than prices. Thus our T-bill in the table is quoted as 8.11 bid, offered at 7.91.

7) The effective annual rate on this bill would annualize the b.y.e. of 8.18% (which uses simple interest) using the familiar formula:

\[ EAR = \left(1 + \frac{\text{quoted rate}}{n}\right)^n - 1 \]

where \( n \) = number of compounding periods per year.

\[
EAR = \left(1 + \frac{.08186}{365/91}\right)^{365/91} - 1
\]

\[ = .0844 = 8.44\% \]