# 316-466 Monetary Economics — Final Exam

1. **Flexible-price monetary economics** (90 marks). Consider a stochastic flexible-price money in the utility function model. Time is discrete and denoted $t = 0, 1, ...$ There is a representative consumer who faces a random sequence of money growth shocks and productivity shocks. The consumer supplies labor to a competitive spot market in return for a wage. Money growth shocks are denoted $\mu_t$ and labor productivity shocks are denoted $A_t$ and both are Markov processes. The state at $t$ is $s_t = (\mu_t, A_t)$ and a finite history is $s^t = (s_t, s_{t-1}, \cdots s_0)$ with $s_0$ known at date zero. As of date zero, the probability of $s^t$ is $f(s^t \mid s_0)$. The consumer has expected utility preferences over consumption $c_t(s^t)$, end of period real balances $m_t(s^t) \equiv M_{t+1}(s^t)/P_t(s^t)$, and leisure $\ell_t(s^t) \equiv 1 - n_t(s^t)$:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t U[c_t(s^t), m_t(s^t), 1 - n_t(s^t)] f(s^t \mid s_0), \qquad 0 < \beta < 1$$

The utility function $U$ is assumed to be strictly increasing and strictly concave in consumption and real balances but decreasing in labor. The representative consumer trades in money $M_{t+1}(s^t)$ and in a complete set of nominal, one-period, state contingent bonds $B_{t+1}(s^t, s')$. She has the flow budget constraint

$$\begin{aligned} & P_t(s^t)c_t(s^t) + M_{t+1}(s^t) + \sum_{s'} Q_t(s^t, s')B_{t+1}(s^t, s') \\ \leq \;& P_t(s^t)w_t(s^t)n_t(s^t) + M_t(s^{t-1}) + B_t(s^{t-1}, s_t) - T_t(s^t) \end{aligned}$$

where $P_t(s^t)$ denotes the price level, $w_t(s^t)$ denotes the real wage, $T_t(s^t)$ denotes lump-sum taxes and where $M_0 > 0$ and $B_0 = 0$ are given initial conditions.

There is also a representative firm with the production function $y_t(s^t) = A_t(s^t)n_t(s^t)$ that seeks to maximize profits by hiring labor. That is, taking as given prices and wages, the firm chooses $n_t(s^t)$ so as to maximize

$$P_t(s^t)A_t(s^t)n_t(s^t) - P_t(s^t)w_t(s^t)n_t(s^t)$$

The government's flow budget constraint is

$$\mathcal{M}_{t+1}(s^t) + T_t(s^t) = \mathcal{M}_t(s^{t-1})$$

where $\mathcal{M}_t(s^{t-1})$ denotes the exogenous money supply. The government's policy is

$$\mathcal{M}_{t+1}(s^t) = \mu_t(s^t)\mathcal{M}_t(s^{t-1})$$

where $\mu_t(s^t)$ denotes an exogenously given stochastic money growth rate.

   (a) (5 marks). Solve the firm's decision problem. What does this imply about the *real* wage? How does the real wage relate to labor productivity?

(b) (20 marks). Form a Lagrangian for the consumer's decision problem. Derive first order conditions characterizing the choice of $c_t(s^t)$, $M_{t+1}(s^t)$, $n_t(s^t)$, for each $t$ and $s^t$ and for $B_{t+1}(s^t, s')$ for each $t$ and $s^t$ and $s'$. Interpret these conditions. Use these first order conditions to derive a formula for the price $Q_t(s^t, s')$ of a state contingent nominal bond.

(c) (20 marks). Suppose that the period utility function is

$$U(c, m, 1-n) = \frac{c^{1-\sigma}}{1-\sigma} + \psi \frac{m^{1-\gamma}}{1-\gamma} - \chi \frac{n^{1+\eta}}{1+\eta}$$

where $\sigma$, $\psi$, $\gamma$, $\chi$ and $\eta$ are all positive parameters. A log-linear version of this model can be written

$$\begin{aligned}
\hat{c}_t &= E_t\{\hat{c}_{t+1}\} - \frac{1}{\sigma}\left(\hat{\imath}_t - E_t\{\hat{\pi}_{t+1}\}\right) \\
\hat{m}_t &= -\frac{\kappa}{\gamma}\hat{\imath}_t + \frac{\sigma}{\gamma}\hat{c}_t \\
\eta\hat{n}_t + \sigma\hat{c}_t &= \hat{w}_t \\
\hat{w}_t &= \hat{A}_t \\
\hat{c}_t &= \hat{y}_t = \hat{A}_t + \hat{n}_t \\
\hat{m}_t &= \hat{m}_{t-1} + \hat{\mu}_t - \hat{\pi}_t
\end{aligned}$$

plus the exogenous shocks. In this system, the constant $\kappa > 0$ comes from the log-linearization. Hats above a variable denote log-deviations from the non-stochastic steady state. Interpret each of the equations in this model (i.e., relate them to the consumer's optimality conditions, the firm's optimality conditions, market clearing, etc). Show that the supply side of the model is completely determined by labor productivity, i.e., that $(\hat{c}_t, \hat{y}_t, \hat{n}_t)$ depend only on $\hat{A}_t$ and not on $\hat{\mu}_t$. Explain how each of these variables responds to a labor productivity shock. Do they increase or decrease? How do the signs of your answers depend on the values of parameters? Give economic intuition for these findings.

(d) (20 marks). Suppose that in log-deviations, the money growth rate and the productivity shock follow mean zero AR(1) processes with independent, mean zero homoskedastic innovations

$$\begin{aligned}
\hat{\mu}_{t+1} &= \rho_\mu \hat{\mu}_t + \epsilon^\mu_{t+1}, \qquad 0 \le \rho_\mu \le 1 \\
\hat{A}_{t+1} &= \rho_A \hat{A}_t + \epsilon^A_{t+1}, \qquad 0 \le \rho_A \le 1
\end{aligned}$$

Show that the equations of the log-linear model (given in part (c) above) can be reduced to a stochastic difference equation in real balances. Solve this difference equation by iterating forward. Express your solution in terms of $\hat{\mu}_t$ and $\hat{A}_t$ alone.

(e) (20 marks). Use your solution from part (d) to explain how nominal interest rates and inflation respond to (i) money growth shocks, and (ii) productivity shocks. Are nominal interest rates more or less volatile than money growth? Discuss the sensitivity of your answers to the serial correlation coefficients $\rho_\mu$ and $\rho_A$.

(f) (5 marks). Roughly speaking, a monetary model exhibits the "classical dichotomy" if the real variables are determined independently of the monetary variables. Does this model exhibit the classical dichotomy? Why or why not? Explain the implications of this finding for monetary policy.

2. **Sticky-price monetary economics** (90 marks). This model is quite similar to that in Question 1, but with imperfect competition and Calvo-style sticky prices. There is a single representative consumer, a representative perfectly competitive final good firm and a large number of monopolistically competitive intermediate firms indexed by $i \in [0,1]$. Final goods firms produce the single consumption good using a CES production function

$$y_t(s^t) = \left[\int_0^1 y_t(i, s^t)^\theta di\right]^{1/\theta}, \qquad 0 < \theta \leq 1$$

While the intermediate firms, as with the firms in Question 1, produce their output using the linear production function

$$y_t(i, s^t) = A_t(s^t) n_t(i, s^t)$$

Denote the price level (in units of account) by $\bar{P}_t(s^t)$ and the price of a unit of intermediate output (also in units of account) by $p_t(i, s^t)$.

Intermediate firms are Calvo price setters. Each period, there is a signal for each intermediate firm. If that signal is "green" — which happens with probability $1-\omega$ — the intermediate firm gets a chance to re-optimize and choose a price $p_t(i, s^t)$. However, if that signal is "red" — which happens with the complementary probability $\omega \in (0,1)$ — the intermediate firm is stuck with its old price for the current period.

(a) (20 marks). Show that profit maximization by final goods firms leads to a demand function for intermediates

$$y_t(i, s^t) = \left[\frac{\bar{P}_t(s^t)}{p_t(i, s^t)}\right]^{1/(1-\theta)} y_t(s^t) \qquad ((1))$$

and an ideal price index

$$\bar{P}_t(s^t) = \left[\int_0^1 p_t(i, s^t)^{\theta/(\theta-1)} di\right]^{(\theta-1)/\theta}$$

(b) (8 marks). Suppose that we have a symmetric equilibrium where all firms that get a green signal in period $t$ set the same price. Call that price $p_t(s^t)$. Explain why the aggregate price level satisfies

$$\bar{P}_t(s^t) = \left[\omega \bar{P}_{t-1}(s^{t-1})^{\theta/(\theta-1)} + (1-\omega) p_t(s^t)^{\theta/(\theta-1)}\right]^{(\theta-1)/\theta}$$

(c) (7 marks). Let $v_t(s^t)$ denote the real marginal cost for an intermediate firm. Explain why static cost minimization by an intermediate firm leads to

$$v_t(s^t) = \frac{w_t(s^t)}{A_t(s^t)}$$

for all $i$.

(d) (10 marks). Given $v_t(s^t)$ as above, the profit maximization problem facing an intermediate firm with a green signal in period $t$ is

$$\max_{p_t(s^t)} \sum_{k=0}^{\infty} \sum_{s^k} \omega^k Q_{t,t+k}(s^{t+k}|s^t)[p_t(s^t) - \bar{P}_{t+k}(s^{t+k})v_{t+k}(s^{t+k})]y_{t+k}(i, s^{t+k})$$

subject to the demand curve (1) for their product. The term $Q_{t,t+k}(s^{t+k}|s^t)$ denotes the price of a unit of account in $s^{t+k}$ discounted back to $s^t$. The profit maximizing price for an intermediate is

$$p_t(s^t) = \frac{1}{\theta} \frac{\sum_{k=0}^{\infty} \sum_{s^k} \omega^k Q_{t,t+k}(s^{t+k}|s^t) \bar{P}_{t+k}(s^{t+k})^{\frac{2-\theta}{1-\theta}} v_{t+k}(s^{t+k}) y_{t+k}(s^{t+k})}{\sum_{k=0}^{\infty} \sum_{s^k} \omega^k Q_{t,t+k}(s^{t+k}|s^t) \bar{P}_{t+k}(s^{t+k})^{\frac{1}{1-\theta}} y_{t+k}(s^{t+k})}$$

Explain how this price relates to the one that this firm would set if prices were flexible. How does this price depend on future real marginal costs and demand conditions? Give intuition for your answers.

(e) (15 marks). Suppose that the representative consumer's period utility function is as in Question 1:

$$U(c, m, 1-n) = \frac{c^{1-\sigma}}{1-\sigma} + \psi \frac{m^{1-\gamma}}{1-\gamma} - \chi \frac{n^{1+\eta}}{1+\eta}$$

A log-linear version of this new Keynesian model can be written

$$\begin{aligned}
\hat{y}_t &= E_t\{\hat{y}_{t+1}\} - \frac{1}{\sigma}(\hat{\imath}_t - E_t\{\hat{\pi}_{t+1}\}) \\
\hat{\pi}_t &= \frac{(1-\omega\beta)(1-\beta)}{\omega}(\eta+\sigma)\left(\hat{y}_t - \frac{1+\eta}{\eta+\sigma}\hat{A}_t\right) + \beta E_t\{\hat{\pi}_{t+1}\} \\
\hat{m}_t &= -\frac{\kappa}{\gamma}\hat{\imath}_t + \frac{\sigma}{\gamma}\hat{y}_t \\
\hat{m}_t &= \hat{m}_{t-1} + \hat{\mu}_t - \hat{\pi}_t
\end{aligned}$$

plus the exogenous shocks. In this system, the constant $\kappa > 0$ comes from the log-linearization. Interpret each of the equations in this model (i.e., relate them to concepts in intermediate macroeconomics). How does this model differ from a standard IS-LM-AS model? How do these equations relate to the ones given in part (c) of Question 1. What is the fundamental difference between these two models? How do you interpret the term

$$\hat{y}_t - \frac{1+\eta}{\eta+\sigma}\hat{A}_t \ ?$$

[Hint: what would output be if prices were flexible?].

(f) (10 marks). Explain what would happen if the monetary authority used the nominal interest rate $\hat{\imath}_t$ as its policy instrument. How would $\hat{m}_t$ be determined? Is money demand relevant in this model?

(g) (5 marks). Does the classical dichotomy (as discussed in part (f) of Question 1) hold in this model? Why or why not?

(h) (15 marks) Outline the monetary transmission mechanism in this economy. To be precise, suppose that the monetary authority has the *feedback rule*

$$\hat{\imath}_t = 1.5\hat{\pi}_t + \hat{\varepsilon}_t$$

where $\hat{\varepsilon}_t$ is a serially uncorrelated control error with very small variance. Suppose that output $\hat{y}_t$ is high relative to trend and that the monetary authority raises the nominal interest rate. Why would the monetary authority do this? What happens to the real interest rate, real output and inflation following this change in the nominal interest rate?

END OF EXAM