High Frequency Quoting: Short-Term Volatility in Bids and Offers

Joel Hasbrouck
Stern School, NYU

Disclaimers

- I teach in an entry-level training program at a large financial firm that is generally thought to engage in high frequency trading.
- I serve on a CFTC advisory committee that discusses issues related to high frequency trading.

Figure 1. AEPI bid and offer, April 29, 2011

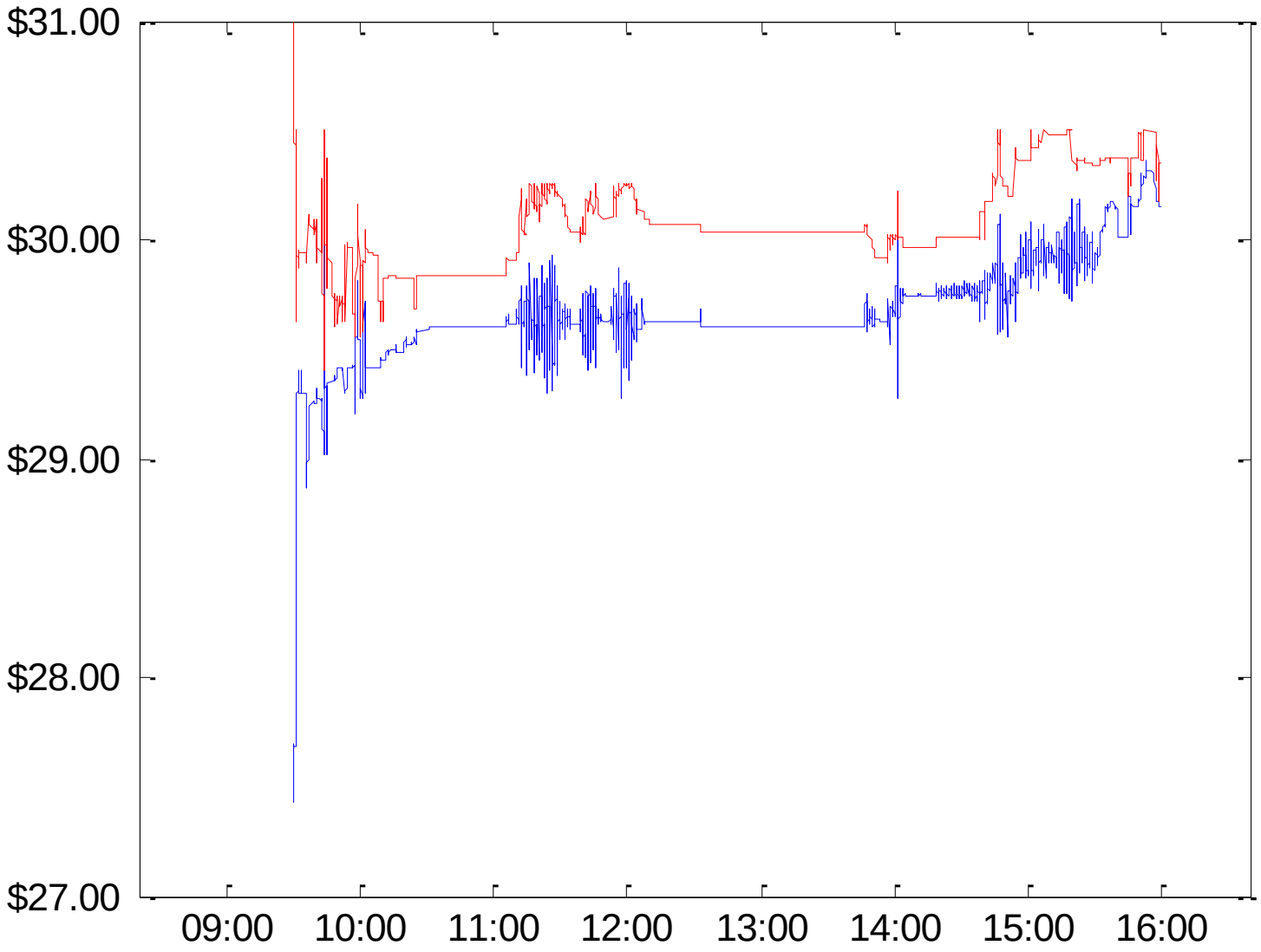
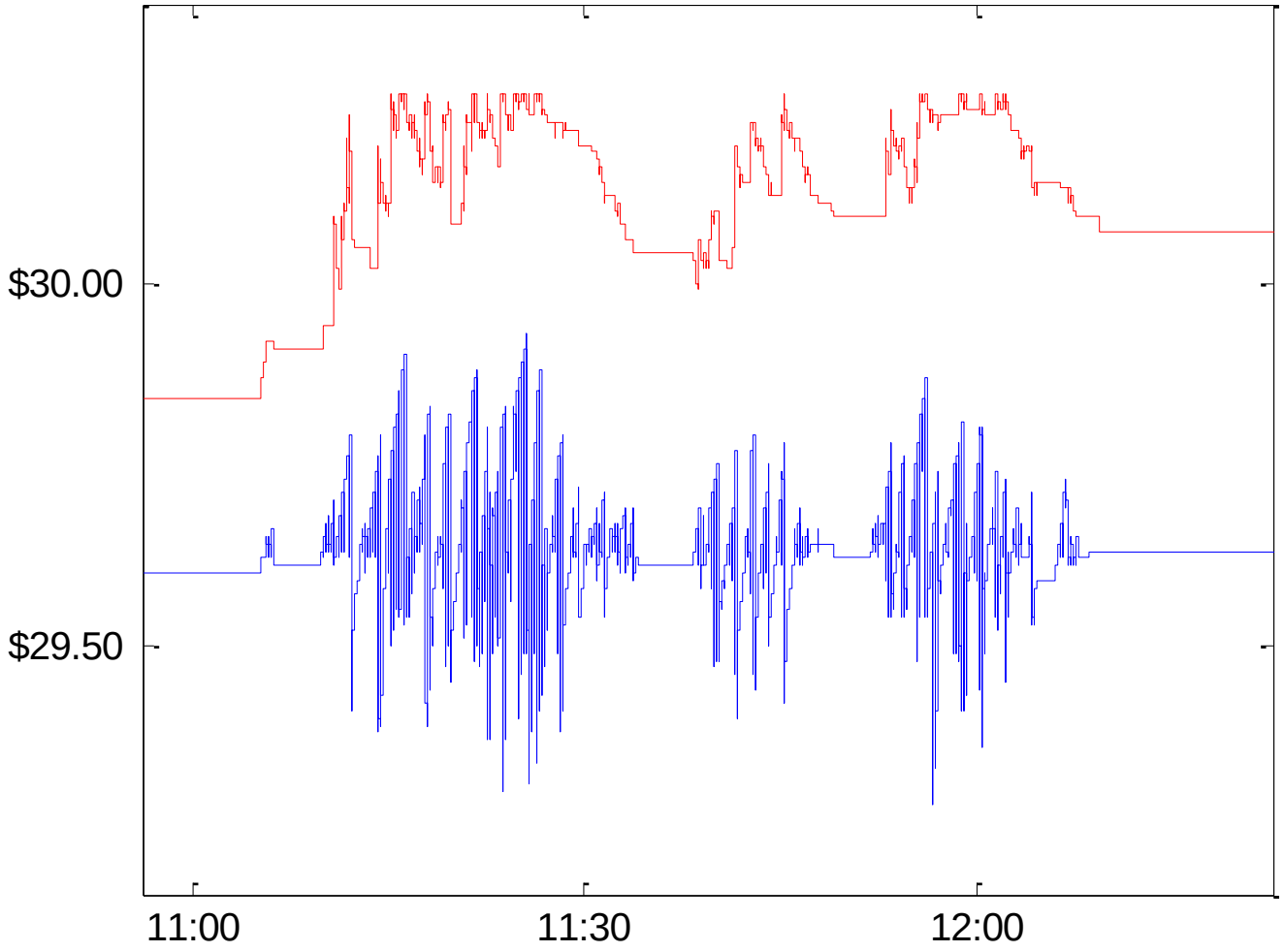
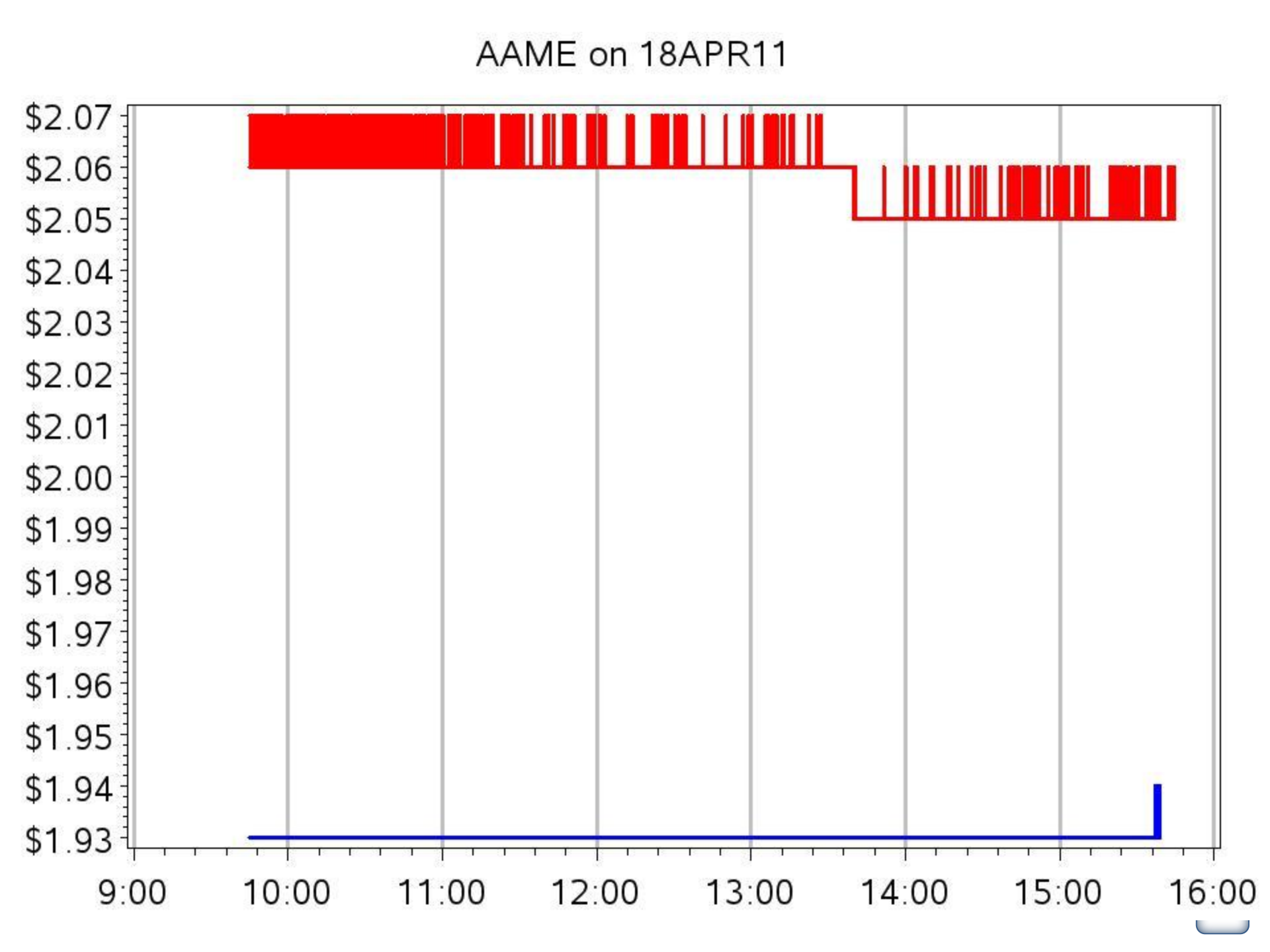
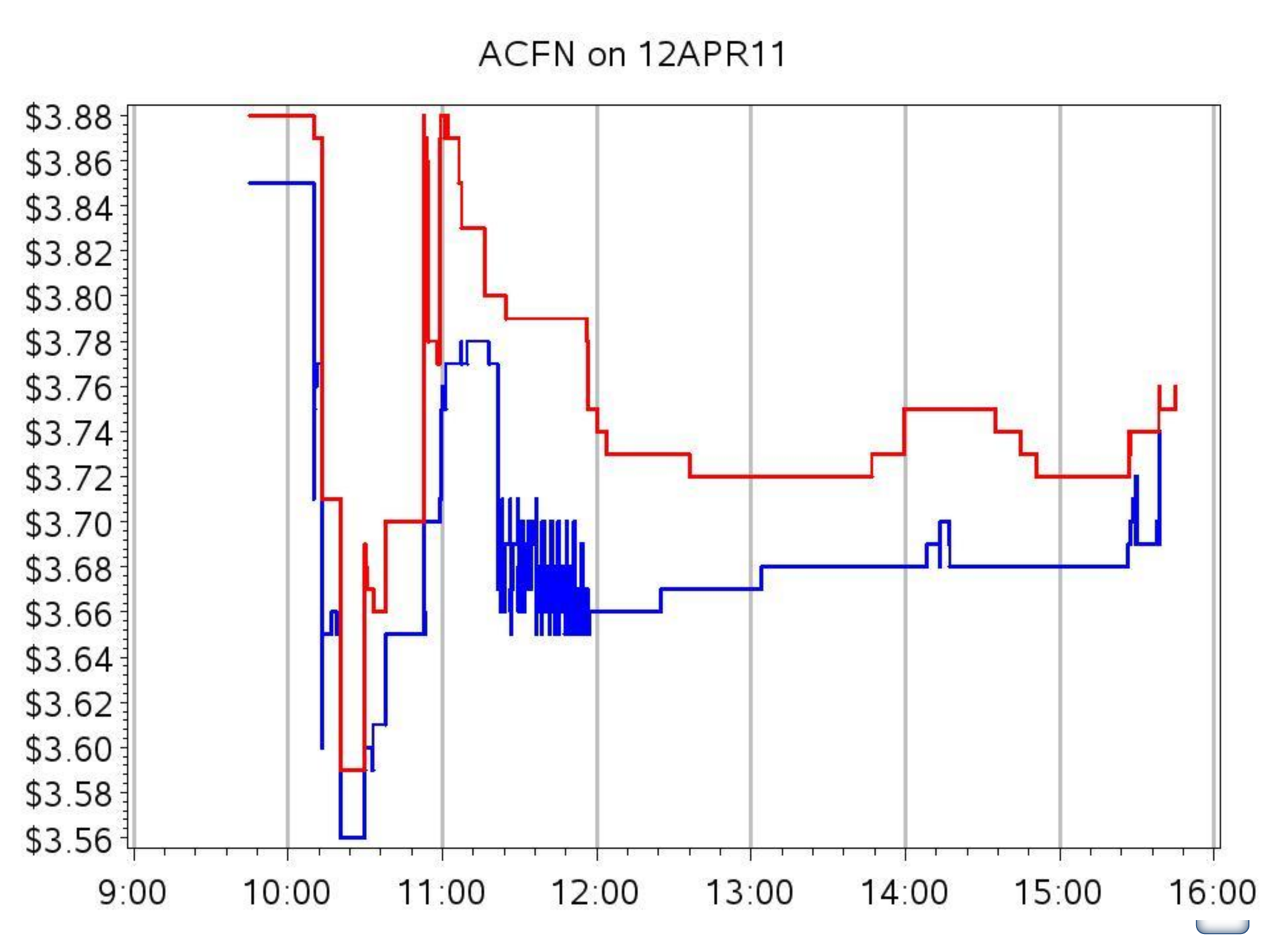


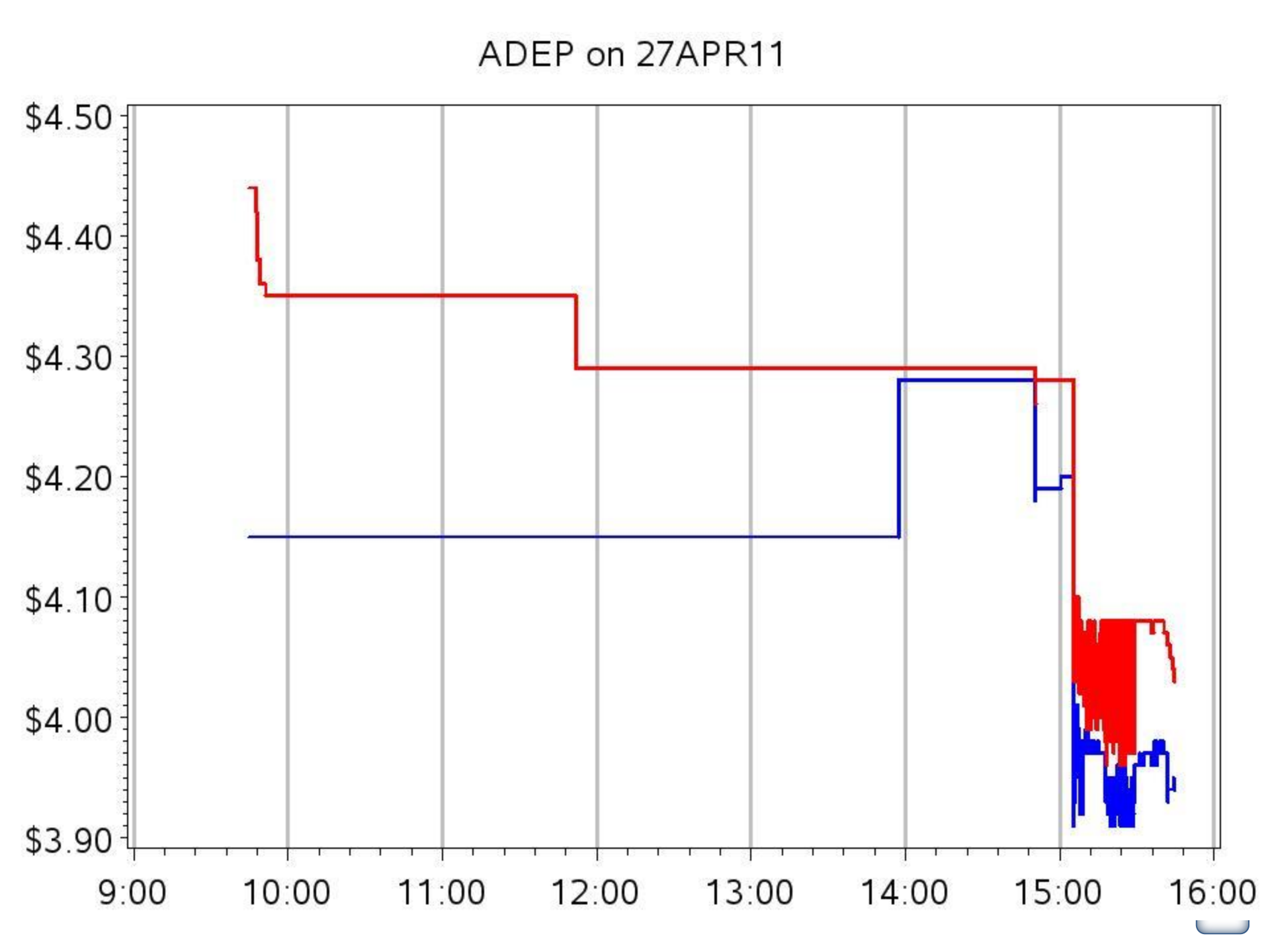
Figure 1. AEPI bid and offer on April 29, 2011 (detail)

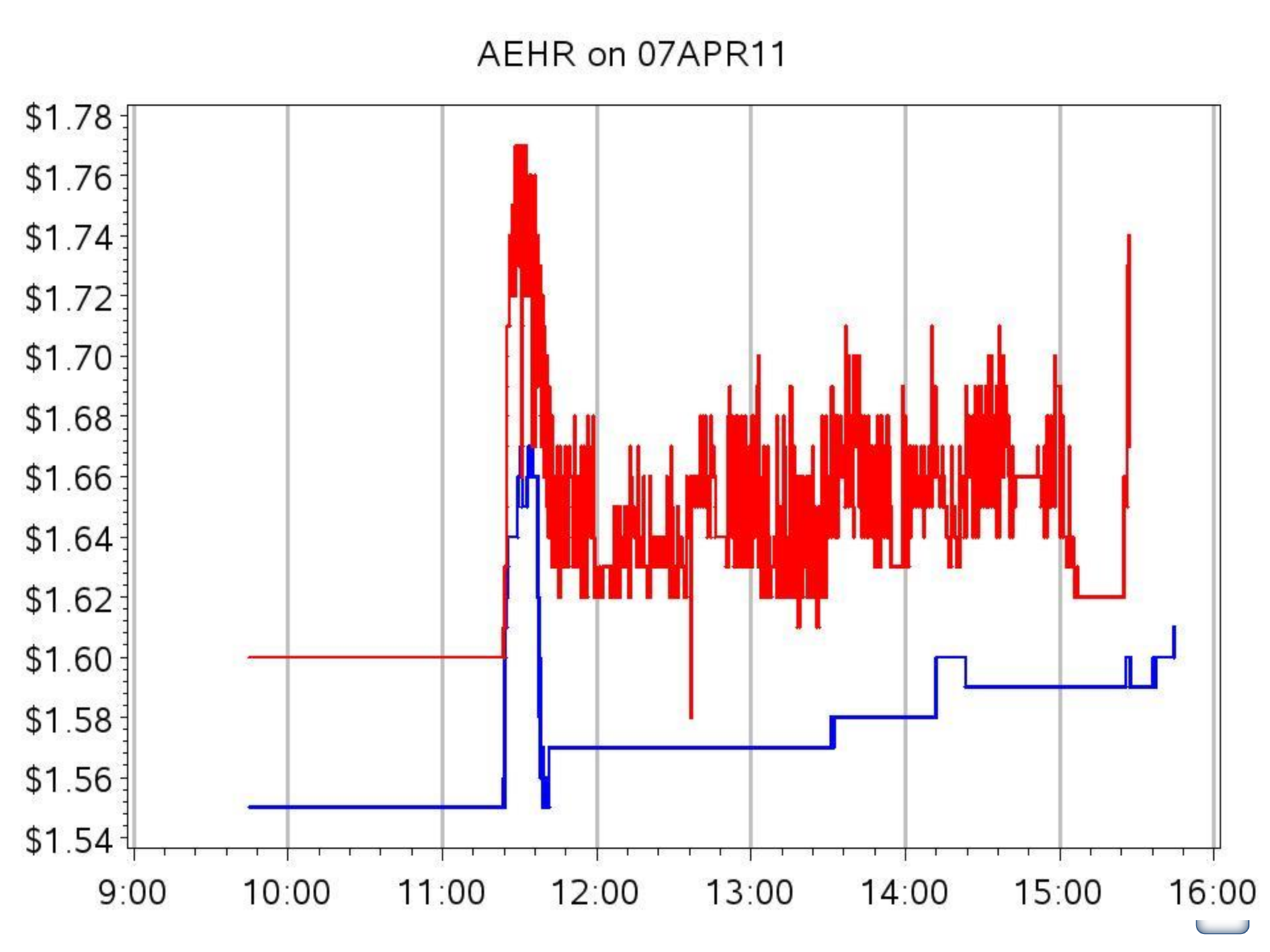


More pictures from the National (High Frequency) Portrait Gallery


AAME on 18APR11
$2.07
$2.06
$2.05
$2.04
$2.03
$2.02
$2.01
$2.00
$1.99
$1.98
$1.97
$1.96
$1.95
$1.94
$1.93
9:00
10:00
11:00
12:00
13:00
14:00
15:00
16:00


ACFN on 12APR11
$3.88
$3.86
$3.84
$3.82
$3.80
$3.78
$3.76
$3.74
$3.72
$3.70
$3.68
$3.66
$3.64
$3.62
$3.60
$3.58
$3.56
9:00
10:00
11:00
12:00
13:00
14:00
15:00
16:00


ADEP on 27APR11
$4.50
$4.40
$4.30
$4.20
$4.10
$4.00
$3.90
9:00
10:00
11:00
12:00
13:00
14:00
15:00
16:00


AEHR on 07APR11
$1.78
$1.76
$1.74
$1.72
$1.70
$1.68
$1.66
$1.64
$1.62
$1.60
$1.58
$1.56
$1.54
9:00
10:00
11:00
12:00
13:00
14:00
15:00
16:00

Quote volatility: the questions

- What is its economic meaning and importance?
- How should we measure it?
- Is it elevated? Relative to what?
- Has it increased over the recent past along with the utilization of advanced trading technology?

Context and connections

- Analyses of high frequency trading
- Volatility modeling
- Methodology: time scale resolution and variance estimation

"HF traders are the new market makers."

- Provide valuable intermediation services.
  - Like traditional designated dealers and specialists.
- Hendershott, Jones and Menkveld (2011): NYSE message traffic
- Hasbrouck and Saar (2012): strategic runs / order chains
- Brogaard, Hendershott and Riordan (2012) use Nasdaq HFT dataset in which trades used to define a set of high frequency traders.
- Studies generally find that HFT activity is associated with (causes?) higher market quality.

"HF traders are predatory."

- They profit from HF information asymmetries at the expense of natural liquidity seekers (hedgers, producers of fundamental information).
- Jarrow and Protter (2011); Foucault and Rosu (2012)
- Baron, Brogaard and Kirilenko (2012); Weller (2012); Clark-Joseph (2012)

Volatility Modeling

- ARCH, GARCH, and similar models focus on fundamental/informational volatility.
  - Statistically: volatility in the unit-root component of prices.
  - Economically important for portfolio allocation, derivatives valuation and hedging.
- Quote volatility is non-informational
  - Statistically: short-term, stationary, transient volatility
  - Economically important for trading and market making.

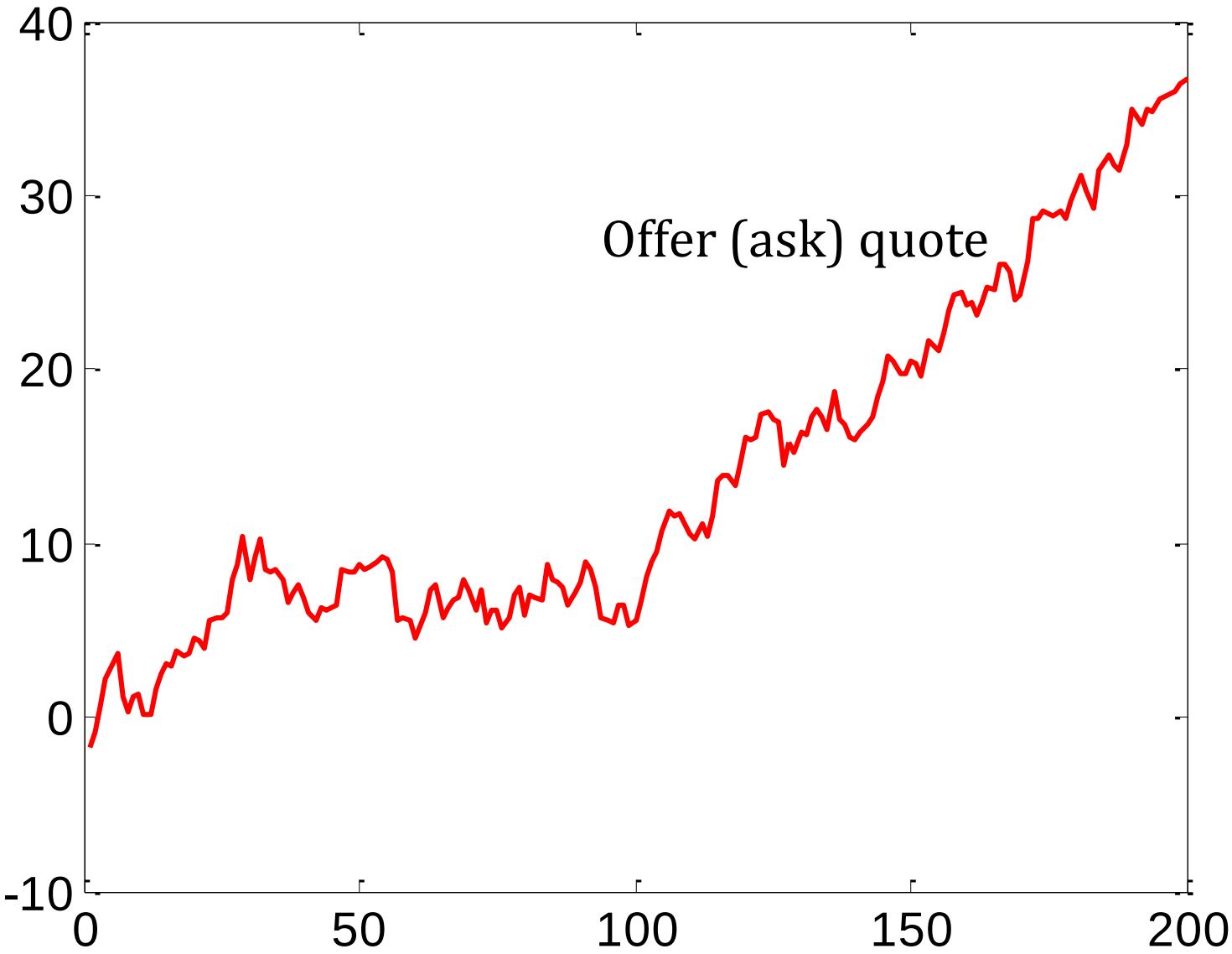
Realized volatility

- Volatility estimates formed from HF data.
  - average (absolute/squared) price changes.
  - Andersen, Bollerslev, Diebold and Ebens (2001), and others
- Hansen and Lunde (2006) advocate using local averaging ("pre-averaging") to eliminate microstructure noise.
- Quote volatility *is* the microstructure noise.

Economics of quote volatility

- Noise degrades the value of any signal.
- Creates execution price risk for
  - marketable orders
  - dark trades
- Creates and increases value of intermediaries' look-back options

Execution price risk for marketable orders




A buyer who has *arrival time uncertainty* can expect to pay the average offer over the arrival period (and has price risk relative to this average).
40
30
20
10
0
-10
0
50
100
150
200

Dark trading

- "Dark" the market executing the order did not previously post a visible bid or offer at the execution price.
  - The trade itself is promptly reported.
- Dark mechanisms
  - Hidden (undisplayed) limit orders
  - Internalized executions
  - Dark pools

Dark trades: internalized execution

- A broker receives a retail buy order.
  - The order is not sent to an exchange or any other venue.
  - The broker sells directly to the customer at the National Best Offer (NBO)
- Volatility in the NBO → volatility in execution price.

Dark trades: dark pools

- Mechanism
  - Traders send buy and sell orders to a computer.
  - The orders are not displayed.
  - If the computer finds a feasible match, a trade occurs.
- The trade is priced at the midpoint of the National Best Bid and Offer (NBBO)
- Volatility in the NBBO causes volatility in the execution price.

Look-back options

- Internalization: a broker receives a retail buy order and executes the order at the NBO.
- Problem: how does the customer know what the NBO is or was?
- Might the dealer take the highest price in the interval of indeterminacy?
  - Stoll and Schenzler (2002)

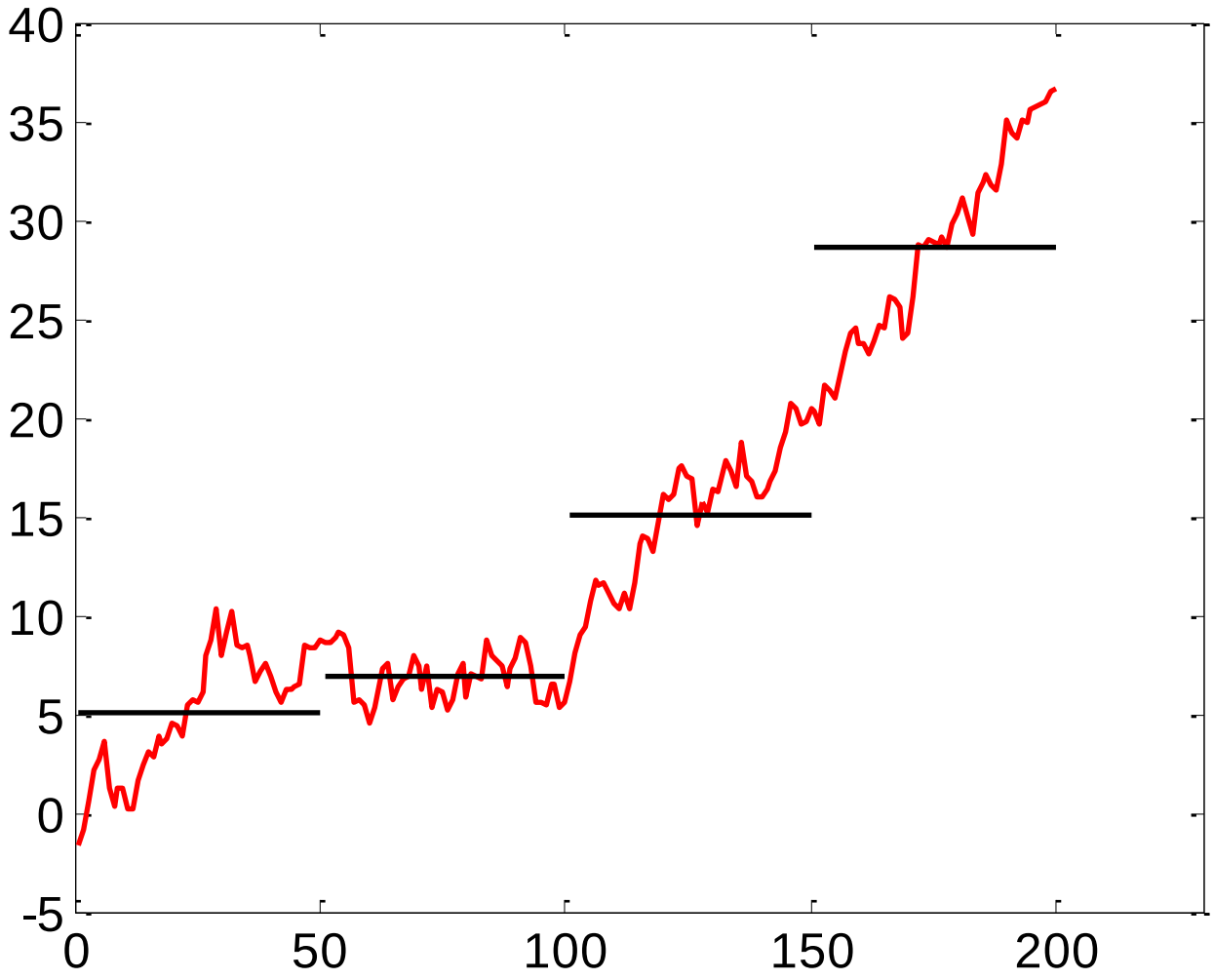
“Spoofing” manipulations

- A dark pool buyer enter a spurious sell order in a visible market.
- The sell order drives down the NBBO midpoint.
- The buyer pays a lower price in the dark pool.

Analyzing quote volatility

- Usual approach
  - *parametric model* for variance of price *changes* (ARCH, GARCH, …)
- This study
  - *Non-parametric* analysis of variances of price *levels*

Variance about a local mean of a random walk



Computational definitions

- p_t is a discrete-time price process
- Local mean of length n ending at time t
  - $S(n,t) = n^{-1}(p_t + p_{t-1} + \cdots + p_{t-n+1})$
- Deviation (for $t - n < s \leq t$)
  - $R(n,t,s) = p_s - S(n,t)$
- Mean square deviation
  - $MSD(n,t) = n^{-1} \sum_{s=t-n+1}^{t} [R(n,t,s)]^2$

- Now assume that p_t has stationary first differences
  - Stationary ⇏ uncorrelated
- Most of the statistics in this paper are simple transformations of
  - $E[MSD(n,t)] = Var[R(n,t,s)] \equiv \sigma_n^2$
- n is set to reflect the horizon of interest
  - This differs across traders, so use a range, such as $\{1\ ms, 5\ ms, 10\ ms, 100\ ms, \ldots\}$
- For computational efficiency, take a dyadic ("powers of two") sequence
  - $n_j = n_0 2^j\ for\ j = 0,1,\ldots$
- In the application $n_0 = 50\ ms$: $\{50, 100, 200, \ldots\}$

Signal processing and time scale decompositions

- The mean $S(n,t)$ is a ***smooth*** [component]
- The deviation series $R(s,n,t)$ is a ***rough*** [component]
- $\sigma_j^2 \equiv \sigma_{n_j}^2$ is the ***rough variance***
  - … at time scale $\tau_j = n_0 2^{j-1}$
  - σ_j^2 reflects variation at time scale τ_j and shorter.
- The incremental change in moving from time scale τ_{j-1} to τ_j is $\nu_j^2 = \sigma_j^2 - \sigma_{j-1}^2$
  - ν_j^2 reflects variation at time scale τ_j only.

The incremental variance ν_j^2. What to call it?

- In frequency domain ("spectral") analysis ν_j^2 is the *spectral variance* (over a particular band of frequencies).
- In modern signal processing, it is a ***wavelet variance***.
- For computational efficiency, it is calculated using wavelet transforms (a relative of Fourier transforms).
- It can be defined, interpreted and computed without invoking wavelets.

Interpretation

- To assess economic importance, I present the (wavelet and rough) variance estimates in three ways.
  - In mils per share
  - In basis points
  - As a short-term/long-term ratio

Mils per share

- Variances are computed on bid and offer prices scaled to $mils/share$.
- One mil = \$0.001
- Variances are $(mils/share)^2$
- Most trading charges are assessed per share.
  - Someone sending a marketable order to a US exchange typically pays an "access fee" of about three mils/share.
  - An executed limit order receives a "liquidity rebate" of about two mils/share.

Basis points

- The variance in basis points:
  - $\frac{\nu^2}{Price^2} \times (10{,}000)^2$
- "One bp is a one cent bid-offer spread on a $100 stock."

The short/long variance ratio

- For a random walk with per period variance σ^2, the variance of the n-period difference is $n\sigma^2$.
- An conventional variance ratio might be something like
  - $V = \frac{60 \times one\ minute\ return\ variance}{one\ hour\ return\ variance}$
- For a random walk, $V = 1$.
  - Due to microstructure effects we usually find $V > 1$.
- Extensively used in microstructure studies: Barnea (1974); Amihud and Mendelson (1987); etc.

Variance ratios (cont'd)

- The wavelet variance ratio is
  - $V_{j,J} = 2^{J-j} \times \frac{\nu_j^2}{\nu_J^2}$
  - J is the highest level (longest time scale) in the analysis (27 minutes).
- The rough variance ratio is
  - $VR_{j,J} = 2^{J-j-1} \times \frac{\sigma_j^2}{\nu_J^2}$
- Like traditional variance ratios, any excess above unity indicates inflation of short-term volatility relative to fundamental volatility.

The empirical analysis

CRSP Universe 2001-2011. (Share code = 10 or 11; average price $2 to $1,000; listing NYSE, Amex or NASDAQ)

In each year, chose 150 firms in a random sample stratified by dollar trading volume

2011 April TAQ with one-*millisecond* time stamps

High-resolution analysis

2001-2011 April TAQ data with one-second time stamps

Lower-resolution analysis

Table 1. Summary Statistics, 2011

		Dollar trading volume quintile				
	Full sample	1 (low)	2	3	4	5 (high)
No. of firms	150	30	30	30	30	30
NYSE	47	0	5	7	16	19
Amex	6	2	2	0	1	1
NASDAQ	97	28	23	23	13	10
Avg. daily trades	1,331	31	431	1,126	3,478	16,987
Avg. daily quotes	23,928	967	7,706	24,026	53,080	181,457
Avg. daily NBBO records	7,138	328	3,029	7,543	16,026	46,050
Avg. daily NBB changes	1,245	120	511	1,351	2,415	4,124
Avg. daily NBO changes	1,164	103	460	1,361	2,421	4,214
Avg. price	$15.62	$4.87	$5.46	$17.86	$27.76	$51.60
Market capitalization of equity, $ Million	$683	$41	$202	$747	$1,502	$8,739

Table 2. Time scale variance estimates, 2011

- Variance estimates (across)
- Time scale (down, shortest to longest)

		Rough variances, σ_j^2			Wavelet variances, ν_j^2			
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
Level, j	Time scale	(mils per share)^2	(basis points)^2	Variance ratio	(mils per share)^2	(basis points)^2	Variance ratio	Bid-Offer Corr
0	< 50 ms	0.17	0.06	4.22	0.17	0.06	4.22	
1	50 ms	0.32	0.11	3.99	0.15	0.05	3.76	0.32
2	100 ms	0.61	0.22	3.79	0.29	0.10	3.58	0.36
3	200 ms	1.17	0.41	3.53	0.55	0.19	3.27	0.41
4	400 ms	2.19	0.75	3.21	1.03	0.34	2.88	0.44
5	800 ms	4.15	1.38	2.90	1.96	0.63	2.59	0.47
6	1,600 ms	7.93	2.56	2.64	3.78	1.18	2.38	0.51
7	3.2 sec	15.27	4.73	2.40	7.35	2.17	2.16	0.55
8	6.4 sec	29.59	8.57	2.12	14.31	3.85	1.84	0.60
9	12.8 sec	57.62	15.49	1.88	28.03	6.91	1.65	0.64
10	25.6 sec	112.38	28.03	1.70	54.76	12.54	1.51	0.69
11	51.2 sec	219.31	51.17	1.54	106.92	23.14	1.39	0.74
12	102.4 sec	428.81	94.11	1.42	209.50	42.94	1.29	0.79
13	3.4 min	842.72	174.70	1.32	413.91	80.60	1.21	0.83
14	6.8 min	1,668.69	328.05	1.23	825.97	153.35	1.15	0.86
15	13.7 min	3,287.68	618.26	1.16	1,618.99	290.21	1.08	0.88
16 (=J)	27.3 min	6,379.91	1,159.03	1.08	3,092.22	540.77	1.00	0.90

		Rough variances, σ_j^2			Wavelet variances, ν_j^2			
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
Level, j	Time scale	(mils per share)^2	(basis points)^2	Variance ratio	(mils per share			
0	< 50 ms	0.17	0.06	4.22	0			
1	50 ms	0.32	0.11	3.99				
2	100 ms	0.61	0.22	3.79				
3	200 ms	1.17	0.41	3.53				
4	400 ms	**2.19**	**0.75**	**3.21**	1			
5	800 ms	4.15	1.38	2.90	1			
6	1,600 ms	7.93	2.56	2.64	3			
7	3.2 sec	15.27	4.73	2.40	7			
8	6.4 sec	29.59	8.57	2.12	14			
9	12.8 sec	57.62	15.49	1.88	28			
10	25.6 sec	112.38	28.03	1.70	54			
11	51.2 sec	219.31	51.17	1.54	106.92			
12	102.4 sec	428.81	94.11	1.42	209.50	42.94	1.29	0.79
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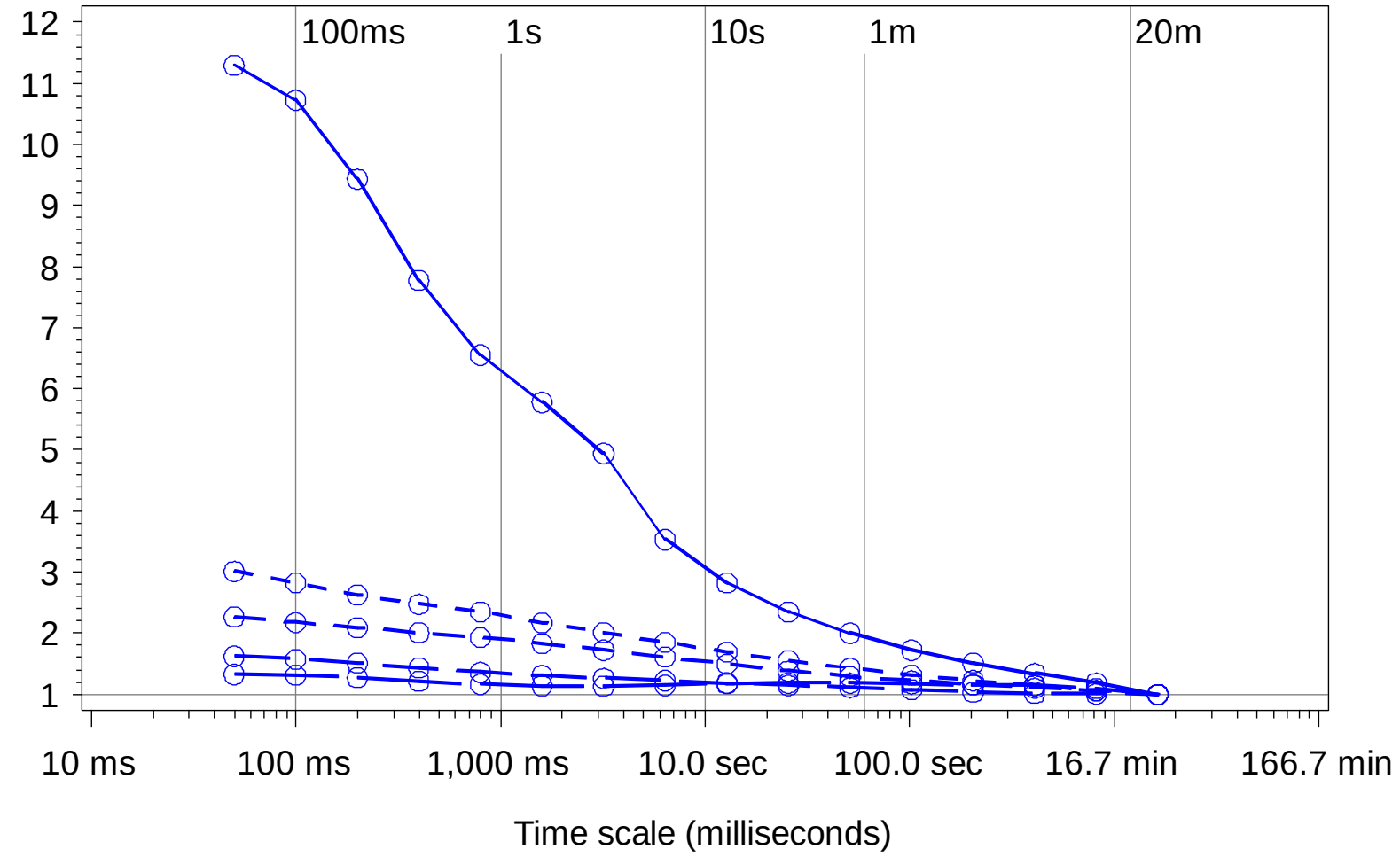
A trader who faces time uncertainty of 400 ms incurs price risk of $\sqrt{2.19} \approx 1.5\ mils/share$ or $\sqrt{0.75} \approx 0.9\ bp$.

At a horizon of 400 ms. The rough variance is 3.21 times the value implied by a random walk with variance calibrated to 27.3 minutes.

Figure 2. Wavelet variance ratios across time scale and dollar volume quintiles



		Rough variances, σ_j^2			Wavelet variances, ν_j^2			
		(1)	(		)		(6)	(7)
Level, j	Time scale	(mils per share)^2	(b poi			sis	Variance	Bid-Offer Corr
0	< 50 ms	0.17				0.06	4.22	
1	50 ms	0.32	0.11	3.99	0.15	0.05	3.76	0.32
2	100 ms	0.61	0.22	3.79	0.29	0.10	3.58	0.36
3	200 ms	1.17	0.41	3.53	0.55	0.19	3.27	0.41
4	400 ms	2.19	0.75	3.21	1.03	0.34	2.88	0.44
5	800 ms	4.15	1.38	2.90	1.96	0.63	2.59	0.47
6	1,600 ms	7.93	2.56	2.64	3.78	1.18	2.38	0.51
7	3.2 sec	15.27	4.73	2.40	7.35	2.17	2.16	0.55
8	6.4 sec	29.59	8.57	2.12	14.31	3.85	1.84	0.60
9	12.8 sec	57.62	15.49	1.88	28.03	6.91	1.65	0.64
10	25.6 sec	112.38	28.03	1.70	54.76	12.54	1.51	0.69
11	51.2 sec	219.31	51.17	1.54	106.92	23.14	1.39	0.74
12	102.4 sec	428.81	94.11	1.42	209.50	42.94	1.29	0.79
13	3.4 min	842.72	174.70	1.32	413.91	80.60	1.21	0.83
14	6.8 min	1,668.69	328.05	1.23	825.97	153.35	1.15	0.86
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16 (=J)	27.3 min	6,379.91	1,159.03	1.08	3,092.22	540.77	1.00	0.90

How closely do the bid and offer track at the indicated time scale?

Figure 3. Wavelet correlations between the National Best Bid and National Best Offer

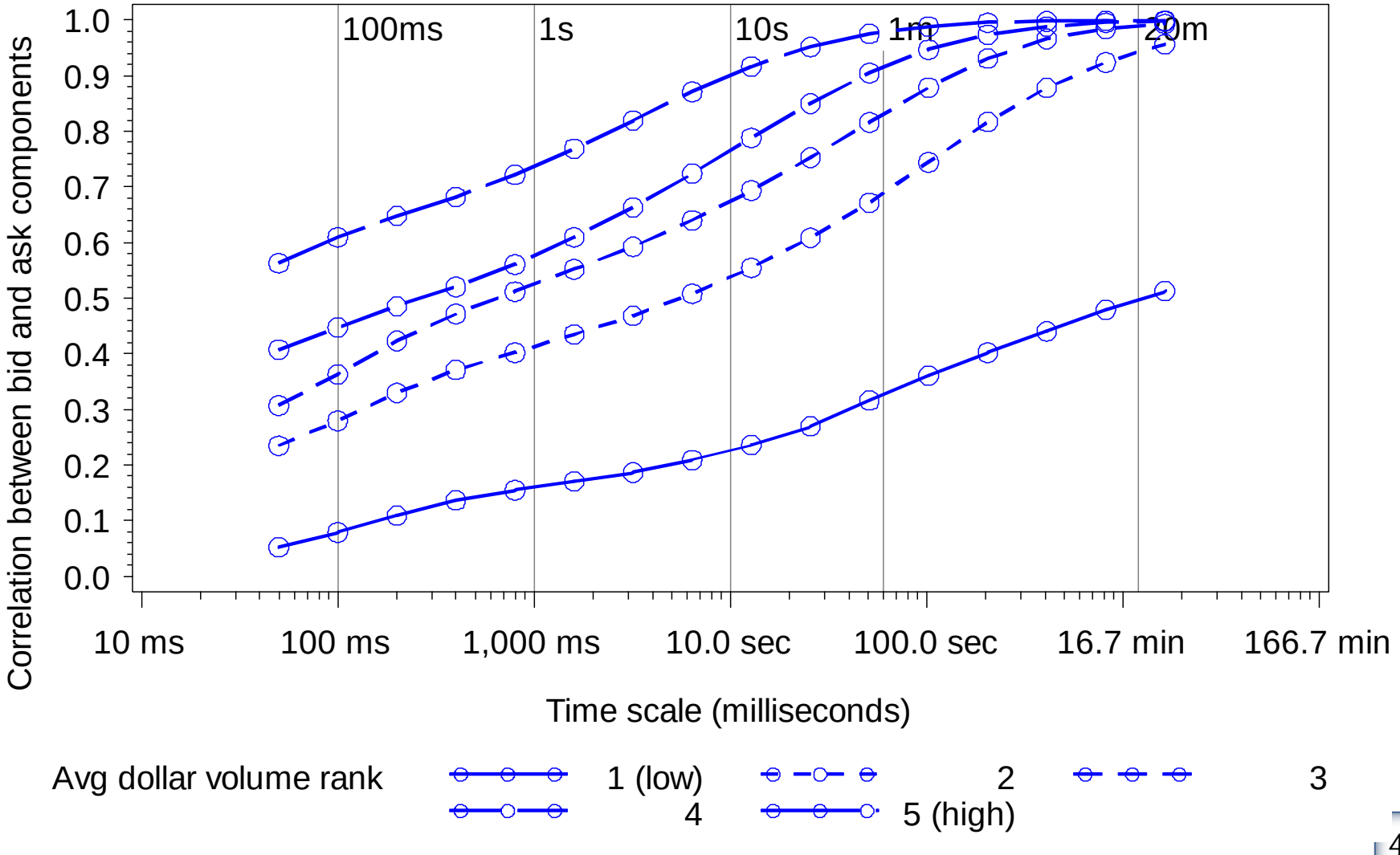

Table 3. Time scale variance across dollar volume quintiles, 2011
Panel A: Rough variance σ_j^2 in mils per share

			Dollar trading volume quintiles				
Level, j	Time scale	Full sample	1 (low)	2	3	4	5 (high)
0	< 50 ms	0.17	0.13	0.10	0.14	0.22	0.25
		(0.01)	(0.07)	(0.02)	(0.01)	(0.01)	(0.01)
1	50 ms	0.15	0.12	0.09	0.12	0.20	0.23
		(0.01)	(0.07)	(0.01)	(0.01)	(0.01)	(0.01)
3	200 ms	0.55	0.38	0.29	0.43	0.75	0.89
		(0.04)	(0.22)	(0.03)	(0.02)	(0.04)	(0.05)
5	800 ms	1.96	1.03	1.00	1.53	2.79	3.35
		(0.09)	(0.40)	(0.09)	(0.07)	(0.14)	(0.20)
7	3.2 sec	7.35	3.33	3.45	5.41	10.73	13.42
		(0.28)	(0.93)	(0.30)	(0.23)	(0.57)	(0.84)
10	25.6 sec	54.76	15.25	20.99	34.87	81.37	117.44
		(2.09)	(2.41)	(1.84)	(1.50)	(4.50)	(8.22)
14	6.8 min	825.97	173.25	242.94	445.93	1,273.22	1,930.34
		(42.96)	(28.14)	(21.87)	(21.43)	104.07)	173.36)
16	27.3 min	3,092.22	533.78	798.24	1,665.48	5,425.58	6,786.46
		(188.75)	(87.40)	(73.69)	(82.94)	(634.46)	(639.18)

Table 3. Time scale variance across dollar volume quintiles, 2011
Panel B, Rough variance σ_j^2 in bp^2

			Dollar trading volume quintiles				
Level, j	Time scale	Full sample	1 (low)	2	3	4	5 (high)
0	< 50 ms	0.06	0.20	0.06	0.03	0.02	0.01
		(0.01)	(0.07)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
1	50 ms	0.11	0.38	0.12	0.05	0.03	0.02
		(0.03)	(0.14)	(0.01)	(<0.01)	(<0.01)	(<0.01)
3	200 ms	0.41	1.33	0.44	0.19	0.11	0.06
		(0.09)	(0.47)	(0.02)	(0.01)	(<0.01)	(<0.01)
5	800 ms	1.38	4.26	1.58	0.70	0.40	0.23
		(0.21)	(1.15)	(0.07)	(0.03)	(0.01)	(0.01)
7	3.2 sec	4.73	13.90	5.65	2.59	1.51	0.87
		(0.49)	(2.60)	(0.26)	(0.12)	(0.04)	(0.03)
10	25.6 sec	28.03	71.52	36.85	17.37	11.39	7.27
		(1.46)	(7.44)	(1.74)	(0.78)	(0.34)	(0.25)
14	6.8 min	328.05	694.55	463.45	225.57	173.41	119.32
		(10.59)	(44.14)	(27.35)	(11.01)	(6.47)	(4.60)
16	27.3 min	1,159.03	2,234.94	1,718.14	815.77	685.54	446.54
		(41.18)	143.62)	138.93)	(42.03)	(27.48)	(18.25)

Table 3. Time scale variance estimates across $ volume quintiles, 2011
Panel C. Rough variance ratios

			Dollar trading volume quintiles				
Level, j	Time scale	Full sample	1 (low)	2	3	4	5 (high)
0	< 50 ms	4.22	12.72	3.45	2.62	1.76	1.37
		(1.28)	(6.96)	(0.18)	(0.07)	(0.04)	(0.02)
1	50 ms	3.99	12.01	3.23	2.44	1.69	1.35
		(1.25)	(6.81)	(0.16)	(0.06)	(0.04)	(0.02)
3	200 ms	3.53	10.40	2.83	2.20	1.57	1.30
		(1.06)	(5.77)	(0.11)	(0.05)	(0.03)	(0.02)
5	800 ms	2.90	7.82	2.50	2.02	1.43	1.21
		(0.66)	(3.56)	(0.08)	(0.04)	(0.03)	(0.02)
7	3.2 sec	2.40	5.87	2.17	1.82	1.32	1.15
		(0.38)	(2.08)	(0.06)	(0.04)	(0.02)	(0.02)
10	25.6 sec	1.70	3.06	1.70	1.49	1.19	1.17
		(0.12)	(0.64)	(0.04)	(0.03)	(0.02)	(0.02)
14	6.8 min	1.23	1.58	1.24	1.17	1.04	1.16
		(0.02)	(0.12)	(0.03)	(0.03)	(0.02)	(0.02)
16	27.3 min	1.08	1.19	1.08	1.06	1.01	1.06
		(0.01)	(0.06)	(0.03)	(0.02)	(0.02)	(0.02)

Table 3. Time scale variance estimates across $ volume quintiles, 2011
Panel D. Bid-offer correlations

			Dollar trading volume quintiles				
Level, j	Time scale	Full sample	1 (low)	2	3	4	5 (high)
1	50 ms	0.32	0.05	0.23	0.31	0.41	0.56
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
3	200 ms	0.41	0.11	0.33	0.42	0.49	0.65
		(0.01)	(0.01)	(0.02)	(0.02)	(0.01)	(0.02)
5	800 ms	0.48	0.15	0.40	0.51	0.56	0.72
		(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)
7	3.2 sec	0.55	0.19	0.47	0.59	0.66	0.82
		(0.01)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
10	25.6 sec	0.70	0.27	0.61	0.75	0.85	0.95
		(0.01)	(0.02)	(0.03)	(0.02)	(0.02)	(0.03)
14	6.8 min	0.86	0.44	0.88	0.97	0.99	1.00
		(0.02)	(0.03)	(0.04)	(0.03)	(0.03)	(0.03)
16	27.3 min	0.90	0.51	0.96	0.99	1.00	1.00
		(0.02)	(0.04)	(0.05)	(0.03)	(0.04)	(0.03)

The 2011 results: a summary

- Variance ratios: short term volatility is much higher than we'd expect relative to a random-walk.
- In mils per share or basis points, *average* short term volatility is economically meaningful, but small.

Back to the empirical analysis

CRSP Universe 2001-2011. (Share code = 10 or 11; average price $2 to $1,000; listing NYSE, Amex or NASDAQ)

In each year, chose 150 firms in a random sample stratified by dollar trading volume

2011 April TAQ with one-*millisecond* time stamps

High-resolution analysis

2001-2011 April TAQ data with one-second time stamps

Lower-resolution analysis

High-resolution analysis … … with low resolution data

- TAQ with millisecond time stamps only available from 2006 onwards
- TAQ with one second time stamps available back to 1993.
- Can we draw inferences about *subsecond* variation from second-stamped data?

The problem

Quote A	10:01:35
Quote B	10:01:35
Quote C	10:01:35

- Where within the second did these quotes actually occur?
- With a few simple assumptions, we know how they are distributed and how they may be simulated.

Recall the constant intensity Poisson process …

- $N(t) =$ no. of events in an interval $(0, t)$
- $s_i =$ arrival time of event i
- If $N(t) = n$, then $s_1, s_2, \ldots, s_n$ have the same distribution as the order statistics in a sample of n independent $U(0, t)$ random variables.
- This suggests a simple procedure…

Quote A	10:01:35
Quote B	10:01:35
Quote C	10:01:35

- Draw three U(0,1) random numbers
- Sort them
- Assign them as the millisecond remainders

Quote A	10:01:35.***243***
Quote B	10:01:35.***347***
Quote C	10:01:35.***912***

- Compute variance estimates using the simulated time stamps.

Formalities

- Assume that
  - The quotes are correctly sequenced.
  - Arrivals within the second are Poisson with (unknown) constant intensity.
  - The bid and offer process is independent of the within-second arrival times.
- Then each calculated statistic constitutes a draw from the corresponding Bayesian posterior.

Does this really work?

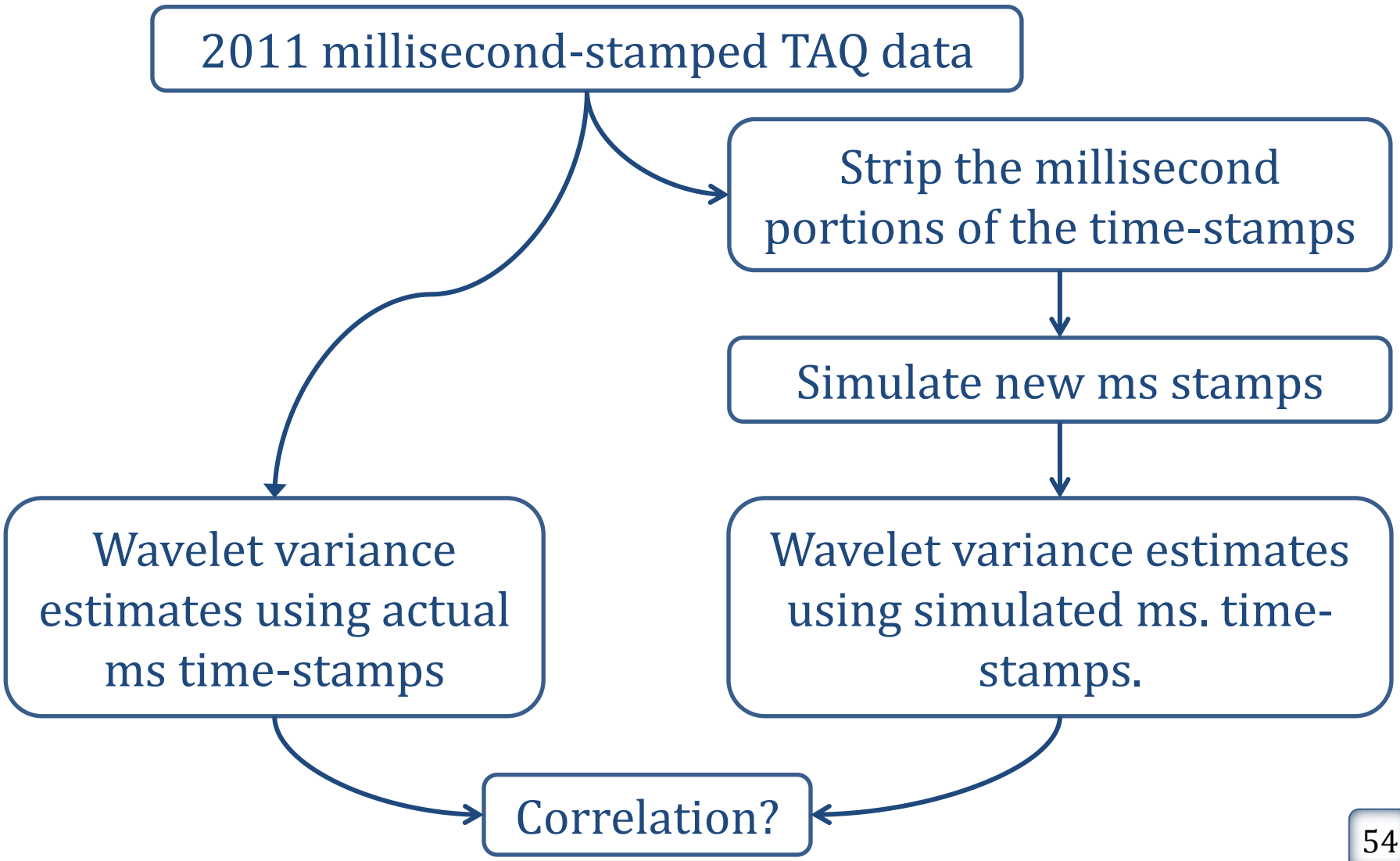

Table 4. Correlations between statistics
based on actual vs. simulated millisecond time stamps.

Panel A: Correlations for wavelet variances, ν_j^2

			Dollar trading volume quintiles				
Level, j	Time scale	Full sample	1 (low)	2	3	4	5 (high)
0	< 50 ms	0.952	0.948	0.960	0.958	0.916	0.979
1	50 ms	0.953	0.944	0.952	0.952	0.937	0.982
3	200 ms	0.975	0.965	0.969	0.975	0.977	0.988
5	800 ms	0.994	0.991	0.989	0.995	0.996	0.998
7	3.2 sec	0.999	0.999	0.999	1.000	1.000	1.000
10	25.6 sec	1.000	1.000	1.000	1.000	1.000	1.000
14	6.8 min	1.000	1.000	1.000	1.000	1.000	1.000
16	27.3 min	1.000	1.000	1.000	1.000	1.000	1.000

The correlations are terrific. *Why?*

- If observations are *sparse* in time, their exact location doesn't matter.
  - Suppose that there is one quote change in the hour ...
- If observations are *dense*, their exact location is known more precisely.
  - Consider the first order statistic (the minimum) in a sample of n Uniform(0,1) draws.
  - Sample of $n = 1$ vs. $n = 1{,}000$

Table 4. Correlations between estimates
based on actual vs. simulated millisecond time stamps.

Panel B: Correlations for bid-offer wavelet correlation estimates

			Dollar trading volume quintiles				
Level, j	Time scale	Full sample	1 (low)	2	3	4	5 (high)
0	< 50 ms	0.775	0.333	0.768	0.896	0.919	0.943
1	50 ms	0.900	0.662	0.926	0.965	0.972	0.978
3	200 ms	0.979	0.921	0.986	0.995	0.995	0.998
5	800 ms	0.999	0.998	0.999	1.000	1.000	1.000
7	3.2 sec	1.000	1.000	1.000	1.000	1.000	1.000
10	25.6 sec	1.000	1.000	1.000	1.000	1.000	1.000
14	6.8 min	1.000	1.000	1.000	1.000	1.000	1.000
16	27.3 min	0.775	0.333	0.768	0.896	0.919	0.943

Table 5. Summary statistics, historical sample, 2001-2011 (*only odd numbered years are shown)*

	2001	2003	2005	2007	2009	2011
No. firms	146	150	150	150	150	150
NYSE	108	51	48	55	56	47
Amex	22	11	8	14	5	6
NASDAQ	16	88	94	81	89	97
Avg. daily trades	142	187	425	970	1,790	1,331
Avg. daily quotes	1,078	1,299	5,828	12,521	39,378	23,928
Avg. daily NBB changes	103	203	596	772	1,618	1,210
Avg. daily NBO changes	103	213	729	789	1,731	1,126
Avg. price	$18.85	$14.83	$16.10	$15.81	$10.72	$15.62
Market capitalization of equity, $ Million	$745	$189	$325	$480	$316	$683

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Amex	22	11	8	14	5	6
NASDAQ	16	88			89	97
Avg. daily trades	142	187			1,790	1,331
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Avg. daily NBB changes	103	203			1,618	1,210
Avg. daily NBO changes	103	213	729	789	1,731	1,126
Avg. price	$18.85	$14.83	$16.10	$15.81	$10.72	$15.62
Market capitalization of equity, $ Million	$745	$189	$325	$480	$316	$683

25% CAGR

36% CAGR

Table 6. Wavelet variance ratios for bids and offers, 2001-2011

Panel A: Computed from *unadjusted* bids and offers

Level, j	Time scale	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
1	50 ms	5.22	7.16	6.03	10.28	6.69	8.57	6.96	6.06	4.52	7.08	4.70
2	100 ms	5.44	6.58	5.28	9.69	6.51	8.07	6.27	5.38	4.12	6.26	4.32
3	200 ms	5.28	6.28	5.13	9.03	6.22	7.34	5.33	4.64	3.68	5.40	3.74
4	400 ms	4.59	5.23	5.00	8.16	5.75	6.30	4.25	3.84	3.21	4.53	3.07
5	800 ms	3.12	4.04	3.93	5.57	5.03	5.10	3.41	3.11	2.76	3.71	2.56
6	1,600 ms	2.11	2.55	3.25	4.11	4.14	4.05	2.89	2.59	2.43	3.04	2.23
7	3.2 sec	1.98	2.24	2.93	3.38	3.48	3.37	2.56	2.29	2.17	2.53	2.01
8	6.4 sec	1.94	2.11	2.62	2.91	2.93	2.92	2.35	2.08	1.95	2.16	1.82

Table 6. Wavelet variance ratios for bids and offers, 2001-2011

Panel A: Computed from *unadjusted* bids and offers

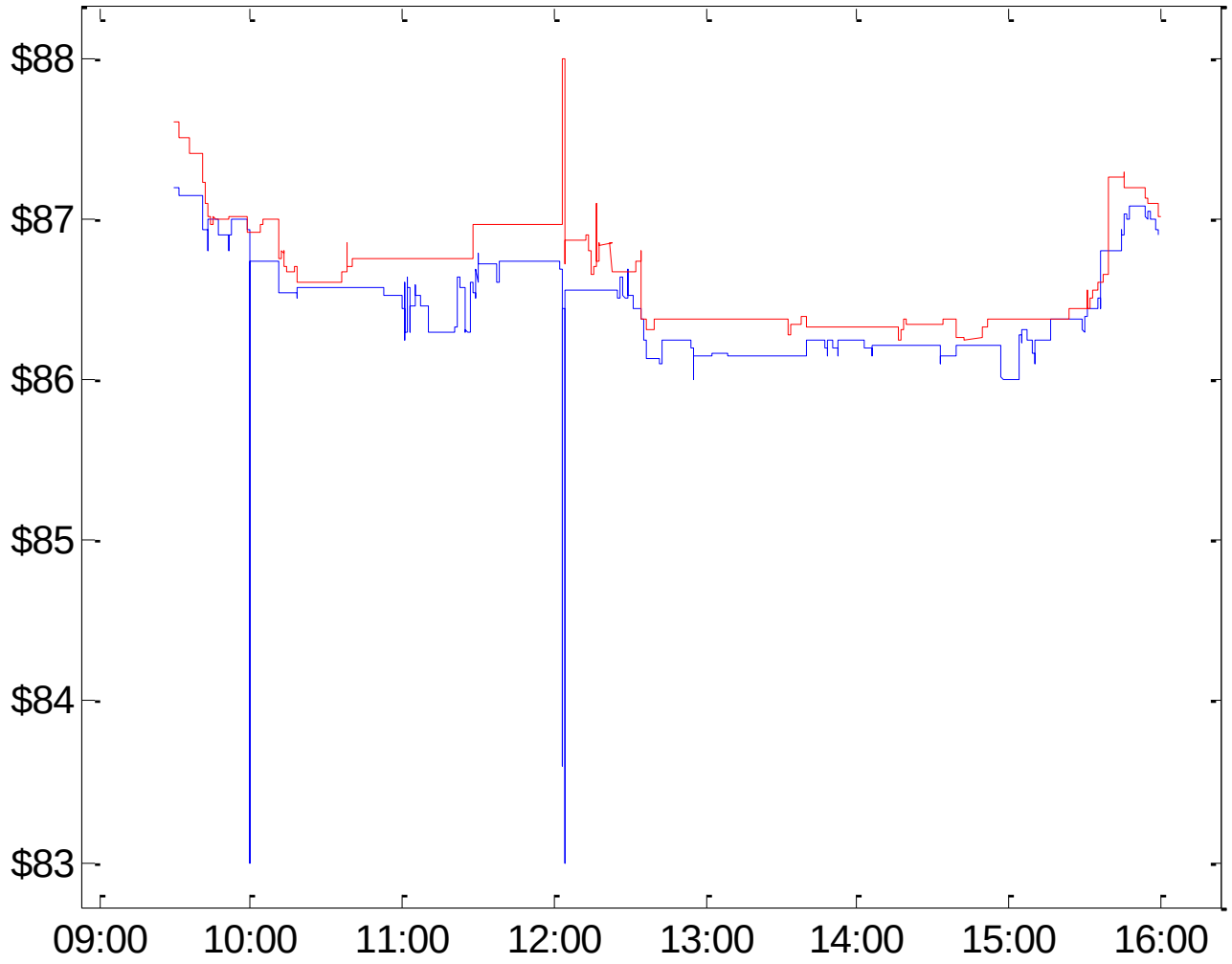
Level, j	Time scale	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
1	50 ms	5.22	7.16	6.03	10.28	6.69	8.57	6.96	6.06	4.52	7.08	4.70
2	100 ms	5.44	6.58	5.28	9.69	6.51	8.07	6.27	5.38	4.12	6.26	4.32
3	200 ms	5.28	6.28	5.13	9.03	6.22	7.34	5.33	4.64	3.68	5.40	3.74
4	400 ms	4.59	5.23	5.00	8.16	5.75	6.30	4.25	3.84	3.21	4.53	3.07
5	800 ms	3.12	4.04	3.93	5.57	5.03	5.10	3.41	3.11	2.76	3.71	2.56
6	1,600 ms	2.11	2.55	3.25	4.11	4.14	4.05	2.89	2.59	2.43	3.04	2.23
7	3.2 sec	1.98	2.24	2.93	3.38	3.48	3.37	2.56	2.29	2.17	2.53	2.01
8	6.4 sec	1.94	2.11	2.62	2.91	2.93	2.92	2.35	2.08	1.95	2.16	1.82

No trend in quote volatilities?

- Maybe …
  - "Flickering quotes" aren't new.
  - Concerns about high frequency trading are all media hype.
  - The good old days weren't really so great after all.

- What *did* quote volatility look like circa 2001?

Figure 4 Panel A. Bid and offer for PRK, April 6, 2001.



Compare

- PRK in 2001 differs from AEPI in 2011
  - PRK: large amplitude, no oscillation.
  - AEPI: low amplitude, intense oscillation.

Denoising (filtering) “pops”

- $bid = rough + smooth$
- Denoised $rough^*$
  - $= rough\ capped\ at\ \pm 1.5 \times spread$
- Reconstruct a denoised bid as
  - $bid^* = rough^* + smooth$
- Construct denoised bids and offers for all stocks in all years.
- Form new variance estimates.

Figure 4 Panel B. PRK, April 6, 2001, Rough component of the bid

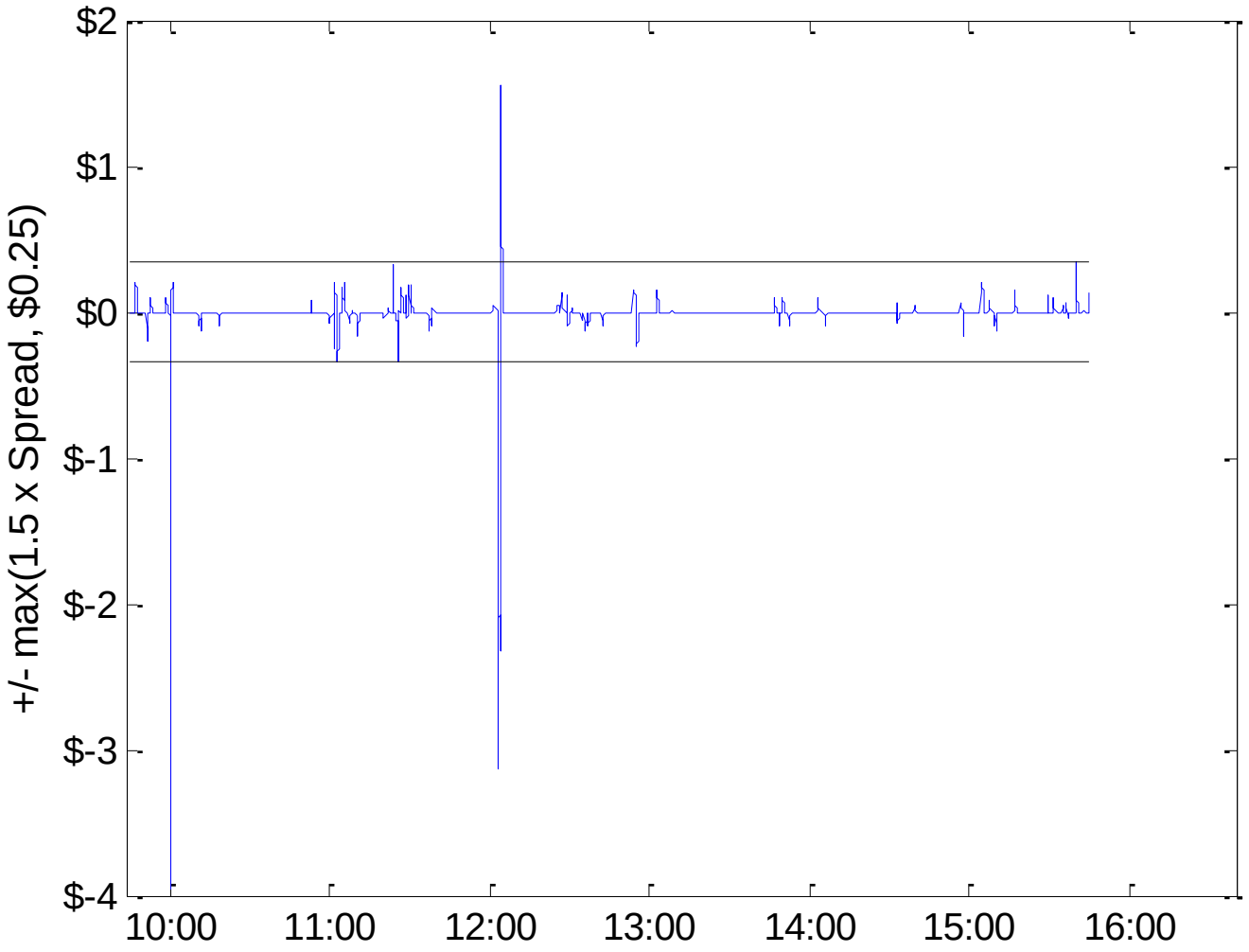


Table 6. Wavelet variance ratios for bids and offers, 2001-2011, *Detail*

Panel A: Computed from *unadjusted* bids and offers

Level, j	Time scale	2001	...	2011
1	50 ms	5.22		4.70
2	100 ms	5.44		4.32
3	200 ms	5.28		3.74
4	400 ms	4.59		3.07
5	800 ms	3.12		2.56
6	1,600 ms	2.11		2.23
7	3.2 sec	1.98		2.01
8	6.4 sec	1.94		1.82

Panel B: Computed from *denoised* bids and offers

Level, j	Time scale	2001	...	2011
1	50 ms	1.60		4.46
2	100 ms	1.57		4.07
3	200 ms	1.56		3.57
4	400 ms	1.55		3.00
5	800 ms	1.57		2.52
6	1,600 ms	1.64		2.20
7	3.2 sec	1.81		2.00
8	6.4 sec	2.11		1.82

Table 6. Wavelet variance ratios for bids and offers, 2001-2011

Panel B. Computed from *denoised* bids and offers

Level, j	Time scale	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
1	50 ms	1.60	2.37	3.15	7.02	6.09	8.24	6.56	5.83	4.20	6.79	4.46
2	100 ms	1.57	2.32	3.09	6.82	5.89	7.76	5.89	5.17	3.83	6.00	4.07
3	200 ms	1.56	2.27	3.03	6.48	5.61	7.04	4.99	4.45	3.41	5.18	3.57
4	400 ms	1.55	2.23	2.94	5.90	5.16	6.02	3.96	3.68	2.97	4.36	3.00
5	800 ms	1.57	2.19	2.83	5.00	4.47	4.82	3.13	2.98	2.56	3.58	2.52
6	1,600 ms	1.64	2.20	2.71	3.99	3.60	3.79	2.63	2.51	2.27	2.94	2.20
7	3.2 sec	1.81	2.30	2.62	3.44	3.02	3.16	2.33	2.23	2.04	2.46	2.00
8	6.4 sec	2.11	2.51	2.59	3.20	2.65	2.75	2.15	2.04	1.86	2.11	1.82

Table 6. Wavelet variance ratios for bids and offers, 2001-2011

Panel B. Computed from *denoised* bids and offers

Level, j	Time scale	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
1	50 ms	1.60	2.37	3.15	7.02	6.09	8.24	6.56	5.83	4.20	6.79	4.46
2	100 ms	1.57	2.32	3.09	6.82	5.89	7.76	5.89	5.17	3.83	6.00	4.07
3	200 ms	1.56	2.27	3.03	6.48	5.61	7.04	4.99	4.45	3.41	5.18	3.57
4	400 ms	**1.55**	**2.23**	**2.94**	**5.90**	**5.16**	**6.02**	**3.96**	**3.68**	**2.97**	**4.36**	**3.00**
5	800 ms	1.57	2.19	2.83	5.00	4.47	4.82	3.13	2.98	2.56	3.58	2.52
6	1,600 ms	1.64	2.20	2.71	3.99	3.60	3.79	2.63	2.51	2.27	2.94	2.20
7	3.2 sec	1.81	2.30	2.62	3.44	3.02	3.16	2.33	2.23	2.04	2.46	2.00
8	6.4 sec	2.11	2.51	2.59	3.20	2.65	2.75	2.15	2.04	1.86	2.11	1.82

The facts

- Pre-filtering, no trend in quote volatility.
- Filtering to remove spikes/pops greatly diminishes quote volatility in the early years, but not later years.
  - Early years: volatility due to spikes
  - Later years: volatility reflects oscillations

Table 8. Wavelet bid and offer variances, 2001-2011

Panel A. Rough variances, mils per share

Level, j	time scale	2001	2003	2005	2007	2009	2011
1	50 ms	0.05	0.07	0.23	0.08	0.25	0.07
		(<0.01)	(<0.01)	(0.03)	(<0.01)	(0.03)	(0.01)
3	200 ms	0.34	0.45	1.52	0.45	1.50	0.40
		(0.01)	(0.02)	(0.17)	(0.02)	(0.19)	(0.08)
5	800 ms	1.50	1.85	5.56	1.42	5.16	1.42
		(0.05)	(0.07)	(0.53)	(0.07)	(0.42)	(0.22)
7	3.2 sec	6.82	6.77	16.03	4.19	16.45	4.74
		(0.52)	(0.22)	(1.25)	(0.16)	(0.96)	(0.49)
10	25.6 sec	80.46	47.03	84.18	25.18	109.42	30.76
		(16.17)	(2.57)	(5.75)	(0.98)	(13.16)	(3.22)
14	6.8 min	735.03	489.43	862.96	302.16	1,638.73	333.50
		(30.23)	(12.26)	(73.94)	(23.79)	492.40)	(12.05)
16	27.3 min	2,511.15	1,554.80	2,872.45	1,046.55	4,623.58	1,164.98
		(80.99)	(39.18)	335.55)	101.74)	849.95)	(41.65)

Table 8. Wavelet bid and offer variances, 2001-2011

Panel B. Rough variance ratios

Level, j	Time scale	2001	2003	2005	2007	2009	2011
1	50 ms	1.60	3.15	6.09	6.56	4.20	4.46
		(0.02)	(0.11)	(0.39)	(0.40)	(0.31)	(1.42)
3	200 ms	1.57	3.06	5.76	5.47	3.65	3.84
		(0.02)	(0.10)	(0.36)	(0.30)	(0.26)	(1.16)
5	800 ms	1.56	2.91	4.94	3.87	2.91	2.94
		(0.03)	(0.09)	(0.27)	(0.17)	(0.16)	(0.69)
7	3.2 sec	1.71	2.71	3.64	2.78	2.31	2.28
		(0.09)	(0.10)	(0.15)	(0.09)	(0.09)	(0.35)
10	25.6 sec	2.36	2.60	2.42	2.03	1.74	1.70
		(0.37)	(0.29)	(0.07)	(0.05)	(0.04)	(0.12)
14	6.8 min	1.37	1.50	1.49	1.37	1.32	1.23
		(0.04)	(0.03)	(0.02)	(0.02)	(0.03)	(0.02)
16	27.3 min	1.12	1.16	1.16	1.12	1.11	1.08
		(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)

SEC's Reg NMS ("National Market System")

- Defined the framework for competition among equity markets.
- Enhanced protection against trade-throughs
  - Example: market A is bidding $10 and market B executes a trade at $9.
- For a market's bid and offer to be protected, they have to accessible instantly (electronically)
  - This requirement essentially forced all markets to become electronic.
- Timing: Proposed in 2004; adopted 2005; implemented in 2006.

Prior to SEC's Reg NMS (2004-2006)

- Most markets were manual and slow.
- Few possibilities of automatic execution.
- Dealers were supposed to display customer limit orders … but sometimes didn't.
- Bid and offer quotes were supposed to be firm … but sometimes weren't.
- Protection against trade-throughs was weak.

And now …

- Quotes are firm, accessible, and (more strongly) protected.
- But quote volatility *has* increased.
- More questions
  - What strategies give rise to these patterns?
  - Are the HFQ episodes unstable algos?
  - Are they sensible strategies to detect and access liquidity?