316-466 Monetary Economics — Homework 3

This problem asks you to work through the setup of the sticky price model in Chari, Kehoe, and McGrattan [2000, *Econometrica*, **68**(5): 1151-1179]. This exercise draws on your knowledge of the Blanchard-Kiyotaki (1987) imperfect competition model and the Taylor (1980) model of overlapping contracts.

Time is discrete and denoted $t = 0, 1, \ldots$ There is a representative consumer who faces a random sequence of Markov shocks. The state at t is s_t and a finite history is

$$s^t = (s_t, s_{t-1}, \cdots s_0)$$

with s_0 known at date zero. As of date zero, the probability of s^t is $f(s^t \mid s_0)$.

The economy is comprised of a representative consumer, a representative perfectly competitive final goods firm and a continuum $[0,1]$ of differentiated monopolistically competitive intermediate firms. The production function of a final goods firm is

$$y_t(s^t) = \left[\int_0^1 y_t(i, s^t)^\theta di\right]^{1/\theta}, \qquad 0 < \theta \leq 1 \tag{1}$$

where $y_t(s^t)$ is output of the final good and $y_t(i, s^t)$ is output of intermediate i. The profit maximization problem of a final good firm is

$$0 = \max_{y_t(i,s^t)} \left\{ \bar{P}_t(s^t) y_t(s^t) - \int_0^1 P_t(i, s^{t-1}) y_t(i, s^t) \right\}$$

subject to the production function (1). The price level in units of account is $\bar{P}_t(s^t)$ while the price of intermediate i in units of account is $P_t(i, s^{t-1})$. The intermediate prices depend only on the history s^{t-1} because the prices set by intermediate firms will be set after the shock s_t has been realized in period t (see below)

1. Show that this profit maximization problem leads to a demand function for intermediates

   $$y_t(i, s^t) = \left[\frac{\bar{P}_t(s^t)}{P_t(i, s^{t-1})}\right]^{1/(1-\theta)} y_t(s^t) \tag{2}$$

   and an ideal price index

   $$\bar{P}_t(s^t) = \left[\int_0^1 P_t(i, s^{t-1})^{\theta/(\theta-1)} di\right]^{(\theta-1)/\theta}$$

   Intermediate goods are set in a staggered, overlapping, fashion. In particular, each period a uniform fraction $1/N$ of intermediates set their price $P_t(i, s^{t-1})$ before s_t is realized. These prices are then fixed for N periods. The interval $[0,1]$ is partitioned into N subsets with firms with names $i \in [0, 1/N)$ setting prices on dates $t = 0, N, 2N, \ldots$, and so on. An intermediate that can set its price on date t following s^{t-1} solves the following maximization problem

   $$\max_{P_t(i,s^{t-1})} \sum_{\tau=t}^{t+N-1} \sum_{s^\tau} Q_{\tau,t-1}\left(s^\tau | s^{t-1}\right) [P_t(i, s^{t-1}) - \bar{P}_\tau(s^\tau) v_\tau(s^\tau)] y_\tau(i, s^\tau) \tag{3}$$

subject to the demand curve (2) for their product. The term $Q_{\tau,t-1}\left(s^\tau|s^{t-1}\right)$ denotes the price of a unit of account in s^τ discounted back to s^{t-1}. The term $v_t(s^\tau)$ denotes the firm's real marginal cost (discussed below).

The production function of an intermediate firm is Cobb-Douglas in capital and labor

$$y_t(i,s^t) = k_t(i,s^t)^\alpha n_t(i,s^t)^{1-\alpha}, \qquad 0<\alpha<1$$

and real marginal cost is given by

$$v_t(s^t) = \min_{k_t(i,s^t),n_t(i,s^t)} \left\{ r_t(s^t)k_t(i,s^t) + w_t(s^t)n_t(i,s^t) \quad | \quad k_t(i,s^t)^\alpha n_t(i,s^t)^{1-\alpha} = 1 \right\}$$

where $r_t(s^t)$ and $w_t(s^t)$ denote competitive rental rates for capital and labor.

2. Show that cost minimization implies

$$\frac{1-\alpha}{\alpha}\frac{k_t(i,s^t)}{n_t(i,s^t)} = \frac{w_t(s^t)}{r_t(s^t)}$$

Explain why this implies that all intermediate use the same capital/labor ration and therefore why all intermediates have the same real marginal cost $v_t(s^t)$.

2. Solve the intermediate price setting problem i.e., that the optimal price to set is

$$P_t(i,s^{t-1}) = \frac{1}{\theta}\frac{\sum_{\tau=t}^{t+N-1}\sum_{s^\tau} Q_{\tau,t-1}\left(s^\tau|s^{t-1}\right)\bar{P}_\tau(s^\tau)^{(2-\theta)/(1-\theta)}v_\tau(s^\tau)y_\tau(s^\tau)}{\sum_{\tau=t}^{t+N-1}\sum_{s^\tau} Q_{\tau,t-1}\left(s^\tau|s^{t-1}\right)\bar{P}_\tau(s^\tau)^{1/(1-\theta)}y_\tau(s^\tau)}$$

Give intuition for this price. [Hint: what does this reduce to if $N=1$?].

The consumer has expected utility preferences over consumption, end of period real balances $m_t(s^t) \equiv M_{t+1}(s^t)/P_t(s^t)$, and leisure

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t U[c_t(s^t), m_t(s^t), \ell_t(s^t)]f(s^t \mid s_0), \qquad 0<\beta<1$$

The utility function U is assumed to be strictly increasing and strictly concave in all arguments. The representative consumer trades in money, a complete set of nominal, one-period, state contingent bonds, and capital. She has the flow budget constraint

$$\begin{aligned} & \bar{P}_t(s^t)[c_t(s^t)+k_{t+1}(s^t)] + M_{t+1}(s^t) + \sum_{s'} Q_{t+1,t}(s^t,s'|s^t)B_{t+1}(s^t,s') \\ \leq\ & \bar{P}_t(s^t)[w_t(s^t)n_t(s^t) + (r_t(s^t)+1-\delta)k_t(s^{t-1})] \\ & +M_t(s^{t-1}) + B_t(s^{t-1},s_t) + \Pi_t(s^t) - T_t(s^t) \end{aligned}$$

where $\Pi_t(s^t)$ denotes lump-sum profits from intermediate firms and and where $k_0>0$, $M_0>0$ and $B_0=0$ are given initial conditions. She also has the constraint

$$\ell_t(s^t) + n_t(s^t) \leq 1$$

on her endowment of time for leisure or labor.

3. Explain in words the representative consumer's flow budget constraints. Explain the dating concepts and any other implicit assumptions.

4. Derive and interpret the following FONC for the consumer's problem

$$\begin{aligned}
\frac{U_{\ell,t}(s^t)}{U_{c,t}(s^t)} &= w_t(s^t) \\
U_{c,t}(s^t) - U_{m,t}(s^t) &= \beta \sum_{s'} U_{c,t+1}(s^t, s') \frac{\bar{P}_t(s^t)}{\bar{P}_{t+1}(s^t, s')} f(s' \mid s_t) \\
U_{c,t}(s^t) &= \beta \sum_{s'} U_{c,t+1}(s^t, s')[r_{t+1}(s^t, s') + 1 - \delta] f(s' \mid s_t) \\
Q_{\tau,t}(s^\tau | s^t) &= \beta^{\tau - t} \frac{U_{c,\tau}(s^\tau)}{U_{c,t}(s^t)} \frac{\bar{P}_t(s^t)}{\bar{P}_\tau(s^\tau)}, \qquad \tau > t
\end{aligned}$$

where the short hand $U_{\ell,t}(s^t) \equiv U_\ell[c_t(s^t), m_t(s^t), \ell_t(s^t)]$, and so on, is used.

Money is introduced into the economy by having the exogenous money supply satisfy $\mathcal{M}_{t+1}(s^t) = \mu_t(s^t)\mathcal{M}_t(s^{t-1})$ where $\mu_t(s^t)$ is an exogenous stochastic process, and where the government's budget constraint is $\mathcal{M}_{t+1}(s^t) + T_t(s^t) \geq \mathcal{M}_t(s^{t-1})$.

5. Explain the following equilibrium conditions

$$\begin{aligned}
k_t(s^{t-1}) &= \int_0^1 k_t(i, s^t) di \\
n_t(s^t) &= \int_0^1 n_t(i, s^t) di \\
c_t(s^t) + k_{t+1}(s^t) &= y_t(s^t) + (1 - \delta) k_t(s^{t-1}) \\
\mathcal{M}_{t+1}(s^t) &= M_{t+1}(s^t) \\
B_{t+1}(s^t, s') &= 0
\end{aligned}$$

6. Use the market clearing conditions and the solution to the intermediates' cost minimization problem to show that the relation ship between the output of the final goods firm and *aggregate* capital and labor is given by

$$\begin{aligned}
y_t(s^t) &= A_t(s^t) k_t(s^{t-1})^\alpha n_t(s^t)^{1-\alpha} \\
A_t(s^t) &\equiv \frac{\bar{P}_t(s^t)^{1/(\theta-1)}}{\int_0^1 P_t(i, s^{t-1})^{1/(\theta-1)} di}
\end{aligned}$$

How do you interpret the factor $A_t(s^t)$? [Hint what would $A_t(s^t)$ be if prices were fully flexible?].

7. Try and reproduce as much as possible of the log-linear analysis of Chari, Kehoe and McGrattan in the case where $N = 2$, $\alpha = 0$ (only labor is used in production), household utility is

$$U(c, m, n) = \frac{1}{1 - \sigma} \left\{ \left[\omega c^{(\eta-1)/\eta} + (1 - \omega) m^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)} (1 - n)^\psi \right\}^{1-\sigma}$$

and real money demand is *assumed* to be interest-inelastic

$$m_t = c_t$$

[Note: I am using $m_t = M_{t+1}/\bar{P}_t$ to denote end-of-period real money balances; CKM's notation is slightly different (it is spelled out on p. 1163), so be careful]. It's probably best to begin by solving for the symmetric non-stochastic steady state.

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