Static Entry Models

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Contents

1 Introduction: Why do we study entry models?

Remember that when we were looking at questions like prices and competition we need to that market structure itself is a decision of firms, and hence is itself endogenous. I will show an example later on in which this will matter.

- Market Structure and Prices. A classic case of accounting for the endogeneity of market structure is the analysis of the Office Depot and Staples merger. When you regress prices on the presence of either

Office Depot, Staples or both firms being present in the market, you find that prices are lower in markets in which both of these firms are present. However, this might just be because marginal costs are lower in some markets than others, and thus firms will enter into these markets with lower marginal costs.

- Is there too much or too little entry? Sometimes factors such as barriers to entry limit the number of firms in a market. For instance Microsoft has an application barrier to entry for Windows: to enter the OS market you need to convince developers to build new applications for your platform. One the other hand, the number of entrants might be above what is socially optimal, as discussed by Mankiw and Whinston RAND 1986. When I enter, I don't take into account the fact that by entering I lower the profits of my rival, hence there is a business-stealing externality to entry!

- Innovation: Which types of products do firms choose to develop. For instance, will product differentiation induce Microsoft and Nintendo to produce very different gaming platforms (XBox 360 and Wii).

- Endogeneity of Product Characteristics: When BLP look at demand for automobiles, they take product characteristics as exogenous. What happens if we try to figure out which type of cars lead to the highest profits for manufacturers? (Take the example of the minivan: it filled a gap between getting a van and a large sedan and yielded huge profits for Chrystler)

- Auctions: how much competition will there be for a good? In the German spectrum auction, the most important factor behind the very high prices was the fact that there were more large wireless firms interested in getting a license than licenses available.

- This is an introduction to looking at the dynamic incentives of firms which compete against each other.

Sutton Example:

Often you will see people assume that something like the Herfindahl appropriately captures the concept of a more competitive market. John Sutton has an interesting example of why more concentrated markets are not necessarily more competitive.

Suppose that demand is characterized by a Cournot model of competition, where demand is:

$$P = a - bQ \tag{1}$$

and assume for that marginal costs are $c(q) = c \cdot q$. The fixed cost of entering a market is F. A firm's profits as a function of the number of competitors is:

$$\pi = (\frac{N}{N+1}[a-c])\frac{a-c}{b(N+1)} - F \tag{2}$$

Taking logs of this expression (for variable profits) we get:

$$\log(\pi) = \log(N) - 2\log(N+1) + 2\log(a-c) + \log(b) \tag{3}$$

We will use this expression to justify some of the functional forms used later on as being additively separable in the number of firms and other parameters, and the log form often used in these models.

So the number of firms in the market will be determined by the free-entry condition, i.e.: $\max N s.t. (\frac{N}{N+1}[a-c])\frac{a-c}{b(N+1)} - F > 0$.

Now think of the Bertrand model of competition, since the price is set to marginal cost for any number of firms (p=c), then firms will never be able to cover their fixed costs of entry if they have a competitor. Thus the Bertrand model predicts either 0 or 1 firms in a market. This means that a Bertrand competitive market always has fewer firms than a Cournot market, while the Herfindahl would say that the Bertrand market has little competition while the Cournot market is more competitive. However, this result is only because of the entry process, not the toughness of product market competition.

2 Bresnahan-Reiss

The Bresnahan and Reiss (1991) model was originally used to try to make inference on the nature of competition in settings where there is no cost or demand data. Instead Bresnahan and Reiss (1991) look at the increase in the number of firms in a market as market size increases. The pattern of entry should tell us about how markups decrease as market size increases. Bresnahan and Reiss (1991) look at entry patterns across geographically differentiated markets:

- Dentists
- Tires

- Car Dealers
- Plumbers

is "Isolated Markets", i.e. towns which are located far away from other towns in the US. Can these entry data tell us about markups and competition?

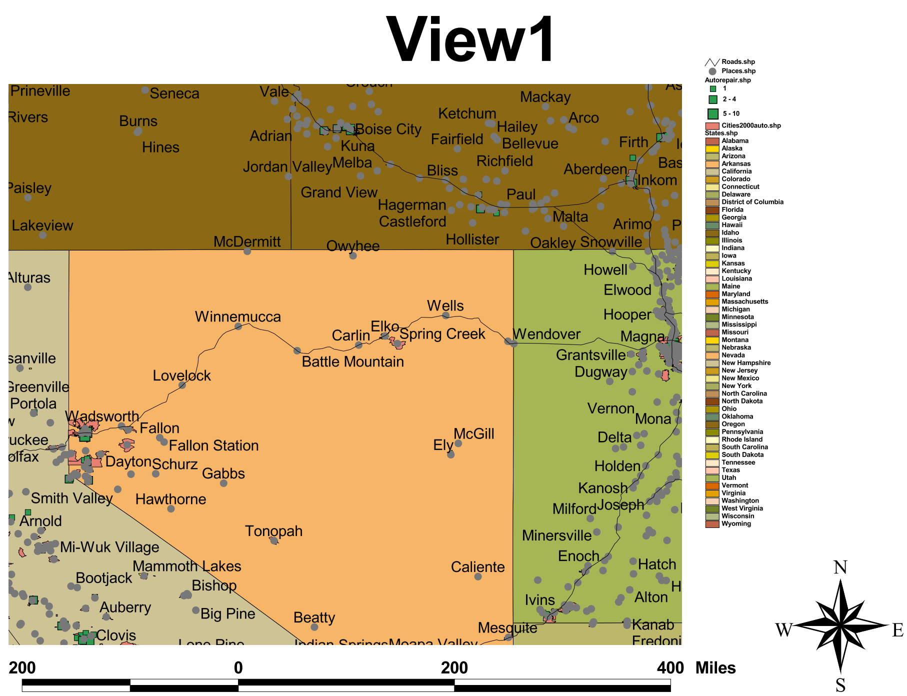


Figure 1: Location of Automobile Repair Shops

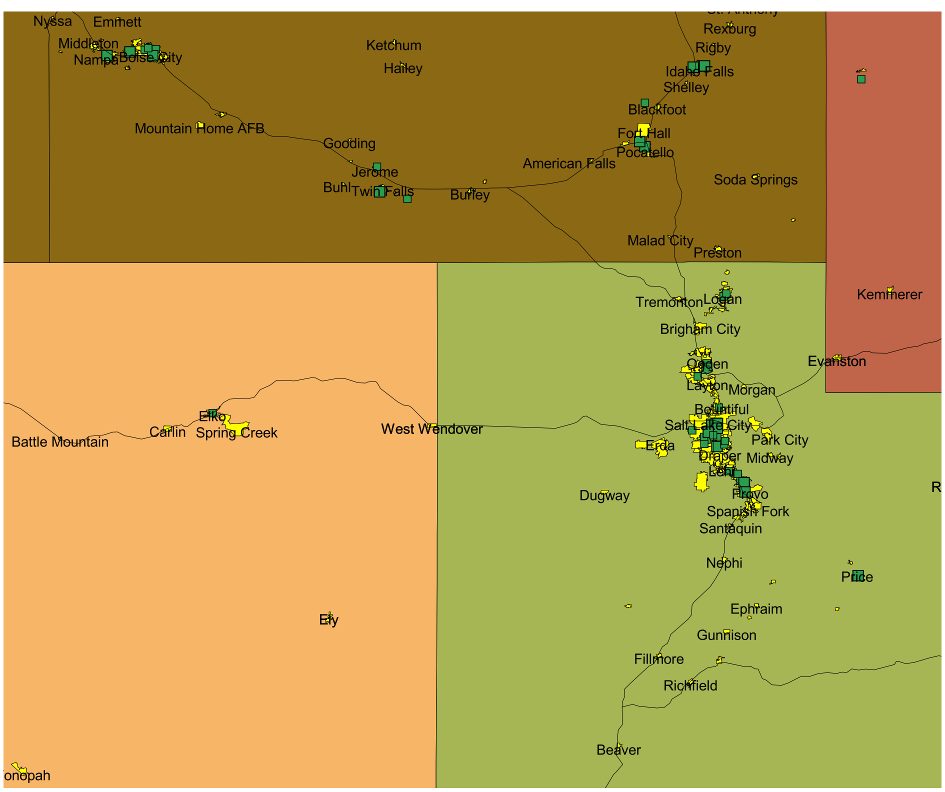


Figure 2: Location of Automobile Repair Shops 2

The firm's entry problem is inherently dynamic. I enter if the continuation value $V(x)$:

$$V(x) = E\sum_{t=0}^{\infty}\beta^t \pi(x^t)\,\mathrm{Pr}[x^t|x^0] \tag{4}$$

is greater than the entry cost, i.e. $V(x) \geq \phi$ (Entry Cost).

However these problems are quite difficult, so let's look at the case where an industry is in "equilibrium", i.e. the state at which you entered and today are roughly the same. This could be very misleading as a large portion of the value of entry for say search engines, is not current profits but the fact that market size is assumed to be increasing.

Timing:

1. Firms simultaneously decide to enter the market.

2. Firms play Cournot in quantities in the subgame.

Note that there are *multiple equilibria* in these games. For instance, suppose firms payoffs are the following:

		Firm 1	
		Out	Enter
Firm 2	Out	0,0	4,0
	Enter	0,5	-11,-10

Thus the only Nash Equilibria in this game involve firm 1 entering and firm 2 staying out, or firm 2 entering and firm 1 staying out. Thus the equilibrium is not pinned down. This is a problem for many estimation techniques since the same outcome of the game could have been generated by two sets of parameters if two different equilibria of the game were being played.

Look at the following example:

		Firm 1	
		Out	Enter
Firm 2	Out	0,0	X_1 ,0
	Enter	0,X_2	$X_1 - 10, X_2 - 10$

The goal is to estimate X_1 and X_2, so which combinations of $\{X_1, X_2\}$ lead to different patterns of entry by firm 1 and 2? The following diagram shows how different entry patterns could have been caused by different parameter configurations.

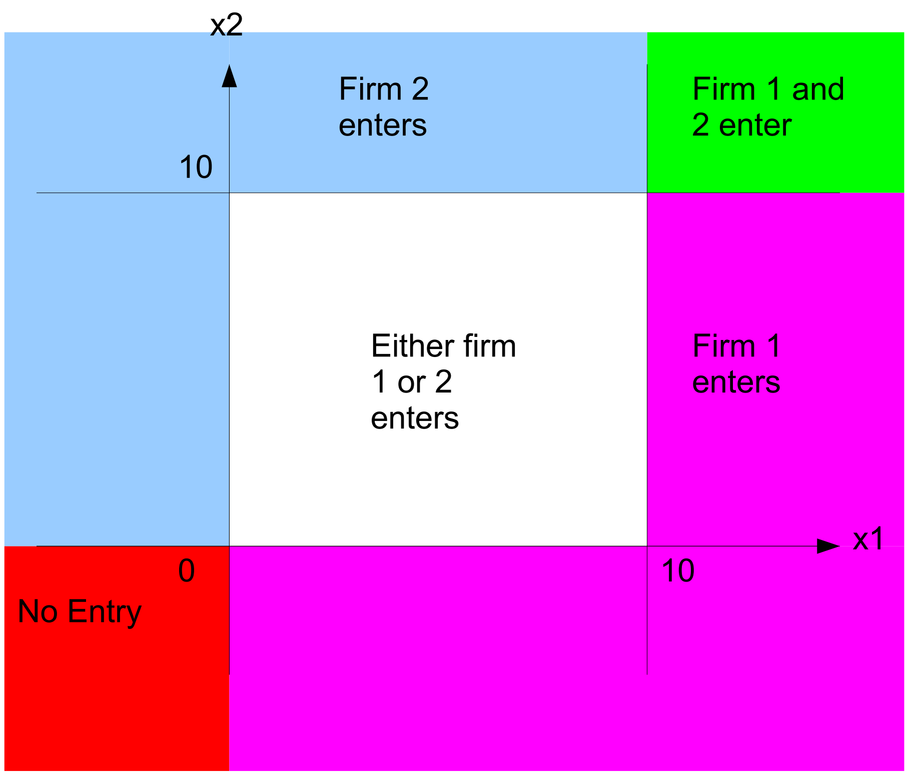


Figure 3: Relationship between observed entry patterns and parameter configuration.

The problem with this setup is that if $\{X_1, X_2\} \in [0, 10] \times [0, 10]$ then it is impossible to predict which entry pattern would be observed. This is a problem for most estimation techniques which try to relate an observed outcome Y to covariate data X and parameters θ. For instance, suppose we are estimating the parameters of the entry game via Maximum Likelihood. There are unobservables ϵ_1 and ϵ_2 which are added to the payoffs X_1 and X_2. Suppose that ϵ_1 and ϵ_2 are distributed as independent normally distributed variables $N(0, 1)$. Then the probability of observing the following outcomes is:

$$\begin{aligned}
\Pr[\text{firm 1 enters, firm 2 enters}] &= \int_{\epsilon_1}\int_{\epsilon_2} 1(X_1+\epsilon_1 > 10, X_2+\epsilon_2 > 10)dF(\epsilon_1,\epsilon_2) \\
\Pr[\text{firm 1 out, firm 2 out}] &= \int_{\epsilon_1}\int_{\epsilon_2} 1(X_1+\epsilon_1 < 0, X_2+\epsilon_2 < 0)dF(\epsilon_1,\epsilon_2) \\
\Pr[\text{firm 1 enters, firm 2 out}] &= ? \\
\Pr[\text{firm 1 out, firm 2 enters}] &= ?
\end{aligned}$$

The problem is that it is impossible to compute the probability of observing either firm 1 or firm 2 entering by itself. All we know is that this probability is greater than zero and smaller than $1 - \Pr[\text{firm 1 enters, firm 2 enters}] - \Pr[\text{firm 1 out, firm 2 out}]$.

There are a number of approaches to fixing this problem of multiplicity:

1. **Refining the set of equilibria**, i.e. picking out an equilibrium which seems more "plausible". In particular, suppose it is the case that the most profitable firm is always the first firm to enter. This leads to the following relationship between parameters and entry: This leads to firm 1 entering alone if and only if ($X_1 \in [0, 10]$ and $X_2 < 0$) or ($X_1 > 10$ and $X_2 < 10$) or ($X_2 \in [0, 10]$ and $X_1 > X_2$ and $X_1 \in [0, 10]$).

   Likewise firm 2 enters alone if and only if ($X_2 \in [0, 10]$ and $X_1 < 0$) or ($X_2 > 10$ and $X_1 < 10$) or ($X_1 \in [0, 10]$ and $X_2 > X_1$ and $X_2 \in [0, 10]$). Both firms enter if and only if $X_1 > 10$ and $X_2 > 10$, while neither firm enters if and and only if $X_1 < 0$ and $X_2 < 0$. Thus the equations used for Maximum likelihood are:

   $$\Pr[\text{firm 1 enters, firm 2 enters}] = \int_{\epsilon_1}\int_{\epsilon_2} 1(X_1+\epsilon_1 > 10, X_2+\epsilon_2 > 10)dF(\epsilon_1,\epsilon_2)$$

$\Pr[\text{firm 1 out, firm 2 out}] = \int_{\epsilon_1}\int_{\epsilon_2} 1(X_1+\epsilon_1 < 0, X_2+\epsilon_2 < 0)dF(\epsilon_1,\epsilon_2)$

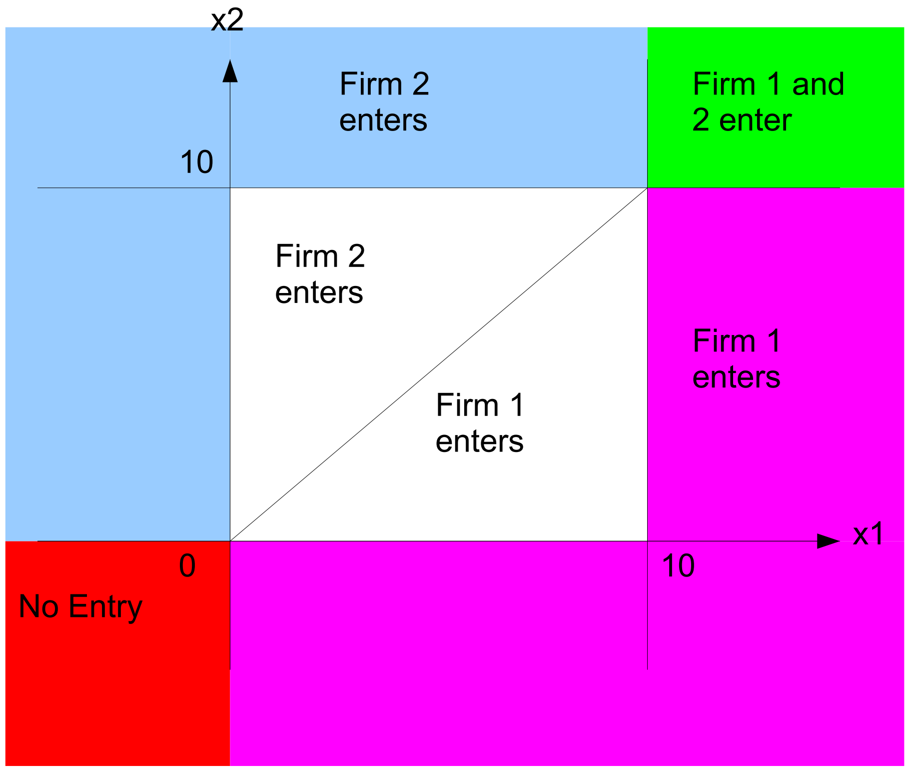


Figure 4: Relationship between observed entry patterns and parameter configuration given the assumption that the most profitable firm moves first.

Pr[firm 1 enters, firm 2 out] $= \int_{\epsilon_1}\int_{\epsilon_2} 1(X_1+\epsilon_1 > 0, X_1+\epsilon_1 > X_2+\epsilon_2, X_2+\epsilon_2 < 10)dF(\epsilon_1,\epsilon_2)$

Pr[firm 1 out, firm 2 enters] $= \int_{\epsilon_1}\int_{\epsilon_2} 1(X_2+\epsilon_2 > 0, X_2+\epsilon_2 > X_1+\epsilon_1, X_1+\epsilon_1 < 10)dF(\epsilon_1,\epsilon_2)$

2. **Partial Identification**: finding implications of the model that could have been generated by some of the equilibria. With enough variation in the data it is possible to identify X_1 and X_2 up to an interval without making any assumptions on equilibrium selection. For instance, I can bound the probability that firm 1 enters by making the appropriate assumptions about the order of entry. So the probability that firm 1 enters is bounded from below by: "I only observe firm 1 entering when firm 2 has already decided not to enter"

$$\text{Pr[firm 1 enters, firm 2 out]} \geq \int_{\epsilon_1}\int_{\epsilon_2} 1(X_1+\epsilon_1 > 0, X_2+\epsilon_2 < 0)dF(\epsilon_1,\epsilon_2) \tag{5}$$

Likewise the probability of firm 1 entering is bounded from above by the assumption that firm 1 always moves first:

$$\text{Pr[firm 1 enters, firm 2 out]} \leq \int_{\epsilon_1}\int_{\epsilon_2} 1(X_1+\epsilon_1 > 0, X_2+\epsilon_2 < 10)dF(\epsilon_1,\epsilon_2) \tag{6}$$

I can represent these inequality constraint in figure 2.

These inequality constraints can be used to estimate a model of entry. I will talk about these methods in the next class, and Ariel will also discuss specific applications of these techniques.

3. Looking a the number of firms that enter, a feature which is pinned down across different equilibria. Suppose that all firms are identical, and thus look at the case where $X_1 = X_2$ *and* where $\epsilon_1 = \epsilon_2 = \epsilon$. There are still many different equilibria in this simplified model. However, the *number of firms* in the market is pinned down: there are no firms in the market if $X < 0$, there is one firm in the market if $X \in [0,1]$ and there are two firms in the market if $X > 10$. This allows Bresnahan and Reiss (1991) to estimate their model of entry.

4. **Model of equilibrium selection**: Following the ideas of Bajari, Hong and Ryan (2006) we can think of an equilibrium selection equa-

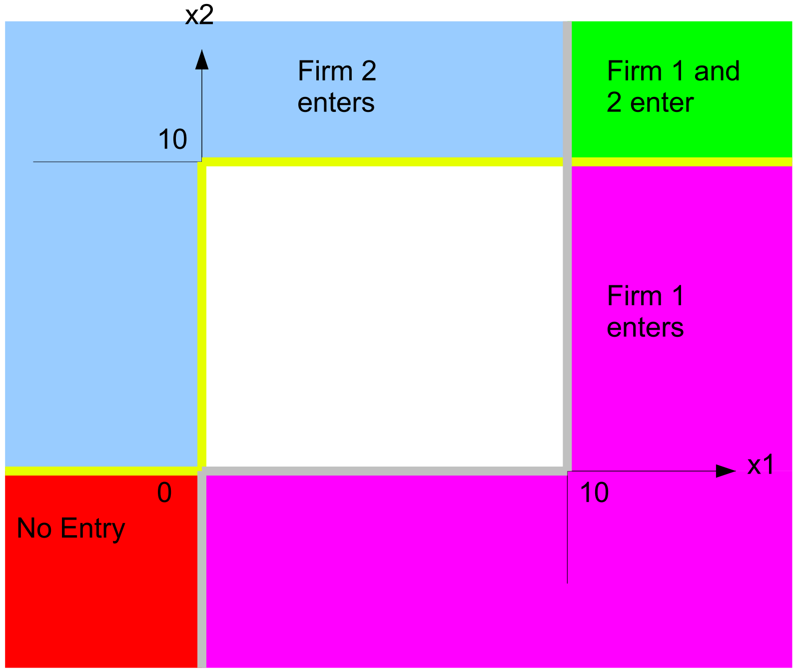


Figure 5: Bounds for observed entry patterns and parameter configuration.

tion where the probability of equilibrium κ is determined by:

$$\Pr[\kappa_j|X] = \frac{\exp(X_j\beta)}{\sum_{\kappa_k}\exp(X_k\beta)} \tag{7}$$

where X_j are characteristics of the equilibrium and the market which might affect which equilibrium gets played. For instance, I might believe that the fact that I played an equilibrium in the last period makes it more likely that this equilibrium gets played in the current period. Alternatively, maybe I am more likely to play a Pareto better equilibria if there are fewer firms in the market (for example because we can communicate better). In any case, whichever covariates you think are important for equilibrium selection can be included into the vector of X's. Then I can estimate the following larger model with the following likelihood (which is simply a mixture model):

$$\mathcal{L}(\theta,\beta) = \prod_m \sum_\kappa \Pr[\kappa_j|X^m,\beta] \prod_t \Pr[N_{mt}|X_{mt},\kappa_j,\theta] \tag{8}$$

We can then estimate the selection equation and the parameters of firm profits at the same time via maximum likelihood.

The original Bresnahan and Reiss (1991) model is based on two behavioral assumptions:

1. Firms that Enter make Positive Profits

$$\pi(N, X_m) + \varepsilon_m > 0 \tag{9}$$

2. If an extra firm entered it would make negative profits:

$$\pi(N+1, X_m) + \varepsilon_m < 0 \tag{10}$$

where $\pi(N, X_m)$ is the observable component of profit depending on demand factors X_m and the number of symmetric competitors in a market N, while ε_m are unobserved components of profitability common to all firms in a market.

Assume market level shocks ε_m have a normal distribution with zero mean and unit variance. The probability of observing a market X_m with N plants is the following:

$$\Pr(N = n|X_m) = \Phi[-\pi(n+1, X_m)] - \Phi[-\pi(n, X_m)]1(n > 0)$$

where $\Phi(.)$ is the cumulative distribution function of the standard normal. I parameterize the profit function as $\pi(\theta, N, X_m)$. Parameters can be estimated via Maximum Likelihood, where the likelihood is the following:

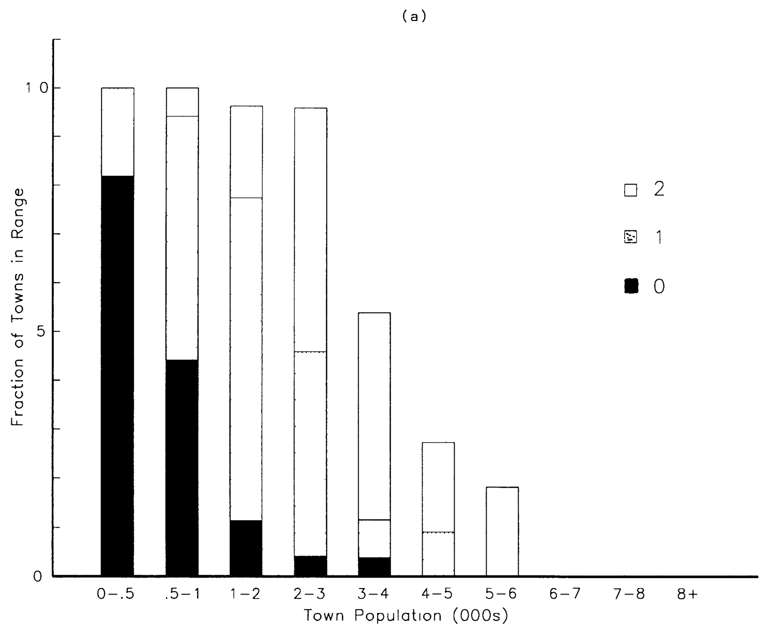
$$\mathcal{L}(\theta) = \prod_{m=1}^{M} \prod_{t=1}^{T} \Pr(N_m^t = n | X_m^t, \theta) \tag{11}$$

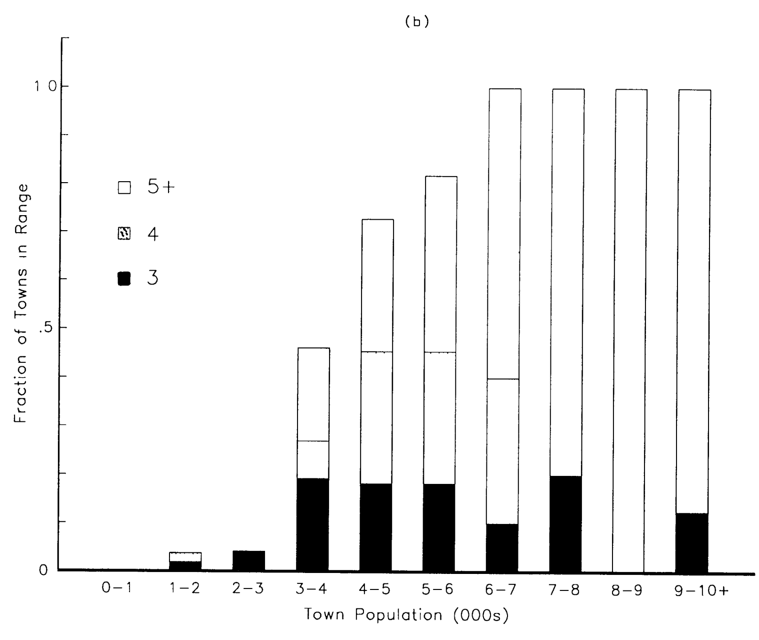
Firms make sunk, unrecoverable investments when they enter a market. The decision of an incumbent firm to remain in a market differs from the decision of an entrant to build a new plant. The next series of models deal with this difference.

MARKET COUNTS BY INDUSTRY AND NUMBER OF INCUMBENTS

	NUMBER OF FIRMS							
INDUSTRY	$N = 0$	$N = 1$	$N = 2$	$N = 3$	$N = 4$	$N = 5$	$N = 6$	$N \geq 7$
Druggists	28	62	68	23	8	6	3	4
Doctors	37	61	36	16	11	7	6	28
Dentists	32	67	39	15	12	12	4	21
Plumbers	71	47	26	21	10	4	6	17
Tire dealers	45	39	39	24	13	15	6	21
Barbers	95	66	23	9	3	6	0	0
Opticians	173	19	5	1	4	0	0	0
Beauticians	10	26	19	24	26	19	11	67
Optometrists	68	85	36	7	3	3	0	0
Electricians	60	54	32	17	10	5	7	17
Veterinarians	53	80	41	21	5	0	1	1
Movie theaters	90	72	25	10	5	0	0	0
Automobile dealers	38	44	54	35	25	2	1	3
Heating contractors	117	40	19	8	4	8	3	3
Cooling contractors	153	30	13	5	1	0	0	0
Farm equipment dealers	90	39	23	19	17	9	1	4

SOURCE.—Authors' tabulations from American Business Lists, Inc.


(a)
1 0
5
0
2
1
0
Fraction of Towns in Range
0-.5
.5-1
1-2
2-3
3-4
4-5
5-6
6-7
7-8
8+
Town Population (000s)


(b)
1 0
.5
0
5+
4
3
Fraction of Towns in Range
0-1
1-2
2-3
3-4
4-5
5-6
6-7
7-8
8-9
9-10+
Town Population (000s)

Sample Market Descriptive Statistics

Variable	Name	Mean	Standard Deviation	Min	Max
Firm counts:					
Doctors	DOCS	3.4	5.4	.0	45.0
Dentists	DENTS	2.6	3.1	.0	17.0
Druggists	DRUG	1.9	1.5	.0	11.0
Plumbers	PLUM	2.2	3.3	.0	25.0
Tire dealers	TIRE	2.6	2.6	.0	13.0
Population variables (in thousands):					
Town population	TPOP	3.74	5.35	.12	45.09
Negative TPOP growth	NGRW	−.06	.14	−1.34	.00
Positive TPOP growth	PGRW	.49	1.05	.00	7.23
Commuters out of the county	OCTY	.32	.69	.00	8.39
Nearby population	OPOP	.41	.74	.01	5.84
Demographic variables:					
Birth ÷ county population	BIRTHS	.02	.01	.01	.04
65 years and older ÷ county population	ELD	.13	.05	.03	.30
Per capita income ($1,000's)	PINC	5.91	1.13	3.16	10.50
Log of heating degree days	LNHDD	8.59	.47	6.83	9.20
Housing units ÷ county population	HUNIT	.46	.11	.29	1.40
Fraction of land in farms	FFRAC	.67	.35	.00	1.27
Value per acre of farmland and buildings ($1,000's)	LANDV	.30	.23	.07	1.64
Median value of owner-occupied houses ($1,000's)	HVAL	32.91	14.29	9.90	106.0

Source.—Firm counts: American Business Lists, Inc.; population variables: U.S. Bureau of the Census (1983) and *Rand McNally Commercial Atlas and Marketing Guide* (annual); demographic variables: U.S. Bureau of the Census (1983).

BASELINE SPECIFICATIONS

Variable Name	Doctors	Dentists	Druggists	Plumbers	Tire Dealers
OPOP (λ_1)	1.15	−.46	.08	.27	−.53
	(.85)	(.32)	(.37)	(.60)	(.43)
NGRW (λ_2)	−1.89	.63	−.30	.68	2.25
	(1.60)	(.85)	(.97)	(1.10)	(.75)
PGRW (λ_3)	1.92	−.35	−.24	−.45	.34
	(1.01)	(.41)	(.41)	(.36)	(.59)
OCTY (λ_4)	.80	2.72	.16	−.28	.23
	(1.26)	(.98)	(.34)	(.71)	(.94)
BIRTHS (β_1)	−.59	9.86	11.34		
	(6.57)	(8.29)	(10.10)		
ELD (β_2)	−.11	.22	2.61		−.49
	(.55)	(.74)	(.78)		(.75)
PINC (β_3)	−.00	.04	.02	.05	−.03
	(.00)	(.03)	(.02)	(.03)	(.04)
LNHDD (β_4)	.013	.28	.08	.003	.004
	(.05)	(.07)	(.06)	(.06)	(.06)
HUNIT (β_5)				.51	
				(.46)	
HVAL (β_6)				.42	
				(.03)	
FFRAC (β_7)					−.02
					(.08)
V_1 (α_1)	.63	−1.85	−.13	.06	.86
	(.46)	(.61)	(.58)	(.52)	(.45)
$V_1 - V_2$ (α_2)	.34		.29		.03
	(.17)		(.21)		(.15)
$V_2 - V_3$ (α_3)		.12	.19	.15	.15
		(.14)	(.17)	(.09)	(.10)
$V_3 - V_4$ (α_4)	.07	.20	.25	.07	
	(.05)	(.06)	(.14)	(.08)	
$V_4 - V_5$ (α_5)			.04	.04	.08
			(.12)	(.07)	(.05)
F_1 (γ_1)	.92	1.10	.91	1.28	.53
	(.30)	(.25)	(.29)	(.26)	(.23)
$F_2 - F_1$ (γ_2)	.65	1.84	1.34	1.04	.76
	(.30)	(.19)	(.35)	(.14)	(.21)
$F_3 - F_2$ (γ_3)	.84	1.14	1.77	.32	.46
	(.13)	(.46)	(.54)	(.28)	(.21)
$F_4 - F_3$ (γ_4)	.18		.06	.40	.60
	(.23)		(.70)	(.35)	(.12)
$F_5 - F_4$ (γ_5)	.42	.66	.51	.25	.12
	(.13)	(.60)	(.95)	(.35)	(.20)
LANDV (γ_L)	−1.02	−1.31	−.84	−1.18	−.74
	(.53)	(.37)	(.51)	(.48)	(.34)
Log likelihood	−233.49	−183.20	−195.16	−228.27	−263.09

NOTE.—Asymptotic standard errors are in parentheses

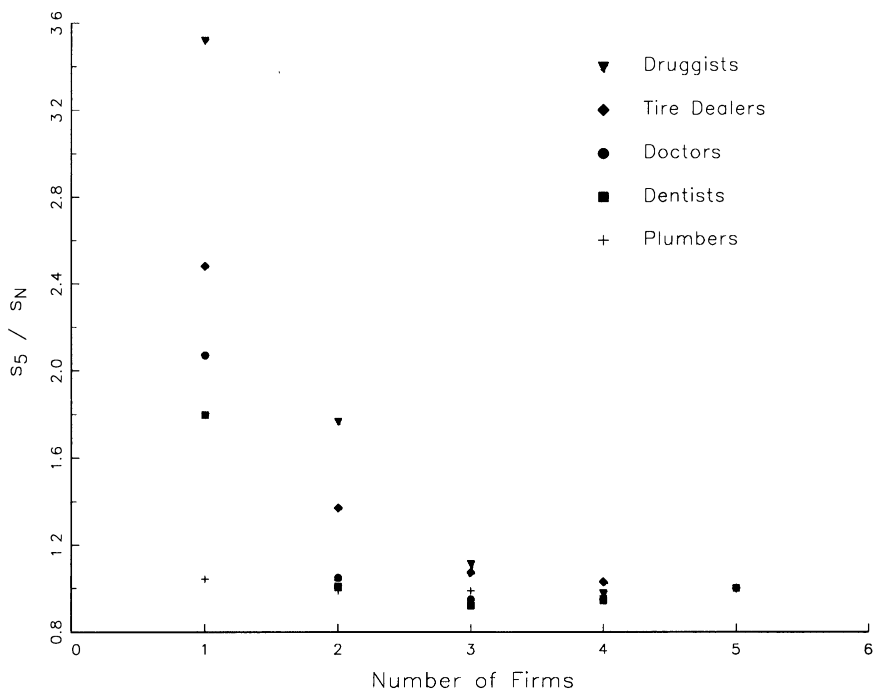
A. ENTRY THRESHOLD ESTIMATES

	Entry Thresholds (000's)					Per Firm Entry Threshold Ratios			
PROFESSION	S_1	S_2	S_3	S_4	S_5	s_2/s_1	s_3/s_2	s_4/s_3	s_5/s_4
Doctors	.88	3.49	5.78	7.72	9.14	1.98	1.10	1.00	.95
Dentists	.71	2.54	4.18	5.43	6.41	1.78	.79	.97	.94
Druggists	.53	2.12	5.04	7.67	9.39	1.99	1.58	1.14	.98
Plumbers	1.43	3.02	4.53	6.20	7.47	1.06	1.00	1.02	.96
Tire dealers	.49	1.78	3.41	4.74	6.10	1.81	1.28	1.04	1.03

B. LIKELIHOOD RATIO TESTS FOR THRESHOLD PROPORTIONALITY

Profession	Test for $s_4 = s_5$	Test for $s_3 = s_4 = s_5$	Test for $s_2 = s_3 = s_4 = s_5$	Test for $s_1 = s_2 = s_3 = s_4 = s_5$
Doctors	1.12 (1)	6.20 (3)	8.33 (4)	45.06* (6)
Dentists	1.59 (1)	12.30* (2)	19.13* (4)	36.67* (5)
Druggists	.43 (2)	7.13 (4)	65.28* (6)	113.92* (8)
Plumbers	1.99 (2)	4.01 (4)	12.07 (6)	15.62* (7)
Tire dealers	3.59 (2)	4.24 (3)	14.52* (5)	20.89* (7)

NOTE.—Estimates are based on the coefficient estimates in table 4. Numbers in parentheses in pt. B are degrees of freedom.
* Significant at the 5 percent level.


Druggists
Tire Dealers
Doctors
Dentists
Plumbers
S5 / SN
3.6
3.2
2.8
2.4
2.0
1.6
1.2
0.8
0
1
2
3
4
5
6
Number of Firms

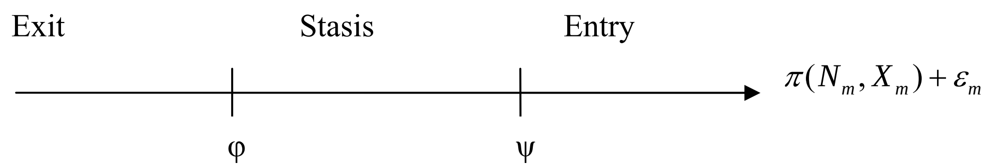


Figure 6: Entry Threshold ψ and Exit Threshold ϕ based on static profits.

3 Bresnahan-Reiss Model of Exit

The Bresnahan and Reiss (1994) model of exit distinguishes between two types of firms: firms which are already active and firms which are deciding to enter the market. Entrants and incumbents have the same profits, and hence the same continuation values. However, entrants always have lower values than incumbents, since they pay an entry cost that incumbents do not, as is shown by Figure 3. This implies that there cannot be simultaneous entry and exit: either firms exit, enter, or nothing happens. This is a feature of all models which do not have firm specific shocks and where firms are symmetric: they cannot rationalize the same type of plant in the same market making different choices. Thus market-years in which there is both entry and exit are dropped. With yearly data and markets with on average less than 3 incumbents there is very little simultaneous entry and exit, less than 5% of markets need to be dropped. Moreover, including these markets in the data does not significantly change estimated parameters. So the selection caused by this procedure does not seem to be of great import for this data. Three regimes need to be considered: *entry, exit and stasis.*

1. Net *Entry*: $N^t > N^{t-1}$

$$\begin{aligned} \pi(N^t, X_m^t) + \varepsilon_m^t &> \psi \\ \pi(N^t + 1, X_m^t) + \varepsilon_m^t &< \psi \end{aligned}$$

2. Net *Exit*: $N^t < N^{t-1}$

$$\begin{aligned} \pi(N^t, X_m^t) + \varepsilon_m^t &> \phi \\ \pi(N^t + 1, X_m^t) + \varepsilon_m^t &< \phi \end{aligned}$$

3. No Net Change: $N^t = N^{t-1}$

$$\begin{aligned} \pi(N^t, X_m^t) + \varepsilon_m^t &> \phi \\ \pi(N^t + 1, X_m^t) + \varepsilon_m^t &< \psi \end{aligned}$$

where ϕ is the entry fee that an existing firm pays to enter the market and ψ is the scrappage value of a firm. Entry fees and scrap value are not identified from fixed costs, since it is always possible to increase fixed costs and decrease entry/exit fees by the same amount without changing the likelihood of observing a particular market configuration. Yet, the *difference* between entry and exit fees is identified and can be compared to other quantities such as the effect of an extra competitor.

These equations can be combined into:

$$\pi(N^t, X_m^t) + \varepsilon_m^t > 1(N^t > N^{t-1})\psi + 1(N^t \leq N^{t-1})\phi \tag{12}$$

$$\pi(N^t + 1, X_m^t) + \varepsilon_m^t < 1(N^t \geq N^{t-1})\psi + 1(N^t < N^{t-1})\phi \tag{13}$$

The probability of observing a market X_m with N^t plants today and N^{t-1} plants in the last period is:

$$\begin{aligned} \Pr(n^t = N^t, n^{t-1} = N^{t-1} | X_m^t) &= \Phi[-\pi(n^t + 1, X_m^t) + 1(n^t + 1 \geq n^{t-1})\psi + 1(n^t + 1 < n^{t-1})\phi] \\ -\Phi[-\pi(n^t, X_m^t) + 1(n^t &> n^{t-1})\psi + 1(n^t \leq n^{t-1})\phi]1(n^t > 0) \end{aligned}$$

which is used to form a maximum likelihood estimator as in equation (**??**).

4 Berry

The Berry (1992) model allows for some degree of heterogeneity between firms entering the market, in particular differences in the fixed costs of entry.

- Airline Markets: Questions about the nature of the barriers to entry for airlines: gates, slots, hub structures (frequent flyer points),
- Deregulation in the industry since the 1970s (before that routes and prices were heavily regulated), which has led to a enormous decrease in the prices and costs of airline travel.

- However, the main firms around since deregulation such as United and American Airlines are still in the market. Why have they not been displaced by Southwest Airlines (for instance), a lower cost and more profitable carrier?
- Very clear market definition: city pairs like Saint-Louis to Savannah.
- Terrific airline data: the origin and destination survey captures 10% of airline tickets from flights with a U.S. airport, both prices and quantity are available, but not the class of ticket (business or coach).

1. Profits: Profits in market i for firm k are given by:

$$\pi_{ik}(s) = v_i(N(s)) + \phi_{ik}$$

   Note that heterogeneity only enters into the entry cost (which is firm specific), not the profits given the type of entrants.

2. Entry: Berry shows that the number of firms is unique, but not which firms will enter!
   - order entry costs:

$$\phi_{i1} > \phi_{i2} > \cdots > \phi_{ik}$$

   - Suppose firms with lowest entry costs enter first: The number of firms N_i is: $N_i = \max_n$ s.t. $v_i(n) + \phi_{in} \geq 0$, i.e. $v_i(n) + \phi_{in} \geq 0$ and $v_i(n+1) + \phi_{in+1} \leq 0$
   - I could also assume that less profitable firms enter first: that's why they have such high fixed costs in the first place!

3. Parametrize entry costs: Fixed costs are parametrized as:

$$\phi_{ik} = \alpha Z_{ik}(\text{covariates}) + \sigma u_{ik}(\text{shocks})$$

   why could there be differences in entry cost: previous presence in the airport to negotiate better slot assignments, not the presence of a hub which affects competition between firms.

4. Parametrize firm profits:

$$\underbrace{v_i(N)}_{\text{common to all firms in the market}} = X_i\beta + \underbrace{h(\delta, N)}_{-\delta\ln(N)} + \rho \underbrace{u_{i0}}_{\text{market level unobservable}}$$

Note that all firm level idiosyncratic stuff gets captured in the entry costs. The total value of entering is just:

$$X_i\beta - \delta \ln(N) + \rho u_{i0} + \alpha Z_{ik} + \sigma u_{ik}$$

So we have two shocks here, and we can define the error term as:

$$\epsilon_{ik} = \rho u_{i0} + \sigma u_{ik}$$

Since we have a discrete choice problem, we need to normalize both the mean of the error term and it's variance. So set

$$\sigma = \sqrt{1-\rho^2}$$

5. Probability of N firms in the market? This is quite difficult since the probability of observing firm 1 enter which I will call $a_{i1} = 1$ depends on the entire vector of shocks $\vec{\epsilon} = \{\epsilon_{i1}, \epsilon_{i2}, \cdots, \epsilon_{ik}\}$ and the parameters θ:

$$\Pr[a_{i1}|\theta] = \int_{\epsilon_{i1}} \int_{\epsilon_{i2}} \cdots \int_{\epsilon_{ik}} 1(a_{i1} = 1|\theta, \vec{\epsilon}) df(\vec{\epsilon})$$

which is the area of the $\vec{\epsilon}$ space where firm 1 decides to enter, so denote the set B_{kj} as the set of ϵ's where 1 firm enters:

$$B_{kj} = \{\epsilon : \epsilon_k \geq -v(1) \& \epsilon_j < -v(2)\}$$

so the probability of observing 1 firm is:

$$\Pr[N^* = 1] = \Pr(\epsilon \in B_{12}) + \Pr(\epsilon \in B_{21}) - \Pr(\epsilon \in B_{12} \cup B_{21})$$

You can already see that it will be very difficult to compute this integral analytically (because of the odd shape of the domain of integration, which can be quite difficult to describe), so we will do what we did with the random coefficient logit model: Integrate by SIMULATION.

6. Algorithm

   (a) Pick θ.

   (b) Draw a vector of ϵ's: $\{\epsilon_{ik}\}_{k=1}^N$ (Remember to keep these the same over the algorithm, just like in BLP.

   (c) Find the fixed costs $\{\phi_{ik}\}_{k=1}^N$ and order them using an order of entry assumption.

(d) Add firms from $n = 1, \cdots, N$ until:

$$v(N^*|\theta) + \phi_{iN} \geq 0$$
$$v(N^* + 1|\theta) + \phi_{iN+1} < 0$$

(e) The predicted number of firms is $N^*(\theta, \vec{\epsilon}^d)$.

(f) Compute the criterion function

$$\xi = \sum_d N^*(\theta, \vec{\epsilon}^d) - \hat{N}$$

(g) Finally you can estimate the model via GMM as usual, with the criterion function (and which ever instruments you want, OLS works fine as well):

$$Q(\theta) = (\xi \mathbf{Z})(\mathbf{Z}'\mathbf{Z})^{-1}(\xi \mathbf{Z})'$$

- Note: If we had tried to estimate this model using Maximum Likelihood, we often could encounter cases where the model has difficulty rationalizing certain outcomes such as 10 firms entering. This leads us to get zero probability events (at least as far as the computer can tell), which lead to the computer stalling out when it encounters the log of 0. In practice, you should include what I like to think of as probability dust, i.e. that the probability of any event in the max of a "dust" constant (like 10e-15) and what is predicted by the model.

- In contrast, GMM is both more stable computationally than Maximum Likelihood, and is weaker than ML since we are only assuming mean zero error term ξ, not a parametric distribution on ξ. Furthermore, a simulation estimator will be biased if the simulation error enters non-linearly.

- In practice you can choose the moments that you use in GMM so that your model fits any particular moment that you really care about. As well, you get to play around with instruments and moments until you get the result you want!

TABLE IV

REGRESSION RESULTS FOR NUMBER OF FIRMS

Var	Est Parm (Std. Error)	Mean Value of Var. (Std. Dev.)
N		1.629
		(1.393)
Const	−0.727	—
	(0.097)	
Pop	2.729	0.558
	(0.255)	(0.114)
Dist	−1.591	1.149
	(0.827)	(0.093)
Dist^2	0.337	0.022
	(1.850)	(0.039)
Tourist	0.134	0.116
	(0.089)	(0.320)
City $N2$	0.338	4.574
	(0.011)	(2.684)
City $N+$	0.084	10.377
	(0.009)	(3.656)
	R-squared is: 0.612	

PROBIT RESULTS[a]

Variable:	(1)	(2)	(3)
Const	−3.53	−3.23	−3.36
	(0.069)	(0.101)	(0.064)
Pop	1.21	0.18	1.21
	(0.082)	(0.100)	(0.082)
Dist	0.03	0.010	1.18
	(0.004)	(0.005)	(0.169)
Dist^2	−0.001	−0.001	—
	(0.0001)	(0.001)	—
Tourist	0.12	−0.071	—
	(0.044)	(0.050)	—
City2	2.05	1.83	2.06
	(0.054)	(0.054)	(0.054)
City share	5.58	—	5.47
	(0.164)	—	(0.162)
# Routes	—	0.732	—
	—	(0.018)	—
−2 log-likelihood:	6799	6220	6834

[a]Observations are 18218 firm/market combinations. Standard errors are in parentheses.

SIMULATION ESTIMATES[a]

Variable	Most Profitable Move First	Incumbents Move First
Constant	−5.32	−3.20
	(0.354)	(0.258)
Population	1.36	5.28
	(0.239)	(0.343)
Dist	1.72	−1.45
	(0.265)	(0.401)
City2	4.89	5.91
	(0.295)	(0.149)
City Share	4.73	5.41
	(0.449)	(0.206)
δ	0.527	4.90
	(0.119)	(0.206)
ρ	0.802	0.050
	(0.105)	(0.048)
Value of the objective fn:	33.3	26.2

[a]Observations are 1219 markets. Standard errors are in parentheses.

5 Mazzeo

The Mazzeo (2002) model extends the Bresnahan and Reiss (1991) model by allowing firms to chose which type of firms they enters as. In many empirical applications, firms can differentiate between each other by choosing to enter in different areas of the product space, so for instance I might build a Chinese restaurant if you decide to build an Italian restaurant. In the specific application Mazzeo considers, firms can enter either as high or low quality motels, where we denote the firm's type as $\theta_i \in \{h, l\}$.

- We want to understand how firms decide the "location" of the products that they produce. This is driven in large part by the incentive to differentiate my product from those of my competitor.
- Allow for different types of producers to compete with each other: not just a homogenous good.
- Mazzeo chooses the Motel Industry: High and Low quality motels, and a clearly defined market: exit on a highway.

A firm's profit function depends on total demand in the market denoted as X and the number of firms that choose to enter as either high or low quality hotels:

$$\pi_{\theta_i}(N_l, N_h, X) = X\beta_{\theta_i} - g_{\theta i}(N_l, N_h) + \epsilon_i \tag{14}$$

with the addition of a market/type unobservable to profits denoted ϵ_{theta_i} which is common to all firms in a market which are of the same type. Note that this ϵ_{theta_i} should be correlated within the market accross firms of the same type.

Note: Note that firms are identical except for their type. So they get the same shocks (which means we don't have to specify the number of potential entrants in a market).

Thus the equilibrium conditions in this market are:

1. Firms that are in the market make positive profits:

$$\begin{aligned} \pi_{\theta_h}(N_l, N_h, X) + \epsilon_{\theta_h} &> 0 \\ \pi_{\theta_l}(N_l, N_h, X) + \epsilon_{\theta_l} &> 0 \end{aligned} \tag{15}$$

2. If an additional firm entered, it would make negative profits:

$$\begin{aligned} \pi_{\theta_h}(N_l, N_h + 1, X) + \epsilon_{\theta_h} &< 0 \\ \pi_{\theta_l}(N_l + 1, N_h, X) + \epsilon_{\theta_l} &< 0 \end{aligned} \tag{16}$$

3. To close the model, Mazzeo needs to impose and additional assumption, that the effect of competition is higher for firms of your own type that the other type:

$$\begin{aligned} \pi_{\theta_h}(N_l, N_h - 1, X) &> \pi_{\theta_l}(N_l, N_h - 1, X) \\ \pi_{\theta_l}(N_l - 1, N_h, X) &> \pi_{\theta_h}(N_l - 1, N_h, X) \end{aligned} \tag{17}$$

Estimation: The model is estimated via simulated maximum likelihood:

$$\mathcal{L}(\theta) = \prod_m \Pr[N_l, N_h | X_m, \theta] = \int_{\epsilon_l} \int_{\epsilon_h} 1(\text{all equations satisfied}) d\epsilon_h d\epsilon_l$$

where we assume that the error term:

$$(\epsilon_h, \epsilon_l) \longrightarrow \mathcal{N}(0, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix})$$

Simulated Maximum Likelihood:

1. Draw:
$$(u_l^k, u_h^k) \longrightarrow \mathcal{N}(0, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix})$$

2. Transform these draws via the Cholesky decomposition:
$$(\epsilon_h, \epsilon_l) = chol(\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix})(u_l^k, u_h^k)$$

3. Get the number of times that the model gets the right answer:
$$\mathcal{L}^S(\theta) = \prod_m \frac{1}{\#K} \sum_k 1(\pi_{\theta_h}(N_l, N_h, X) + \epsilon_{\theta_h} > 0, \cdots, \pi_{\theta_l}(N_l - 1, N_h, X) > \pi_{\theta_h}(N_l - 1, N_h, X))$$

4. Maximize $\mathcal{L}^S(\theta)$ over θ.

One of the issues with Mazzeo's estimator is that it is an accept-reject simulator, which is no smooth. Thus, if I change θ, my objective function is a step function which can be tough to maximize over. In constrast, BLP was set up to have a smooth simulator, i.e. very small changes in θ always lead to changes in the criterion function.

FIGURE 1
PARTITIONING FOR EQUILIBRIUM OUTCOMES

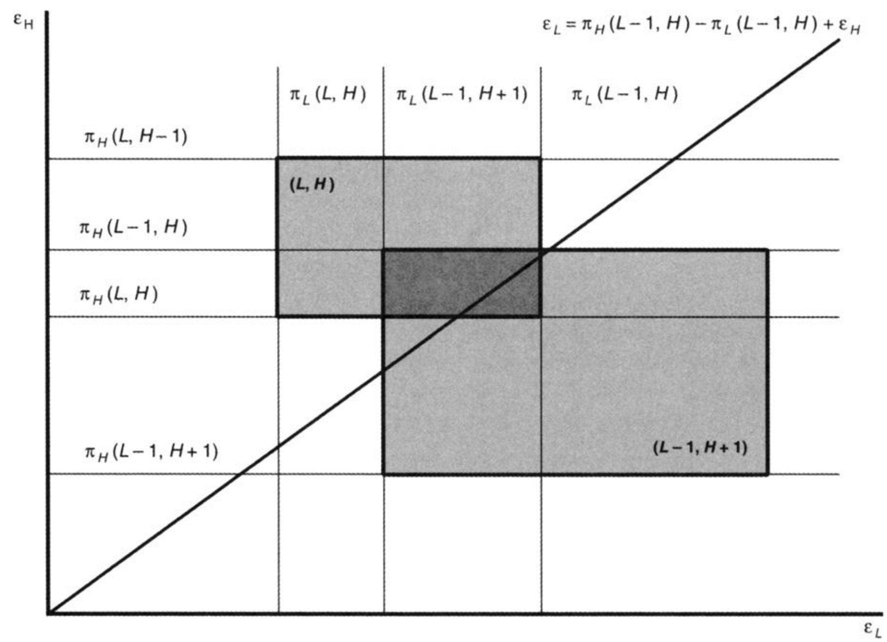


TABLE 4 Observed Market Configurations for the Two-Product-Type Models

Market Configuration	Number of Markets	Percent of Total (%)
(1, 0)	61	12.4
(0, 1)	67	13.6
(2, 0)	26	5.3
(1, 1)	40	8.1
(0, 2)	30	6.1
(3, 0)	10	2.0
(2, 1)	22	4.5
(1, 2)	30	6.1
(0, 3)	33	6.7
(3, 1)	13	2.6
(2, 2)	17	3.5
(1, 3)	35	7.1
(3, 2)	20	4.1
(2, 3)	30	6.1
(3, 3)	58	11.8
Total	492	

TABLE 6 **Estimated Parameters: Two-Product-Type Models**

Parameter		Two-Substage Version Estimate	Two-Substage Version Standard Error	Stackelberg Version Estimate	Stackelberg Version Standard Error
Effect on low-type payoffs					
Constant	C_L	1.6254	.9450	1.5420	.9192
Low competitor #1	θ_{LL1}	−1.7744	.9229	−1.6954	.8931
Low competitor #2	θ_{LL2}	−.6497	.0927	−.6460	.0922
High competitor #1 (0 lows)	θ_{L0H1}	−.8552	.9449	−.7975	.9258
Additional high competitors (0 lows)	θ_{L0HA}	−.1247	.0982	−.1023	.0857
Number of high competitors (1 low)	θ_{L1H}	−.0122	.1407	−.0154	.0444
Number of high competitors (2 lows)	θ_{L2H}	−.0000	.0000	−1.12E-6	.0001
PLACEPOP	β_{L-P}	.2711	.0550	.2688	.0554
TRAFFIC	β_{L-T}	−.0616	.1070	−.0621	.1069
SPACING	β_{L-S}	.3724	.1271	.3700	.1271
WEST	β_{L-W}	.5281	.1515	.5246	.1511
Effect on high-type payoffs					
Constant	C_H	2.5252	.9395	2.5303	.8925
High competitor #1	θ_{HH1}	−2.0270	.9280	−2.0346	.8810
High competitor #2	θ_{HH2}	−.6841	.0627	−.6841	.0627
Low competitor #1 (0 highs)	θ_{H0L1}	−1.2261	.9314	−1.2176	.8841
Additional low competitors (0 highs)	θ_{H0LA}	−5.25E-6	.0006	−.0000	.0000
Number of low competitors (1 high)	θ_{H1L}	−2.82E-7	.0001	.0000	.0001
Number of low competitors (2 high)	θ_{H2L}	−.0000	.0000	−5.34E-6	.0003
PLACEPOP	β_{H-P}	.6768	.0551	.6801	.0570
TRAFFIC	β_{H-T}	.2419	.1137	.2419	.1142
SPACING	β_{H-S}	.5157	.1332	.5159	.1328
WEST	β_{H-W}	.2562	.1585	.2588	.1592
Log-likelihood		−1143.01		−1143.12	

6 Seim

An issue with the Mazzeo (2002) model is that it is difficult to estimate the model when there are multiple "types" of firms, since the number of inequalities which need to be satisfied rises exponentially. We can either use a moment inequality model to deal with this problem (and Ariel will talk to you about these models a bit more), or change the structure of the game in such a way that makes the estimation of models with many types of entrants feasible.

The way that Seim does this is by introducing private information into the model, i.e. firms have information about the payoffs of entering a market that other firms do not see. This is by now a very common strategy in empirical I.O. (and Daniel will bring it up extensively in the context of auctions): assume that unobservables about other firms are also unobserved by other firms. To make this point as ridiculous as possible, this implies that you know as much about Wal-Mart's probability of entry as K-Mart does!

Second, private information shocks will lead to "mistakes" in the sense that two firms enter when only one firm would make positive profits. This could also happen in the perfect information case if firms are playing a mixed strategy equilibria.

Questions:

- How do firms choose their geographic locations: tradeoff between density of demand (lots of consumers) versus competition.
- Choice of different type of modems technologies (Greenstein, Augereau and Rysman have a paper on this).

The model that Seim uses has:

- Simultaneous Moves.
- Asymettric Information: I don't know my competitors ϵ's.
- The profits of entering into location i:

  $$\Pi_i = X_i^m\beta + \underbrace{\xi^m}_{\text{market shocks}} + h(\theta_i^m, n^m) + \underbrace{\eta_i^{mx}}_{\text{private information ideosyncratic shocsk}}$$

- Note that firms can want to exit ex-post, so this might be an unstable configuration in the market.

$$\Pi_i = X_i^m\beta + \xi^m + \sum_b \theta_{bi} r_{bi} + \eta_i^{mx}$$

- What are the expected payoffs of entering into location i?

$$E^x[\Pi_i^x] = \int_{\eta^{-x}} \left(X_i^m \beta + \xi^m + \sum_b \theta_{bi} r_{bi} + \eta_i^{mx} \right) df(\eta^{-x})$$

Because this a linear function, we just care about the probability of other firms entering in location b:

$$E^x[\Pi_i^x] = X_i^m \beta + \xi^m + \sum_b \theta_b (\hat{N} - 1) p_b^* + \eta_i^{mx}$$

- Assume that the private information shocks η_i^{mx} are distributed as i.i.d. logit draws. To get this probability of entry in location i:

$$p_i^* = \frac{\exp(X_i^m \beta + \xi^m + \theta_0 + (\hat{N} - 1) \sum_b \theta_b \sum_j 1_{ij}^b p_j^*)}{1 + \sum_k \exp(X_k^m \beta + \xi^m + \theta_0 + (\hat{N} - 1) \sum_b \theta_b \sum_j 1_{kj}^b p_j^*)}$$

- We need to solve for the fixed point of this equation. We can do this just by iterating on this equation since it is a contraction mapping.

- We can recover the market level shock ξ^m in exactly the same way that we did using BLP (with no non-linear parameters). Notice that:

$$\xi^m = \ln(\hat{N}) - \ln(E - \hat{N}) - \ln(\sum_k \exp(X_k^m \beta + \theta_0 + (\hat{N} - 1) \sum_b \theta_b \sum_j 1_{kj}^b p_j^*))$$

- Suppose $\xi \leftarrow \mathcal{N}(\mu, \sigma)$. How do you estimate this model? Do it by maximum likelihood:

$$\mathcal{L}(\theta) = \prod_m \phi(\frac{\xi^m(\theta) - \mu}{\sigma}) \frac{n_i^m}{\hat{N}^m} p_{im}^*$$

Figure 2: Sample Market - Great Falls, MT

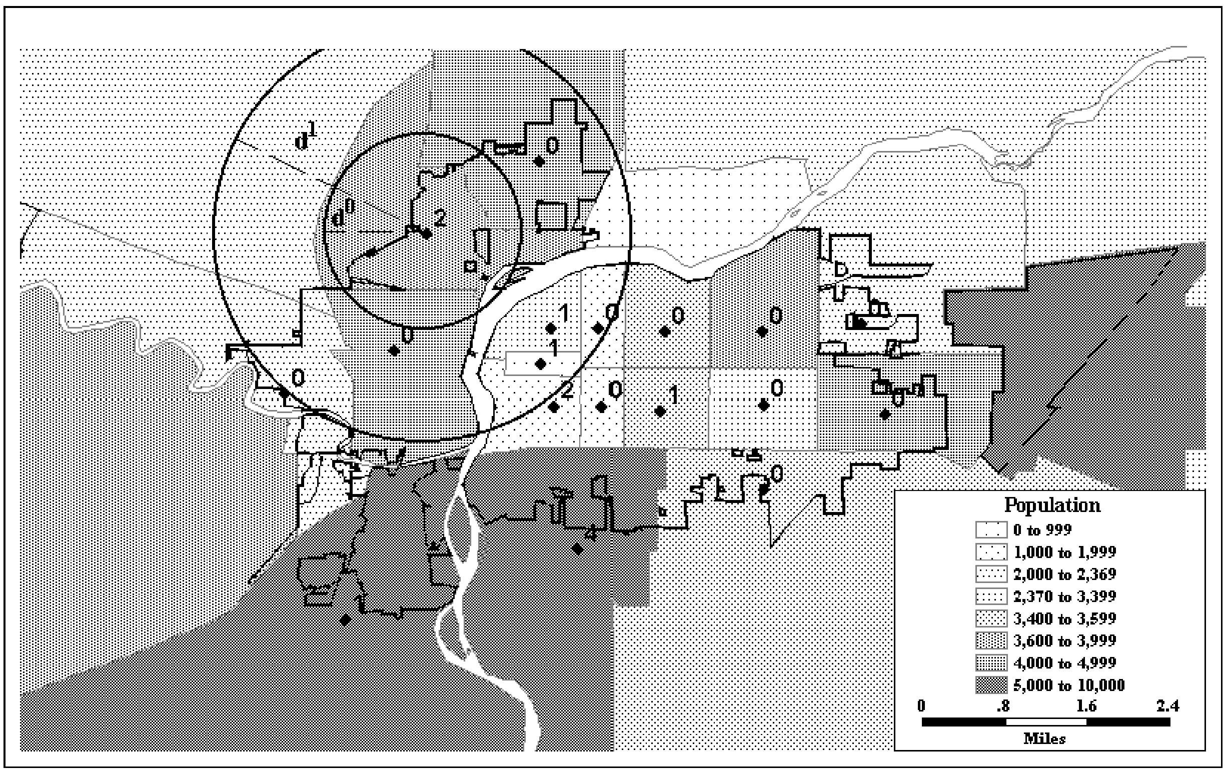


Table 4: Parameter Estimates, Entry and Location Choice Model

	Potential Entrant Pool =			
	2 x Total Entrants		50 Firms	
Variable	Coefficient (Std. Error)	Marginal Effect	Coefficient (Std. Error)	Marginal Effect
Population_0 (000)	1.8191 (0.1534)	0.0333	2.1258 (0.1764)	0.0393
Population_1 (000)	1.3109 (0.1200)	0.0236	1.7349 (0.1498)	0.0314
Population_2 (000)	0.6070 (0.1192)	0.0121	1.1348 (0.1486)	0.0227
Business density	-0.8077 (0.1458)	-0.0155	-0.8889 (0.1477)	-0.0173
Avg. Per-Cap. Income_0 (0000)	0.9309 (0.1136)	0.0180	1.0380 (0.1233)	0.0204
Avg. Per-Cap. Income_1 (0000)	1.0081 (0.2081)	0.0193	0.9188 (0.2043)	0.0178
Avg. Per-Cap. Income_2 (0000)	0.4851 (0.2512)	0.0092	0.4884 (0.2601)	0.0094
γ_0	-3.4520 (0.3111)		-3.3853 (0.3266)	
γ_1	-1.0103 (0.0745)		-1.0087 (0.0923)	
γ_2	-0.3448 (0.0738)		-0.4870 (0.0934)	
σ	3.5829 (0.3110)		4.6760 (0.4316)	
μ	-2.8764 (1.3425)		-7.0364 (1.5801)	

Notes:

Results based on 1999 demographic and firm data. Subscript 0 denotes the immediately adjacent locations to the chosen tract, within 0.5 miles in distance; subscript 1 denotes tracts at 0.5 to 3 miles in distance from the chosen tract; and subscript 2 denotes tracts at more than 3 miles distance from the chosen tract. Tract-level business density is defined as the number of establishments (0000) per square mile. γ denotes competitive effects, and σ and μ the estimates of the parameters of the distribution of ξ.

Table 1: Descriptive Statistics, Markets and Locations

	151 Sample Markets		
	Mean	Minimum	Maximum
Market level			
Population, market	74,367	41,352	142,303
Population, main city	59,428	40,495	140,949
Population, all tracts in market	92,563	41,614	193,322
Largest Incorporated Place within 10 mi	2,618	-	9,972
Largest Incorporated Place within 20 mi	7,916	-	24,725
Tract level			
Number of tracts	21.13	8	49
Number of store locations	18.72	7	44
Tract population	4,380	247	32,468
Area (sqmi)	10.10	0.10	181.50
Average distance (mi) to other locations in market	3.49	1.08	8.05

Notes:

The largest incorporated place within 10 and 20 miles is relative to the centroid of the market's main city. The distance between locations within a market is computed as the distance between the tracts' population-weighted centroids. Demographic data is as of 1999.

7 Rationalizeability

I will estimate a simple entry model in the spirit of Bresnahan and Reiss (1991) using only the restrictions that players use rationalizeable strategies, on data from the ready-mix concrete industry. While this empirical exercise is fairly stripped down, it can be adapted for greater realism, such as allowing for different types of entrants, or correlation in the unobserved component of firm profits. In the second section, I discuss the realism of Nash Equilibrium in applied work.

7.1 Data

I use data on entry patterns of ready-mix concrete manufacturers in isolated towns across the United States. In previous work such as **?** I have studied entry patterns in the ready-mix concrete market. Concrete is a material that cannot be transported for much more than an hour, and thus it makes sense to study entry in local markets. I construct "isolated markets" by selecting all cities in the United States which are at least 20 miles away from any other city of at least 2000 inhabitants. I then count the number of ready-mix concrete establishments in the U.S. Census Bureau's Zip Business Patterns for zip codes at most 5 miles away from the town. [1] [2]

I estimate the probability of entry using nonparametric regression:

$$\hat{P}(x_i) = \frac{1}{N^e} \sum_{x_j \neq x_i} N_j K(\frac{x_j - x_i}{h}) \tag{18}$$

I use a normal density as a kernel, and I choose a smoothing parameter $h = 0.43$ in order to minimize the sum of squared errors from the regression. Moreover, the number of potential entrants in the model (denoted N^e) is set to 6, the maximum number of firms in the data. Figure 7 presents the number of ready-mix concrete establishments in a town plotted against town population, along with a non-parametric regression of this relationship where $\hat{N}(x_i) = N^e \hat{P}(x_i)$.

[1] A description of the Zip Business Patterns dataset can be found at `http://www.census.gov/epcd/www/zbpbase.html`, accessed September 1, 2007.

[2] More information on the construction of data on isolated towns as well as the set of towns and zip codes used to construct the dataset used in this discussion can be found at `http://pages.stern.nyu.edu/ acollard/Data%20Sets%20and%20Code.html`.

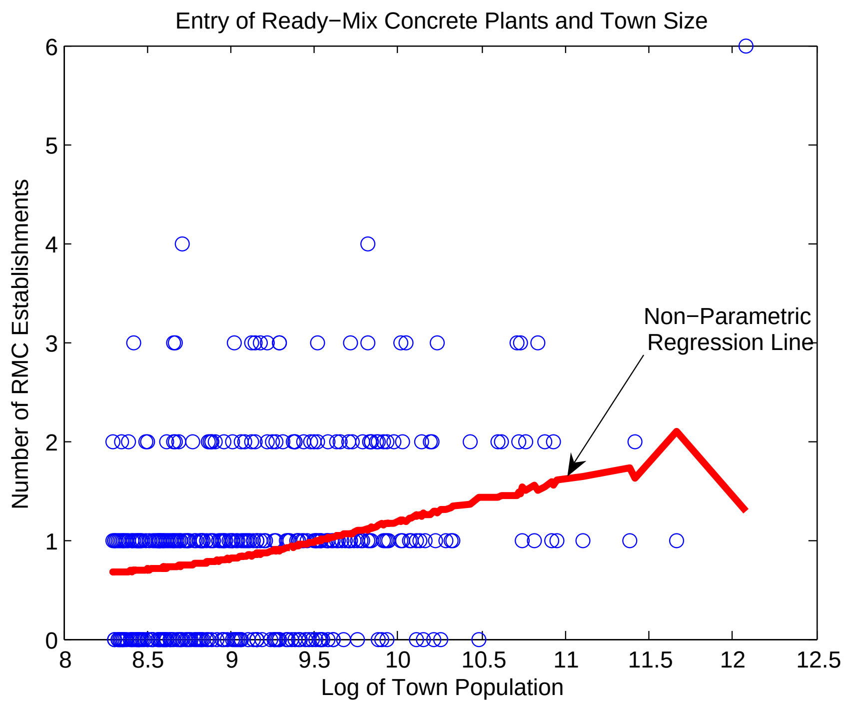


Figure 7: Entry Patterns of Ready-Mix Concrete Plants in Isolated Markets.

7.2 Estimator

I use a entry model similar to the one discussed in Tamer and Aradillas-Lòpez. Firms are ex-ante identical, but receive different private information shocks to the profits they will receive upon entry. I parametrize a firm's profits as:

$$\pi_i = \beta \underbrace{x_i}_{\text{Log Population}} + \alpha \underbrace{N_i}_{\text{Number of Entrants}} + \underbrace{\epsilon_i}_{\text{private information shock}}$$

where β measures the effect of population on profits and α is the effect of an additional competitor on profits. Initially, the highest prior I can assign to the entry probability of my opponents is that all opponents enter, i.e. $\bar{e}(x_i)^0 = 1$. Likewise, the lowest possible prior I can have is that no other firms enter the market, i.e. $\underline{e}(x_i)^0 = 0$. Given these upper and lower bounds on beliefs, a firm choose to enter if it makes positive profits. Thus the bounds on a firm's expected profits π_i^k (where k denotes the K-level of rationality) given the assumption that effect of additional firms is to decrease profits are:

$$x_i\beta + \alpha N^e \bar{e}(x_i)^0 + \epsilon_i \leq \pi_i^0 \leq x_i\beta + \alpha N^e \underline{e}(x_i)^0 + \epsilon_i$$

The bounds on the probability that a firm will enter given $K = 0$, denoted $e_i(\theta)$ follow directly:

$$F_\epsilon(x_i\beta + \alpha N^e \bar{e}(x_i)^0) \leq e_i(\theta) \leq F_\epsilon(x_i\beta + \alpha N^e \underline{e}(x_i)^0)$$

where F_ϵ is the c.d.f. of ϵ.

From here it is straightforward to iterate on the upper and lower bounds for entry probabilities for levels of rationality higher than $K = 0$. The upper bound on the entry probability for a firm is denote $\bar{e}(x_i)^k$ and the lower bound is denoted $\underline{e}(x_i)^k$, and these are given recursively by:

$$\bar{e}(x_i)^{k+1} = F_\epsilon(x_i\beta + \alpha N^e \underline{e}(x_i)^k) \quad (19)$$

$$\underline{e}(x_i)^{k+1} = F_\epsilon(x_i\beta + \alpha N^e \bar{e}(x_i)^k) \quad (20)$$

In my application I will just assume that ϵ has a standard normal distribution, i.e. $\epsilon \sim N(0,1)$.

A natural estimator for this model can be derived from looking for cases when the entry probabilities in the data are outside of the upper and lower bounds. The criterion for one such estimator is presented in equation (21):

$$Q^k(\theta) = \sum_i ([\hat{P}(x_i) - \bar{e}^k(\theta, x_i)]^+)^2 + ([\underline{e}^k(\theta, x_i) - \hat{P}(x_i)]^+)^2 \quad (21)$$

The identified set is just the set of parameters θ for which there are no violations of the upper and lower bounds:

$$\hat{\Theta}^I = \{\theta \in \Theta : Q^k(\theta) = 0\} \tag{22}$$

In contrast, the standard estimator using a symmetric Nash Equilibrium, such as the model of **?**, would look for an entry probability e^* which is a fixed point to the Best-Response mapping, i.e. e^* such that:

$$e^* = F_\epsilon(x_i\beta + \alpha N^e e^*)$$

This would lead to an estimator with the following criterion function, the distance between the symmetric Nash solution and the data:

$$Q^N(\theta) = \sum_i [\hat{P}(x_i) - e^*(\theta, x_i)]^2 \tag{23}$$

Note that the estimator which minimizes the Nash Criterion in equation (23) will be in general point identified. The estimated parameter for the Nash Criterion is $\hat{\theta}^N = \text{argmin}_\theta Q^N(\theta)$, the (generically) unique parameter which minimizes the deviations of the Nash prediction from the data.

Figure 8 presents the prediction of both the Rationalizable model for up to 100 levels of iterated deletion of dominated strategies and the Nash Equilibrium model; for the parameters $\alpha = -0.42$ and $\beta = 0.1$. The green lines show the upper bound on the number of firms which enter, corresponding to the smallest possible belief about the number of other firms that might enter. Likewise, the red lines correspond to the lower bound on the the number of expected entrants, if I held the greatest belief about the entry probability of opponents. The middle dashed line shows the prediction from the symmetric Nash Model. Note that while the upper and lower bounds get closer to each other as we increase the K-level of iterated deletion of strategies, they do not necessarily converge to the symmetric Nash model. Indeed, it is possible to sustain asymettric equilibria in this model of the type: 1-firms enter because they expect other firms not to enter and 2-firms stay out of the market because they expect other firms to enter. The larger the competitive interaction parameter α, the larger the split between the upper and lower bounds. In fact it is this effect of competitive interaction α on the spread between the upper and lower bound that will make it hard to reject very high competitive interactions.

Figure 9 presents the identified set described by equation (22) for the Tamer and Aradillas-Lòpez model using Ready-Mix Concrete data where I let K go from 0 to 100. As K increase above 30, the blue shaded area

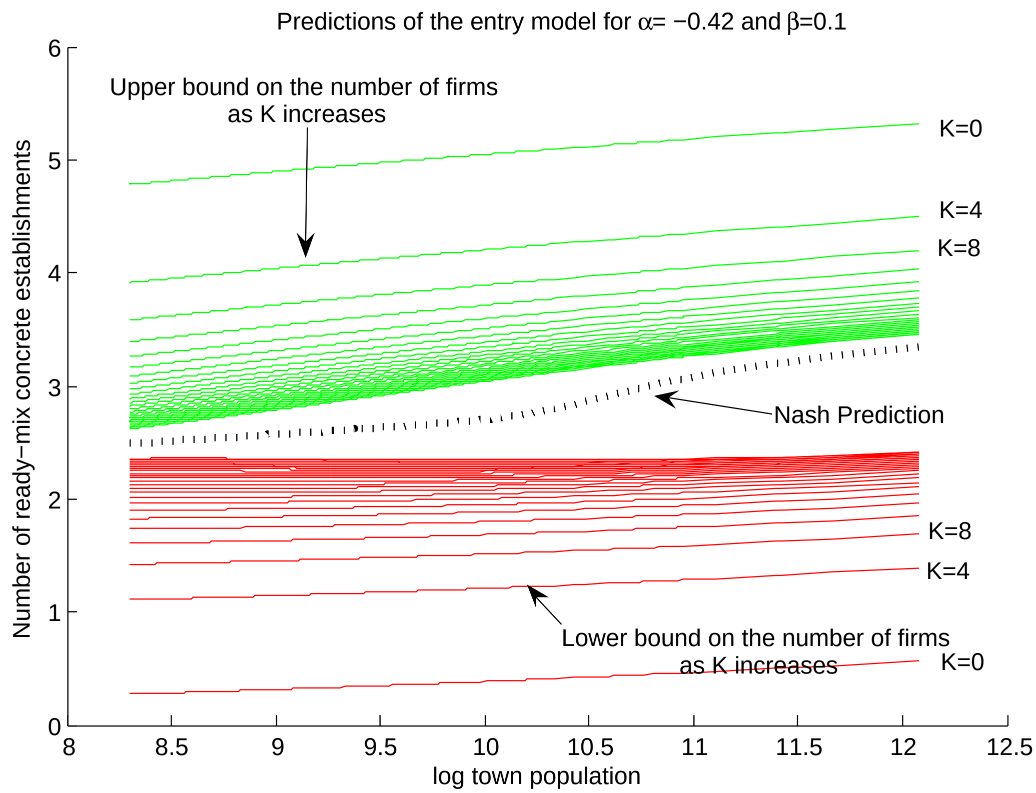


Figure 8: Model Predictions for K Levels of Iterated Deletion of Dominated Strategies and Nash Equilibrium.

in the top left disappears from the identified set indicating that assuming a higher K-level of rationality shrinks the identified set. In particular, the highest possible α in the identified set decreases from -0.2 to -0.5 as K goes from 0 to about 30. Above $K = 30$ the identified set stays about the same, which we should expect given that in a finite number of iterated deletion of dominated strategies gives the set of rationalizable strategies. The upper bound on the effect of competition on profits is about $\alpha = -0.50$, so we can state conclusively that there is an effect of competition on profits in the ready-mix concrete industry. However, there is no lower bound on the effect of competition on profits, so we cannot reject the assertion that competition reduces profits by and arbitrarily large amount. To understand this result, it is worth remembering the increasing the effect of competition on profits pushes out the upper and lower bounds in Figure 8, since a big effect of competition on profits makes it possible to sustain asymettric equilibria of the type, if I expect no other firm to enter, I will enter for sure, and if I expect other firms to enter, I will choose not to enter myself. Thus, increasing the competitive parameter α will enlarge the set of permissible entry probabilities, which makes it impossible to form a lower bound on α. This seems to be a fairly generic results which casts some doubt on estimates from the entry literature on the strength of competition. Figure 9 also shows the location of the parameter that minimizes the Nash criterion function presented in equation (23). Notice that this parameter gives no incling of the size of the identified set.

8 More References on Entry Games

Here are several papers I would recommend you read on entry games.

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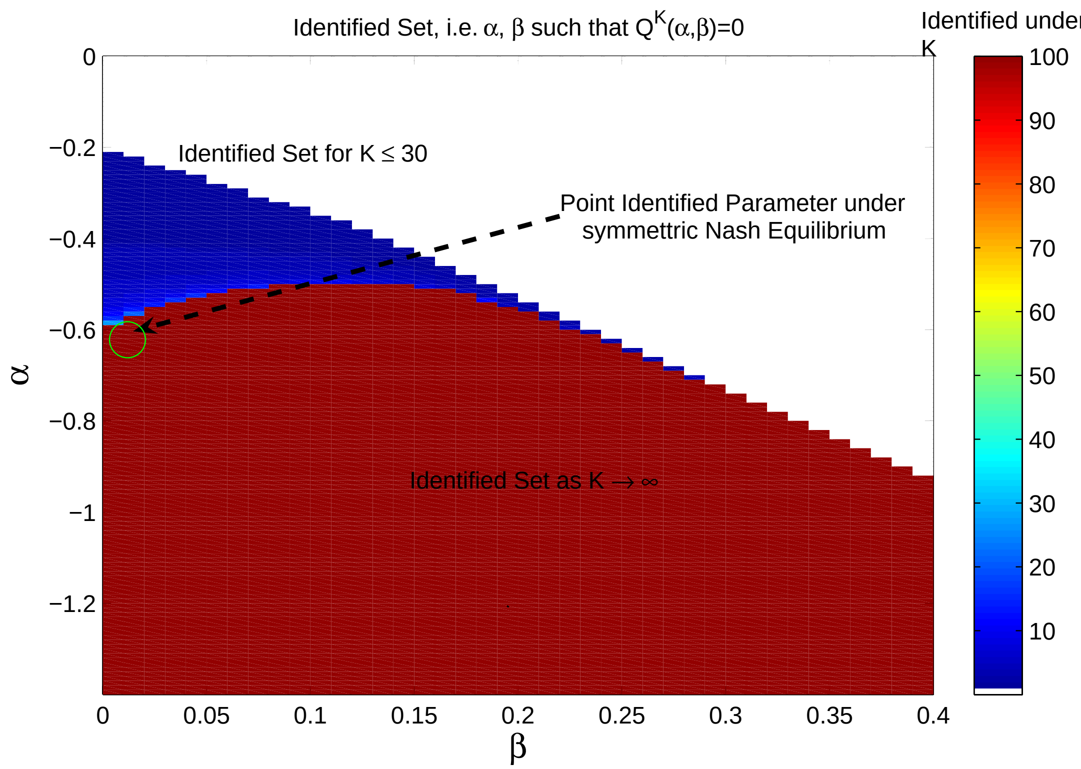

Figure 9: Identified Set for the Tamer and Aradillas-Lòpez Model using Ready-Mix Concrete Data.

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