316-632 International Monetary Economics

Problem Set #1 *Due 23 August, 2004*

Question 1. Let there be two dates, $t = 0, 1$ and let there be S possible states of nature that may be realized at date $t = 1$. Index the states by $s \in \{1, 2, ..., S\}$. Let there be I countries with $i \in \{1, 2, ..., I\}$. Let each country have a representative consumer with identical preferences

$$u(c^i) = U(c_0^i) + \beta \sum_s U[c_1^i(s)]\pi(s)$$

with common constant time discount factor $0 < \beta < 1$ and subjective probabilities $\pi(s)$. Let country i have endowments y_0^i and $y_1^i(s)$ and denote world output by the sums

$$\begin{aligned} Y_0 &\equiv \sum_i y_0^i \\ Y_1(s) &\equiv \sum_i y_1^i(s) \end{aligned}$$

Each country faces the budget constraint

$$c_0^i + \sum_s q_1(s)c_1^i(s) = y_0^i + \sum_s q_1(s)y_1^i(s)$$

where $q_1(s)$ are the prices of Arrow securities.

a). Let period utility be

$$U(c) = \log(c)$$

Solve for equilibrium prices $q_1(s)$ and consumption allocations c_0^i and $c_1^i(s)$. Use these results to solve for the world interest rate on riskless bonds.

b). Let period utility be

$$U(c) = -\exp(-\gamma c)$$

(this is constant **absolute** risk aversion). Again, solve for equilibrium prices $q_1(s)$ and consumption allocations c_0^i and $c_1^i(s)$ and use these results to solve for the world interest rate on riskless bonds

c). Now let $I = 2$. Suppose that the two countries have different risk aversion coefficients, say $\gamma_1 > \gamma_2$. Again solve for equilibrium prices $q_1(s)$ and consumption allocations c_0^i and $c_1^i(s)$ for $i = 1, 2$. How do the prices and consumption allocations change relative to the case where both countries have the same preferences? Give **economic** intuition for your results.

Question 2. Now consider the following model, due to Backus (JIE, 1993). Let there be 2 countries with $i = 1, 2$. Let there be countable dates, $t = 0, 1, 2...$ and let there be Z possible states of nature that may be realized at each date $t \geq 1$. Index the states by $z_t \in \{1, 2, ..., Z\}$. A history is a vector $z^t = (z_0, z_1, ..., z_t)$. The unconditional probability of a history z^t being realized as of date zero is denoted $\pi_t(z^t)$. The initial state z_0 is known as of date zero. At each date and state, there are two commodities **both of which are traded**. Country $i = 1$ produces one good and has endowments $x_t(z^t)$ while country $i = 2$ produces the other good and has endowments $y_t(z^t)$. Each country consumes both goods and has a representative consumer with preferences

$$u(a^i, b^i) = \sum_{t=0}^{\infty} \sum_{z^t} \beta^t \left[\frac{a_t^i(z^t)^{1-\alpha} + b_t^i(z^t)^{1-\alpha}}{1-\alpha} \right] \pi_t(z^t), \qquad 0 < \beta < 1, \quad \alpha > 0$$

The resource constraints for this world are

$$\begin{aligned} a_t^1(z^t) + a_t^2(z^t) &\leq x_t(z^t) \\ b_t^1(z^t) + b_t^2(z^t) &\leq y_t(z^t) \end{aligned}$$

And the date zero budget constraint for the two countries are

$$\sum_{t=0}^{\infty} \sum_{z^t} [Q_t^x(z^t) a_t^1(z^t) + Q_t^y(z^t) b_t^1(z^t)] \leq \sum_{t=0}^{\infty} \sum_{z^t} Q_t^x(z^t) x_t(z^t)$$

and

$$\sum_{t=0}^{\infty} \sum_{z^t} [Q_t^x(z^t) a_t^2(z^t) + Q_t^y(z^t) b_t^2(z^t)] \leq \sum_{t=0}^{\infty} \sum_{z^t} Q_t^y(z^t) y_t(z^t)$$

where $Q_t^x(z^t) \equiv \beta^t \pi_t(z^t) q_t^x(z^t)$ and $Q_t^y(z^t) \equiv \beta^t \pi_t(z^t) q_t^y(z^t)$ are the date-zero prices of the x and y goods. The associated spot prices are $q_t^x(z^t)$ and $q_t^y(z^t)$. Since country 1 exports the x good and imports the y good while country 2 does the reverse, we can define the **terms of trade** for country 1 as

$$\mathsf{tot}_t(z^t) \equiv \frac{Q_t^y(z^t)}{Q_t^x(z^t)} = \frac{q_t^y(z^t)}{q_t^x(z^t)}$$

a). Set up the equivalent social planning problem with welfare weights ω_i. Obtain the first order conditions that characterize the allocations $a_t^i(z^t)$ and $b_t^i(z^t)$ for $i = 1, 2$ and the (shadow) prices.

b). Taking as given the welfare weights ω_i, use the planner's optimality conditions to solve for the allocations and shadow prices. Use the solutions for the shadow prices to solve for the terms of trade. Give economic intuition for all your answers.

c). Find the implied welfare weights associated with a market economy when country 1 has endowments $x_t(z^t)$ and country 2 has endowments $y_t(z^t)$.

d). The **trade balance** for country 1 is

$$\mathsf{tb}_t(z^t) \equiv x_t(z^t) - a_t^1(z^t) - \frac{q_t^y(z^t)}{q_t^x(z^t)} b_t^1(z^t)$$

Solve for the trade balance. How does the trade balance co-vary with the terms of trade? [Hint: what common factors do both the terms of trade and the trade balance depend on?]

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