

**Question 1.** (*Deterministic Dynamic Programming*). Since answering the first question just involved changing some settings in the "dp\_demo.m" program, these answers are brief. With the given parameter values:

$$\begin{aligned}\bar{c} &= 1.60 \\ \bar{k} &= 6.66\end{aligned}$$

The grid is then a vector with a 1000 elements given by

$$k \in \mathcal{K} \equiv [0 < \dots < 33.30]$$

With a high discount factor ( $\beta = 0.95$ ) and a low depreciation rate ( $\delta = 0.04$ ) it takes a little while for the value function iteration to converge. For example, my code took 265 iterations and 3 minutes 20 seconds to complete. (I hasten to add that there are much smarter ways to solve dynamic programming problems; the way that "dp\_demo.m" does it is very naive and can be substantially improved upon).

The value and policy functions are shown in Figure 1a and Figure 1b, attached. Notice that  $V(k)$  is a nice strictly increasing strictly concave function. It has negative values but that doesn't matter because utility is an ordinal concept. The policy function  $g(k)$  is nearly linear and crosses the 45-degree line at the steady state  $\bar{k} = 6.66$ .

**Question 2.** (*Stochastic dynamic programming*). The main difficulty is setting up the big transition matrix on  $\mathcal{K} \times \mathcal{Z}$ . This endogenous Markov chain has transition probabilities from  $(k, z)$  to  $(k', z')$  given by the formula

$$\Pr(k_{t+1} = k', z_{t+1} = z' | k_t = k, z_t = z) = \Pr(k_{t+1} = k' | k_t = k, z_t = z) \Pr(z_{t+1} = z' | z_t = z)$$

But the probability  $\Pr(k_{t+1} = k' | k_t = k, z_t = z)$  is either 1 if  $k' = g(k, z)$  or 0 otherwise. So if we write an indicator function

$$I_g(k', k, z) \equiv \begin{cases} 1, & \text{if } k' = g(k, z) \text{ and} \\ 0, & \text{otherwise} \end{cases}$$

then we can compute the transition probabilities on  $\mathcal{K} \times \mathcal{Z}$  using

$$\Pr(k_{t+1} = k', z_{t+1} = z' | k_t = k, z_t = z) = I_g(k', k, z) \pi(z', z)$$

where  $\pi(z', z)$  are read off the transition matrix  $P$ .

*Note:* I made an error of judgement in writing the problem set. When you include  $k = 0$  as the smallest capital stock in  $\mathcal{K}$ , you will get **TWO** stationary distributions for the Markov chain on  $\mathcal{K} \times \mathcal{Z}$ . One stationary distribution corresponds to the usual interior steady state of the deterministic model, while the other corresponds to the trivial steady state of the deterministic model (which is relevant only if the initial capital stock is zero so that output is zero for all time independent of the realizations of the technology shock). You can either disregard the trivial stationary distribution or set, say,  $k_{\min} = 10^{-6}$  so that there is only one stationary distribution.

That said, the Matlab code required to answer this problem is given as "ps4\_solutions.m". In the attached figures, I consider an alternative parameterization to give you some sense of how the model works. To begin with, suppose that the spread in the technology shocks is relatively large, say

$$z \in \mathcal{Z} = \{z_L, z_H\} \equiv \{0.8, 1.2\}$$

and that the transition probabilities are

$$P = \begin{pmatrix} 0.50 & 0.50 \\ 0.50 & 0.50 \end{pmatrix}$$

Finally, let the discount factor be  $\beta = 0.50$ . The value and policy functions conditional on  $z_H$  are uniformly greater than the value and policy functions conditional on  $z_L$ . Figure 2a shows the associated policy functions. Both are quite concave (due to the low  $\beta$ ). The crossing points of the two policy functions with the 45-degree line implicitly define the subset, say  $[\underline{k}, \bar{k}] \subset \mathcal{K}$ , that gets given any probability mass by the stationary distribution. This is shown in Figures 2b and 2c. The general "spikiness" of the probability mass function is due to the relatively naive way we discretized the state space. Everything looks a lot smoother when we integrate to get the cumulative distribution. In these figures, the distribution conditional on  $z_L$  and the distribution conditional on  $z_H$  lie on top of each other because of the IID transitions.

The answers when we have the parameterization originally given in the problem set are shown in Figures 3a-3d. The main difference is that with a high discount factor, the policy functions have much less curvature and with a small spread between  $z_L$  and  $z_H$ , everything is much closer together. With persistence in the transitions, the conditional distributions no longer lie on top of each other. There are big spikes near where the policy functions cross the 45-degree line.