

Problem Set 1: Math Review and Basics of Supply and Demand

1. SHORT ANSWER QUESTIONS

1. Taylor believes that people are irrational; that each person is irrational in his or her own special way; and that the nature of a person's irrationality sometimes changes from one moment to the next. Is her belief a theory? Does it have implications that can be verified or falsified? Is it a useful theory?
2. Name an example of each of the following:
 - (a) Goods that are provided centrally, by the government.
 - (b) Goods that are provided by firms operating in decentralized markets.
 - (c) Goods that are provided both centrally by the government and by firms operating in decentralized markets.
 - (d) Goods that are provided through some decentralize procedures other than markets.
3. Which of the following is an example of a normative statement?
 - A) Cap and Trade Legislation would reduce Global Warming.
 - B) Cap and Trade Legislation would raise prices for Electricity.
 - C) Cap and Trade Legislation's benefits outweigh it's effect on electricity prices.
 - D) Cap and Trade Legislation would have a lower impact on electricity prices than a carbon tax.
4. Legislators argue that a minimum wage law is instituted to help poor people. Economists can attack the minimum wage law on two fronts. First, some argue that government should not help the poor. Second, some argue that minimum wage laws actually hurt the poor because it creates unemployment. Which argument is normative and which is positive?

2. MATH REVIEW QUESTIONS

1. What is the partial derivative, $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ of:
 - (a) $f(x, y) = 3y^2 + 2x^3$.
 - (b) $f(x, y) = 10x^2y^4$.
 - (c) $f(x, y) = 2\ln(x) + \frac{1}{2}y^2$

(d) $f(x, y) = 10 \ln(x) \ln(y) + 5x$.

2. Consider the following function. Graph them, along with labeled axis and show at what x and y values the function intersects the axis.

(a) $f(x) = 10 - 2x$

(b) $f(x) = \frac{10}{x+5}$.

(c) $q(p) = 8 - 2p^2$.

(d) $5p - 20 = 10q + 3p$.

3. The following functions need to be maximized (show (i) the first-order conditions i.e. $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$ and (ii) solve them for the solutions):

(a) $f(x) = 15 - 3x^2$

(b) $f(y) = 10 + 9y - \frac{1}{3}y^3$

(c) $f(x, y) = -2(2x - 10)^2 - 10(5 - y)^2$

(d) $f(x, y) = xy - \frac{x^2}{2} - y^2 + 10x - 30y + 50$

3. PRACTICE QUESTION: PROFIT MAXIMIZATION (THIS WILL BE DONE IN TA SECTION)

A large company produces chairs and bench next to a sawmill, and they produce these two items together since some of the short pieces of wood which are cut while making benches can be reused while making chairs. The two decisions they need to make are:

- The number of benches produced (which we will denote as b).
- The number of chairs produced (which we will denote as c).

A team of advanced wood engineers and marketing specialists do a study and find that the firm's profits π as a function of c and b . The firm's profit function end up looking like:

$$\pi(c, b) = 70b + 50c - b^2 - c^2 - cb$$

You've been brought in to consult on the optimal production of benches and chairs, and produce a production plan for the company.

- (a) What are the first-order conditions that characterize the profit maximizing bench and chair production.

Answer: The two first-order conditions are: ($\frac{\partial \pi}{\partial b} = 0$):

$$\begin{aligned} \frac{\partial \pi}{\partial b} &= \frac{\partial [70b + 50c - b^2 - c^2 - cb]}{\partial b} \\ &= \frac{\partial [70b + 50\Delta - b^2 - \spadesuit - \clubsuit b]}{\partial b} \quad (\text{we really need to do our substitutions here!}) \\ &= \frac{\partial [70b]}{\partial b} + \frac{\partial [50\Delta]}{\partial b} + \frac{\partial [-b^2]}{\partial b} + \frac{\partial [-\spadesuit]}{\partial b} + \frac{\partial [-\clubsuit b]}{\partial b} \\ &= 70 + 0 - 2b + 0 - \clubsuit \\ &= 70 - 2b - c \end{aligned}$$

So $\frac{\partial \pi}{\partial b} = 0$, gives us $c = 70 - 2b$. The next FOC is ($\frac{\partial \pi}{\partial c} = 0$):

$$\begin{aligned} \frac{\partial \pi}{\partial c} &= \frac{\partial [70b + 50c - b^2 - c^2 - cb]}{\partial c} \\ &= \frac{\partial [70\Re + 50c - \Im - c^2 - c\heartsuit]}{\partial c} \\ &= \frac{\partial [70\Re]}{\partial c} + \frac{\partial [50c]}{\partial c} + \frac{\partial [-\Im]}{\partial c} + \frac{\partial [-c^2]}{\partial c} + \frac{\partial [-c\heartsuit]}{\partial c} \\ &= 0 + 50 + 0 - 2c - \heartsuit \\ &= 50 - 2c - b \end{aligned}$$

So $\frac{\partial \pi}{\partial c} = 0$, gives us $b = 50 - 2c$.

- (b) How many benches and chairs should this company produce?

Answer: Now we can put the two focs together and solve them:

$$c = 70 - 2b$$

$$b = 50 - 2c$$

which has a solution for $c = 10$, and $b = 30$. This is the recommendation we will submit to the company.

- (c) What are their profits if they implement your production of bench and chair recommendation?

Answer: Let's just plug in our recommendation into the profit function:

$$\begin{aligned}\pi(c, b) &= 70b + 50c - b^2 - c^2 - cb \\ &= 70 \times 30 + 50 \times 10 - 30^2 - 10^2 - 10 \times 30 \\ &= 2300\end{aligned}$$

PROBLEM SET 2: BASICS OF SUPPLY AND DEMAND

1. SHORT ANSWER QUESTIONS

1. Explain the difference between a change in quantity demanded and a change in demand. Give an example of each.
2. Explain how we can account for ‘bads’ (such as pollution) in analysis of consumer preferences.

2. SUPPLY AND DEMAND- SHORT-ANSWER STYLE QUESTION 1

Suppose that a new and better microprocessor is developed for laptop computers. What would be the effect on prices and quantity of laptops if the new microprocessor is better in the following ways (illustrate using well labeled graphs and discuss).

1. It is cheaper to produce at the same speed as the old microprocessor.
2. It is faster but costs the same to produce as the old microprocessor.
3. It is both cheaper to produce, and faster than the old microprocessor.

3. SUPPLY AND DEMAND- SHORT-ANSWER STYLE QUESTION 2

After terrorists destroyed the World Trade Center and surrounding office buildings on September 11, 2001, some business people worried about the risks of remaining in Manhattan. What effect would you expect this concern to have in the short run (before any of the destroyed office buildings are rebuilt) on prices of office space in Manhattan? What factors does your answer depend on? What about the effects in the long run? Suppose the area around the World Trade Center is built into a park, so that the destroyed office buildings are never rebuilt. Economically, who would gain and who would lose from such a plan?

4. SUPPLY AND DEMAND QUESTION

Suppose we are looking at the demand and supply for housing. The supply curve for housing, i.e. the number of houses and apartments that builders will produce given a price for housing p is $S(p) = 80p - 50$. The demand curve for housing, i.e. the number of housing units that are demanded by individuals, is given by $D(p) = 220 - 10p$.

1. Draw the supply and the demand curve on a well labeled graph, indicating the price and quantity axis, as well as the intersection of the supply and demand curve with each other and with the horizontal

and vertical axis.

2. What is the slope of the demand curve, and what is the slope of the supply curve? Why do the demand and supply curves have different slopes, and can you think of cases where the slopes would be different, i.e. where demand curves would be upward sloping, or where supply curves would be downward sloping?
3. Suppose we increase the number of immigrants that can enter the country which are a large part of the building trade. What would happen to the demand and supply curves? Draw it on a graph and explain in a sentence or two.
4. Suppose the supply curve is now $S(p) = 80p - 5$. What is the new market clearing price and quantity?

5. PREFERENCES AND INDIFFERENCE CURVES

Suppose that there two types of food, meat and bread. Draw indifference curves for the following consumers.

1. Ed likes variety and prefers to eat meat and bread together.
2. Taka, a sumo wrestler, cares only about the number of calories he consumes; he wants to consume as many calories as possible. Suppose that Beef has 100 calories per unit and Bread has 200 calories per unit (since protein has fewer calories per gram than carbohydrates). What do Taka's indifference curves look like?
3. Mia dislikes variety; she likes to eat the same thing all the time.
4. Rahul is a vegetarian who doesn't care (one way or the other) about meat. What if Rahul actually dislikes meat?

Problem Set 3: Budget Set, Utility and Consumer choice

1 Short Answer Questions

1. Suppose that Ellen has $\$I$ to spend on goods x and y . The price of good x is p_x and the price of good y is p_y . Write down Ellen's budget line and draw it. Indicate the slope, as well as the vertical and the horizontal intercept.

Answer: See your notes.

2. Explain why utility maximization requires that a consumer's budget is allocated so that the marginal utility per dollar of expenditure is the same for each good.

Answer: Otherwise, the consumer can readjust the quantities he consumes and increase his utility while respecting his budget constraint.

3. Teresa likes to talk on the telephone. We can represent her preferences with the utility function $U(B, C) = 10B + 5J$, where B and J are minutes of conversation per month with Bill and Jackie respectively. What is the formula for her indifference curve to get U units of utility? What is her marginal rate of substitution of minutes talking to Jackie for minutes talking to Bill?

Answer: Her indifference curve to get U units is $U = 10B + 5J$ which we can rearrange to $10B = U - 5J$ or $B = \frac{U}{10} - 0.5J$. Thus the marginal rate of substitution of minutes talking to Jackie for minutes talking to Bill is $MRS = 0.5$; i.e. Teresa would give up 0.5 minutes talking to Bill for one minute talking to Jackie.

4. Explain what is meant by a corner solution? Can you give an example?

Answer: We mean a situation where a consumer maximizes utility by spending all her income on just one good. For example, consider Anna whose utility function over blue and green pens is $U(B, G) = B + G$. Then her MRS of blue for green pens is 1 at all bundles so she always would trade one blue for one green pen. Suppose that $P_B = 1$ whereas $P_G = 2$. Since blue pens are cheaper and she is always as happy with one blue pen as one green pen, she maximizes her utility by spending all her income on blue pens.

5. True or false. Explain your answer.

Naj's preferences for hours of microeconomics study are represented by the utility function

$$u(h) = h^3 + 2h + 5$$

where h is the number of hours he studies. It is also the case that Naj's preferences for hours of microeconomics study are represented by the utility function

$$u(h) = h^3 + 2h + 27$$

True. Utility represents preferences, but the level of utility is not important, only relative comparisons between different bundles (in this case the single good hours of microeconomics study). Hence, adding a number (in this case 22) to a utility function does not change any comparisons so the second utility function also represents Naj's preferences.

2 Problem 1: MRS and Consumer Choice

A Consumer has preferences for apples (a) and oranges (o) which can be represented by the following utility function:

$$U(a, o) = 12a^{\frac{3}{4}}o^{\frac{1}{4}} + 7$$

The price of apples (p_a) is \$16 per unit while the price of orange (p_o) is \$4 per unit, and the consumer has income of \$64.

1.1 Compute the $MRS_{a \text{ for } o}$ for this consumer if she has

(A) nine units of apples ($a = 9$) and three units of oranges ($o = 3$).

(B) three units of apples ($a = 3$) and nine units of oranges ($o = 9$).

Marginal utility of apples

$$MU_a = \frac{\partial U}{\partial a} = 12 \times \frac{3}{4} a^{-\frac{1}{4}} o^{\frac{1}{4}}$$

Marginal utility of oranges

$$MU_o = \frac{\partial U}{\partial o} = 12 \times \frac{1}{4} a^{\frac{3}{4}} o^{-\frac{3}{4}}$$

Then, at option (A) nine apples ($a = 9$) and three oranges ($o = 3$)

$$MRS_{a \text{ for } o} = \frac{MU_a}{MU_o} = \frac{12 \times \frac{3}{4} a^{-\frac{1}{4}} o^{\frac{1}{4}}}{12 \times \frac{1}{4} a^{\frac{3}{4}} o^{-\frac{3}{4}}} = 3 \frac{o}{a} = 3 * \frac{3}{9} = 1$$

Then, at option (B) three apples ($a = 3$) and nine oranges ($o = 9$)

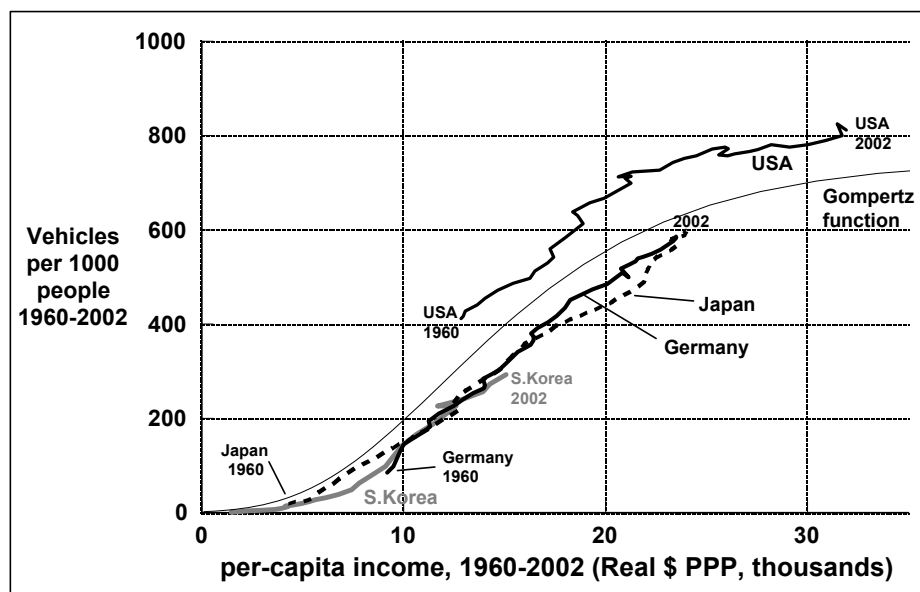
$$MRS_{a \text{ for } o} = \frac{MU_a}{MU_o} = \frac{12 \times \frac{3}{4} a^{-\frac{1}{4}} o^{\frac{1}{4}}}{12 \times \frac{1}{4} a^{\frac{3}{4}} o^{-\frac{3}{4}}} = 3 \frac{o}{a} = 3 * \frac{9}{3} = 9$$

Problem Set 4: Individual, Market Demand & Elasticity

PART I: SHORT ANSWER QUESTIONS

1. The graph below shows the relationship between vehicle ownership and average income in the USA, Germany, Japan and South Korea. Based on this graph, are vehicles a normal or inferior good? What would the graph look like in the opposite case?

Figure 1. Vehicle Ownership and Per-Capita Income for USA, Germany, Japan, and South Korea, with an Illustrative Gompertz Function, 1960-2002



2. Explain what is meant by a Giffen good? Which goods might be Giffen goods?
3. In the third quarter of 2011, Netflix announced a 60% price increase in their subscription fees. As a result, the total number of subscribers decreased from 24.59 million at the end of the second quarter to 23.79 million by the end of the third quarter. Based on these numbers, what do you compute to be the price elasticity of demand for Netflix subscriptions?
4. A study by Caulkins (1996) found that the price elasticity of demand for cocaine and heroin in the U.S. are -2.5 and -1.5 respectively. Provide a full interpretation of these elasticities.

What reason(s) can you give for the size of the elasticities? Suppose the government wants to spend money on raising the cost of cocaine or heroin by making it harder to smuggle drugs into the country. Which drugs should it target if it wants to lower total drug consumption?

5. Explain what is meant by isoelastic demand?
6. True or false, explain. The aggregation of individual demands into market demands is just a theoretical exercise.

PART II: PROBLEM

In the advanced planet of Aspin inhabitants take all their vitamins from the consumption of magic juice J and stardust S .

All inhabitants of Aspin have the same preferences over stardust and juice. The typical individual's preferences are represented by

$$U(S, J) = J^{\frac{1}{3}}S^{\frac{2}{3}}.$$

Let $p_S > 0$, $p_J > 0$ denote the price for stardust and J —juice respectively. The income of a typical inhabitant of Aspin is denoted by I .

- 1.1 Find the demand for S and J of a typical individual.
- 1.2 Suppose that $I = 12$ and $P_S = 1$ and that there are 500 individuals living in Aspin. Derive the market demand for J .
- 1.3 Suppose that $I = 75$ and $P_J = 2$ and that there are 60 individuals living in Aspin. Derive the market demand for S .
- 1.4 What is the price elasticity of S ?
- 1.5 What is the price elasticity of J ?
- 1.6 Suppose the price of stardust goes up to \$2. What is the demand for J ?
- 1.7 Draw a graph of the market demand for J indicating the consumer surplus when the price is \$2 and the price is \$1. Indicate the change in consumer surplus.

Problem Set 5: The Basics of Production

Question 1: Consider the following production functions where $F(L, K)$ is the number of cars produced given K units of capital and L units of labor. For each of these production functions determine whether it has constant, decreasing or increasing returns to scale.

1. $F(L, K) = 4K^{\frac{2}{3}}L^{\frac{1}{3}}$
2. $F(L, K) = 2K^{\frac{1}{2}}L^{\frac{3}{4}}$
3. $F(L, K) = 2K + 1L$
4. $F(L, K) = (2K + L)^{\frac{1}{2}}$

Question 2: Consider the following production function:

$$F(K) = 3K^{\frac{1}{2}}$$

where $F(K)$ is a number of computer chips produced given K units of capital.

1. Draw this production function (that is draw the number of computer chips produced given K units of capital).
2. Find the marginal product function MP_K and draw it.
3. Find the average product function AP_K and draw it.

Question 3: Consider the following production function:

$$F(L, K) = K^{\frac{1}{3}}L^{\frac{2}{3}}$$

where $F(L, K)$ is the number of cars produced given K units of capital and L units of labor.

1. Draw the isoquant function assuming that $F(L, K) = 1$:

2. Compute the marginal rate of technical substitution (MRTS): Interpret.
3. Compute MRTS when $K = 10$ and $L = 5$. Interpret.

Question 4: Below is a figure from an article on the effects of climate change on global food production¹. Suppose we performed a similar exercise by writing a production function:

$$F = AL^a T^b C^c$$

Where F is total food production, L is the amount of land used to produce food, T is temperature, C is the concentration of CO_2 in the atmosphere and A represents current technology. Suppose that we knew this function (i.e. we knew A, a, b and c), and used forecasts on changes in temperature (T) and CO_2 (C) to predict how much food production will change in the coming decades. Is there a potential problem with using only this information to forecast future production? What other important factor in our equation above are we ignoring?

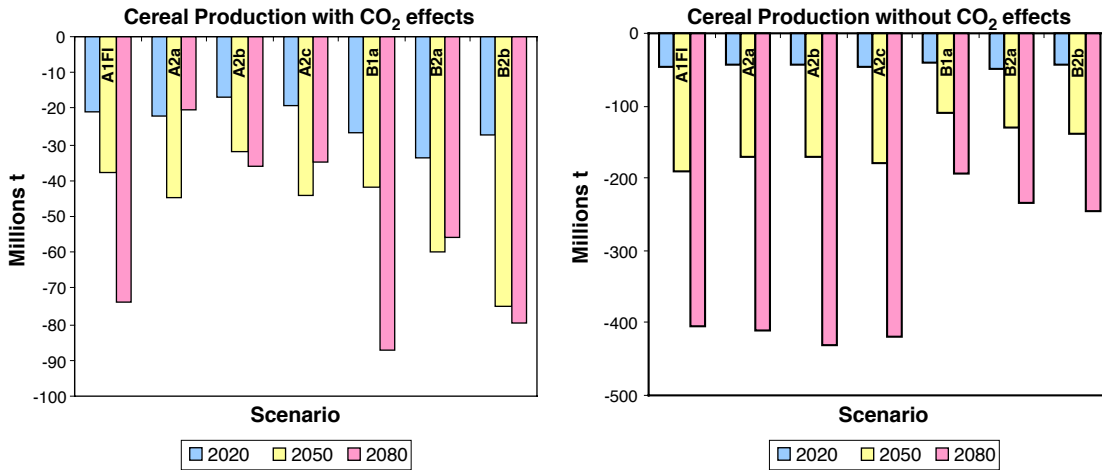


Fig. 12. Changes in global cereal production due to anthropogenic climate change under seven SRES scenarios with and without CO_2 effects, relative to the reference scenario (no climate change).

¹Parry, Rosenzweig, Iglesias, Livermore and Fisher (2004), *Effects of climate change on global food production under SRES emissions and socio-economic scenarios*, Global Environmental Change, 14

Question 5:

1. True or false, explain. When the firm's production function exhibits decreasing returns to scale, the isoquants come closer together as we move away from the origin.
2. True or false, explain. The law of diminishing marginal returns implies that when the use of an input increases with the other input fixed, we will always end up with a negative marginal product for the variable input.

Problem Set 6: Costs, Cost Minimization and Profit Maximization

PART I: SHORT ANSWER QUESTIONS

1. Ted is deciding whether or not to attend community college. Tuition costs \$1,000 a year and attending college will allow him 10 hours a week of leisure time. Ted's next best option is to work at his uncle's factory, which pays \$10,000 a year and would afford him 11 hours a week in leisure time. Assume that Ted values leisure at \$3 an hour. What are the accounting costs of Ted attending college for a year? What are the opportunity costs? *Hint: not all of the information given here is needed for your calculation.*
2. When output increases, the firm's average cost of production is likely to fall, at least to a point. Give three possible reasons for this.
3. What is the difference between a sunk and a fixed cost?
4. McSorley's Pub is making \$5,000 per month in sales and pays \$4,000 a month on an annually renewed lease, as well as \$500 per month in wages. In September, the cost of wages permanently increases to \$1,500 per month. Will McSorley's shut down immediately? Explain.
5. Write down the first-order conditions that must be satisfied for a profit-maximizing firm.
6. True or false, explain. The demand curve facing a perfectly competitive firm is a horizontal line, whilst the demand curve facing the industry as a whole is downward-sloping.
7. Explain the difference between producer surplus and producer profit.
8. Why might a firm continue to produce at a loss? Under what circumstances will it decide to shut down?
9. Explain why a competitive firm's short-run supply curve is upward sloping.

PART II: MAIN QUESTIONS

QUESTION 1

Suppose an electric utility has the following production function for Megawatts of electricity:

$$Q = F(L, K) = L^{\frac{1}{2}} K^{\frac{1}{2}}$$

Moreover, the firm pays \$ 4 per unit of capital and \$ 1 per unit of labor. Remember that a firms costs are determined by the fact that it pays rK for its capital and wL for its labor.

- 1.1 Does this production function exhibit increasing, decreasing or constant returns to scale?
- 1.2 What is the marginal product of capital and labor for this firm?
- 1.3 Find the cost minimizing amounts of capital and labor this firm will use, given that it must produce Q units of output (Remember the condition that must be satisfied at the cost minimizing level of output).
- 1.4 What is the cost of producing Q units of output (given that the firm is using the cost minimizing combination of labor and capital)?
- 1.5 What is the cost of producing 50 units of output?
- 1.6 Does this firm have economies or diseconomies of scale?

QUESTION 2

Suppose we face an industry in which each firm has the following cost function:

$$C(q_i) = 30 + 2q_i^2$$

where q_i is the output produced by each individual firm. There are exactly 40 firms in this industry, and this industry is perfectly competitive.

- 2.1 What is the marginal cost for a producer? What are average variable costs, what are fixed costs?

- 2.2 How much will an individual firm supply given that the market price is P ?
- 2.3 What is total supply in this industry, i.e. the total supply produced by all firms at a price of P ? (call total supply Q)
- 2.4 What are the profits for each firm as a function of P ?