316-406 ADVANCED MACROECONOMIC TECHNIQUES **Samples**

Chris Edmond ⟨cpedmond@unimelb.edu.au⟩

The midterm will last 90 minutes and will have three questions, each of equal marks. Within each question there will be a number of parts (say 4-6 parts per question) and the weight given to each part will also be indicated.

Here are two sample questions.

Question 1. *Solow Growth Model* (30 marks): Let time be discrete, $t = 0, 1, \ldots$.. Let the national resource constraint be

$$C_t + I_t = Y_t = F(K_t, A_t N_t)$$

where C_t denotes aggregate consumption, I_t denotes aggregate investment, Y_t denotes aggregate output, K_t denotes aggregate capital, and N_t denotes the level of the working age population. Labor-augmenting technological progress is denoted A_t. The production function is constant returns in capital and labor with always positive but diminishing marginal product. In particular,

$$\lim_{K\to 0} F_K(K, AN) = \infty$$
$$\lim_{K\to \infty} F_K(K, AN) = 0$$

Both labor and technology grow exogenously

$$N_{t+1} - N_t = nN_t, \qquad n > 0, \qquad N_0 \text{ given}$$
$$A_{t+1} - A_t = gA_t, \qquad g > 0, \qquad A_0 \text{ given}$$

Let capital accumulation be given by

$$K_{t+1} = (1-\delta)K_t + I_t, \qquad 0 < \delta < 1, \qquad K_0 \text{ given}$$

where δ is the physical depreciation rate. Finally, let consumption be a fixed fraction of national output

$$C_t = (1-s)Y_t, \qquad 0 < s < 1$$

where s denotes the national saving rate.

(a) (5 marks): Show that this model can be reduced to a single non-linear difference equation in a state variable x_t, i.e., that you can write the model as

$$x_{t+1} = \psi(x_t), \qquad x_0 \text{ given}$$

What is x_t? Provide an explicit formula for the function ψ.

(b) (5 marks): Let $\bar{x}$ denote a fixed point of ψ. How many fixed points does ψ have? Linearize the function ψ around each of its fixed points and determine the local stability or instability of each such point.

(c) (5 marks): Suppose that the aggregate production function is Cobb-Douglas with capital share α,

$$F(K, AN) \equiv K^{\alpha}(AN)^{1-\alpha}, \qquad 0 < \alpha < 1$$

Provide an explicit solution for each of the function ψ's fixed points. Explain how the fixed points depend on the economic parameters of the model (especially α, s, δ, g, n). Give economic intuition.

(d) (5 marks): Use a log-linearization argument to determine the approximate speed of convergence to each steady state. Explain how the speed of convergence depends on the economic parameters of the model.

(e). (10 marks): Suppose that the production function is hit by random shocks

$$F(K_t, A_t N_t) \equiv z_t K_t^{\alpha}(A_t N_t)^{1-\alpha}$$

where the $\log(z_t)$ are IID Gaussian with mean 0 and variance σ^2. Use a log-linearization to derive an approximate linear stochastic difference equation in the state $\hat{x}_t$ and the shocks $\hat{z}_t$. Solve for the stationary distribution of $\hat{x}_t$ and explain how its mean and variance depend on the parameters of the model.

Question 2. *Stochastic Labor Supply* (30 marks): Let time be discrete, $t = 0, 1, ...$ Suppose that a worker faces a stochastic real wage rate each period which follows an n-state Markov chain (w, P, π_0) where w is an n-vector, P is a transition matrix and π_0 is an initial distribution. Suppose that each period, the worker solves the static utility maximization problem over consumption c and leisure ℓ

$$\max_{c_t,\ell_t} \quad U(c_t, \ell_t)$$

subject to a budget constraint and a constraint on the time endowment

$$\begin{aligned} c_t &= w_t n_t \\ n_t + \ell_t &= 1 \end{aligned}$$

where w_t is this period's random wage realization and n_t is the fraction of time allocated to working. Suppose that $U(c, \ell)$ is strictly increasing and strictly concave in each argument.

(a) (5 marks): Explain how a labor supply schedule of the form

$$n = \varphi(w)$$

can be derived from the optimization problem. Explain how you characterize the function φ. What assumption do preferences have to satisfy in order for $\varphi'(w) > 0$ always?

(b) (5 marks): Explain the stochastic dynamics that n_t exhibits. Carefully explain how you could simulate the optimal labor supply choices.

(c) (3 marks): Suppose that the utility function is

$$U(c, \ell) = \log(c) + \eta \log(\ell), \qquad \eta > 0$$

What pattern of labor supply would one observe given the fluctuations in w? How does your answer depend on the preference weight η? What stochastic dynamics does this imply for consumption? Explain your answers.

(d) (5 marks): Suppose instead that the utility function is

$$U(c, \ell) = \log\left[c - v(1 - \ell)\right], \qquad v(1 - \ell) = \frac{(1 - \ell)^{1+\gamma}}{1 + \gamma}, \qquad \gamma > 0$$

What pattern of labor supply would one observe given the fluctuations in w? How does your answer depend on the parameter γ? What does γ measure? What stochastic dynamics does this imply for consumption? Explain your answers.

(e). (12 marks): Let the utility function be as in part (d). Suppose that the Markov chain has 4 states with

$$w = \begin{pmatrix} w_1 & w_2 & w_3 & w_4 \end{pmatrix}$$

and transition matrix

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ 0 & p_{22} & p_{23} & p_{24} \\ 0 & 0 & p_{33} & p_{34} \\ 0 & 0 & p_{43} & p_{44} \end{pmatrix}$$

(each $0 < p_{ij} < 1$ unless otherwise indicated). Finally, the initial distribution is

$$\pi_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}$$

Solve for the stationary distribution of wages. Compute the implied stationary distribution of labor supply. Explain how the stationary distributions depend on the transition probabilities and on the parameter γ. Suppose we have the numbers

$$\begin{aligned} w &= \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix} \\ P &= \begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0 & 0.50 & 0.25 & 0.25 \\ 0 & 0 & 0.75 & 0.25 \\ 0 & 0 & 0.10 & 0.90 \end{pmatrix} \\ \gamma &= 2 \end{aligned}$$

Compute the average and standard deviation of the wage and the labor supply in the stationary distribution.