Problem Set Zero

This is a model problem set, you do not have to turn it in and it does not count for anything. Instead, you should read over the questions, and in particular study the associated Matlab code, to make sure that you understand everything. I hope you find this useful.

Question 1. A discrete-time version of the Solow model leads to a non-linear difference equation in a single state variable, k_t the capital stock per efficiency unit of labor

$$(1+g)(1+n)k_{t+1} = sk_t^{\alpha} + (1-\delta)k_t \qquad k_0 > 0 \text{ given}$$

The meaning of the parameters in this expression are listed in the following table. The far column lists some suggestive values of these parameters.

Symbol	Meaning	Value
g	annual growth rate of labor-augmenting factor productivity	0.03
n	annual growth rate of working age population	0.01
s	national savings rate	0.20
α	capital's share in national output	0.33
δ	annual depreciation rate of physical capital	0.04
k_0	initial capital per effective working age person	varies

Write the non-linear difference equation as

$$k_{t+1} = \frac{s}{(1+g)(1+n)}k_t^{\alpha} + \frac{(1-\delta)}{(1+g)(1+n)}k_t \equiv \psi(k_t)$$

Verify that there are two-steady states, points that solve $\bar{k} = \psi(\bar{k})$, and that the non-trivial steady-state can be expressed as

$$\log(\bar{k}) = \frac{1}{1-\alpha}\log\left(\frac{s}{g+n+\delta+gn}\right)$$

(In continuous time the second order term gn would vanish). Clearly, steady state effective capital per worker is increasing in capital's share α, increasing in the savings rate s, and decreasing in the growth rates of productivity g and work force n and in the physical depreciation rate. With the parameter values given above, verify that

$$\bar{k} = \exp\left[\frac{1}{0.67}\log\left(\frac{0.20}{0.0803}\right)\right] = 3.9040$$

Also verify that the steady state capital/output ratio is about 2.5 and that the marginal product of capital (gross of depreciation) is about 13%.

Question 2. Now compute a sequence $\{k_t\}$ until

$$\left|k_t - \bar{k}\right| < 10^{-6}$$

Plot the trajectory of $\{k_t\}$ against time t, and plot transitions k_t against k_{t+1} using $\log(k_0/\bar{k}) = -0.5$, so that initially the capital stock is approximately 50% below its steady state level.

Question 3. The (net) growth rate between t and $t+1$ of a variable x is defined by

$$\frac{x_{t+1} - x_t}{x_t} = \frac{x_{t+1}}{x_t} - 1$$

Net growth rates are often approximated by log-differences, defined by

$$\log\left(\frac{x_{t+1}}{x_t}\right) = \log(x_{t+1}) - \log(x_t)$$

The basis for approximating growth rates by log-differences is the result that

$$\log\left(\frac{x_{t+1}}{x_t}\right) = \log\left(1 + \frac{x_{t+1} - x_t}{x_t}\right) \simeq \frac{x_{t+1} - x_t}{x_t}$$

which is equivalent to

$$\log(1+z) \simeq z, \qquad \text{for small } z$$

This assertion is based on the linearization (first order Taylor series expansion)

$$\log(1+z) \simeq \log(1+z_0) + \frac{1}{1+z_0}(z - z_0)$$

around the point $z_0 = 0$. Use Matlab to investigate how big z has to be until this approximation is bad. Does the appropriateness of the approximation depend on the "frequency" (i.e., how long a time period is) of the model? Why or why not?

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