CHAPTER FIVE
RETURN, RISK, AND CO-MOVEMENT

5.1 RETURN ON INVESTMENT

(Background reading: section 2.8 and application 2.4)

While this chapter is primarily concerned with the measurement and variability of security returns, it is also intended to provide an introduction to certain statistical measures. Investors are most concerned with the economic efficiency and riskiness of their investments, while the statistics branch of mathematics provides us with a variety of important measures of economic efficiency and risk.

Measuring an investment return is intended to enable one to determine the economic efficiency of that investment. An investment's return will express the profits generated by an initial cash outlay relative to the amount of the outlay required to secure that profit. This efficiency measure is, in a sense, the output from an investment relative to the investment's input. There are several methods to determine the return on an investment. The methods presented in this chapter are return on investment and internal rate of return. Holding period, arithmetic mean, and geometric mean rates of return on investment will be discussed along with internal rate of return and bond return measures. These methods differ in their ease of computation and how they account for the timeliness and compounding of cash flows.

Perhaps the easiest method to determine the economic efficiency of an investment is to add all of the investment profits π_t accruing at each time period t and divide this sum by the amount of the initial cash outlay P_0. This measure is called a holding period return:

$$ROI_{\mathrm{H}} = \frac{\sum_{t=1}^{n} \pi_t}{P_0}. \tag{5.1}$$

A \$100 investment that generates \$20 profits in each of three years would have a holding period return of 0.60, determined as follows:

$$ROI_{\mathrm{H}} = \frac{\sum_{t=1}^{n} \pi_t}{P_0} = \frac{20+20+20}{100} = \frac{60}{100} = 0.60.$$

To ease comparisons between investments with different life expectancies, one can compute an arithmetic mean return on investment, ROI_{A}, by dividing the holding period return, ROI_{H}, by the life expectancy of the investment, n, as follows:

$$ROI_{\mathrm{A}} = \frac{\sum_{t=1}^{n} \pi_t}{P_0} \div n. \tag{5.2}$$

The subscript A after ROI designates that the return value expressed is an arithmetic mean return and the variable π_t is the profit generated by the investment in year t. Since it is not always clear exactly what the profit on an investment is in a given year, one can compute a return based on periodic cash flows. Therefore, this arithmetic mean rate of return formula can be written as follows:

$$ROI_{\mathrm{A}} = \frac{\sum_{t=1}^{n} CF_t - P_0}{P_0} \div n = \frac{\sum_{t=1}^{n} CF_t - P_0}{nP_0} = \frac{\sum_{t=0}^{n} CF_t}{nP_0}. \tag{5.3}$$

The numerator in each term represents the investment profits. Notice that the summation in the third expression begins at time zero, ensuring that the initial cash outlay is deducted from the numerator. (The cash flow CF_0 associated with any initial cash outlay or investment will be negative.) The primary advantage of equation (5.3) over equation (5.2) is that a profit level need not be determined each year for the investment; that is, the annual cash flows generated by an investment do not have to be classified as to whether they are profits or merely return of capital. Multiplying P_0 by n in the denominator of equation (5.3) to annualize the return has the same effect as dividing the entire fraction by n as in equation (5.2). In the first two expressions in equation (5.3), the summation begins at time one. The initial outlay is recognized by subtracting P_0 in the numerator. For example, consider a stock whose purchase price three years ago was \$100. This stock paid a dividend of \$10 in each of the three years and was sold for \$130. If time zero is the stock's date of purchase, its arithmetic mean annual return is 20%:

$$ROI_{\mathrm{A}} = \frac{\sum_{t=0}^{n} CF_t}{nP_0} = \frac{-100+10+10+10+130}{3 \cdot 100} = \frac{60}{300} = 0.20.$$

Identically, the stock's annual return is determined by equation (5.4):

$$
\begin{aligned}
ROI_{\mathrm{A}} &= \frac{\sum_{t=1}^{n} DIV_t}{nP_0} + \frac{P_n - P_0}{nP_0} \\
&= \frac{10+10+10}{3 \cdot 100} + \frac{130-100}{nP_0} = \frac{30}{300} + \frac{30}{300} = 0.20,
\end{aligned} \tag{5.4}
$$

where DIV_t is the dividend payment for the stock in time t, P_0 is the purchase price of the stock, and P_n is the selling price of the stock. The difference $P_n - P_0$ is the capital gain realized from the sale of the stock.

Consider a second stock held over the same period whose purchase price was also \$100. If this stock paid no dividends and was sold for \$160, its annual return would also be 20%:

$$
ROI_{\mathrm{A}} = \frac{\sum_{t=1}^{n} DIV_t}{nP_0} + \frac{P_n - P_0}{nP_0} = \frac{0}{3 \cdot 100} + \frac{160-100}{nP_0} = \frac{60}{300} = 0.20.
$$

Therefore, both the first and second stocks have realized arithmetic mean returns of 20%. The total cash flows generated by each, net of their original \$100 investments, is \$60. Yet the first stock must be preferred to the second, since its cash flows are realized sooner. The arithmetic mean return (ROI_{A}) does not account for the timing of these cash flows. Therefore, it evaluates the two stocks identically, even though the first should be preferred to the second. Because this measure of economic efficiency does not account for the timeliness of cash flows, another measure must be developed.

APPLICATION 5.1: FUND PERFORMANCE
(Background reading: sections 2.8 and 5.1 and application 2.4)

This application is concerned with how one might collect price data for a fund and compute returns from that data. Suppose that one is interested in a given publically traded fund for which prices and dividends are reported in standard news sources. First, retrieve fund price and dividend data for each period under consideration. This is easily done with the *Wall Street Journal* or *Barrons*, as well as a variety of online data sources. Calculate periodic holding returns based on the following:

$$
r_t = \frac{P_t - P_{t-1} + DIV_t}{P_{t-1}}.
$$

Note the similarity of this expression with equation (5.4). The appropriate period to record the dividend is the period during which the fund went "ex-dividend." Suppose that we wished to compute monthly returns for a fund over a period of five months. We collect relevant end-of-month prices along with any dividends. The following table lists prices and dividends collected for a fund from June 30 to November 30. Following the price data are sample return calculations:

Date	t	P_t	P_{t-1}	DIV_t	r_t	Notes
June 30	1	50	–	0	–	First month
July 31	2	55	50	0	0.100	$(55 \div 50) - 1 = 0.10$
August 31	3	50	55	0	−0.091	$(50 \div 55) - 1 = -0.091$
September 30	4	54	50	0	0.080	$(54 \div 50) - 1 = 0.08$
October 31	5	47	54	2	−0.092	ex-$2 dividend; $[(47 + 2) \div 54)] - 1 = -0.092$
November 30	6	51	47	0	0.081	$(51 \div 47) - 1 = 0.081$

Now, five-monthly holding period returns have been computed for the fund. The five-month holding period return might be approximated as the sum of individual monthly returns (although we will discuss some problems with this in section 5.2 and in application 5.2). The holding period and arithmetic mean returns for the fund can be computed with variations of equations (5.1) and (5.2) as follows:

$$ROI_{\mathrm{H}} = \sum_{t=1}^{n} r_t = 0.10 - 0.091 + 0.08 - 0.092 + 0.081 = 0.078,$$

$$ROI_{\mathrm{A}} = \frac{\sum_{t=1}^{n} r_t}{n} = \frac{0.10 - 0.091 + 0.08 - 0.092 + 0.081}{5} = 0.0156.$$

5.2 Geometric Mean Return on Investment

(Background reading: sections 2.10 and 5.1 and application 2.5)

The arithmetic mean return on investment does not account for any difference between dividends (intermediate cash flows) and capital gains (profits realized at the end of the investment holding period). That is, ROI_{A} does not account for the time value of money or the ability to reinvest cash flows received prior to the end of the investment's life. In reality, if an investor receives profits in the form of dividends, he has the option to reinvest them as they are received. If profits are received in the form of capital gains, the investor must wait until the end of his investment holding period to reinvest them. The difference between these two forms of profits can be accounted for by expressing compounded returns. That is, the geometric mean return on investment will account for the fact that any earnings that are retained by the firm will be automatically reinvested, and thus compounded. The geometric mean return is computed as follows:

$$ROI_g = \sqrt[n]{\prod_{t=1}^{n}(1+r_t)} - 1. \tag{5.5}$$

A \$100 investment that generates \$20 profits in each of three years would have a holding period return equal to 20% in each of those three years. The geometric mean return on this stock would be computed as follows:

$$ROI_g = \sqrt[n]{\prod_{t=1}^{n}(1+r_t)} - 1 = \sqrt[3]{\prod_{t=1}^{3}(1+r_t)} - 1$$

$$= \sqrt[3]{(1+0.2)(1+0.2)(1+0.2)} - 1 = 0.20.$$

Consider a stock that pays no dividends but appreciates from \$100 to \$160 over three years. If each of the end-of-year prices is not given, holding period returns for each of the years may not be determined. However, the three-year geometric return may be computed from the following:

$$ROI_g = \sqrt[n]{P_n \div P_0} - 1. \tag{5.6}$$

Thus, the stock's geometric mean return is determined as follows:

$$ROI_g = \sqrt[n]{P_n \div P_0} - 1 = \sqrt[3]{160 \div 100} - 1 = 0.1696.$$

Notice that both of the stocks in this section generated a total of \$60 in profits. The first stock generated \$20 in each of the three years, while the second stock generated \$60 when the stock is to be sold in the third year. The waiting period for the profits is less for the first stock. Therefore, its geometric mean rate of return is higher.

APPLICATION 5.2: FUND PERFORMANCE, PART II
(Background reading: section 5.2 and application 5.1)

Consider the fund from application 5.1 (we will call it fund A) and compare its monthly returns and prices to a second fund B:

		Fund A				Fund B			
Date	t	P_t	P_{t-1}	DIV_t	r_t	P_t	P_{t-1}	DIV_t	r_t
June 30	1	50	–	0	–	50	–	0	–
July 31	2	55	50	0	0.100	80	50	0	0.60
August 31	3	50	55	0	–0.091	40	80	0	–0.50
September 30	4	54	50	0	0.080	60	40	0	0.50
October 31	5	47	54	2	–0.092	30	60	0	–0.50
November 30	6	51	47	0	0.081	45	30	0	0.50

Recall that the arithmetic mean return on investment for fund A is 0.0156:

$$ROI_A = \frac{\sum_{t=1}^{n} r_t}{n} = \frac{0.10 - 0.091 + 0.08 - 0.092 + 0.081}{5} = 0.0156.$$

The arithmetic mean return on investment on fund B might be computed as follows:

$$ROI_A = \frac{\sum_{t=1}^{n} r_t}{n} = \frac{0.60 - 0.50 + 0.50 - 0.50 + 0.50}{5} = 0.12.$$

This computational procedure is obviously quite misleading. Fund B paid no dividends and ended the five-year period worth \$5 less than at the beginning of the five-year period. Despite the fact that fund B obviously lost money, its arithmetic mean return was computed to be 0.12 over the five years. Fund A clearly outperformed fund B, yet its arithmetic mean return appears to be lower. Alternatively, one could compute the arithmetic mean rate of return for fund B using the following:

$$ROI_A = \frac{P_n - P_0}{n \cdot P_0} = \frac{45 - 50}{5 \cdot 50} = -0.02.$$

While this seems more intuitively acceptable, the return for fund A cannot be computed in exactly the same way. However, the geometric mean return on investment can be computed for both funds:

$$ROI_g = \sqrt[n]{\prod_{t=1}^{n} (1 + r_t)} - 1 = \sqrt[5]{\prod_{t=1}^{5} (1 + r_t)} - 1,$$

$$ROI_g \text{ for A} = \sqrt[5]{(1 + 0.10)(1 - 0.091)(1 + 0.08)(1 - 0.092)(1 + 0.081)} - 1 = 0.0117,$$

$$ROI_g \text{ for B} = \sqrt[5]{(1 + 0.60)(1 - 0.5)(1 + 0.5)(1 - 0.5)(1 + 0.5)} - 1 = -0.0208.$$

With the geometric mean return computation, changes in the funds' investment bases are accounted for. That is, as the values of the funds change, the amounts on which returns are computed vary. Hence, the geometric mean rate of return can account for compounding and the comparison of fund returns is more meaningful.

5.3 Internal Rate of Return
(Background reading: sections 4.7, 4.8, and 4.9)

The primary strength of the internal rate of return (*IRR*) as a measure of the economic efficiency of an investment is that it accounts for the timeliness of all cash flows generated by that investment. The *IRR* of an investment is calculated by solving for *r* in the present-value series model discussed in section 4.8:

$$NPV = 0 = \sum_{t=0}^{n} \frac{CF_t}{(1+r)^t} = -P_0 + \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t}, \tag{5.7}$$

where net present value NPV is the present value of the series net of the initial cash outlay, and r is the return or discount rate that sets the investment's NPV equal to zero. The investment's internal rate of return is that value for r that equates NPV with zero.

There exists no general closed-form solution for r in equation (5.7). This means that there is no general equation that enables us to set aside r on one side of equation (5.7) and other terms on the other side. Therefore, we must substitute values for r until we find one that sets NPV equal to zero. Often, this substitution process is very time-consuming, but with experience calculating internal rates of return, one can find shortcuts to solutions in various types of problems. Perhaps the most important shortcut will be to find an easy method for deriving an initial value to substitute for r, resulting in an NPV fairly close to zero. One easy method for generating an initial value to substitute for r is by first calculating the investment's return on investment. Consider again the stock from section 5.1, whose purchase price three years ago was \$100, paid a dividend of \$10 in each of the three years, and was sold for \$130. If an investor wanted to calculate the internal rate of return for this stock, she should substitute for r in the following:

$$0 = NPV = \sum_{t=0}^{n} \frac{CF_t}{(1+r)^t} = \frac{-100}{(1+r)^0} + \frac{10}{(1+r)^1} + \frac{10}{(1+r)^2} + \frac{130+10}{(1+r)^3}.$$

Since we found the arithmetic mean return on investment to be 0.20 in section 5.1, we may try this value as our initial trial solution for r:

$$NPV = \sum_{t=0}^{n} \frac{CF_t}{(1+0.2)^t} = \frac{-100}{(1+0.2)^0} + \frac{10}{(1+0.2)^1} + \frac{10}{(1+0.2)^2} + \frac{140}{(1+0.2)^3} = -3.7.$$

Since this NPV is less than zero, a smaller r value should be substituted. A smaller r value will decrease the right-hand side denominators, increasing the size of the fractions and NPV. Perhaps a feasible value to substitute for r is 10%. The same calculations will be repeated with the new r value of 10%:

$$NPV = \sum_{t=0}^{n} \frac{CF_t}{(1+0.1)^t} = \frac{-100}{(1+0.1)^0} + \frac{10}{(1+0.1)^1} + \frac{10}{(1+0.1)^2} + \frac{140}{(1+0.1)^3} = 22.54.$$

Since this new NPV exceeds zero, the r value of 10% is too small. However, because −3.7 is closer to zero than 22.54, the next value to substitute for r might be closer to 20% than to 10%. Perhaps a better estimate for the IRR will be 18%. Substituting this value for r results in an NPV of 0.86:

$$NPV = \sum_{t=0}^{n} \frac{CF_t}{(1+0.1)^t} = \frac{-100}{(1+0.18)^0} + \frac{10}{(1+0.18)^1} + \frac{10}{(1+0.18)^2} + \frac{140}{(1+0.18)^3} = 0.86.$$

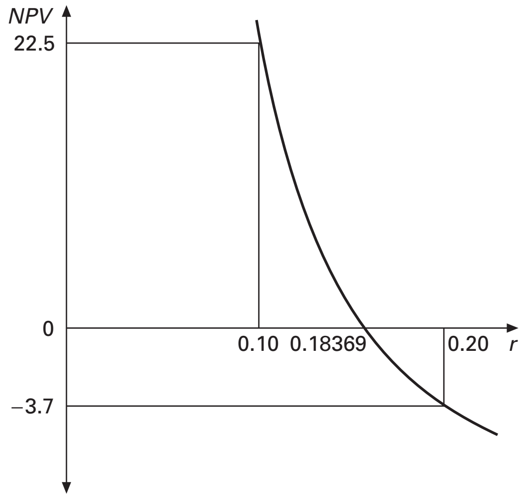

Figure 5.1 The relationship between *NPV* and *r*.

This *NPV* is quite close to zero; in fact, further substitutions will indicate that the true stock internal rate of return is approximately 18.369%. These iterations have a pattern: when *NPV* is less than zero, decrease *r* for the next substitution; when *NPV* exceeds zero, increase *r* for the next substitution. This process of iterations need only be repeated until the desired accuracy of calculations is reached. Figure 5.1 depicts the relationship between *NPV* and *r*.

The primary advantage of the internal rate of return over the return on investment measures is that it accounts for the timeliness of all cash flows generated by that investment. However, *IRR* does have three major weaknesses:

1 As we have seen, *IRR* takes considerably longer to calculate than does *ROI*. Therefore, if ease of calculation is of primary importance in a situation, the investor may prefer to use *ROI* as his measure of efficiency. Of course, calculators, spreadsheets, and other computer programs will compute *IRR* very quickly.
2 Sometimes an investment will generate multiple rates of return; that is, more than one *r* value will equate *NPV* with zero. This will occur when that investment has associated with it more than one negative cash flow. When multiple rates are generated, there is often no method to determine which is the true *IRR*. In fact, none of the rates generated may make any sense. When the *IRR* is infeasible as a method for comparing two investments, and the investor still wishes to consider the time value of money in his calculations, he may simply compare the present values of the investments. This approach and its weaknesses will be discussed in later chapters.
3 The internal rate of return is based on the assumption that cash flows received prior to the expiration of the investment will be reinvested at the internal rate of return. That is, it is assumed that future investment rates are constant and equal to the *IRR*. Obviously, this assumption may not hold in reality.

5.4 Bond Yields

By convention, rates of return on bonds are often expressed in terms that are somewhat different from those of other investments. For example, the coupon rate of a bond is the annual interest payment associated with the bond divided by the bond's face value. Thus, a four-year \$1,000 corporate bond making \$60 annual interest payments has a coupon rate of 6%. However, the coupon rate does not account for the actual purchase price of the bond. Corporate bonds are usually traded at prices that differ from their face values. The bond's current yield accounts for the actual purchase price of the bond:

$$cy = \frac{INT}{P_0}. \tag{5.8}$$

If the 6% bond described above were purchased for \$800, its current yield would be 7.5%:

$$cy = \frac{60}{800} = 0.075.$$

The formula for current yield, while easy to work with, does not account for any capital gains (or losses) that may be realized when the bond matures. Furthermore, current yields do not account for the timeliness of cash flows associated with bonds. The bond's yield to maturity, which is essentially its internal rate of return, does account for any capital gains (or losses) that may be realized at maturity in addition to the timeliness of all associated cash flows:

$$NPV = 0 = \sum_{t=0}^{n} \frac{CF_t}{(1+y)^t} = -P_0 + \left[\sum_{t=1}^{n} \frac{INT}{(1+y)^t}\right] + \frac{F}{(1+y)^n}. \tag{5.9}$$

The yield to maturity (y) of the bond described above would be 12.679% . The value of y that sets the bond's NPV equal to zero is the bond's internal rate of return. If the bond makes semiannual interest payments, its yield to maturity can be more accurately expressed as

$$NPV = 0 = -P_0 + \left[\sum_{t=1}^{2n} \frac{INT/2}{\left(1+\frac{y}{2}\right)^t}\right] + \frac{F}{\left(1+\frac{y}{2}\right)^{2n}}. \tag{5.10}$$

Here, we are concerned with semiannual interest payments equal to half the annual sum and $2 \cdot n$ six-month time periods, where n is the number of years to the bond's maturity. The yield to maturity of the bond described above making semiannual payments equal to \$30 for eight six-month periods would equal 0.125.

5.5 An Introduction to Risk

When individuals and firms invest, they become subjected to at least some level of uncertainty with respect to future cash flows and returns. Investors and firm managers cannot know with certainty what investment payoffs will be. This chapter is concerned with forecasting investment payoffs and returns and the uncertainty associated with these forecasts. We will define expected return in this chapter, focusing on it as a return forecast. This expected return will be expressed as a function of the investment's potential return outcomes and the probabilities associated with these potential outcomes. The riskiness of an investment is simply the potential for deviation from the investment's expected return. The *risk of an investment is defined here as the uncertainty* associated with returns on that investment. Although other definitions for risk, such as the probability of losing money or going bankrupt, can be very useful, they are often less complete or more difficult to measure. Our definition of risk does have some drawbacks as well. For example, an investment which is certain to be a complete loss is not regarded here to be risky, since its return is known to be −100% (although we note that it would not be regarded to be a particularly good investment).

In addition to providing an introduction to investment risk, the remainder of this chapter will provide introductions to important statistical measures of expected value, variability and co-movement. Each of these statistical measures is very important in finance.

5.6 Expected Return
(Background reading: sections 2.8 and 5.5)

Consider an economy with three potential states of nature in the next year and stock A, whose return is dependent on these states. If the economy performs well, state one is realized and the stock earns a return of 25%. If the economy performs only satisfactorily, state two is realized and the stock earns a return of 10%. If the economy performs poorly, state three is realized and the stock achieves a return of −10%. Thus, there are three potential return outcomes for the stock; the stock will earn either 25%, 10%, or −10%. Only one of these states may occur and we cannot know in advance which will occur. However, assume that there, is a 20% chance that state one will occur, a 50% chance that state two will occur, and a 30% chance that state three will occur. The expected return on the stock will be 7%, determined by:

$$\mathrm{E}[R_{\mathrm{A}}] = \sum_{i=1}^{n} R_{\mathrm{A},i} P_i, \tag{5.11}$$

$$\mathrm{E}[R_{\mathrm{A}}] = (R_{\mathrm{A},1} \cdot P_1) + (R_{\mathrm{A},2} \cdot P_2) + (R_{\mathrm{A},3} \cdot P_3),$$

$$\mathrm{E}[R_{\mathrm{A}}] = (0.25 \cdot 0.20) + (0.10 \cdot 0.50) + (-0.10 \cdot 0.30) = 0.07,$$

where $R_{\mathrm{A},i}$ is return outcome i for stock A and P_i is the probability associated with that outcome. Therefore, our forecasted return is 7%. The expected return considers all potential returns and weights more heavily those returns that are more likely to actually occur. Although our forecasted return level is 7%, it is obvious that there is potential for the actual return outcome to deviate from this figure. This potential for deviation (variation) will be measured in the following section.

5.7 Variance and Standard Deviation

The statistical concept of variance is a useful measure of risk. Variance accounts for the likelihood that the actual return outcome will vary from its expected value; furthermore, it accounts for the magnitude of the difference between potential return outcomes and the expected return. Variance can be computed with

$$\sigma^2 = \sum_{i=1}^{n} (R_i - \mathrm{E}[R])^2 P_i. \tag{5.12}$$

Table 5.1 lists potential return outcomes for the stock discussed in section 5.5. The variance of stock returns presented in section 5.5 is 0.0156, computed below and in table 5.1:

$$\sigma^2 = (0.25 - 0.07)^2 \cdot 0.2 + (0.10 - 0.07)^2 \cdot 0.5 + (-0.10 - 0.07)^2 \cdot 0.3 = 0.0156.$$

The statistical concept of standard deviation is also a useful measure of risk. The standard deviation of a stock's returns is simply the square root of its variance:

$$\sigma = \sqrt{\sum_{i=1}^{n} (R_i - \mathrm{E}[R])^2 P_i}. \tag{5.13}$$

Thus, the standard deviation of returns on the stock in table 5.1 is 12.49%.

Consider a second security, stock B, whose return outcomes are also dependent on economy outcomes one, two, and three. If outcome one is realized, stock B attains a return of 45%; in outcomes two and three, the stock attains returns of 5% and −15%, respectively. From table 5.2, we see that the expected return on stock B is 7%, the same as for stock A. However, the actual return outcome of stock B is subject to more

Table 5.1 Expected return, variance, and standard deviation of returns for stock A

i	R_i	P_i	R_iP_i	$R_i - \mathrm{E}[R_a]$	$(R_i - \mathrm{E}[R_a])^2$	$(R_i - \mathrm{E}[R_a])^2P_i$
1	0.25	0.20	0.05	0.18	0.0324	0.00648
2	0.10	0.50	0.05	0.03	0.0009	0.00045
3	−0.10	0.30	−0.03	−0.17	0.0289	0.00867
			$\mathrm{E}[R_a] = 0.07$			$\sigma_a^2 = 0.01560$
						$\sigma_a = 0.1249$

Table 5.2 Expected return, variance, and standard deviation of returns for stock B

i	R_i	P_i	R_iP_i	$R_i - \text{E}[R_b]$	$(R_i - \text{E}[R_b])^2$	$(R_i - \text{E}[R_b])^2P_i$
1	0.45	0.20	0.09	0.38	0.1444	0.02888
2	0.05	0.50	0.025	0.02	0.0004	0.00020
3	–0.15	0.30	–0.045	0.22	0.0484	0.01452
			$\text{E}[R_b] = 0.070$			$\sigma_b^2 = 0.04360$
						$\sigma_b = 0.2088$

uncertainty. Stock B has the potential of receiving either a much higher or much a lower actual return than does stock A. For example, an investment in stock B could lose as much as 15%, whereas an equal investment in stock A cannot lose more than 10%. An investment in stock B also has the potential of attaining a much higher return than an identical investment in stock A. Therefore, returns on stock B are subject to greater variability (or risk) than returns on stock A. The concept of variance (or standard deviation) accounts for this increased variability. The variance of stock B (0.0436) exceeds that of stock A (0.0156), indicating that stock B is riskier than stock A.

Suppose that a stock has a very large or infinite number of potential return outcomes. A function or curve representing a distribution of values may be used to find the probability that the actual return will fall within a specified range. One of the most commonly used probability distributions in finance is the normal distribution. The curve representing the normal distribution is continuous (any fractional value may be selected), symmetric and ranges from $-\infty$ to $+\infty$. The normal curve (representing the normal distribution) is "bell-shaped" (see the z-table on page 266 of appendix B). The normal curve may be used to find the probability that a randomly selected data point from a data set which is approximately normally distributed will fall within a specified range. For example, suppose that we are considering the purchase of a stock whose monthly return is approximately normally distributed with an expected return of 0.01 and a standard deviation of 0.02. We may use the normal curve to find the probability that the stock's return in a given month falls within a given range or exceeds any given value. Suppose that we wish to compute the probability that the stock's return is positive ($\Pr[R_i > 0]$) in a given month. First, we define a normal deviate for outcome i (in this case zero) as follows:

$$z_i = \frac{R_i - \text{E}[R]}{\sigma_R},$$

$$z_i = \frac{0 - 0.01}{0.02} = -0.5. \tag{5.14}$$

The normal deviate for a zero return equals –0.5; it is one-half of a standard deviation less than the expected value associated with the distribution. Determining the probability that the stock's return exceeds zero is identical to determining the probability that the normal deviate for the stock's returns exceeds –0.5; $\Pr[R_i > 0] = \Pr[z_i > -0.5]$. Our next step is to find the value on a z-table corresponding to 0.5. By matching the appropriate row and column, we find this to be 0.1915. We actually want the value corresponding with −0.5 (which does not appear on the table), so we take advantage of the symmetry characteristic of the normal curve and associate a value of −0.1915 with our normal deviate of −0.5. We then find the probability that $R_i > 0$ and $z_i > -0.5$ by adding 50% (the area to the right of the mean on the normal curve) to −0.1915. Thus the probability that $z_i > -0.5$ or that the stock return will exceed zero is found to be 0.3085.

Consider another stock with normally distributed returns with an expected level of 7% and a standard deviation of 10%. From the z-table in appendix B, we see that there is a 68% probability that the actual return outcome on this stock will fall between −0.03 and 0.17, one standard deviation less than or greater than the mean value of the distribution:

$$\mathrm{E}[R] - 1 \cdot \sigma < R_i < \mathrm{E}[R] + 1 \cdot \sigma,$$
$$0.07 - 0.10 < R_i < 0.07 + 0.10.$$

We find in the z-table in appendix B that the z-value corresponding to a normal deviate equal to one is approximately 0.34. Thus, the probability that the stock's return will fall between the mean and one standard deviation greater than the mean equals 0.34. Using the symmetry characteristic of the normal distribution, we find that the probability that the stock's return will be within one standard deviation of its expected value equals two times 0.34, or 68%. A similar analysis indicates a 95% probability that the actual return outcome will fall between −0.13 and 0.27, two standard deviations from the expected value:

$$\mathrm{E}[R] - 2 \cdot \sigma < R_i < \mathrm{E}[R] + 2 \cdot \sigma,$$
$$0.07 - 0.20 < R_i < 0.07 + 0.20.$$

Obviously, a smaller standard deviation of returns will lead to a narrower range of potential outcomes, given any level of probability. If a security has a standard deviation of returns equal to zero, it has no risk. Such a security is referred to as the risk-free security with a return of r_f. Therefore, the only potential return level of the risk-free security is r_f. No such security exists in reality; however, short-term United States Treasury bills are quite close. The U.S. government has proven to be an extremely reliable debtor. When investors purchase Treasury bills and hold them to maturity, they do receive their expected returns. Therefore, short-term Treasury bills are probably the safest of all securities. For this reason, financial analysts often use the Treasury bill rate (of return) as their estimate for r_f in many important calculations.

5.8 Historical Variance and Standard Deviation

Empirical evidence suggests that historical stock return variances (standard deviations) are excellent indicators of future variances (standard deviations). That is, a stock whose previous returns have been subject to substantial variability probably will continue to realize returns of a highly volatile nature. Therefore, past riskiness is often a good indicator of future riskiness. A stock's historical return variability can be measured with a historical population variance:

$$\sigma^2 = \sum_{t=1}^{n} (R_t - \bar{R})^2 \frac{1}{n}, \tag{5.15}$$

where R_t is the stock return in time t and R is the historical average return over the n time periods. The variance of a sample drawn from a population of potential returns is computed as

$$\sigma^2 = \sum_{t=1}^{n} (R_t - \bar{R})^2 \frac{1}{n-1}. \tag{5.16}$$

The stock's historical standard deviation of returns is simply the square root of its variance. If an investor determines that the historical variance is a good indicator of its future variance, he may need not to calculate potential future returns and their associated probabilities for risk estimates; he may prefer to simply measure the stock's riskiness with its historical variance or standard deviation. The historical standard deviation for a given population (the denominator is the population size) or sample (the denominator is one less than the sample size) may be computed as follows:

$$\sigma_{\mathrm{H}} = \sqrt{\frac{\sum_{t=1}^{n} (R_i - \bar{R})^2}{n}}, \qquad \sigma_{\mathrm{H}} = \sqrt{\frac{\sum_{t=1}^{n} (R_i - \bar{R})^2}{n-1}}. \tag{5.17}$$

Table 5.3 demonstrates historical variance and standard deviation computations for stock D.

Table 5.3 Historical variance and standard deviation of returns of stock D

t	R_t	$R_t - \bar{R}_d$	$(R_t - \bar{R}_d)^2$	$(R_t - \bar{R}_d)^2\ 1/n$
1	0.10	–0.06	0.0036	0.00072
2	0.15	–0.01	0.0001	0.00002
3	0.20	0.04	0.0016	0.00032
4	0.10	–0.06	0.0036	0.00072
5	0.25	0.09	0.0081	0.00162
	$\bar{R}_d = 0.16$			$\sigma_d^2 = 0.00310$
				$\sigma_d = 0.05568$

5.9 COVARIANCE

Standard deviation and variance provide us with measures of the absolute risk levels of securities. However, in many instances, it is useful to measure the risk of one security relative to the risk of another or relative to the market as a whole. The concept of covariance is integral to the development of relative risk measures. Covariance provides us with a measure of the relationship between the returns of two securities. That is, given that two securities' returns are likely to vary, covariance indicates whether they will vary in the same direction or in opposite directions. The likelihood that two securities will covary in the same direction (or, more accurately, the strength of the relationship between returns on two securities) is measured by

$$\sigma_{k,j} = \sum_{i=1}^{n} (R_{k,i} - \mathrm{E}[R_k])(R_{j,i} - \mathrm{E}[R_j])P_i, \tag{5.18}$$

where R_{ki} and R_{ji} are the return of stocks k and j if outcome i is realized, and P_i is the probability of outcome i. $\mathrm{E}[R_k]$ and $\mathrm{E}[R_j]$ are simply the expected returns of securities k and j. For example, the covariance between returns of stocks A and B is

$$\begin{aligned} \mathrm{cov}(A,B) = {} & \{(0.25 - 0.07) \cdot (0.45 - 0.07) \cdot 0.20\} \\ & + \{(0.10 - 0.07) \cdot (0.05 - 0.07) \cdot 0.50\} \\ & + \{(-0.10 - 0.07) \cdot (-0.15 - 0.07) \cdot 0.30\} \\ = {} & \{0.01368\} + \{-0.0003\} + \{0.01122\} = 0.0246. \end{aligned}$$

Since this covariance is positive, the relationship between return variability on these two securities is positive. That is, the larger the positive value of covariance, the more likely it is one security will perform well given that the second will perform well. A negative covariance indicates that strong performance by one security implies likely poor performance by the second security. A covariance of zero implies that there is no relationship between returns on the two securities. Table 5.4 details the solution method for this example.

Empirical evidence suggests that historical covariances are strong indicators of future covariance levels. Thus, if an investor is unable to associate probabilities with

Table 5.4 Covariance between returns on stocks A and B

i	R_{ai}	R_{bi}	P_i	$R_{ai} - \mathrm{E}[R_a]$	$R_{bi} - \mathrm{E}[R_b]$	$(R_{ai} - \mathrm{E}[R_a])(R_{bi} - \mathrm{E}[R_b])P_i$
1	0.25	0.45	0.20	0.18	0.38	0.01368
2	0.10	0.05	0.50	0.03	−0.02	−0.00030
3	−0.10	−0.15	0.30	−0.17	−0.22	0.01122
						COV(A,B) = 0.0246

Table 5.5 Historical covariance between returns on stocks D and E

t	R_{dt}	R_{et}	$(R_{dt}-\bar{R}_d)$	$(R_{et}-\bar{R}_e)$	$(R_{dt}-\bar{R}_d)(R_{et}-\bar{R}_e)1/n$
1	0.10	0.15	−0.06	−0.05	0.00060
2	0.15	0.18	−0.01	−0.02	0.00004
3	0.20	0.25	0.04	0.05	0.00040
4	0.10	0.20	−0.06	0	0
5	0.25	0.22	0.09	0.02	0.00036
	$\bar{R}_d = 0.16$	$0.20 = \bar{R}_e$			COV(D,E) = 0.00140

potential outcome levels, in many cases he may use historical covariance as his estimate for future covariance. Table 5.5 demonstrates how to determine historical covariance for two hypothetical stocks D and E.

5.10 The Coefficient of Correlation and the Coefficient of Determination

The coefficient of correlation provides us with a means of standardizing the covariance between returns on two securities. For example, how large must covariance be to indicate a strong relationship between returns? Covariance will be smaller given low returns on the two securities than given high-security returns. The coefficient of correlation ρ_{kj} between returns on two securities will always fall between −1 and +1.[1] If security returns are directly related, the correlation coefficient will be positive. If the two security returns always covary in the same direction by the same proportions, the coefficient of correlation will equal one. If the two security returns always covary in opposite directions by the same proportions, $\rho_{k,j}$ will equal −1. The stronger the inverse relationship between returns on the two securities, the closer $\rho_{k,j}$ will be to −1. If $\rho_{k,j}$ equals zero, there is no relationship between returns on the two securities. The coefficient of correlation $\rho_{k,j}$ between returns is simply the covariance between returns on the two securities divided by the product of their standard deviations:

$$\rho_{k,j} = \frac{\mathrm{COV}(k,j)}{\sigma_k \cdot \sigma_j}. \tag{5.19}$$

Equation (5.19) implies that the covariance formula can be rewritten as

$$\mathrm{COV}(k,j) = \sigma_k \sigma_j \rho_{k,j}. \tag{5.20}$$

[1] Many statistics textbooks use the notation $r_{i,j}$ to designate the correlation coefficient between variables i and j. Because the letter r is used in this text to designate return, it will use the lower-case rho, ρ_{ij}, to designate the correlation coefficient.

If an investor can access only raw data pertaining to security returns, he might first find security covariances then divide by the products of their standard deviations to find correlation coefficients. However, if for some reason the investor knows the correlation coefficients between returns on securities, he can use this value along with standard deviations to find covariances.

The coefficient of correlation between returns on stocks A and B is 0.94:

$$\rho_{a,b} = \frac{0.0246}{0.1249 \cdot 0.21} = 0.94.$$

This value can be squared to determine the coefficient of determination between returns of the two securities. The coefficient of determination $\rho^2_{k,j}$ measures the proportion of variability in one security's returns that can be explained by or be associated with variability of returns on the second security. Thus, approximately 88% of the variability of stock A returns can be explained by or associated with variability of stock B returns. The concepts of covariance and correlation are crucial to the development of portfolio risk and relative risk models presented in later chapters.

Historical evidence suggests that covariances and correlations between stock returns remain relatively constant over time. Thus, an investor can use historical covariances and correlations as his forecasted values. However, it is important to realize that these historical relationships apply to standard deviations, variances, covariances, and correlations, but not to the actual returns themselves. That is, we can often forecast future risk levels and relationships on the basis of historical data, but we cannot forecast returns on the basis of historical returns. Thus, last year's return for a given stock implies almost nothing about next year's return for that stock. The historical covariance between returns on securities *i* and *j* can be found by solving equation (5.21) for a population or equation (5.22) for a sample:

$$\sigma_{i,j} = \sum_{t=1}^{n} (R_{i,t} - \bar{R}_i)(R_{j,t} - \bar{R}_j)\frac{1}{n}, \tag{5.21}$$

$$\sigma_{i,j} = \sum_{t=1}^{n} (R_{i,t} - \bar{R}_i)(R_{j,t} - \bar{R}_j)\frac{1}{n-2}. \tag{5.22}$$

Historical correlation coefficients may be computed from these covariance results.

EXERCISES

5.1. An investor purchased one share of Mathewson stock for $1,000 in 2002 and sold it exactly one year later for $1,200. Calculate the investor's arithmetic mean return on investment.

5.2. An investor purchased one share of Johnson Company stock in 1976 for $200 and sold it in 1983 for $400. Calculate the following for the investor:

(a) the arithmetic mean return on investment;
(b) the geometric mean return on investment;
(c) the internal rate of return.

5.3. An investor purchased 100 shares of Alexander Company stock for $75 apiece in 1975 and sold each share for $80 exactly six years later. The Alexander Company paid annual dividends of $8 per share in each of the six years the investor held the stock. Calculate the following for the investor:

(a) the arithmetic mean return on investment;
(b) the internal rate of return.

5.4. The Young Corporation is considering the purchase of a machine for $100,000, whose output will yield the company $20,000 in annual after-tax cash flows for each of the next five years. At the end of the fifth year, the machine will be sold for its $40,000 salvage value. Calculate the following for the machine that Young is considering purchasing:

(a) the arithmetic mean return on investment;
(b) the internal rate of return.

5.5. What is the net present value of an investment whose internal rate of return equals its discount rate?

5.6. An investor purchased 100 shares each of Grove Company stock and Dean Company stock for $10 per share. The Grove Company paid an annual dividend of one dollar per share in each of the eight years during which the investor held the stock. The Dean Company paid an annual dividend of $0.25 per share in each of the eight years during which the investor held the stock. At the end of the eight-year period, the investor sold each of his shares of Grove Company stock for $11 and sold each of his shares of Dean Company stock for $18.

(a) Calculate the sum of dividends received by the investor from each of the companies.
(b) Calculate the capital gains realized on the sale of stock of each of the companies.
(c) Calculate the return on investment for each of the two companies' stock using an arithmetic mean return.
(d) Calculate the internal rate of return for each of the two stocks.
(e) Which of the two stocks performed better during their holding periods?

5.7. The Feller Company is considering the purchase of an investment for \$100,000 that is expected to pay off \$50,000 in two years, \$75,000 in four years, and \$75,000 in six years. In the third year, Feller must make an additional payment of \$50,000 to sustain the investment. Calculate the following for the Feller investment:

(a) Return on investment, using an arithmetic mean return.
(b) The investment internal rate of return.
(c) Describe any complications you encountered in part (b).

5.8. A \$1,000 face value bond is currently selling at a premium for \$1,200. The coupon rate of this bond is 12% and it matures in three years. Calculate the following for this bond, assuming that its interest payments are made annually:

(a) its annual interest payments;
(b) its current yield;
(c) its yield to maturity.

5.9. Work through each of the calculations in problem 5.8 assuming that interest payments are made semiannually.

5.10* The Radbourne Company invested \$100,000 into a small business 20 years ago. Its investment generated a cash flow equal to \$3,000 in its first year of operation. Each subsequent year, the business generated a cash flow which was 10% larger than in the prior year; that is, the business generated a cash flow equal to \$3,300 in the second year, \$3,630 in the third year, and so on for 19 years after the first. The Radbourne Company sold the business for \$500,000 after its twentieth year of operation. What was the internal rate of return for this investment?

5.11. Megabyte Products management is considering the investment in one of two projects available to the company. The returns on the two projects, A and B, are dependent on the sales outcome of the company. Megabyte management has determined three potential sales outcomes for the company. The highest potential sales outcome for Megabyte is outcome 1 or \$800,000. If this sales outcome were realized, project A would realize a return outcome of 30%; project B would realize a return of 20%. If outcome 2 were realized, the company's sales level would be \$500,000. In this case, project A would yield 15%, and project B would yield 13%. The worst outcome 3 will result in a sales level of \$400,000, and return levels for projects A and B of 1% and 9% respectively. If each sales outcome has an equal probability of occurring, determine the following for the Megabyte Company:

(a) the probabilities of outcomes 1, 2, and 3;
(b) its expected sales level;

(c) the variance associated with potential sales levels;
(d) the expected return of project A;
(e) the variance of potential returns for project A;
(f) the expected return and variance for project B;
(g) standard deviations associated with company sales, returns on project A and returns on project B;
(h) the covariance between company sales and returns on project A;
(i) the coefficient of correlation between company sales and returns on project A;
(j) the coefficient of correlation between company sales and returns on project B;
(k) the coefficient of determination between company sales and returns on project B.

5.12. Which of the projects in problem 5.11 represents the better investment for Megabyte Products?

5.13. The following table provides historical *percentage* returns for the Patterson and Liston Companies along with percentage returns on the market portfolio (index or fund):

Year	Patterson	Liston	Market
1998	4	19	15
1999	7	4	10
2000	11	−4	3
2001	4	21	12
2002	5	13	9

Calculate the following based on the preceding table:

(a) mean historical returns for the two companies and the market portfolio;
(b) variances associated with Patterson Company returns and Liston Company returns along with returns on the market portfolio;
(c) the historical covariance and coefficient of correlation between returns of the two securities;
(d) the historical covariance and coefficient of correlation between returns of the Patterson Company and returns on the market portfolio;

(e) the historical covariance and coefficient of correlation between returns of the Liston Company and returns on the market portfolio.

5.14. Project the following for both the Patterson and Liston Companies using your results from problem 5.13:

(a) the variance and standard deviation of returns;
(b) the coefficient of correlation between each of the companies' returns and returns on the market portfolio.

5.15. Windsor Company management projects a return level of 15% for the upcoming year. Management is uncertain as to what the actual sales level will be; therefore, it associates a standard deviation of 10% with this sales level. Managers assume that sales will be normally distributed. What is the probability that the actual return level will:

(a) fall between 5% and 25%?
(b) fall between 15% and 25%?
(c) exceed 25%?
(d) exceed 30%?

5.16. What would be each of the probabilities in problem 5.15 if Windsor Company management were certain enough of its forecast to associate a 5% standard deviation with its sales projection?

5.17. An investor has the opportunity to purchase a risk-free Treasury bill yielding a return of 10%. He also has the opportunity to purchase a stock which will yield either 7% or 17%. Either outcome is equally likely to occur. Compute the following:

(a) the variance of returns on the stock;
(b) the coefficient of correlation between returns on the stock and returns on the Treasury bill.

APPENDIX 5.A RETURN AND RISK SPREADSHEET APPLICATIONS

Table 5.A.1 contains spreadsheet entries for computing stock variances, standard deviations, and covariances. The table lists prices for stocks X, Y, and Z from January 9 to January 20 in cells B3:B14, E3:E14, and H3:H14. From these prices, we compute returns in cells B19:B29, E19:E29, and H19:H29.

Formulas for computing returns are given in rows 19–29 in table 5.A.2. Means, variances, standard deviations, covariances, and correlation coefficients are computed in

Table 5.A.1 Stock prices, returns, risk, and co-movement

	A	B	C	D	E	F	G	H
1	CORP. X			CORP. Y			CORP. Z	
2	DATE	PRICE		DATE	PRICE		DATE	PRICE
3	9-Jan	50.125		9-Jan	20		9-Jan	60.375
4	10-Jan	50.125		10-Jan	20		10-Jan	60.5
5	11-Jan	50.25		11-Jan	20.125		11-Jan	60.25
6	12-Jan	50.25		12-Jan	20.25		12-Jan	60.125
7	13-Jan	50.375		13-Jan	20.375		13-Jan	60
8	14-Jan	50.25		14-Jan	20.375		14-Jan	60.125
9	15-Jan	50.25		15-Jan	21.375		15-Jan	62.625
10	16-Jan	52.375		16-Jan	21.25		16-Jan	69.75
11	17-Jan	52.25		17-Jan	21.375		17-Jan	60.75
12	18-Jan	52.375		18-Jan	21.5		18-Jan	60.875
13	19-Jan	52.5		19-Jan	21.375		19-Jan	60.875
14	20-Jan	52.375		20-Jan	21.5		20-Jan	60.875
15								
16	CORP. X			CORP. Y			CORP. Z	
17	DATE	RETURN		DATE	RETURN		DATE	RETURN
18	9-Jan	N/A		9-Jan	N/A		9-Jan	N/A
19	10-Jan	0		10-Jan	0		10-Jan	0.00207
20	11-Jan	0.002494		11-Jan	0.00625		11-Jan	–0.00413
21	12-Jan	0		12-Jan	0.006211		12-Jan	–0.00207
22	13-Jan	0.002488		13-Jan	0.006173		13-Jan	–0.00208
23	14-Jan	–0.00248		14-Jan	0		14-Jan	0.002083
24	15-Jan	0.039801		15-Jan	0.04908		15-Jan	0.04158
25	16-Jan	0.002392		16-Jan	–0.00585		16-Jan	–0.02994
26	17-Jan	–0.00239		17-Jan	0.005882		17-Jan	0
27	18-Jan	0.002392		18-Jan	0.005848		18-Jan	0.002058
28	19-Jan	0.002387		19-Jan	–0.00581		19-Jan	0
29	20-Jan	–0.00238		20-Jan	0.005848		20-Jan	0
30	Mean	0.004064		Mean	0.006694		Mean	0.00087
31	Variance	0.000145		Variance	0.00022		Variance	0.000266
32	Variance (P)	0.000132		Variance (P)	0.0002		Variance (P)	0.000241
33	St.D.	0.01204		St.D.	0.014842		St.D.	0.016296
34	St.D. (P)	0.011479		St.D. (P)	0.014151		St.D. (P)	0.015538
35		COV(X,Y)=		0.0001494	COV(Y,Z)=		0.000192	
36		COV(X,Z)=		0.000139				
37		CORR(X,Y)=		0.9196541	CORR(Y,Z)=		0.8733657	
38		CORR(X,Z)=		0.7791748				

Table 5.A.2 Stock prices, returns, risk, and co-movement: formula entries

	A	B	C	D	E	F	G	H
16	CORP. X			CORP. Y			CORP. Z	
17	DATE	RETURN		DATE	RETURN		DATE	RETURN
18	9-Jan	N/A		9-Jan	N/A		9-Jan	N/A
19	10-Jan	=B4/B3-1		10-Jan	=E4/E3-1		10-Jan	=H4/H3-1
20	11-Jan	=B5/B4-1		11-Jan	=E5/E4-1		11-Jan	=H5/H4-1
21	12-Jan	=B6/B5-1		12-Jan	=E6/E5-1		12-Jan	=H6/H5-1
22	13-Jan	=B7/B6-1		13-Jan	=E7/E6-1		13-Jan	=H7/H6-1
23	14-Jan	=B8/B7-1		14-Jan	=E8/E7-1		14-Jan	=H8/H7-1
24	15-Jan	=B9/B8-1		15-Jan	=E9/E8-1		15-Jan	=H9/H8-1
25	16-Jan	=B10/B9-1		16-Jan	=E10/E9-1		16-Jan	=H10/H9-1
26	17-Jan	=B11/B10-1		17-Jan	=E11/E10-1		17-Jan	=H11/H10-1
27	18-Jan	=B12/B11-1		18-Jan	=E12/E11-1		18-Jan	=H12/H11-1
28	19-Jan	=B13/B12-1		19-Jan	=E13/E12-1		19-Jan	=H13/H12-1
29	20-Jan	=B14/B13-1		20-Jan	=E14/E13-1		20-Jan	=H14/H13-1
30	Mean	=AVERAGE(B19:B29)		Mean	=AVERAGE(E19:E29)		Mean	=AVERAGE(H19:H29)
31	Variance	=VAR(B19:B29)		Variance	=VAR(E19:E29)		Variance	=VAR(H19:H29)
32	Variance (P)	=VARP(B19:B29)		Variance (P)	=VARP(E19:E29)		Variance (P)	=VARP(H19:H29)
33	St.D.	=STDEV(B19:B29)		St.D.	=STDEV(E19:E29)		St.D.	=STDEV(H19:H29)
34	St.D. (P)	=STDEVP(B19:B29)		St.D. (P)	=STDEVP(E19:E29)		St.D. (P)	=STDEVP(H19:H29)
35		COV(X,Y)=		=COVAR(B19:B29,E19:E29)	COV(Y,Z)=		=COVAR(E19:E29,H19:H29)	
36		COV(X,Z)=		=COVAR(B19:B29,H19:H29)				
37		CORR(X,Y)=		=CORREL(B19:B29,E19:E29)	CORR(Y,Z)=		=CORREL(E19:E29,H19:H29)	
38		CORR(X,Z)=		=CORREL(B19:B29,H19:H29)				

rows 30–38. Row 30 computes the arithmetic mean return for each of the three stocks. Table 5.A.2 lists formulas associated with the values in cells A30:H30. The =(Average) function may be typed in directly as listed in table 5.A.2, row 30, or obtained from the Paste Function button (f_x) menu under the Statistical sub-menu. Entry instructions are given in the dialogue box obtained when the Average function is selected. The variance formulas in row 31 are based on the Sample formula; the Variance (P) formulas in row 32 are based on the population formula. Standard deviation sample and population results are given in rows 33 and 34. Covariances and correlation coefficients are given in rows 35–38.