Dynamics of Trading Volume, Price, and Duration of Contractual Relationship

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First Draft: March 2001
This Version: August 2003

Abstract

Agency theorists consider a firm as a nexus of contractual relationships. Contracting parties such as shareholders, debt holders, managers, input suppliers, advertising agencies and retailers may have information superior to outsiders concerning the firm’s quality. In this paper, we show that the duration of contractual relationship within a firm has important implications on the dynamics of the price and trading volume of the firm’s stock. If the duration of information asymmetry corresponds to that of contractual relationship, insiders with long-term relationship is shown to marginally prefer less aggressive trading strategies than those who can only access superior information temporarily. Such behavior makes it more difficult for the market maker

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to infer the existence of informed traders. The market maker will henceforth provide narrower bid-ask spreads, which attract discretionary liquidity traders to gather their trades in that period.

With the long-term contractual relationship, uncertainty is resolved more slowly because the insider trades on her information advantage gradually. The long-term relationship also enlarges the liquidity traders’ loss and discourages them from holding the firm’s stock, and therefore reduces the amount of proceeds that the entrepreneur gathers when she makes financing. On the other hand, with the long-term relationship, a more efficient effort level can be expected, and thus the outcome of the investment plan will be higher. We conclude that the entrepreneur’s profit as well as a firm’s fundamental value will be influenced by the duration of the firm’s internal contract. The optimal contractual duration depends on the adverse selection cost and the investment efficiency.

1 Introduction

In Madhavan[20], current trading mechanisms are categorized as two types, the order-driven type and the quote-driven one. In an order-driven mechanism, traders submit their orders without specifying any price-related actions; the market maker will aggregate market orders and set a price to clear the market. In a quote-driven market such as NASDAQ, however, traders determine their orders after observing the posted bid and ask prices. Bid price is the price at which the market maker accepts a selling order, while ask price is for a buying order. In general these two prices are not the same, and the difference between them is called a spread. The prices can be contingent on the number of units that the trader would like to trade.

The bid-ask spread has been a mainstream research topic in both theoretic and empirical papers. The literature on this topic can be classified into two streams. Papers of the
first stream discuss the relationship between the bid-ask spread and inventory cost, such as Garman[10], Amihud and Mendelson[3], Stoll[27], Ho and Stoll[15][16], Cohen, et. al.[6], and O’Hara and Oldfield[22]. They argue that the bid-ask spread serves as a tool to avoid the market failure and reflect the market power of the monopoly specialist. They also emphasize that the bid-ask spread can bring only short-term influence on investors’ trading behavior. In the long run, the market maker is able to adjust her inventory positions, and the only influential factor is the asset’s fundamental value. However, these papers cannot give a persuasive and unambiguous explanation about the optimal inventory policy of the market maker.

The other stream of literature starts from the adverse-selection problem, such as Bagehot[4], Copeland and Galai[7], Glosten and Milgrom[11], and Easley and O’Hara[8]. They consider the existence of the bid-ask spread as the consequence of the information asymmetry between the market maker and insiders, investors who can access the superior information. Trading with insiders causes the market maker to lose money since she will always trade on the wrong side, and therefore she cannot ensure her break-even if there were no bid-ask spread.

This paper commences with a contract design problem within a firm, and bridges between the traders’ strategic behavior and the duration of contractual relationship. To our knowledge, the established literature lacks of such interactive discussions involving both corporate finance and market microstructure. Traditionally, corporate theorists take as given the stock value while investigating the managerial contract and ownership design, and microstructure economists disregard the influence of stock price on the firm’s fundamental value. However, the real business is an integration of these two parts. Deriving a partial equilibrium can barely provide a full characterization of the real world.

Why will a firm’s internal contract influence its stock market? Jensen and Meckling[17] consider a firm as a nexus of contractual relationships, where contracting parties such as shareholders, debt holders, managers, input suppliers, advertising agencies and retailers
may have information superior to outsiders concerning the firm’s quality, see also Rajan-Raghuram [24], Bolton and Dewatripont [5], and Villas-Boas [28]. Therefore, the duration of contractual relationship thus implies the duration of the information advantage that the insider will hold.

On the other hand, how does a firm’s stock market affect its fundamental value? Manove[21] points out that when buying the newly issued securities from a firm, a rational investor will based on the adverse selection problem adjust the proceeds she provides. She will take into account the fact that, if subsequently she has to engage in liquidity trading, she might have to trade with insiders. Thus if rational investors’ expectation of insider trading is high, their willingness to pay for the securities will be low. Without enough cash inflow, the quality of the investment plan may not achieve its first-best level.

The purpose of this paper is to investigate the impact of the duration of contractual relationship on commonly arrestive market criteria: stock price, trading volume, informational efficiency, traders’ welfare, and the firm’s value. In this paper, an entrepreneur has an investment plan in hand, and she is seeking contracting parties as well as the capital from outside investors to implement it.\(^1\) In order to make the best of this investment plan, the entrepreneur can change her contracting counterpart regularly or have a long-term cooperation with a specific partner. Here choosing a long-term cooperation does not mean simply signing a long-term collaborator, but to commit a long-term contract instead. As the entrepreneur resigns a contract with a specific partner period by period, we recognize this as a short-term case. Within the duration of the contractual relationship, this position may bring superior information to the contracting party.

Our model for the financial market is extended from the single-period framework in Easley and O’Hara [8]. They discuss the trading behavior of insiders with short-term information advantage in a quote-driven market in which the market maker and liquidity traders also

\(^{1}\)Such a scenario is commonly adopted in corporate literature, see Hart[13].
participate. In that model, there exist two mutually exclusive market equilibria. In the separating equilibrium, insiders trade aggressively to exploit their private information. In the pooling equilibrium, however, they disguise themselves by trading randomly as liquidity traders. Easley and O’Hara also claim that in the extended multi-period model, insiders will repeat their single-period optimal strategy.

While the insider possesses a long-term position of acquiring superior information, to simply maximize her profit in each period is no longer optimal even though the acquired information stays private for only one period. If the entrepreneur signs a long-term contract with her partner, this contracting party will intentionally turn to trade randomly in the early stage of the contractual relationship and make the market maker unable to clearly identify who the trader is. In the sequel the market maker adjusts downward her belief of the information asymmetry and sets a narrower bid-ask spread in the latter stage, which creates profitable room for the long-term insider.

In order to discuss the dynamics of trading volume in this quote-driven setting, we introduce discretionary liquidity traders, investors who can choose the appropriate time to fulfill their liquidity demand before a specific deadline. 2 We find that, if the entrepreneur signs a long-term contract, discretionary liquidity traders will intend to participate in the latter stage due to the relatively favorable prices there. This movement increases the market depth in the latter stage, and further enhances the incentive of insiders to imitate the uninformed traders in the initial stage. However, due to the quote-driven mechanism, the participation of discretionary liquidity traders reduces the trading opportunities of incumbent traders. Insiders may benefit from the narrower bid-ask spread, but this benefit realizes with a lower probability.3

2See Admati and Pfeiderer[1][2], Foster and Viswanathan[9], Seppi[25], and Spiegel and Subramanyam[26].
3The result is different from that in an order-driven market. In Kyle[19], the liquidity traders will cover insiders’ trading, and hence as the number of liquidity traders increase, insiders’ profit will be raised up
We demonstrate that the duration of contractual relationship will also influence the informational efficiency, traders’ welfare, and the firm’s fundamental value. Comparing a long-term regime to a short-term regime, due to the random trading of the long-term insider, the market maker cannot by orders distinguish from insiders and liquidity traders, so the information reveals more gradually, i.e., the informational efficiency will be lower. Moreover, a short-term insider’s aggressive trading will reveal rapidly not only the information content but the existence of the information asymmetry, and therefore it will reduce insiders’ profit in the latter stage. Due to this externality, a long-term insider’s profit will be higher than the aggregate profit of short-term insiders, and the expected loss of uninformed traders will be higher because the security trading is a zero-sum game.

To hold a long-term contract relationship will in general induce the better level of managerial effort, and therefore the firm’s fundamental value will be higher. The entrepreneur can also pay less to the contracting party and still make her individual rationality condition satisfied since the insider trading profit can be expected ex ante. However, the adverse selection problem will also be more severe, and therefore a long-term contract brings less proceeds to the entrepreneur. While deciding the contract’s duration, the entrepreneur has to make a trade-off between these conflicting factors.

This paper is organized as follows. In Section 2, the multi-period model is introduced. Section 3 lists some results of the single-period model already established in Easley and O’Hara [8]. We derive the market equilibria of our model in Section 4, and summarize main results and discuss several modifications in Section 5. Finally, we conclude this paper in Sec. 6. Detailed proofs are presented in the Appendix.

Hart and Moore [14] provide another explanation why people tend to sign long-term contracts. They argue that when one party constructs a relationship with a specific party, the long-term contract prevents outside competitors from harming either side of them. This is called the binding effect.
2 The Model

In this section we introduce our model setup, including the firm and the financial market.

2.1 The Firm

We consider an economy with one contracting stage (period 0) and two trading stages (periods 1 and 2).

At the beginning of the world, an entrepreneur has an investment plan in hand, and she is looking for both the capital and the managers to implement it. In the contracting stage, the entrepreneur chooses to change the managers period by period or have a long-term manager.\footnote{Although we discuss only the managers in our model, the deduction can also apply to other contracting parties.} If the entrepreneur chooses to change the managers regularly, we denote these managers in periods 1 and 2 by $I_1$ and $I_2$ respectively. In contrast, $I_L$ denotes the long-term manager if a long-term contract is signed.

As long as the entrepreneur signs contracts with her partners, the duration of contractual relationship becomes common knowledge to all outsiders. Then the entrepreneur issues new securities to those outside investors. Her payoff is the proceeds she receives minus the managers’ salary.

With probability $\alpha$, the managerial position will bring the managers private information concerning the outcome of this investment plan, where $0 < \alpha < 1$. We must clarify that here information advantage does not guarantee a manager to acquire private information; instead, it is merely an access through which the managers may become informed. Since this superior information comes from the position, if information asymmetry occurs, it will last for all trading periods. That is, either managers in both periods possess private information or they are both uninformed.

If the manager receives information at the beginning of the period, she is able to do insider
trading in the financial market. If information asymmetry does not occur, she behave as uninformed liquidity traders, whose behavior will be described momentarily. Here we assume that the insider trading of the manager cannot be fully prohibited by the entrepreneur or other outsiders. Since the manager can do insider trading via her relatives or friends’ accounts, to fully prohibit it might be extremely costly and cannot be implemented in the real world. Moreover, managers may come across liquidity shock when they are at the position, to prohibit managers’ trading may make them unwilling to provide the first-best effort level.

The manager’s payoff is the combination of her compensation from the entrepreneur and the insider trading profit that she can earn within her contract’s duration. So a manager with short-term contract cares about her single-period profit, and a long-term manager cares about her two-period profit.

2.2 The Financial Market

After the contracting stage, traders can exchange their shares in the financial market. We consider a two-period quote-driven market modified from Easley and O’Hara[8].

Two assets are held: the stock and the cash. For simplicity we assume that the risk-free rate is 0, and it is simple to get the time preference involved. The value of the stock is $(\xi_1 + \xi_2 + z)$, where $z$ represents the investment efficiency. Since a long-term corporation will in general bring a better level of managers’ effort or learning benefit, $z$ should exhibit this dominance property. For simplicity, $z=0$ if the entrepreneur changes the managers period by period, and $z=1$ if she hires a long-term manager.

$\xi_i$ will become public at the end of this period. If the information asymmetry does occur, it is acquired by the informed trader at the beginning of period $i$. Except herself, other traders do not know whether the manager receives the private information or not; if the manager does receive it, they cannot know what its content is. $\xi_i$ has two equally probable
outcomes, states L and H. In state L, \( \xi_1 = 0 \), while in state H, \( \xi = 1 \).

There are three types of participants in the market, including insiders, liquidity traders, and the market maker. In this model, an insider, also called an informed trader, is the manager who receives the private information. Based on her private information, the insider will trade with the market maker so as to maximize her profit.

A liquidity trader will have liquidity demand, and therefore she has to trade in the market to fulfill her demand. The demand will be one of the following four cases, buying one unit, buying two units, selling one unit, or selling two units, denoted by \( B^1, B^2, S^1, \) and \( S^2 \) respectively. We assume that the liquidity trader would like to buy or sell stock with equal probability. Given that she wants to buy or sell the stock, she would like to trade one unit with probability \( \epsilon \), and two units with probability \( 1 - \epsilon \).

To discuss the dynamics of traders distribution, we introduce two types of liquidity traders, the discretionary liquidity traders and the nondiscretionary ones. A discretionary liquidity trader is allowed to choose the time to submit her orders among several periods. Therefore, besides fulfilling her demand, she also aims at minimizing her trading loss. So she will intend to trade this firm’s stock if at that time the price is favorable; otherwise she may wait for subsequent periods or switch to other securities. In this model, for simplicity we assume that a discretionary trader will know her demand between periods 1 and 2, and therefore she is considering to attend the market in the second period or switch to other security markets. The number of discretionary liquidity traders is \( n_D \).

Unlike the discretionary liquidity trader, a nondiscretionary liquidity trader has to trade right after her liquidity demand reveals. Henceforth, if a nondiscretionary liquidity trader knows her demand at the beginning of period \( i \), she will participate the market in period \( i \); she will leave the market whether she fulfills her demand or not. \(^6\) The population of

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\(^6\) As the same as in Easley and O’Hara[8], the only crucial assumption we need is the bounded set of liquidity traders’ orders.

\(^7\) We can regard this liquidity demand as an emergency event for the nondiscretionary liquidity trader. If
nondiscretionary liquidity traders in two periods are the same, and we denote it by \( n \). Their demands are independently identically distributed, however.

In a quote-driven market, the market maker posts out the bid-ask spread and trades with one person in each period. Since a liquidity trader will submit only two different-sized orders, the market maker shall set prices when she buys one unit or two units (denoted by \( B^1 \) and \( B^2 \)), or sells one unit or two units (denoted by \( S^1 \) and \( S^2 \)). For all other cases, all traders know that the trading counterpart is an insider. The market maker is competitive, and therefore her spread makes her break-even. All players in this model are risk-neutral.

For convenience we set the proportion of the insider to nondiscretionary liquidity traders is \( \mu : 1 - \mu \). That is, \( \mu = \frac{1}{1+n} \). We say that it is a small-quantity order when they trade one unit, and a large-quantity order when they trade two units. Fig. 1 shows the time flow of this model.

### 3 A Review of the Single-Period Equilibrium

In this section we review the results of the single-period model derived by Easley and O’Hara [8]. They demonstrate that based on the population ratio of liquidity traders, there exist two mutually exclusive market equilibria, presented as the following two propositions. Please see their paper for the proofs.

**Proposition 1.** If \( 2 > \frac{2(1-\alpha\mu)(1-\epsilon)}{(1-\alpha\mu)(1-\epsilon)} \), there exists a separating equilibrium, in which the insider will buy 2 units when the signal is \( H \), and sell 2 units when the signal is \( L \). The stock prices of buying or selling a small quantity are both \( 1/2 \) and those of a large quantity are described below:

\[
b(S^2) = \frac{1}{2}(1 - \alpha\mu)(1 - \epsilon), \quad a(B^2) = \frac{\alpha\mu + \frac{1}{2}(1 - \alpha\mu)(1 - \epsilon)}{\alpha\mu + (1 - \alpha\mu)(1 - \epsilon)}.
\]

she fails to trade the security in the oncoming period, the penalty is imposed in no time, and therefore to postpone her trade does not make any sense.
The expected profit of the informed trader $\pi_s$ is 
\[
\frac{(1-\alpha\mu)(1-\epsilon)}{\alpha\mu+(1-\alpha\mu)(1-\epsilon)}.
\]

When the number of liquidity traders is large or the probability with which the information asymmetry exists is small, the insider will adopt a pure strategy. Knowing this, the market maker accepts a one-unit order at price 1/2 because it is certainly submitted by a liquidity trader. The price of a large-quantity order is set according to the probability with which it comes from the informed trader.

**Proposition 2.** A pooling equilibrium exists when no separating one can be sustained. In a pooling equilibrium, the insider randomizes between submitting one-unit or two-unit orders with probabilities $\beta$ and $1-\beta$, where $\beta$ makes her indifferent to do so. The bid and ask prices are

Figure 1: Time flow diagram of relevant events.
The expected profit of the informed trader $\pi_p$ is
$$\frac{(1 - \alpha \mu)\epsilon}{\alpha \mu \beta + (1 - \alpha \mu)\epsilon} = (1 - \alpha \mu)(2 - \epsilon).$$

If the inequality in Prop. 1 does not hold, the adverse selection problem is so severe that the insider is worse off if she adopts a pure strategy. As a result, in equilibrium she will use a mixed strategy which makes her look like a liquidity trader. In response the market maker sets fair bid and ask prices based upon the ratio at which the submitted order comes from the insider.

### 4 Equilibrium Analysis under Different Contractual Relationships

**Equilibrium Concept**: PBE

Since we are considering a multi-period model with incomplete information, the appropriate equilibrium concept should be perfect Bayesian equilibrium, abbreviated as PBE. PBE is a bundle of strategies and beliefs held by players which satisfies Bayesian updating and Sequential rationality. In words, players should hold correct initial beliefs and at each stage update by Bayes’ rule only the beliefs corresponding to the movements they observe. Furthermore, given their beliefs, their actions must be the best responses. Readers are encouraged to read Osborne and Rubinstein [23] for more details.
4.1 The Economy Without the Discretionary Liquidity Trader

We first consider the economy without the presence of the discretionary liquidity trader. In the next section we will introduce this player and see what will happen. We will apply the backward induction to obtain the equilibrium for this three-period game.

4.1.1 The Trading Stages

If the entrepreneur changes her managers period by period, we call it the short-term regime. The long-term regime refers to the case when a long-term contract is signed. In this section, we will discuss insiders’ trading strategies in the financial market under these two regimes, and we will also evaluate how other market participants respond to their strategies.

The key difference between this model and a repeated multi-period model in Easley and O’Hara[8] is that the market maker will update her belief of the existence of information asymmetry after observing the submitted order. Since all players’ actions before period 1 do not affect the probability of information asymmetry, we can denote it by $\alpha$ regardless of the contract’s duration. However, unlike Easley and O’Hara model, now we denote by $\alpha_j^i(X)$ this belief at the beginning of period 2, where the superscript $j \in \{L, S\}$ represents the long-term or short-term regime, the subscript $i \in \{s, p\}$ shows that the first-period equilibrium is separating or pooling respectively, and $X \in \{S^1, S^2, B^1, B^2\}$ is the order submitted in period 1. Sometimes we will omit the superscript or subscript if this results in no ambiguity there.

Under the short-term regime, since the insiders care about only their single-period payoffs, given the posted bid-ask spread, they will follow the optimal single-period strategy according to Easley and O’Hara. While it is the long-term regime, a long-term insider, however, aims at maximizing her two-period profit. Although in the second period, she will follow the optimal single-period strategy since there is no subsequent period, her behavior in the first period will be distorted. Now the first-period trading will affect the market maker’s posterior belief and in turn the insider’s second-period profit.
We present our major results by several lemmas and propositions:

**Lemma 1.** $\alpha_s(S^2) > \alpha_s(S^1), \alpha_s(B^2) > \alpha_s(B^1)$.

*Proof.* All proofs are in the Appendix.

Lemma 1 demonstrates that in a separating equilibrium, the probability with which the information asymmetry exists will be higher when a large-quantity order is submitted. Due to symmetry of our model, $\alpha^i_j(S^1) = \alpha^i_j(B^1), \alpha^i_j(S^2) = \alpha^i_j(B^2), \forall i, j$, and therefore in the following we mention only the selling orders.

**Lemma 2.** *The single-period expected profit of the insider decreases in $\alpha$.***

Intuitively, if the adverse selection problem is more severe, the expected profit of the insider will be lower.

**Lemma 3.** *If $\alpha$ increases, the pooling equilibrium will come out more likely.*

As $\alpha$ increases, the adverse selection cost becomes higher, and therefore the insider has to follow a mixed strategy in equilibrium. Combining all three lemmas, we obtain the following proposition right away.

**Proposition 3.** *Given that the first-period equilibrium is separating, if in the first period a one-unit order is submitted, the bid-ask spread in the next period will be narrower and sustain a separating equilibrium.*

*In contrast, if the trading counterpart submits a two-unit order in the first period, the market maker will set worse prices in the next period. In such a case, the equilibrium in the next period may be turned into a pooling one.*

The following proposition is the core of this paper.

**Proposition 4.** *Comparing the long-term regime to the short-term regime, a pooling equilibrium will come out more likely in the first period.*
This proposition points out that the threshold of sustaining a pooling equilibrium under a long-term regime is strictly lower than that under a short-term one. Henceforth there exists a range within which a separating equilibrium is sustained under a short-term regime while a pooling equilibrium survives under a long-term regime. Outside this range, traders’ behavior will be exactly the same regardless of the contract’s duration. The range, which we call Condition C, is as follows:

**Condition C:**

\[
\frac{\alpha\mu + (1 - \alpha\mu)(1 - \epsilon)}{(1 - \alpha\mu)(1 - \epsilon)} < 2 < \frac{\alpha\mu + (1 - \alpha\mu)(1 - \epsilon)}{(1 - \alpha\mu)(1 - \epsilon)} + \frac{\alpha\mu + (1 - \alpha\mu)(1 - \epsilon)}{1/2(1 - \alpha\mu)(1 - \epsilon)} \mu [\pi(\alpha_s(S^1)) - \pi(\alpha_s(S^2))]
\]

In the following discussion we will focus on the innovations within this range, and see what consequence the different first-period equilibria will result in.

**Lemma 4.** \(\alpha_s(S^1) < \alpha_p(S^1), \alpha_p(S^2) < \alpha_s(S^2)\).

Since a separating equilibrium brings more information to the market maker, after observing the order she adjusts her belief by a larger magnitude than when a pooling equilibrium exists.

Let \(E_X(Y)\) denote the second-period equilibrium when the equilibrium in the first period is \(X\), and the executed order is \(Y\), where \(X \in \{s(Separating), p(Pooling)\}\) and \(Y \in \{B^1, B^2, S^1, S^2\}\). With the help of these parameters we introduce the following proposition:

**Proposition 5.** If both \(E_s(S^1)\) and \(E_s(S^2)\) are separating, then \(E_p(S^1)\) and \(E_p(S^2)\) shall also be separating. If both \(E_s(S^1)\) and \(E_s(S^2)\) are pooling, \(E_p(S^1)\) and \(E_p(S^2)\) shall be pooling, too. However, the converse does not hold.

Proposition 5 depicts that the trading behavior of the insider in period 2 will be more volatile when the first-period equilibrium is separating.

**Proposition 6.** Suppose Condition C holds. The first-period expected profit of \(I_L\) is higher than that of \(I_1\).
Proposition 6 may look weird at the first glance. Many papers argue that, in order to earn the profit in the subsequent periods, a long-term player is willing to give up her profit at initial stages, see Kreps and Wilson [18] for example. Here we interpret this result in another way. Other things being equal, the probability with which a two-unit order comes from the insider is higher in a separating equilibrium than in a pooling one, and hence its price in a separating equilibrium shall be less favorable from a trader’s view. Note also that the existence of the second period provides a stronger support to a first-period pooling equilibrium because the insider’s single-period profits of submitting a small-quantity order and a large-quantity one need not be the same.

**Proposition 7.** Suppose Condition $C$ holds. Given that the first-period order is submitted by the insider, the expected second-period profit of $I_L$ is higher than that of $I_2$.

This proposition depicts that, if the first-period insider concerns only her single-period profit, her trading strategy will bring externality to her follower.

**Proposition 8.** Suppose Condition $C$ holds. Within a wide range the two-period profit of $I_L$ is higher than the aggregate profits of $I_1$ and $I_2$.

The myopic behavior of two short-term insiders will make their profit less than that of a long-term insider who maximizes her two-period profit. Now we stray from Condition $C$ and provide a result to finish this section.

**Lemma 5.** If $\beta < \frac{1}{2}, \alpha_p(S^1) < \alpha_p(S^2)$.

**Proposition 9.** Suppose that $2 < \frac{\alpha_p(1-\alpha_p)(1-\epsilon)}{(1-\alpha_p)(1-\epsilon)}$. The first-period equilibrium will be pooling regardless of the contract’s duration. Let $\beta^S$ and $\beta^L$ denote respectively the probabilities with which in the first period $I_1$ and $I_L$ will submit a small-quantity order. If $\beta^S < \frac{1}{2}$, then $\beta^S > \beta^L$. If $\beta^S < \frac{1}{2}$, $\alpha_p^L(S^1) > \alpha_p^S(S^1)$ and $\alpha_p^L(S^2) > \alpha_p^S(S^2)$.
4.1.2 The Contracting Stage

After analyzing players’ behavior in the trading stages, we come back to the contracting stage. Even though the entrepreneur cannot observe if the manager is doing insider trading, the expected managers’ profit in the financial market is common knowledge. In this model the entrepreneur has full bargaining power, and therefore her optimal strategy is to simply provide a take-it-or-leave-it offer which makes managers’ individual rationality condition just satisfied. Since all players are risk-neutral, a fixed compensation will do the job. We can expect that a fixed compensation of a long-term manager will be lower because she is able to earn more in the financial market according to Proposition 8.

Now the proceeds that the entrepreneur is able to collect depends on the investment efficiency factor $z$ and the expected trading loss of an uninformed trader. Under a long-term regime, $z$ is higher, but the expected trading loss is also higher. These two pulling forces conflict with each other, and therefore no unambiguous result can be provided without further assumptions. Which regime the entrepreneur will choose depends on all these three factors.

Some readers may conjecture that the optimal incentive scheme is contingent on the stock price so that the manager will be enforced not to do insider trading in the financial market. However, because the security trading is zero-sum, the amount by which the proceeds will be subtracted equals the profit that insiders can earn, which will be directly reduced from the managers’ compensation at the very beginning. Hence the entrepreneur does not bother to provide such a complicated compensation scheme.

4.2 The Economy with Discretionary Liquidity Traders

In this section, we introduce discretionary liquidity traders. We will focus on the interaction between those market participants.
4.2.1 The Trading Stages

Recalling that discretionary liquidity traders are active only in the second period, we immediately obtain the following proposition.

**Proposition 10.** Under a short-term regime, the first-period insider’s optimal strategy is independent of discretionary liquidity traders.

Since the first-period insider only has short-term interest, the presence of discretionary liquidity traders will not affect her profit. Therefore, the insider faces exactly the same optimization problem with or without discretionary liquidity traders. More generally, she does not care about all participants in the second period.

As we have mentioned, whether discretionary liquidity traders will participate the market or not depends on the bid-ask spread that the market maker posts. According to the analysis in the previous section, the second-period bid-ask spread will in general be narrower under the long-term regime than under the short-term regime. Therefore, we can expect that under the long-term regime, the number of liquidity traders in the second period will be larger. This result is formally stated in the next proposition.

**Proposition 11.** Under the long-term regime, if there are discretionary liquidity traders and their population is not too large, in the first period a pooling equilibrium will come out with a higher probability.

The reason that a pooling equilibrium will exist more likely is that the presence of the discretionary liquidity trader brings more profit for the long-term insider in the second period, and henceforth the threshold of sustaining a pooling equilibrium is even lower. The restriction on the population of the discretionary liquidity trader is due to the competition of trading opportunity in the second period. Note that this problem results completely from the characteristics of the quote-driven market. In a quote-driven market, there is no price
uncertainty for traders because they know the price at which they are going to trade, but on the other hand, their trading opportunity is not guaranteed.

**Proposition 12.** Suppose Condition C holds. Given that the first-period order is submitted by the insider, comparing a long-term regime to a short-term regime, discretionary liquidity traders will participate in the market more likely.

This proposition follows from Lemma 4. Given that the first-period order is submitted by the insider, the second-period price will be more favorable if the first-period equilibrium is pooling, and the discretionary liquidity trader will therefore be willing to trade there.

Because a discretionary liquidity trader knows her demand before she chooses the market, we can also discuss the case when only the traders with one-unit demand are willing to trade.

**Proposition 13.** If only the discretionary liquidity traders with small-quantity demand are attracted to join the market, the second-period equilibrium is the same as that without the existence of the discretionary liquidity traders regardless of the contract’s duration. However, under the long-term regime, the first-period equilibrium may be changed.

### 4.2.2 The Contracting Stage

Since under the long-term regime discretionary liquidity traders are more willing to participate in the market, the long-term insider’s profit shall be even higher. Therefore, the compensation that makes the manager’s individual rationality condition binding can be even lower.

### 5 Discussion

In this section, we summarize the observations we have found, and explicitly answer the research questions of this paper. We also provide some possible modifications of our model and discuss their effects.
5.1 Discussion of Main Results

5.1.1 Traders’ Behavior

We now discuss how the traders’ behavior will be affected by the duration of contractual relationship. Comparing a long-term regime to a short-term regime, the long-term insider will have stronger incentive to pool with the liquidity traders in the first period. Since if the long-term insider randomizes between small- or large-quantity orders, the market maker can not by orders infer the existence of information asymmetry, and henceforth she will not adjust up her belief so much. So the market maker will set more favorable second-period spreads, which create a profitable room for the long-term insider, and therefore in the first period a long-term insider will have stronger incentive to randomize between small- and large-quantity orders than a short-term insider. As a result, a pooling equilibrium turns up more likely.

This narrower bid-ask spread in the second period will attract the discretionary liquidity traders to participate in the market. This fact further enhances the incentive of insiders to camouflage the liquidity traders.

5.1.2 Market Criteria

Eventually we are ready to evaluate how these market criteria will be affected by the duration of contractual relationship. We will discuss point by point the following commonly-concerned criteria: stock price, trading volume, information efficiency, traders’ welfare, and the firm’s value.

Theorem 1. The duration of contractual relationship changes the bid-ask spreads.

Comparing a long-term regime to a short-term regime, as we have discussed above, the second-period spread is narrower. More interestingly, the first-period spread of large-quantity orders becomes narrower too due to the random trading of insiders. Spreads in other cases
may be wider or narrower depending on the market parameters.

**Theorem 2.** Under a long-term regime, the population of traders is larger.

There are more traders in the second period due to the participation of discretionary liquidity traders. This result corresponds to the trading volume in an order-driven market because there the more the participants are, the higher the trading volume will be.

**Theorem 3.** The long-term contract harms informational efficiency.

Due to the random trading of the long-term insider, a rational investor can obtain less information when observing the price and volume patterns of the first period. In other words, the information reveals more gradually.

**Theorem 4.** The long-term contract reduces liquidity traders’ welfare.

A short-term insider’s aggressive trading in the first period will reveal not only the information content but also the existence of information asymmetry. Therefore her trading brings negative externality to the insider in the second period. Due to this sort of insider’s competition, the total profit of two short-term insiders will be lower than that of a long-term insider. Since an insider takes advantage of the market maker and the liquidity traders, their trading loss is higher if a firm signs a long-term contract.

**Theorem 5.** The duration of contractual relationship changes the firm’s value.

The expected trading loss will discourage outside investors from holding the stock, and henceforth reduce the amount of cash inflow when the entrepreneur makes financing. To hold a long-term contract relationship will in general induce a better level of managers’ effort, so on average the firm value will be higher. But the adverse selection problem will also be more severe, and therefore a long-term contract brings less proceeds to the entrepreneur.

We can also regard the proceeds as a short-term interest, and the manager’s effort as a long-term interest since it affects the firm’s fundamental value. The entrepreneur has to make a trade-off between these two factors.
5.2 Modifications

5.2.1 Multi-period Extension

It is straightforward to extend this model to that with multiple trading stages. In that case, the incentive for a long-term manager to pretend as a liquidity trader in the initial stages will be strengthened since more subsequent periods imply more profit that a randomizing strategy can bring.

Readers may ask what will change if we cascade several identical 3-period games so as to construct a multi-period model. To answer this question, we first recall that the period 0 represents the startup stage of a company. Therefore, a cascade of multiple rounds, each of which is composed of a contracting stage and trading stages, does not have any connection between different rounds. Because the entrepreneur has no type in this scenario, even if the entrepreneur as well as investors are the same throughout the whole story, no relevant information of either side can be learned by the other players. That the entrepreneur holds investment plans with different qualities is completely another story which we would not like to get involved here.

5.2.2 Discretionary Liquidity Traders

In this paper we assume that discretionary liquidity traders can join the market only in the second period. If they realize their liquidity demand before period 1, they are able to choose either period 1 or 2 to trade, or switch to another market. In that scenario, the first-period trading will change the traders population in the second period. Suppose a discretionary liquidity trader with two-unit demand wins the opportunity to trade in the first period, she may choose to trade both units right away, or trade one unit and leave one unit for the next period depending on the posted bid-ask spread. In either case, neither the market maker nor the insider knows if this order is submitted by a discretionary liquidity or a nondiscretionary trader, and this will cause computational burden.
As long as the discretionary liquidity traders may switch to another market, insiders will always intend to seize them because insiders’ profit is increasing in liquidity traders’ population. Therefore, we can expect that under the long-term regime, the threshold of sustaining a pooling equilibrium is still lower because a long-term insider cares about the second-period population.

5.2.3 Investment Efficiency

Now we consider the investment efficiency factor \( z \). As we have mentioned, \( z \) can be any random variable that preserves the dominance relation. The insider trading profit may be affected if while changing the setting of \( z \) we also change the magnitude of information advantage, which will be the variance of stock value in this model. If, for example, under a long-term regime, \( \xi_2 + z = 2 \) with probability 1, then the long-term insider cannot earn anything in the second period, and her incentive to pool with liquidity traders disappears.

On the other hand, if the long-term contract only raises up the stock value of state H, e.g., \( \xi_2 + z > 2 \) in state H, then the insider profit will be higher because now the second-period information asymmetry becomes larger. In this case, the long-term insider has stronger incentive to trade randomly in the first period. Other cases are also possible, and therefore how investment efficiency affects the insiders does depend on its specific format.

5.2.4 Risk Attitude

The assumption that all players are risk-neutral is merely for computational convenience. The market maker needs the spread to ensure her break-even as well if she is risk averse, and the fact that a long-term insider cares about her tow-period profit will still hold regardless of her risk attitude. The optimal compensation scheme may be changed so as to provide some sort of risk insurance for either the entrepreneur or managers.
6 Conclusion

This paper demonstrates the connection between the firm’s contractual relationship and the capital market. We point out that the duration of contractual relationship may coincide with the period of information advantage held by the contracting party, and explain changes of insiders’ strategic behavior as well as several market criteria. The interaction between market microstructure and corporate finance will refill new lives to the research in finance, and this paper is just the beginning.

Proof of Lemma 1:

Since
\[ \alpha(S^2) = \frac{\alpha \mu + \alpha (1 - \mu) (1 - \epsilon)}{\alpha \mu + (1 - \mu) (1 - \epsilon)}, \alpha(S^1) = \frac{\alpha (1 - \mu)}{1 - \alpha \mu}, \]

Subtracting \( \alpha(S^2) \) by \( \alpha(S^1) \), we obtain that
\[ \alpha(S^2) - \alpha(S^1) = \frac{(1 - \alpha)(1 - \alpha \mu)(1 - \mu)(1 - \epsilon)}{(1 - \alpha \mu)[\alpha \mu + (1 - \mu)(1 - \epsilon)]} > 0. \]

Proof of Lemma 2:

Suppose that \( \alpha_2 > \alpha_1 \), and \( E_1 \) and \( E_2 \) are the corresponding equilibria when \( \alpha = \alpha_1 \) and \( \alpha = \alpha_2 \). We now discuss this case by case.

(a) Both \( E_1 \) and \( E_2 \) are separating.

Since
\[ \frac{\partial}{\partial \alpha} \pi_s(\alpha) = \frac{-\mu (1 - \epsilon) (\alpha \mu + 1 - \epsilon) - \mu \epsilon (1 - \alpha \mu)(1 - \epsilon)}{[\alpha \mu + (1 - \alpha \mu)(1 - \epsilon)]^2} < 0, \]
we obtain that \( \pi(\alpha_2) < \pi(\alpha_1) \).

(b) Both \( E_1 \) and \( E_2 \) are pooling.

Since \( \frac{\partial}{\partial \alpha} \pi_s(\alpha) = -\mu (2 - \epsilon) < 0, \pi(\alpha_2) < \pi(\alpha_1) \).

(c) \( (E_1, E_2) = (S,P) \).
Suppose $\alpha_3$ is the critical value which satisfies $\frac{\alpha\mu+(1-\alpha\mu)(1-\epsilon)}{(1-\alpha\mu)(1-\epsilon)} = 2$, then $\pi(\alpha_2) \leq \pi(\alpha_3) \leq \pi(\alpha_1)$ by the above discussion.

Note that $(E_1, E_2)$ will never be (S,P) because $\alpha_2 > \alpha_1$, and therefore the lemma holds for all possible conditions.

**Proof of Lemma 3:**

Since a pooling equilibrium will exist when $2 \leq \frac{\alpha\mu+(1-\alpha\mu)(1-\epsilon)}{(1-\alpha\mu)(1-\epsilon)}$ and

$$\frac{\partial}{\partial\alpha} \left[ \frac{\alpha\mu+(1-\alpha\mu)(1-\epsilon)}{(1-\alpha\mu)(1-\epsilon)} \right] = \frac{\mu(1-\alpha\mu)(1-\epsilon)+\mu(1-\epsilon)(1-\epsilon+\epsilon\alpha\mu)}{[(1-\alpha\mu)(1-\epsilon)]^2} > 0,$$

the lemma holds.

**Proof of Proposition 3:**

This proposition is a combination of Lemmas 1 and 2.

**Proof of Proposition 4:**

Suppose that a separating equilibrium exists in the first period. If the long-term manager submits a one-unit order, her total expected profit is $1 \times \frac{1}{2} + \mu\pi(\alpha(S_1))$; if instead a two-unit order is submitted, her total profit is $2 \times \frac{1/2(1-\alpha\mu)(1-\epsilon)}{\alpha\mu+(1-\alpha\mu)(1-\epsilon)} + \mu\pi(\alpha(S_2))$. The necessary condition for a separating equilibrium to survive is

$$2 \times \frac{1/2(1-\alpha\mu)(1-\epsilon)}{\alpha\mu+(1-\alpha\mu)(1-\epsilon)} + \mu\pi(\alpha(S_2)) > 1 \times \frac{1}{2} + \mu\pi(\alpha(S_1))$$

Rewriting it, we obtain

$$2 > \frac{\alpha\mu+(1-\alpha\mu)(1-\epsilon)}{(1-\alpha\mu)(1-\epsilon)} + \frac{1/2(1-\alpha\mu)(1-\epsilon)}{\alpha\mu+(1-\alpha\mu)(1-\epsilon)} \mu[\pi(\alpha(S_1)) - \pi(\alpha(S_2))]. \quad (1)$$

By Lemma 1, $\alpha(S_2) > \alpha(S_1)$, and Lemma 2 ensures that $\pi(\alpha(S_2)) < \pi(\alpha(S_1))$. Since the second term of the right-hand side in Eq. (1) is strictly positive, if for some $\alpha$ the condition holds, it shall satisfy $2 > \frac{\alpha\mu+(1-\alpha\mu)(1-\epsilon)}{(1-\alpha\mu)(1-\epsilon)}$. Thus the range within which a separating
equilibrium will survive under the long-term contractual relationship is narrower than that under the short-term one, i.e., the pooling equilibrium will come up more likely.

**Proof of Lemma 4:**

It is easy to verify that $$\alpha_p(S^1) = \frac{\alpha \mu \beta + \alpha(1 - \mu)\epsilon}{\alpha \mu \beta + (1 - \alpha \mu)\epsilon}, \alpha_s(S^1) = \frac{\alpha(1 - \mu)}{1 - \alpha \mu}.$$ Subtracting them, we obtain that $$\alpha_p(S^1) - \alpha_s(S^1) = \frac{\alpha \mu \beta (\alpha - 1)}{(1 - \alpha \mu)[\alpha \mu \beta + (1 - \alpha \mu)\epsilon]} < 0,$$ and henceforth $$\alpha_p(S^1) > \alpha_s(S^1).$$ Similarly,

$$\alpha_p(S^2) = \frac{\alpha \mu (1 - \beta) + \alpha(1 - \mu)(1 - \epsilon)}{\alpha \mu (1 - \beta) + (1 - \alpha \mu)(1 - \epsilon)}, \alpha_s(S^2) = \frac{\alpha \mu + \alpha(1 - \alpha \mu)(1 - \epsilon)}{\alpha \mu + (1 - \alpha \mu)(1 - \epsilon)}.$$ and

$$\alpha_s(S^2) - \alpha_p(S^2) = \frac{\alpha \mu \beta (1 - \alpha)(1 - \epsilon)}{\alpha \mu + (1 - \alpha \mu)(1 - \epsilon)[\alpha \mu (1 - \beta) + (1 - \alpha \mu)(1 - \epsilon)]} > 0.$$ Thus we conclude that $$\alpha_p(S^2) < \alpha_s(S^2).$$ The other two claims can be shown analogously.

**Proof of Proposition 5:**

By Lemmas 3 and 4, this proposition holds.

**Proof of Proposition 6:**

The expected first-period profit of $$I_L$$ is $$2\mu \frac{1/2(1 - \alpha \mu)(1 - \epsilon)}{\alpha \mu (1 - \beta) + (1 - \alpha \mu)(1 - \epsilon)},$$ and that of $$I_1$$ is $$2\mu \frac{1/2(1 - \alpha \mu)(1 - \epsilon)}{\alpha \mu + (1 - \alpha \mu)(1 - \epsilon)}.$$ Due to the larger denominator, the first-period profit of $$I_L$$ is higher.
Proof of Proposition 7:

According to Lemma 2, the expected second-period profit of the insider is decreasing in $\alpha$, and by Lemma 4, $\alpha_p(S^1), \alpha_p(S^2) < \alpha_s(S^2)$. The expected profit of $I_2$ is

$$\pi(\alpha_s(S^2)) = \beta \pi(\alpha_s(S^2)) + (1 - \beta)\pi(\alpha_s(S^2)) < \beta \pi(\alpha_p(S^1)) + (1 - \beta)\pi(\alpha_p(S^2)),$$

where the right-hand side of the inequality is the expected profit of $I_L$.

Proof of Proposition 8:

The expected two-period profit of the insiders is

$$\mu^2(\pi_1 + E[\pi_2|1_I]) + \mu(1 - \mu)\pi_1 + (1 - \mu)\mu E[\pi_2|1_N],$$

where $E[\pi_2|1_I], E[\pi_2|1_N]$ represent the expected second-period profit given that the first-period order is submitted by, respectively, an insider and a liquidity trader. By Propositions 5 and 6, the first two terms of $I_L$ are higher than that of $I_1$ and $I_2$. $E[\pi_2|1_N]$ can be explicitly expressed as $\epsilon \pi(\alpha(S^1)) + (1 - \epsilon)\pi(\alpha(S^2))$, and we have obtained that $\pi(\alpha_s(S^1)) > \pi(\alpha_p(S^1)).$ Unless $(1 - \mu)\mu(1 - \epsilon)\pi(\alpha(S^2))$ outweighs the sum of the other terms in Eq.(2), the expected profit of $I_L$ will be higher.

Proof of Lemma 5:

It is easy to verify that

$$\alpha_p(S^2) - \alpha_p(S^1) = \frac{\alpha \mu(1 - 2\beta)(1 - \alpha)(1 - \epsilon)}{[\alpha \mu \beta + (1 - \alpha \mu)\epsilon][\alpha \mu (1 - \beta) + (1 - \alpha \mu)(1 - \epsilon)],}$$

and the lemma follows directly.

Proof of Proposition 9:

First note that if $2 < \frac{\alpha \mu + (1 - \alpha \mu)(1 - \epsilon)}{(1 - \alpha \mu)(1 - \epsilon)}$, the first-period equilibrium under the short-term contractual relationship will be a pooling one. According to Proposition 4, $I_L$ will also randomize.
her order in the first period, and her incentive compatibility condition is as follows:

\[ 2 \times \frac{1/2(1 - \alpha \mu)(1 - \epsilon)}{\alpha \mu (1 - \beta) + (1 - \alpha \mu)(1 - \epsilon)} + \mu \pi(\alpha_p(S_2)) > 1 \times \frac{1/2(1 - \alpha \mu)\epsilon}{\alpha \mu \beta + (1 - \alpha \mu)\epsilon} + \mu \pi(\alpha_p(S_1)). \]

By continuity of the above equation, there exists a solution \( \beta' \). Since \( \beta' < \frac{1}{2} \), \( \pi(\alpha_p(S_2)) < \pi(\alpha_p(S_1)) \) by Lemma 5. One can easily verify that \( \beta' > \beta^* \). The last claim follows directly.

**Proof of Proposition 11:**

Since \( \alpha_s(S_1) < \alpha_s(S_2) \), by Lemma 2 the price under \( \alpha_s(S_1) \) is strictly preferred. Discretionary liquidity traders will therefore show up in the second period more likely when the order executed in the first period is a small-quantity one. In other words, the second-period profit of insider is higher than that without the discretionary liquidity traders. Moreover, the expected profit difference \( \pi(\alpha_s(S_1)) - \pi(\alpha_s(S_2)) \) is higher. Thus Eq.(1) holds with a higher probability. \( I_L \)'s incentive to randomize her order is strengthened by the discretionary liquidity traders if they do not occupy a significant part of the second-period trading opportunity.

**Proof of Proposition 12:**

By Lemma 4, \( \alpha_p(S_1), \alpha_p(S_2) < \alpha_s(S_2) \). Therefore the bid-ask spreads in the second period under \( \alpha_p(S_1) \) and \( \alpha_p(S_2) \) are strictly favorable to traders than that under \( \alpha_s(S_2) \). Given that the first-period order is submitted by the insider, \( \alpha_2 = \alpha_p(S_1) \) or \( \alpha_2 = \alpha_p(S_2) \) when the first-period equilibrium is pooling and \( \alpha_2 = \alpha_s(S_2) \) when it is separating. The proposition follows immediately.

**Proof of Proposition 13:**

It is easy to verify that the necessary condition of sustaining the second-period separating equilibrium is still

\[ 2 \times \frac{1/2(1 - \alpha_2 \mu)(1 - \epsilon)}{\alpha_2 \mu + (1 - \alpha_2 \mu)(1 - \epsilon)} > 1 \times \frac{1}{2} \]
independent of the presence of the discretionary liquidity traders. However, if the second-period equilibrium is pooling, both the small-quantity and large-quantity spreads are different from those without the presence of them. Therefore the expected second-period profit of the insider may vary, and it may cause a change of the first-period equilibrium.

References


