A P P E N D I X   A
SOLUTIONS TO EXERCISES

CHAPTER 2

2.1. (a) 25;   (e) 0.04;   (i) 2.2360;   (m) 11.1803;
     (b) 125;   (f) 0.008;   (j) 1.7099;   (n) 0.5848;
     (c) 625;   (g) 0.0016;   (k) 1.4953;   (o) 3.3437;
     (d) 625;   (h) 0.0016;   (l) 5;   (p) 11.1803.

2.2. (a) 1;   (e) add the exponents, $5^2 = 25$;
     (b) 1;   (f) add the exponents, $5^{-8/3} = 0.0136798$;
     (c) 1;   (g) add the exponents, $5^1 = 5$;
     (d) 1;   (h) add the exponents, $25^1 = 25$.

2.3. $FV = 50(1 + 0.3)^4 = 50(1.3)^4 = 50 \cdot 2.8561 = 142.805$.

2.4. $PV = 180(1 + 0.2)^{-0.5} = 180(1.2)^{-0.5} = 180 \cdot 0.9128709 = 164.31677$.

2.5. (a) $y = 24x^3(12x) = 288x^4$;
     (b) $y = 7(-3x^2 + 3x) + 4x = -21x^2 + 21x + 4x = -21x^2 + 25x$;
     (c) $y = 25 + \{2x[3x + 4(x + 5)(x + 5) \cdot 6] - 5\} - 7x^2$
$$= 25 + \{2x[3x + 4(x^2 + 10x + 25) \cdot 6] - 5\} - 7x^2$$
$$= 25 + \{2x[3x + 24x^2 + 240x + 600] - 5\} - 7x^2$$
$$= 25 + \{6x^2 + 48x^3 + 480x^2 + 1{,}200x - 5\} - 7x^2 = 48x^3 + 479x^2 + 1{,}200x + 20.$$

2.6. (a) $y = 9(4(7 + 9)^2)^{0.5} + 216 \cdot 5(9) = 9(4(16)^2)^{0.5} + 9{,}720$
$$= 9(4 \cdot 256)^{0.5} + 9{,}720 = 9 \cdot 1{,}024^{0.5} + 9{,}720 = 9 \cdot 32 + 9{,}720 = 10{,}008;$$
     (b) $y = ((7 + 16)(27 \cdot 16 + 2)^3)^{-0.5} = (23(432 + 2)^3)^{-0.5} = (23 \cdot 434^3)^{-0.5}$
$$= (23 \cdot 81{,}746{,}504)^{-0.5} = 1{,}880{,}169{,}600^{-0.5} = 0.000023.$$

2.7. Simplify as follows:

$$PVA = \$250 \cdot \left[\frac{1}{0.08} - \frac{1}{0.08 \cdot (1 + 0.08)^7} \right] = \$250 \cdot \left[\frac{1}{0.08} - \frac{1}{0.08 \cdot (1.08)^7} \right]$$

$$= \$250 \cdot \left[\frac{1}{0.08} - \frac{1}{0.08 \cdot 1.7138243} \right] = \$250 \cdot \left[\frac{1}{0.08} - \frac{1}{0.1371059} \right]$$

$$= \$250 \cdot \left[\frac{1}{0.08} - 7.29363\right] = \$250 \cdot [12.5 - 7.29363]$$

$$= \$250 \cdot 5.20637 = \$1,301.59.$$

2.8. (a) −13.8155; (c) 0; (e) 0.99325; (g) 1.0986; (i) 4.60517;
     (b) −0.693147; (d) 0.09531; (f) 0.99989; (h) 2.3025; (j) 6.90775.

2.9. Assume for each of the accounts that the initial deposit is 1 for each account and its future
      value is 2. Then, the general procedure for each account is to solve the following for n
      given each interest rate i:

$$2 = 1(1 + i)^n,$$

$$\ln(2) = \ln(1) + n \cdot \ln(1 + i),$$

$$n = \frac{\ln(2) - \ln(1)}{\ln(1 + i)} = \frac{0.69314718 - 0}{\ln(1 + i)}.$$

(a) $0.69314718 \div \ln(1.03) = 23.44977$ years;
(b) $0.69314718 \div \ln(1.05) = 14.20699$ years;
(c) $0.69314718 \div \ln(1.07) = 10.24476$ years;
(d) $0.69314718 \div \ln(1.10) = 7.27254$ years;
(e) $0.69314718 \div \ln(1.12) = 6.11625$ years;
(f) $0.69314718 \div \ln(1.15) = 4.95948$ years.

2.10. (a) $72 \div 3 = 24$ years;
       (b) $72 \div 5 = 14.4$ years;
       (c) $72 \div 7 = 10.286$ years;
       (d) $72 \div 10 = 7.2$ years;
       (e) $72 \div 12 = 6$ years;
       (f) $72 \div 15 = 4.8$ years.

2.11. First, for sake of simplicity, assume PV to be 1 and FV_n to be 2:

$$2 = 1 \cdot e^{rn} = e^{rn}.$$

Now, find the natural logs of both sides:

$$\ln(2) = rn,$$
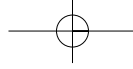
$$\ln(2) \approx 0.693 \approx 0.72 \approx rn,$$

$$\frac{0.72}{r} \approx n.$$

Thus, for example, if the interest rate equals 0.10, it will take approximately 7.2 years
for an account to double.

2.12. Our demonstration is as follows:

$$FV_n = PV \cdot e^{rn} = 1 \cdot e^{0.1 \cdot 10} = e.$$

2.13. (a) $X_1 = 15, X_2 = 25, X_3 = 35, X_4 = 45$;
       (b) $15 + 25 + 35 + 45 = 120$;
       (c) $3 \cdot 15 + 3 \cdot 25 + 3 \cdot 35 + 3 \cdot 45 = 3 \cdot 120 = 360$;
       (d) $3 \cdot 15^2 + 3 \cdot 25^2 + 3 \cdot 35^2 + 3 \cdot 45^2 = 3 \cdot 225 + 3 \cdot 625 + 3 \cdot 1225 + 3 \cdot 2025$
            $= 12,300$;

(e) $\prod_{t=1}^4 3X_t^2 = (3 \cdot 15^2) \cdot (3 \cdot 25^2) \cdot (3 \cdot 35^2) \cdot (3 \cdot 45^2)$
$= (3 \cdot 225) \cdot (3 \cdot 625) \cdot (3 \cdot 1225) \cdot (3 \cdot 2025) \approx 2.8255869 \cdot 10^{13}$
$= 28{,}255{,}869{,}000{,}000;$

(f) $\sum_{t=1}^4 X_t/n = (15 + 25 + 35 + 45)/4 = 120/4 = 30;$

(g) $(\prod_{t=1}^4 X_t)^{1/n} = (15 \cdot 25 \cdot 35 \cdot 45)^{1/4} = 27.722217.$

2.14. (a) $\sum_{i=1}^3 w_i = 0.4 + 0.5 + 0.1 = 1;$

(b) $\sum_{i=1}^3\sum_{j=1}^3 w_i w_j x_{i,j} = [0.4 \cdot 0.4 \cdot 7] + [0.4 \cdot 0.5 \cdot 4] + [0.4 \cdot 0.1 \cdot 9] + [0.5 \cdot 0.4 \cdot 6]$
$+ [0.5 \cdot 0.5 \cdot 4] + [0.5 \cdot 0.1 \cdot 12] + [0.1 \cdot 0.4 \cdot 3] + [0.1 \cdot 0.5 \cdot 2] + [0.1 \cdot 0.1 \cdot 17]$
$= 1.12 + 0.8 + 0.36 + 0.12 + 1 + 0.6 + 1.2 + 0.1 + 0.17 = 5.47.$

2.15. The three-year rate is based on a geometric mean of the short-term spot rates as follows:

$$y_{0,n} = \sqrt[n]{\prod_{t=1}^n (1 + y_{t-1,t})} - 1 = \sqrt[3]{(1 + y_{0,1})(1 + y_{1,2})(1 + y_{2,3})} - 1$$

$$= \sqrt[3]{(1 + 0.05)(1 + 0.06)(1 + 0.07)} - 1 = 0.05997.$$

2.16. $FV = \sum_{t=1}^4 25(1 + 0.10)^t = 25(1.1)^1 + 25(1.1)^2 + 25(1.1)^3 + 25(1.1)^4$
$= 25(1.1) + 25(1.21) + 25(1.331) + 25(1.4641) = 127.6275.$

2.17. (a) $x = 8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40{,}320;$

(b) $x = 9! = 9 \cdot 8! = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 362{,}880;$

(c) $x = 0! = 1.$

2.18. (a) $(6 \cdot 5 \cdot 4 \cdot 3) = 360;$ (d) $(6 \cdot 5)/(2 \cdot 1) = 15;$

(b) $(4 \cdot 3 \cdot 2 \cdot 1) = 24;$ (e) $1;$

(c) $(8 \cdot 7) = 56;$ (f) $(8 \cdot 7)/(2 \cdot 1) = 28.$

Chapter 3

3.1. (a) $90 = 5x,\ 90/5 = x = 18;$

(b) $-2{,}500 = -2x,\ (-2{,}500)/(-2) = 1{,}250;$

(c) $85 = 8x,\ 85/8 = x = 10.625;$

(d) $50 = 60 + 5x,\ -10 = 5x,\ -10/5 = x = -2;$

(e) $20 - 10 - 25 = -15/x,\ -15 = -15/x,\ x = 1;$

(f) $25 - 100 = 3x - 2x + 5x,\ -75 = 6x,\ -75/6 = x = -12.5.$

3.2. (a) $100 \div 90 = (1 + x)^2,\ 1.1111111 = (1 + x)^2,\ (1.1111111)^{1/2} = 1 + x,$
$1.05409 = 1 + x,\ x = 0.05409;$

(b) $100 \div 90 = (1 + x)^3,\ 1.1111111 = (1 + x)^3,\ (1.1111111)^{1/3} = 1 + x,$
$1.03574 = 1 + x,\ x = 0.03574;$

(c) $100 \div 90 = (1 + x)^{-2},\ 1.1111111 = (1 + x)^{-2},\ (1.1111111)^{-1/2} = 1 + x,$
$0.94868 = 1 + x,\ x = -0.051316;$

(d) $90 \div 100 = (1 + x)^{-2},\ 0.9 = (1 + x)^{-2},\ 0.9^{-1/2} = 1 + x,\ 1.05409 = 1 + x,\ x = 0.05409.$

3.3. (a) The value of \$1 is equal to the value of £0.6. Thus, the value of £1 must be
\$1/0.6 = \$1.6667. Since the value of \$1 is ¥108, the value of £1 must be
1.6667 · ¥108 = ¥180. Thus, we could have solved as follows: £1 = (\$1 ÷ 0.6) ·
¥108 = ¥180.

(b) The value of \$1 is equal to the value of ¥108. Thus, the value of ¥1 must be
\$1/¥108 = \$0.0092593. Since the value of £1 is \$1.6667, the value of ¥1 must

be $1 ÷ (1.6667 · ¥108) = £0.0055556. Thus, we could have solved as follows:
¥1 = $1 ÷ (1/0.6 · ¥108) = £0.0055556.
(c) $300 = £0.6 · 300 = £180. $300 = ¥108 · 300 = ¥32,400.

3.4. The firm's break-even production level is determined by solving for Q when profits π equal zero:

$$0 = 80Q - (500,000 + 50 \cdot Q).$$

First, we will add 500,000 to both sides:

$$500,000 = 80Q - 50Q.$$

Next, note that $80Q - 50Q = 30Q$, so that we will divide both sides of the above equation by 30 to obtain:

$$500,000 ÷ 30 = Q^* = 16,666.67,$$

where Q^* is the break-even production level (the asterisk does not represent a product symbol here). Thus, the firm must produce 16,666.67 units to recover its fixed costs in order to break even.

3.5. The three-year rate is based on a geometric mean of the short-term spot rates as follows:

$$(1 + y_{0,3})^3 = \prod_{t=1}^{3}(1 + y_{t-1,t}) = (1 + 0.05)(1 + 0.06)(1 + 0.07) = 1.19091,$$

$$y_{0,3} = [(1 + 0.05)(1 + 0.06)(1 + 0.07)]^{1/3} - 1 = \sqrt[3]{1.19091} - 1 = 0.0599686.$$

3.6. The three-year rate is based on a geometric mean of the short-term spot rates as follows:

$$(1 + y_{0,3})^3 = (1.07)^3 = 1.22504 = \prod_{t=1}^{3}(1 + y_{t-1,t}) = (1 + 0.05)(1 + 0.07)(1 + y_{2,3}).$$

We solve for $y_{2,3}$ as follows:

$$1.22504 ÷ [(1 + 0.05)(1 + 0.07)] - 1 = y_{2,3} = 0.07.$$

3.7. (a) The firm's profit function, total revenues minus total costs, is

$$\pi = 50Q - 0.00002Q^2 - (500,000 + 20Q + 0.00001Q^2),$$

which simplifies to

$$\pi = -0.00003Q^2 + 30Q - 500,000.$$

(b) Note that the terms are arranged in descending order of exponents for Q. Our coefficients for this quadratic equation are $a = -0.00003$, $b = 30$, and $c = -500,000$. We can solve for the break-even production level by setting π equal to zero, using the quadratic formula as follows:

$$Q = \frac{-30 \pm \sqrt{30^2 - 4 \cdot (-0.00003) \cdot (-500,000)}}{2 \cdot (-0.00003)} = \frac{-30 \pm \sqrt{900 - (60)}}{-0.00006}$$

$$= \frac{-30 \pm \sqrt{840}}{-0.00006} = \frac{-30 \pm 28.982753}{-0.00006} = \frac{-1.0172466}{-0.00006} \text{ and } \frac{-58.982753}{-0.00006}$$

$$= 16,954.11 \text{ and } 983,045.88 \text{ units.}$$
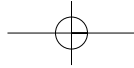
Either of the above production levels will enable the firm to break even with a profit level equal to zero.

3.8.   We need to solve this equation for w_1 using the quadratic formula. This is accomplished as follows:

$$w_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \qquad \text{where } a = 0.25, \, b = -0.3, \text{ and } c = 0.09.$$

We fill in our coefficients' values to determine the proportion of the investor's money to be invested in stock 1:

$$w_1 = \frac{0.3 \pm \sqrt{(-0.3)^2 - 4 \cdot 0.25 \cdot 0.09}}{2 \cdot 0.25} = \frac{0.3 \pm \sqrt{0}}{0.5} = 0.6.$$

The value under the square root sign (the radical) will be zero. Hence, there will be only one value for $w_1 = 0.6$. Therefore, we find that the portfolio is riskless when $w_1 = 0.6$. Thus, 60% of the riskless portfolio should be invested in stock 1 and 40% of the portfolio should be invested in stock 2.

3.9.   First, number the equations as follows:

$$0.04x_1 + 0.04x_2 = 0.01, \tag{A1}$$

$$0.04x_1 + 0.16x_2 = 0.11. \tag{A2}$$

Now, write equation (A1) in terms of x_1:

$$0.04x_1 = 0.01 - 0.04x_2,$$

$$x_1 = 0.01/0.04 - 0.04x_2/0.04,$$

$$x_1 = 0.25 - 1x_2.$$

Next, we substitute our revised version of x_1 back into equation (A2) to obtain the following:

$$0.11 = 0.04(0.25 - 1x_2) + 0.16x_2,$$

which simplifies to

$$0.10 = -0.12x_2. \tag{B2}$$

Thus, $x_2 = 0.83333$. Substitute x_2 back into either equation (A1) or equation (A2) and we find that $x_1 = -0.58333$.

3.10.

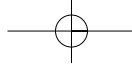|	
$20x - 5\lambda \quad = -3$	(1a)
$-5x - 0\lambda \quad = -100$	(2a)
$x = 20$	(2b) = (2a) ÷ −5
$20 \cdot 20 - 5\lambda = -3$	(2a)
$-5\lambda = -403$	
$\lambda = 403/5 = 80\frac{3}{5}, \, x = 20.$	

3.11.   First, number the equations as follows:

$$962 = 100x_1 + 100x_2 + 1{,}100x_3, \tag{A1}$$

$$1{,}010.4 = 120x_1 + 120x_2 + 1{,}120x_3, \tag{A2}$$

$$970 = 100x_1 + 1{,}100x_2. \tag{A3}$$

Now, write equation (A3) in terms of x_1:

$$100x_1 = 970 - 1{,}100x_2,$$

$$x_1 = 970/100 - 1{,}100x_2/100,$$

$$x_1 = 9.7 - 11x_2.$$

Next, we plug our revised version of x_1 back into equations (A1) and (A2) to obtain the following:

$$962 = 100(9.7 - 11x_2) + 100x_2 + 1{,}100x_3,$$

$$1{,}010.4 = 120(9.7 - 11x_2) + 120x_2 + 1{,}120x_3,$$

which simplifies to

$$-8 = -1{,}000x_2 + 1{,}100x_3, \qquad\qquad\qquad (B1)$$

$$-153.6 = -1{,}200x_2 + 1{,}120x_3. \qquad\qquad\qquad (B2)$$

Now solve equations (B1) and (B2) for x_3 by multiplying equation (B1) by 1.2 and subtracting the result from equation (B2):

$$
\begin{array}{lr}
-153.6 = -1{,}200x_2 + 1{,}120x_3 & (B2) \\
\underline{(-9.6 = -1{,}200x_2 + 1{,}320x_3)} & (B1) \cdot 1.2 \\
-144 = \qquad\qquad -200x_3 &
\end{array}
$$

Thus, $x_3 = 0.72$. Plug x_3 back into either equation (B1) or equation (B2) and we find that $x_2 = 0.8$. Now, substitute $x_3 = 0.72$ and $x_2 = 0.8$ back into any of equations (A1), (A2), or (A3) and we find that $x_1 = 0.9$. Thus, we obtain $x_1 = 0.9$, $x_2 = 0.8$, and $x_3 = 0.72$.

3.12. (a) $16.80 = 4 \cdot GNP + 80 \cdot i,$
  　　　 $9.00 = 2 \cdot GNP + 50 \cdot i;$
  (b) $16.80 = 4 \cdot GNP + 80 \cdot i,$
  　　　 $9.00 = 2 \cdot GNP + 50 \cdot i.$

$$
\begin{array}{l}
16.80 = 4 \cdot GNP + 80 \cdot i \\
\underline{18.00 = 4 \cdot GNP + 100 \cdot i} \\
-1.20 = \qquad\qquad -20i;
\end{array}
$$

$$i = 0.06, \; GNP = 3.$$

3.13. Our first problem is to complete a pro-forma income statement for 2001. However, we don't know what the company's interest expenditure in 2001 will be until we know how much money it will borrow (*EFN*). At the same time, we cannot determine how much money the firm needs to borrow until we know its interest expenditure (so we can solve for retained earnings). Therefore, we must solve simultaneously for *EFN* and interest expenditure. We know that *EFN* can be found as follows:

$$EFN = \Delta \, Assets - \Delta CL - RE,$$

$$EFN = \$400{,}000 - \$60{,}000 - RE.$$

Retained earnings (*RE*) can be found using the following pro-forma income statement for 2001:

> *Pro-forma Income Statement, 2001*
>
> | | |
> |---|---|
> | Sales (*TR*) .... | $1,050,000 |
> | Cost of Goods Sold .... | 450,000 |
> | Gross Margin .... | 600,000 |
> | Fixed Costs .... | 150,000 |
> | *EBIT* .... | 450,000 |
> | Interest Payments .... | 50,000 + (0.10 · EFN) |
> | Earnings Before Tax .... | 400,000 − (0.10 · EFN) |
> | Taxes (@ 40%) .... | 160,000 − (0.04 · EFN) |
> | Net Income After Tax .... | 240,000 − (0.06 · EFN) |
> | Dividends (@ 33%) .... | 80,000 − (0.02 · EFN) |
> | Retained Earnings .... | 160,000 − (0.04 · EFN) |

Now, rewrite the *EFN* expression, substituting in for *RE*:

$$EFN = \$400{,}000 - \$60{,}000 - (\$160{,}000 - (0.04 \cdot EFN)),$$

$$EFN = \$180{,}000 + 0.04 \cdot EFN,$$

$$0.96 \cdot EFN = \$180{,}000,$$

$$EFN = \$187{,}500.$$

Our *EFN* problem is complete. We now know that the firm must borrow $187,500. Thus, the firm's total interest payments for 2001 must be $50,000 plus 10% of $187,500, or $68,750.

3.14. $$1{,}200 = 100(1 + x + x^2 + x^3 + \ldots + x^9),$$

$$1{,}200x = 100(x + x^2 + x^3 + \ldots + x^{10}),$$

$$1{,}200x - 1{,}200 = 100(x + x^2 + x^3 + \ldots + x^{10} - 1 - x - x^2 - x^3 - \ldots - x^9),$$

$$1{,}200(x - 1) = 100(x^{10} - 1),$$

$$12 = (x^{10} - 1)/(x - 1).$$

Substitute for *x*:

$$x = 1.0398905.$$

3.15.   The expansion is performed as follows:

$$\Delta Y = \Delta \overline{C} + Y(c + c^2 + c^3 + \ldots + c^\infty), \tag{B}$$

$$c\Delta Y = c\Delta \overline{C} + Y(c^2 + c^3 + c^4 + \ldots + c^{\infty+1}), \tag{C}$$

$$(1 - c)\Delta Y = (1 - c)\Delta \overline{C} + Y(c - c^{\infty+1}) \qquad \text{where } 0 < c \leq 1, \tag{D}$$

$$\Delta Y = \Delta \overline{C} + Y[c \div (1 - c)] \qquad \text{where } c^{\infty+1} = 0. \tag{E}$$

Thus, the income multiplier equals $c/(1 - c) = c/s$, where *s* represents the proportion of marginal income saved by individuals.

3.16.   Plot several points for *x* and *y* for the functions, one function at a time. When enough points have been plotted to determine the shape of the curve, map out the graph. Functions (a) and (b) will plot similarly to the two functions in figure 3.2 in section 3.5. Functions (c) and (d) will plot similarly to the two functions in figure 3.1 in section 3.5.

CHAPTER 4

4.1. $TV_8 = 10,500(1 + 8 \cdot 0.09) = 10,500 \cdot 1.72 = 18,060.$

4.2.   (a) $[10\% \cdot \$10,000,000]/2 = \$500,000;$
       (b) $10\% \cdot \$10,000,000 = 2 \cdot \$500,000 = \$1,000,000;$
       (c) $\$10,000,000 + \$1,000,000 =$ principal + interest in year five $= \$11,000,000.$

4.3. $\$10,000(1 + 0.055/365)^{5 \cdot 365} = \$13,165.03.$

4.4.   (a) $TV_8 = 10,500(1 + 0.09)^8 = 10,500 \cdot 1.99256 = 20,921.908;$

        (b) $TV_8 = 10,500\left(1 + \dfrac{0.09}{2}\right)^{2 \cdot 8} = 10,500 \cdot 2.0223702 = 21,234.887;$

        (c) $TV_8 = 10,500\left(1 + \dfrac{0.09}{12}\right)^{12 \cdot 8} = 10,500 \cdot 2.0489212 = 21,513.673;$

        (d) $TV_8 = 10,500\left(1 + \dfrac{0.09}{365}\right)^{365 \cdot 8} = 10,500 \cdot 2.0542506 = 21,569.632;$

        (e) $TV_8 = 10,500e^{0.09 \cdot 8} = 10,500 \cdot 2.0544332 = 21,571.549.$

4.5. $\$10,000(1 + 0.055/365)^{5 \cdot 365} = \$13,165.03.$

4.6.   For example, let $X_0 = \$1,000$ in each case:

        for CD_1, $TV_5 = 1,000(1 + 0.12)^5 = 1,762.3417;$

        for CD_2, $TV_5 = 1,000\left(1 + \dfrac{0.10}{365}\right)^{365 \cdot 5} = 1,648.6005.$

4.7.   Solve for X_0:

$$X_0 = \frac{TV_n}{(1 + i)^n} = \frac{10,000}{(1 + 0.08)^3} = 7,938.322.$$

4.8   Solve the following for APY:

$$APY = \left(1 + \frac{i}{m}\right)^m - 1 = \left(1 + \frac{0.03}{4}\right)^4 - 1 = 0.0303391.$$

4.9.   In all cases here, $FV_n = 2X_0$. Thus, let $FV_n = 2,000$ and $X_0 = 1,000$:

        (a) $2,000 = 1,000(1 + n \cdot 0.1)$, $2 = (1 + n \cdot 0.1)$, $1 = 0.1n$, $n = 10$ years.
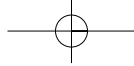   (b) $2,000 = 1,000(1.1)^n$. Using logs:

$$\log 2,000 = (\log 1,000) + n \cdot \log(1.1),$$

$$3.30103 = 3 + n \cdot (0.04139),$$

$$0.30103 = n(0.04139), \qquad n = 7.2725 \text{ years.}$$

        (c) $2,000 = 1,000\left(1 + \dfrac{0.10}{12}\right)^{12n}:$

$$\log 2{,}000 = (\log 1{,}000) + 12n \cdot \log(1.008333),$$

$$n = 6.96059 \text{ years.}$$

(d) $2{,}000 = 1{,}000e^{0.1 \cdot n}$. Use natural logs:

$$\ln 2{,}000 = (\ln 1{,}000) + 0.1n, \qquad n = 6.93148 \text{ years.}$$

4.10. (a) $PV = \dfrac{CF_n}{(1+k)^n} = \dfrac{10{,}000}{(1+0.20)^5} = \dfrac{10{,}000}{1.2^5} = \dfrac{10{,}000}{2.48832} = 4{,}018.775;$

(b) $PV = \dfrac{10{,}000}{1.10^5} = \dfrac{10{,}000}{1.61051} = 6{,}209.213;$

(c) $PV = \dfrac{10{,}000}{1.01^5} = \dfrac{10{,}000}{1.0510101} = 9{,}514.656;$

(d) $PV = \dfrac{10{,}000}{1.0^5} = \dfrac{10{,}000}{1} = 10{,}000.$

4.11. (a) $PV = \dfrac{10{,}000}{1.1^{20}} = \dfrac{10{,}000}{6.7275} = 1{,}486.436;$

(b) $PV = \dfrac{10{,}000}{1.1^{10}} = \dfrac{10{,}000}{2.5937425} = 3{,}855.432;$

(c) $PV = \dfrac{10{,}000}{1.1^1} = \dfrac{10{,}000}{1.1} = 9{,}090.909;$

(d) $PV = \dfrac{10{,}000}{1.1^{0.5}} = \dfrac{10{,}000}{1.1^{0.5}} = \dfrac{10{,}000}{1.0488088} = 9{,}534.625$

(note that six months is 0.5 of one year);

(e) $PV = \dfrac{10{,}000}{1.1^{0.2}} = \dfrac{10{,}000}{1.0192449} = 9{,}811.184$

(note that 73 days is 0.2 of one year).

4.12. $PV = \displaystyle\sum_{t=1}^{n} \dfrac{CF_t}{(1+k)^t} = \dfrac{2{,}000}{1.08^1} + \dfrac{3{,}000}{1.08^2} + \dfrac{7{,}000}{1.08^3};$

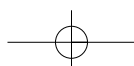$PV = 1{,}851.85 + 2{,}572.02 = 5{,}556.83 = 9{,}980.70;$

$$10{,}000 > 9{,}980.70.$$

Since $P_0 > PV$, the investment should not be purchased.

4.13. $PV_n = CF\left[\dfrac{1}{k} - \dfrac{1}{k(1+k)^n}\right]:$

(a) $PV_A = 2{,}000\left[\dfrac{1}{0.05} - \dfrac{1}{0.05(1.05)^9}\right] = 2{,}000(20 - 12.892178) = 14{,}215.643;$

(b) $PV_A = 2{,}000\left[\dfrac{1}{0.10} - \dfrac{1}{0.10(1.10)^9}\right] = 2{,}000(10 - 4.2409762) = 11{,}518.048;$

(c) $PV_A = \left[\dfrac{1}{0.2} - \dfrac{1}{0.2(1.2)^9}\right] = 2{,}000(5 - 0.9690335) = 8{,}061.933.$

4.14. $PV_p = \dfrac{CF}{K} = \dfrac{50}{0.08} = 625.$

4.15. $CF_n = CF_1(1 + g)^{n-1}$:
   (a) $CF_2 = 10,000(1 + 0.1)^{2-1} = 10,000(1 + 0.1) = 10,000 \cdot 1.1 = 11,000;$
   (b) $CF_3 = 10,000(1 + 0.1)^{3-1} = 10,000 \cdot 1.21 = 12,100;$
   (c) $CF_5 = 10,000(1 + 0.1)^{5-1} = 10,000 \cdot 1.4641 = 14,641;$
   (d) $CF_{10} = 10,000(1 + 0.1)^{10-1} = 10,000 \cdot 2.3579477 = 23,579.477.$

4.16. $PV_{ga} = CF_1 \cdot \left| \dfrac{1}{k - g} - \dfrac{(1 + g)}{(k - g)(1 + k)^n} \right|$

   $= 5,000 \cdot \left| \dfrac{1}{0.02} - \dfrac{(1 + 0.10)^7}{(0.12 - 0.10)(1 + 0.12)^7} \right|;$

   $PV_{ga} = 5,000 \cdot [50 - 44.075033] = 29,624.837.$

4.17. $PV_{gp} = \dfrac{CF_1}{k - g} = \dfrac{100}{0.12 - 0.05} = 1,428.5714.$

4.18.   \$60,000 per year for 20 years:

   (a) $PV = 500,000;$

   (b) $PV = 100,000 \left[\dfrac{1}{0.05} - \dfrac{1}{0.05(1.05)^8} \right] = 646,321.27;$

   (c) $PV = 60,000 \left[\dfrac{1}{0.05} - \dfrac{1}{0.05(1.05)^{20}} \right] = 747,732.62;$

   (d) $PV = \dfrac{30,000}{0.05} = 600,000.$

   Series (c) has the highest present value.

4.19.   \$500,000 now:

   (a) $PV = 500,000;$

   (b) $PV = 100,000 \left[\dfrac{1}{0.2} - \dfrac{1}{0.2(1.2)^8} \right] = 383,715.98;$

   (c) $PV = 60,000 \left[\dfrac{1}{0.2} - \dfrac{1}{0.2(1.2)^{20}} \right] = 292,174.78;$

   (d) $PV = \dfrac{30,000}{0.2} = 150,000.$

4.20. $\text{Pay} = \text{Principal} \Big/ \left[\dfrac{1}{i} - \dfrac{1}{i(1 + i)^n} \right];$

   Principal $= 200,000 - 50,000 = 150,000$:

(a) $\text{Pay} = 150{,}000 \Big/ \left[\dfrac{1}{0.1} - \dfrac{1}{0.1(1+0.1)^{20}} \right]$

$= 150{,}000/8.5135637 = 17{,}618.944;$

(b) $\text{Pay} = 150{,}000 \Big/ \left[\dfrac{1}{0.008333} - \dfrac{1}{0.008333(1.008333)^{240}} \right]$

$= 150{,}000/103.62442 = 1{,}447.5352$

(note that $10\%/12 = 0.008333$; $20 \cdot 12 = 240$.

4.21. Substitute discount rates into the present-value annuity function until you find one that sets *PV* equal to the purchase price:

try 15%, $PV = 9{,}543.1685 < 10{,}000;$
try 13%, $PV = 10{,}803.31 > 10{,}000;$
try 14%, $PV = 9{,}892.8294 < 10{,}000;$
try 13.7%, $PV = 10{,}001.638 > 10{,}000;$
try 13.71%, $PV = 9{,}997.977 < 10{,}000;$
try 13.704%, $PV = 10{,}000.174 > 10{,}000.$

Thus, *K* is approximately 13.704%.

4.22. (a) $PV = \dfrac{10{,}000}{1.1^{20}} = \dfrac{10{,}000}{6.7275} = 1{,}486.436;$

(b) $PV = \dfrac{10{,}000}{(1+0.1/12)^{12 \cdot 20}} = \dfrac{10{,}000}{7.328074} = 1{,}364.615;$

(c) $PV = \dfrac{10{,}000}{(1+0.1/365)^{365 \cdot 20}} = \dfrac{10{,}000}{7.3870321} = 1{,}353.7236;$

(d) $PV = 10{,}000 \cdot e^{-0.1 \cdot 20} = 1{,}353.3528.$

4.23. (a) First, the monthly discount rate is $0.1 \div 12 = 0.008333$:

$$PV = 1{,}000 \cdot \left[\dfrac{1}{0.008333} - \dfrac{1}{0.008333(1+0.008333)^{360}} \right]$$

$= 1{,}000 \cdot 113.95082 = \$113{,}950.82.$

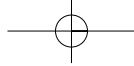(b) Yes, since the *PV* exceeds the \$100,000 price.

(c) $100{,}000 = 1{,}000 \cdot \left[\dfrac{1}{(k/12)} - \dfrac{1}{(k/12) \cdot (1+k/12)^{360}} \right].$

Solve for *k*; by a process of substitution, we find that $k = 0.11627.$

4.24. Use the present-value annuity function to amortize the loan. The payment is \$2,637.97.

4.25. $PV_{\text{ga}} = CF \left[\dfrac{(1+g)^0}{(1+k)^1} + \dfrac{(1+g)^1}{(1+k)^2} + \ldots + \dfrac{(1+g)^{n-1}}{(1+k)^n} \right];$

$PV_{\text{ga}} \cdot \dfrac{(1+k)}{(1+g)} = CF \left[\dfrac{(1+g)^{-1}}{(1+k)^0} + \dfrac{(1+g)^0}{(1+k)^1} + \ldots + \dfrac{(1+g)^{n-2}}{(1+k)^{n-1}} \right];$

$$PV_{ga} \cdot \frac{(1+k)}{(1+g)} - PV_{ga} = CF\left[\frac{(1+g)^{-1}}{(1+k)^0} - \frac{(1+g)^{n-1}}{(1+k)^n}\right];$$

$$PV_{ga} \cdot \frac{(1+k)-(1+g)}{(1+g)} = PV_{ga}\frac{(k-g)}{(1+g)} = CF\left[\frac{(1+g)^{-1}}{(1+k)^0} - \frac{(1+g)^{n-1}}{(1+k)^n}\right];$$

$$PV_{ga} = CF\left[\frac{1}{k-g} - \frac{(1+g)^n}{(k-g)(1+k)^n}\right].$$

4.26. This problem can be solved with either of the following:

$$PV = \$5,000 \cdot \frac{\dfrac{1}{0.12} - \dfrac{1}{0.12(1+0.12)^{50}}}{(1+0.12)^9} = \$14,973.42;$$

$$PV = \$5,000 \cdot \left(\frac{1}{0.12} - \frac{1}{0.12(1+0.12)^{59}}\right) - \$5,000 \cdot \left(\frac{1}{0.12} - \frac{1}{(1+0.12)^9}\right) = \$14,973.42.$$

4.27. The following Single-Stage Growth Model can be used to evaluate this stock:

$$p_0 = \frac{DIV_1}{k-g},$$

$$p_0 = \frac{\$1.80}{0.06 - 0.04} = \$90.$$

Since the $100 purchase price of the stock is less than its $90 model value, the stock should not be purchased.

4.28. The following Three-Stage Growth Model can be used to evaluate this stock:

$$p_0 = DIV_1\left[\frac{1}{k-g_1} - \frac{(1+g_1)^{n(1)}}{(k-g_1)(1+k)^{n(1)}}\right]$$

$$+ DIV_1\left[\frac{(1+g_1)^{n(1)-1}(1+g_2)}{(1+k)^{n(1)}(k-g_2)} - \frac{(1+g_1)^{n(1)-1}(1+g_2)^{n(2)-n(1)+1}}{(k-g_2)(1+k)^{n(2)}}\right]$$

$$+ \frac{DIV_1(1+g_1)^{n(1)-1}(1+g_2)^{n(2)-n(1)}(1+g_3)}{(k-g_3)(1+k)^{n(2)}},$$

$$p_0 = \$5\left[\frac{1}{0.08-0.15} - \frac{(1+0.15)^3}{(0.08-0.15)(1+0.08)^3}\right]$$

$$+ \$5\left[\frac{(1+0.15)^{3-1}(1+0.06)}{(1+0.08)^3(0.08-0.06)} - \frac{(1+0.15)^{3-1}(1+0.06)^{6-3+1}}{(0.08-0.06)(1+0.08)^6}\right]$$

$$+ \frac{\$5(1+0.15)^{3-1}(1+0.06)^{6-3}(1+0)}{(0.08-0)(1+0.08)^6} = 92.0171078.$$

Since the $100 purchase price of the stock exceeds its $92.0171 value, the stock should not be purchased.

CHAPTER 5

5.1. $ROI = \sum_{t=0}^{n} CF_t \div nP_0$

   $= 200/(1(1,000)) = 0.20$, or 20%.

5.2.   (a) $ROI = (400 - 200)/(7(200)) = 0.1428$, or 14.28%;

   (b) $ROI = (400/200)^{1/7} - 1 = (2)^{1/7} - 1 = 0.1041$, or 10.41%;

   (c) $IRR = 10.41\%$ (note that $ROI_{AG} = IRR$ when there is only capital gain profit).

5.3.   (a) $ROI = (500 + 4,800)/6(7,500) = 0.1178$, or 11.78%;

   (b) $IRR = 11.499\%$.

5.4.   (a) $ROI = (-100,000 + 20,000 + 20,000 + 20,000 + 20,000 + 60,000)/5(100,000)$

   $= 40,000/500,000$

   $= 0.08$, or 8%;

   (b) $IRR = 10.21\%$.

5.5. $NPV = 0$, by the definition of IRR.

5.6.   (a)   Dividends:

   Grove = \$800,

   Dean = \$200.

   (b)   Capital gains:

   Grove = \$1,100 − \$1,000 = \$100,

   Dean = \$1,800 − \$1,000 = \$800.

   (c)   Arithmetic mean capital gain return:

   Grove = (100 + 800)/8(1,000)

   = 0.1125, or 11.25%;

   Dean = (800 + 200)/8(1,000)

   = 0.125, or 12.5%.

   (d) IRR:

   Grove = 11.08477%,

   Dean = 9.598%.

   Summary of results

Company	Dividends	Capital gains	ROI_a	IRR
Grove	800	100	11.25%	10.0%
Dean	200	800	12.50%	9.5%

   (e)   Under ROI_a = Dean,
   under IRR = Grove.

The performance evaluation depends on the measure used. This depends on the investor's time value of money. A higher time value indicates that *IRR* is more useful.

5.7. (a) $ROI_A = \sum_{t=0}^{n} CF_t \div nP_0$

$$= (-100{,}000 + 50{,}000 - 50{,}000 + 75{,}000 + 75{,}000)/6(100{,}000)$$

$$= 0.083, \text{ or } 8.3\%.$$

$100{,}000 = 50{,}000/(1+r)^2 - 50{,}000/(1+r)^3 + 75{,}000/(1+r)^4 + 75{,}000/(1+r)^6,$

$IRR = 9.32487405\%, \qquad IRR = -227.776188859\%.$

(c) There are actually two internal rates of return for this problem. However, 9.32487% seems to be a reasonable rate.

5.8. (a) Its annual interest payments:

$$i_y = Int/F_0,$$

$$Int = i_y(F_0)$$

$$= (0.12) \cdot (1{,}000)$$

$$= \$120.$$

(b) Its current yield:

$$cy = Int/P_0$$

$$= 120/1{,}200$$

$$= 0.10.$$

(c) With equation (4.8), yield to maturity is found to be 0.04697429, or 4.697429%.

5.9. (a) Its annual interest payments: $120, or $60 every six months.
(b) $120 \div 1{,}200 =$ its current yield $= 0.10$, or 10%.
(c) Its yield to maturity:

$$-1{,}200 + 60/[1+(r/2)]^1 + 60/[1+(r/2)]^2 + \ldots + 60/[1+(r/2)]^5 + 1{,}060/[1+(r/2)]^6,$$

$$r = 0.0476634.$$

5.10. $PV_{ga} = CF_1 \cdot \left[\dfrac{1}{r-g} - \dfrac{(1+g)^n}{(r-g) \cdot (1+r)^n} \right] + \dfrac{CF_n}{(1+r)^n};$

$CF_1 = \$3{,}000, \qquad n = 20, \qquad g = 0.10.$

Solve for r above to obtain $IRR = 0.11794166365$.

5.11. (a) Each outcome has a one-third or 0.333 probability of being realized, since the probabilities are equal and must sum to one.
(b) E[SALES] $= (800{,}000 \cdot 0.333) + (500{,}000 \cdot 0.333) + (400{,}000 \cdot 0.333),$
E[SALES] $= 566{,}667.$

(c) Compute as follows:
VAR[*Sales*] $= [(800{,}000 - 566{,}667)^2 \cdot 0.333 + (500{,}000 - 566{,}667)^2 \cdot 0.333$

$$(400{,}000 - 566{,}667) \cdot 0.333] = 28{,}888{,}000{,}000 = \sigma^2_{sales}.$$
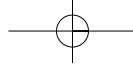
(d) Expected return of project A:

$$(0.3 \cdot 0.333) + (0.15 \cdot 0.333) + (0.01 \cdot 0.333) = 0.15333.$$

(e) Variance of A's returns:

$$[(0.3 - 0.1533)^2 \cdot 0.333 + (0.15 - 0.1533)^2 \cdot 0.333$$
$$+ (0.01 - 0.1533)^2 \cdot 0.333] = 0.0140222 = \sigma_A^2.$$

(f) Expected return of project B:

$$(0.2 \cdot 0.333) + (0.13 \cdot 0.333) + (0.09 \cdot 0.333) = 0.14.$$

Variance of B's returns:

$$[(0.2 - 0.14)^2 \cdot 0.333 + (0.13 - 0.14)^2 \cdot 0.333$$
$$+ (0.09 - 0.14)^2 \cdot 0.333] = 0.0020666 = \sigma_B^2.$$

(g) Standard deviations are square roots of variances:

$$\sigma_{sales} = 169,964,$$
$$\sigma_A = 0.1184154,$$
$$\sigma_B = 0.0454606.$$

(h) Compute as follows:

$$COV[Sales] = \sum_{i=1}^{n} (Sales_i - E[Sales]) \cdot (R_{Ai} - E[R_A]) \cdot P_i,$$

$$COV[Sales, A] = (800,000 - 566,667) \cdot (0.3 - 0.1533) \cdot 0.333$$
$$+ (500,000 - 566,667) \cdot (0.15 - 0.1533) \cdot 0.333$$
$$+ (400,000 - 566,667) \cdot (0.01 - 0.1533) \cdot 0.333$$
$$= 19,444 = \sigma_{sales,A}.$$

(i) $\rho_{s,A} = \dfrac{\sigma_{sales,A}}{\sigma_{sales} \cdot \sigma_A} = \dfrac{19,444}{169,964 \cdot 0.118} = 0.97.$

(j) First, find the covariance between sales and returns on B:

$$COV[Sales, B] = (800,000 - 566,667) \cdot (0.20 - 0.14) \cdot 0.333$$
$$+ (500,000 - 566,667) \cdot (0.13 - 0.14) \cdot 0.333$$
$$+ (400,000 - 566,667) \cdot (0.09 - 0.14) \cdot 0.333$$
$$= 7,666.67 = \sigma_{sales,B};$$

$$\rho_{sales,B} = \frac{\sigma_{sales,B}}{\sigma_{sales} \cdot \sigma_B} = \frac{7,666}{169,964 \cdot 0.0454} = 0.993.$$

(k) The coefficient of determination is the correlation coefficient squared: $0.993^2 = 0.986$.

5.12. Project A has a higher expected return; however, it is riskier. Therefore, it does not clearly dominate project B. Similarly, B does not dominate A. Therefore, we have insufficient evidence to determine which of the projects is better.

5.13. (a) $\overline{R}_P = 0.062,$
$\overline{R}_L = 0.106,$
$\overline{R}_M = 0.098.$

(b) $\sigma_P^2 = 0.000696$
(remember to convert returns to percentages),

$$\sigma_L^2 = 0.008824,$$

$$\sigma_M^2 = 0.001576.$$

(c) COV[L,Y] = [(0.04 − 0.062) · (0.19 − 0.106) + (0.07 − 0.062) · (0.04 − 0.106)

+ (0.11 − 0.062) · (−0.04 − 0.106) + (0.04 − 0.062) · (0.21 − 0.106)

+ (0.05 − 0.062) · (0.13 − 0.106)]/5 = −0.002392,

$$\rho_{P,L} = \frac{COV[L,Y]}{\sigma_P \sigma_L} = \frac{-0.002392}{0.0264 \cdot 0.094} = -0.96521.$$

(d) COV[L,M] = [(0.04 − 0.062) · (0.15 − 0.098) + (0.07 − 0.062) · (0.10 − 0.098)

+ (0.11 − 0.062) · (0.03 − 0.098) + (0.04 − 0.062) · (0.12 − 0.098)

+ (0.05 − 0.062) · (0.09 − 0.098)] ÷ 5 = −0.000956,

$$\rho_{P,M} = \frac{COV[L,M]}{\sigma_P \sigma_M} = \frac{-0.000956}{0.0264 \cdot 0.039} = -0.912.$$

(e) COV[M,Y] = [(0.15 − 0.098) · (0.19 − 0.106) + (0.10 − 0.098) · (0.04 − 0.106)

+ (0.03 − 0.098) · (−0.04 − 0.106) + (0.12 − 0.098) · (0.21 − 0.106)

+ (0.09 − 0.098) · (0.13 − 0.106)] ÷ 5 = 0.003252,

$$\rho_{M,L} = \frac{COV[M,Y]}{\sigma_M \sigma_Y} = \frac{0.003252}{0.039 \cdot 0.094} = 0.872.$$

5.14. Assuming variance and correlation stability, the forecasted values would be the same as the historical values in problem 5.13.

5.15. Standardize returns by standard deviations and consult "z" tables: $(R_i - E[R]) \div \sigma_i = z_i$. Only use positive values for z.

(a) $(0.05 - 0.15) \div 0.10 = z(low) = -1$, $(0.25 - 0.15) \div 0.10 = z(high) = 1$.
From the z-table in appendix B, we see that the probability that the security's return will fall between 0.05 and 0.15 is 0.34. 0.34 is also the probability that the security's return will fall between 0.15 and 0.25. Therefore, the probability that the security's return will fall between 0.05 and 0.25 is 0.68.
(b) From part (a), we see that the probability is 0.34.
(c) 0.16.
(d) 0.0668.

5.16. Simply reduce the standard deviations in the z-scores in problem 5.15 to 0.05:
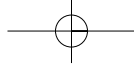
(a) 0.95;
(b) 0.47;
(c) 0.0228;
(d) 0.0013.

5.17. (a) VAR = 0.0025.
(b) 0. The coefficient of correlation between returns on any asset and returns on a riskless asset must be zero. Riskless asset returns do not vary.
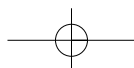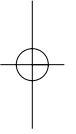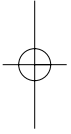
CHAPTER 6

6.1. (a) $\bar{R}_p = (w_M \cdot \bar{R}_M) = (w_H \cdot \bar{R}_H) = (0.25 \cdot 0.20) + (0.75 \cdot 0.06) = 0.09;$

(b) $\sigma_p^2 = w_H^2 \cdot \sigma_H^2 + w_M^2 \cdot \sigma_M^2 + 2 \cdot w_H \cdot w_M \cdot \sigma_H \cdot \sigma_M \cdot \rho_{H,M};$

$\sigma_p^2 = 0.75^2 \cdot 0.09^2 + 0.25^2 \cdot 0.30^2 + 2 \cdot 0.75 \cdot 0.25 \cdot 0.09 \cdot 0.30 \cdot 0.4 = 0.05068;$

(c) $\sigma_p = \sqrt{0.05068} = 0.2251,$

since the standard deviation is the square root of the variance.

6.2. $\sigma_p = [0.5^2 \cdot 0.09 + 0.5^2 \cdot 0.09 + 2(0.5 \cdot 0.5 \cdot 0)]^{1/2} = [0.0225 + 0.0225]^{1/2} = 0.2121.$

6.3. The following equation will be used for each part of this problem:

$$\sigma_p^2 = w_1^2 \cdot \sigma_1^2 + w_2^2 \cdot \sigma_2^2 + 2(w_1 \cdot w_2 \cdot \sigma_1 \cdot \sigma_2 \cdot \rho_{1,2}).$$

(a) $0.36 = (0.6^2 \cdot 0.6^2) + (0.4^2 \cdot 0.6^2) + 2(0.6 \cdot 0.4 \cdot 0.36), \qquad \sigma_p = 0.6;$
(b) $0.2736 = (0.6^2 \cdot 0.6^2) + (0.4^2 \cdot 0.6^2) + 2(0.6 \cdot 0.4 \cdot 0.36), \qquad \sigma_p = 0.523;$
(c) $0.1872 = (0.6^2 \cdot 0.6^2) + (0.4^2 \cdot 0.6^2) + 2(0.6 \cdot 0.4), \qquad \sigma_p = 0.433;$
(d) $0.1008 = (0.6^2 \cdot 0.6^2) + (0.4^2 \cdot 0.6^2) + 2[0.6 \cdot 0.4 \cdot (-0.18)], \qquad \sigma_p = 0.3175;$
(e) $0.0144 = (0.6^2 \cdot 0.6^2) + (0.4^2 \cdot 0.6^2) + 2[0.6 \cdot 0.4 \cdot (-0.36)], \qquad \sigma_p = 0.12.$

6.4. (a) $\bar{R}_p = 0.075, \qquad \sigma_p = 0.16;$
(b) $\bar{R}_p = 0.075, \qquad \sigma_p = 0.116619;$
(c) $\bar{R}_p = 0.075, \qquad \sigma_p = 0.04.$

6.5. The correlation coefficients have no effect on the expected return of the portfolio. However, a decrease in the correlation coefficients between security returns will decrease the variance or risk of the portfolio.

6.6. $E[R_p] = 0.33333 \cdot 0.25 + 0.16667 \cdot 0.15 + 0.5 \cdot 0.05 = 0.13333;$

$\sigma_p^2 = w_1^2 \cdot \sigma_1^2 + w_2^2 \cdot \sigma_2^2 + w_3^2 \cdot \sigma_3^2 + 2w_1 \cdot w_2 \cdot \sigma_{1,2} + 2w_1 \cdot w_3 \cdot \sigma_{1,3} + 2w_2 \cdot w_3 \cdot \sigma_{2,3};$

$0.02444 = 0.33333^2 \cdot 0.40^2 + 0.16667^2 \cdot 0.20^2 + 0.5^2 \cdot 0^2$
$+ 2 \cdot 0.16667 \cdot 0.33333 \cdot 0.05 + 2 \cdot 0.16667 \cdot 0.5 \cdot 0 + 2 \cdot 0.33333 \cdot 0.5 \cdot 0;$

$\sigma_p^2 = 0.02444, \qquad \sigma_p = 0.15635.$

6.7. $0.08 = w_{risky}^2 \cdot 0.10 + w_{riskless}^2 \cdot 0, \qquad w_{risky} = 0.8944;$

$E[R_p] = 0.8944 \cdot 0.25 + 0.1056 \cdot 0.10 = 0.2342.$

6.8. (a) Recall that correlation coefficients (and covariances) equal zero:

$\sigma_p^2 = w_1^2 \cdot \sigma_1^2 + w_2^2 \cdot \sigma_2^2 = 0.5^2 \cdot 0.8^2 + 0.5^2 \cdot 0.8^2 = 0.32, \qquad \sigma_p = 0.565685.$

(b) $\sigma_p^2 = w_1^2 \cdot \sigma_1^2 + w_2^2 \cdot \sigma_2^2 + w_3^2 \cdot \sigma_3^2 + w_4^2 \cdot \sigma_4^2 = 0.25^2 \cdot 0.8^2 + 0.25^2 \cdot 0.8^2$
$+ 0.25^2 \cdot 0.8^2 + 0.25^2 \cdot 0.8^2 = 0.16, \qquad \sigma_p = 0.4.$

(c) $\sigma_p^2 = 0.125^2 \cdot 0.8^2 + 0.125^2 \cdot 0.8^2 + 0.125^2 \cdot 0.8^2 + 0.125^2 \cdot 0.8^2 + 0.125^2 \cdot 0.8^2$
$+ 0.125^2 \cdot 0.8^2 + 0.125^2 \cdot 0.8^2 + 0.125^2 \cdot 0.8^2 = 0.08, \qquad \sigma_p = 0.282843.$

(d) $\sigma_p^2 = 0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.065^2 \cdot$
$0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 +$
$0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.065^2 \cdot 0.8^2$
$+ 0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 = 0.04, \qquad \sigma_p = 0.2.$

(e) They don't differ. Expected portfolio returns are always a weighted average of component security expected returns, which is always 0.10 in this example.

CHAPTER 7

7.1. (a) $\begin{bmatrix} 7 \\ 0 \\ 10 \end{bmatrix}$;

(b) $\begin{bmatrix} 7 & 0 \\ 10 & 2.5 \end{bmatrix}$.

7.2. (a) $[4 \quad 6 \quad 5]$;

(b) $\begin{bmatrix} 4 & 6 \\ 5 & 2 \end{bmatrix}$.

7.3. $\begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix} = -\begin{bmatrix} 30 \\ 40 \\ 60 \\ 70 \end{bmatrix} + \begin{bmatrix} 40 \\ 40 \\ 40 \\ 40 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 20 \\ 30 \end{bmatrix}$,

$\mathbf{P} \quad = \quad -\mathbf{S} \quad + \quad \mathbf{X} \quad + \quad \mathbf{C}$.

7.4. (a) $\begin{bmatrix} 2 & 4 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -4 + \frac{12}{2} & 2 - \frac{4}{2} \\ -6 + \frac{12}{2} & 3 - \frac{4}{2} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$;

(b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$;

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$;

(d) $[4 \quad 5 \quad 6]\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = 77$;

(e) $\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}[4 \quad 5 \quad 6] = \begin{bmatrix} 16 & 20 & 24 \\ 20 & 25 & 30 \\ 24 & 30 & 36 \end{bmatrix}$.

7.5. (a) The weights matrix is given as follows:

$$\mathbf{W} = \begin{bmatrix} 0.15 & 0.25 & 0.60 \\ 0.40 & 0.30 & 0.30 \\ 0.30 & 0.25 & 0.45 \end{bmatrix}.$$

(b) The returns vector is given as follows:

$$\mathbf{r} = \begin{bmatrix} 0.12 \\ 0.18 \\ 0.24 \end{bmatrix}.$$

(c) The funds' returns are computed as follows:

$$\begin{bmatrix} 0.15 & 0.25 & 0.60 \\ 0.40 & 0.30 & 0.30 \\ 0.30 & 0.25 & 0.45 \end{bmatrix} \begin{bmatrix} 0.12 \\ 0.18 \\ 0.24 \end{bmatrix} = \begin{bmatrix} 0.207 \\ 0.174 \\ 0.189 \end{bmatrix}.$$

(d) $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 0.207 \\ 0.174 \\ 0.189 \end{bmatrix} = 0.19.$

7.6. (a) Form a returns vector as follows:

$$\mathbf{r} = \begin{bmatrix} 0.07 \\ 0.09 \\ 0.13 \end{bmatrix}.$$

(b) The covariance matrix is as follows:

$$\mathbf{V} = \begin{bmatrix} 0.04 & 0.01 & 0.02 \\ 0.01 & 0.16 & 0.08 \\ 0.02 & 0.08 & 0.36 \end{bmatrix}.$$

(c) The weights vector is as follows:

$$\mathbf{w} = \begin{bmatrix} 0.30 \\ 0.50 \\ 0.20 \end{bmatrix}.$$

(d) (a) 3×1; (b) 3×3; (c) 3×1.

(e) The expected portfolio return is given as follows:

$$E[R_p] = [0.30 \quad 0.50 \quad 0.20] \begin{bmatrix} 0.07 \\ 0.09 \\ 0.13 \end{bmatrix} = 0.092,$$

$$E[R_p] = \qquad \mathbf{w}' \qquad \mathbf{r}.$$

(f) The portfolio variance is found as follows:

$$\sigma_p^2 = [0.30 \quad 0.50 \quad 0.20] \begin{bmatrix} 0.04 & 0.01 & 0.02 \\ 0.01 & 0.16 & 0.08 \\ 0.02 & 0.08 & 0.36 \end{bmatrix} \begin{bmatrix} 0.30 \\ 0.50 \\ 0.20 \end{bmatrix},$$

$$\sigma_p^2 = \qquad \mathbf{w}' \qquad\qquad \mathbf{V} \qquad\qquad \mathbf{w};$$

$$\sigma_p^2 = [0.021 \quad 0.099 \quad 0.118] \begin{bmatrix} 0.30 \\ 0.50 \\ 0.20 \end{bmatrix} = 0.0794,$$

$$\sigma_p^2 = \quad \mathbf{w'} \ \mathbf{V} \quad \mathbf{w} \quad = \sigma_p^2.$$

7.7. (a) $1/8 = 0.125$.

(b) The inverse of the identity matrix is the identity matrix:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

(c) The inverse of a diagonal matrix is found by inverting each of the principle diagonal elements:

$$\begin{bmatrix} 0.25 & 0 \\ 0 & 2 \end{bmatrix}.$$

(d) First, augment the matrix with the identity matrix:

$$\begin{array}{c} \text{row 1} \\ \text{row 2} \end{array} \begin{bmatrix} 1 & 2 & \vdots & 1 & 0 \\ 3 & 4 & \vdots & 0 & 1 \end{bmatrix}.$$

Now use the Gauss–Jordan method to transform the original matrix to an identity matrix; the resulting right-hand side will be the inverse of the original matrix:

$$\begin{array}{c} \text{1a} \\ \text{1b} \end{array} \begin{bmatrix} 1 & 2 & \vdots & 1 & 0 \\ 0 & -0.6 & \vdots & -1 & \frac{1}{3} \end{bmatrix} \begin{array}{l} \text{row } 1 \times 1 \\ \text{row } 2 \times \frac{1}{3} - (1a) \end{array}$$

$$\begin{array}{c} \text{2a} \\ \text{2b} \end{array} \begin{bmatrix} 1 & 0 & \vdots & -2 & 1 \\ 0 & 1 & \vdots & 1.5 & -0.5 \end{bmatrix} \begin{array}{l} (1a) - 2 \times (2b) \\ (2a) \times 1/(-0.6) \end{array}$$

Thus, the inverse matrix is

$$\begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}.$$

(e) Augment the matrix with the identity matrix and perform elementary row operations to obtain the inverse matrix as follows:

$$\begin{bmatrix} 0.02 & 0.04 & \vdots & 1 & 0 \\ 0.06 & 0.08 & \vdots & 0 & 1 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 2 & \vdots & 50 & 0 \\ 0 & \frac{2}{3} & \vdots & 50 & -16\frac{2}{3} \end{bmatrix},$$

$$\begin{bmatrix} 1 & 0 & \vdots & -100 & 50 \\ 0 & 1 & \vdots & 75 & -25 \end{bmatrix}.$$

The inverse matrix is

$$\begin{bmatrix} -100 & 50 \\ 75 & -25 \end{bmatrix}.$$

(f) The inverse matrix is

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

(g) The inverse matrix is

$$\begin{bmatrix} 0.04 & 0.04 \\ 0.04 & 0.16 \end{bmatrix}.$$

(h) Augment the matrix with the identity matrix and perform elementary row operations to obtain the inverse matrix as follows:

$$\begin{bmatrix} 2 & 0 & 0 & \vdots & 1 & 0 & 0 \\ 2 & 4 & 0 & \vdots & 0 & 1 & 0 \\ 4 & 8 & 20 & \vdots & 0 & 0 & 1 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & 0.5 & 0 & 0 \\ 0 & 4 & 0 & \vdots & -1 & 1 & 0 \\ 0 & 8 & 20 & \vdots & -2 & 0 & 1 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & 0.5 & 0 & 0 \\ 0 & 1 & 0 & \vdots & -0.25 & 0.25 & 0 \\ 0 & 0 & 2 & \vdots & 0 & -2 & 1 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & 0.5 & 0 & 0 \\ 0 & 1 & 0 & \vdots & -0.25 & 0.25 & 0 \\ 0 & 0 & 1 & \vdots & 0 & -0.1 & 0.05 \end{bmatrix}.$$

The inverse matrix is

$$\begin{bmatrix} 0.5 & 0 & 0 \\ -0.25 & 0.25 & 0 \\ 0 & -0.1 & 0.05 \end{bmatrix}.$$

7.8.
$$\mathbf{C}^{-1} = \begin{bmatrix} 0.04 & 0.04 \\ 0.04 & 0.16 \end{bmatrix};$$

$$\begin{bmatrix} 0.04 & 0.04 \\ 0.04 & 0.16 \end{bmatrix}\begin{bmatrix} 0.05 \\ 0.10 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0.006 \\ 0.018 \end{bmatrix},$$

$$\mathbf{C}^{-1} \quad \cdot \quad \mathbf{s} \quad = \quad \mathbf{x} \quad = \quad \mathbf{x}.$$

See 7.4(b) above for the inverse matrix.

7.9. Our original system of equations is represented as follows:

$$
\begin{bmatrix} 0.08 & 0.08 & 0.1 & 1 \\ 0.08 & 0.32 & 0.2 & 1 \\ 0.1 & 0.2 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \end{bmatrix},
$$

$$
\mathbf{C} \qquad \cdot \quad \mathbf{x} \ = \ \mathbf{s}.
$$

The elements of $\mathbf{C}$ and $\mathbf{s}$ are known; our problem is to find the weights in vector $\mathbf{x}$. Thus we will rearrange the system from $\mathbf{Cx} = \mathbf{s}$ to $\mathbf{C}^{-1}\mathbf{s} = \mathbf{x}$, where $\mathbf{C}^{-1}$ is the inverse of matrix $\mathbf{C}$. So, the time-consuming part of our problem is to find $\mathbf{C}^{-1}$. We will begin by augmenting matrix $\mathbf{C}$ with the identity matrix $\mathbf{I}$:

$$
\begin{array}{l} \text{row 1} \\ \text{row 2} \\ \text{row 3} \\ \text{row 4} \end{array}
\left[\begin{array}{cccc:cccc} 0.08 & 0.08 & 0.1 & 1 & 1 & 0 & 0 & 0 \\ 0.08 & 0.32 & 0.2 & 1 & 0 & 1 & 0 & 0 \\ 0.1 & 0.2 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \quad \text{original system}
$$

$$
\begin{array}{l} \text{1a} \\ \text{2a} \\ \text{3a} \\ \text{4a} \end{array}
\left[\begin{array}{cccc:cccc} 1 & 1 & 1.25 & 12.5 & 12.5 & 0 & 0 & 0 \\ 0 & 3 & 1.25 & 0 & -12.5 & 12.5 & 0 & 0 \\ 0 & 1 & -1.25 & -12.5 & -12.5 & 0 & 10 & 0 \\ 0 & 0 & -1.25 & -12.5 & -12.5 & 0 & 0 & 1 \end{array}\right]
\begin{array}{l} (\text{row 1}) \cdot 12.5 \\ (\text{row 2}) \cdot 12.5 - (\text{1a}) \\ (\text{row 3}) \cdot 10(\text{1a}) \\ (\text{row 4}) \cdot 1 - (\text{1a}) \end{array}
$$

$$
\begin{array}{l} \text{1b} \\ \text{2b} \\ \text{3b} \\ \text{4b} \end{array}
\left[\begin{array}{cccc:cccc} 1 & 0 & 0.8\overline{3} & 12.5 & 16.\overline{6} & -4.1\overline{6} & 0 & 0 \\ 0 & 1 & 0.41\overline{6} & 0 & -4.1\overline{6} & 4.1\overline{6} & 0 & 0 \\ 0 & 0 & -1.\overline{6} & -12.5 & -8.\overline{3} & -4.1\overline{6} & 10 & 0 \\ 0 & 0 & -1.25 & -12.5 & -12.5 & 0 & 0 & 1 \end{array}\right]
\begin{array}{l} (\text{1a}) - (\text{2b}) \\ (\text{2a}) \cdot \frac{1}{3} \\ (\text{3a}) - (\text{2b}) \\ (\text{4a}) \end{array}
$$

$$
\begin{array}{l} \text{1c} \\ \text{2c} \\ \text{3c} \\ \text{4c} \end{array}
\left[\begin{array}{cccc:cccc} 1 & 0 & 0 & 6.25 & 12.5 & -6.25 & 5 & 0 \\ 0 & 1 & 0 & -3.125 & -6.25 & 3.125 & 2.5 & 0 \\ 0 & 0 & 1 & 7.5 & 5 & 2.5 & -6 & 0 \\ 0 & 0 & 0 & -3.125 & -6.25 & 3.125 & -7.5 & 1 \end{array}\right]
\begin{array}{l} (\text{1b}) - (\text{3c}) \cdot 0.83 \\ (\text{2b}) - (\text{3c}) \cdot 0.416 \\ (\text{3b}) - (\text{3c}) \cdot -1/1.6 \\ (\text{4b}) \cdot (\text{3c}) \cdot (-1.25) \end{array}
$$

$$
\begin{array}{l} \text{1d} \\ \text{2d} \\ \text{3d} \\ \text{4d} \end{array}
\left[\begin{array}{cccc:cccc} 1 & 0 & 0 & 0 & 0 & 0 & -10 & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 & 10 & -1 \\ 0 & 0 & 1 & 0 & -10 & 10 & -24 & 2.4 \\ 0 & 0 & 0 & 1 & 2 & -1 & 2.4 & -0.32 \end{array}\right]
\begin{array}{l} (\text{1d}) - (\text{4a}) \cdot 6.2 \\ (\text{2c}) - (\text{4d}) \cdot (-3.125) \\ (\text{3c}) - (\text{4d}) \cdot 7.5 \\ (\text{4c}) \cdot (-1)/3.125 \end{array}
$$

$$
\qquad \mathbf{I} \qquad\qquad\qquad \mathbf{C}^{-1}
$$

$$
\begin{bmatrix} 0 & 0 & -10 & 2 \\ 0 & 0 & 10 & -1 \\ -10 & 10 & -24 & 2.4 \\ 2 & -1 & 2.4 & -0.32 \end{bmatrix} \cdot \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -0.8 \\ 0.9 \\ -2.16 \\ 0.308 \end{bmatrix}
$$

$$
\mathbf{C}^{-1} \qquad\qquad \cdot \quad \mathbf{s} \ = \ \mathbf{x} \ = \quad \mathbf{x}.
$$

Now it is clear that:

$$x_1 = (0 \cdot 0.1) + (0 \cdot 0.1) + (-10 \cdot 0.1) + (2 \cdot 0.1) = -0.8,$$

$$x_2 = (0 \cdot 0.1) + (0 \cdot 0.1) + (10 \cdot 0.1) + (-1 \cdot 0.1) = 0.9,$$

$$x_3 = (-10 \cdot 0.1) + (10 \cdot 0.1) + (-24 \cdot 0.1) + (2.4 \cdot 0.1) = -2.16,$$

$$x_4 = (2 \cdot 0.1) + (-1 \cdot 0.1) + (2.4 \cdot 0.1) + (-0.32 \cdot 0.1) = 0.308.$$

7.10.   Our first problem is to complete a pro-forma income statement for 2001. However, we don't know what the company's interest expenditure in 2001 will be until we know how much money it will borrow (*EFN*). At the same time, we cannot determine how much money the firm needs to borrow until we know its interest expenditure (so that we can solve for retained earnings). Therefore, we must solve simultaneously for *EFN* and interest expenditure. We know that *EFN* can be found as follows:

$$EFN = \Delta Assets - \Delta CL - RE,$$

$$EFN = \$500,000 - \$75,000 - RE.$$

Retained earnings (*RE*) can be found using the following pro-forma income statement for 2001:

Pro-forma income statement, 2001	
Sales (*TR*)	$1,125,000
Cost of Goods Sold	450,000
Gross Margin	675,000
Fixed Costs	150,000
EBIT	525,000
Interest Payments	$50,000 + (0.10 \cdot EFN)$
Earnings Before Tax	$475,000 - (0.10 \cdot EFN)$
Taxes (@ 40%)	$190,000 - (0.04 \cdot EFN)$
Net Income After Tax	$285,000 - (0.06 \cdot EFN)$
Dividends (@ 33%)	$95,000 - (0.02 \cdot EFN)$
Retained Earnings	$190,000 - (0.04 \cdot EFN)$

$$\begin{bmatrix} 1 & [0.1 \cdot (1-0.4) \cdot (1-0.333)] \\ 1 & 1 \end{bmatrix} \begin{bmatrix} RE \\ EFN \end{bmatrix} = \begin{bmatrix} [(525,000 - 50,000) \cdot (1-0.4) \cdot (1-0.333)] \\ (500,000 - 75,000) \end{bmatrix},$$

$$\mathbf{C} \qquad\qquad\quad \cdot \quad \mathbf{x} \quad = \qquad\qquad\qquad \mathbf{s};$$

$$\begin{bmatrix} 1 & 0.04 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} RE \\ EFN \end{bmatrix} = \begin{bmatrix} 190,000 \\ 425,000 \end{bmatrix},$$

$$\mathbf{C} \quad \cdot \quad \mathbf{x} \quad = \quad \mathbf{s};$$

$$\begin{bmatrix} 1.0416667 & -0.0416667 \\ -1.0416667 & 1.0416667 \end{bmatrix} \begin{bmatrix} 190,000 \\ 425,000 \end{bmatrix} = \begin{bmatrix} 180,208.45 \\ 244,791.78 \end{bmatrix},$$

$$\mathbf{C} \qquad\qquad \cdot \qquad \mathbf{x} \quad = \quad \mathbf{s}.$$

Our *EFN* problem is complete. We now know that the firm must borrow $244,791.78.

7.11. (a) Solve the following system:

$$
\begin{bmatrix}
1{,}100 & 0 & 0 & 0 \\
100 & 1{,}100 & 0 & 0 \\
100 & 100 & 1{,}100 & 0 \\
100 & 100 & 100 & 1{,}100
\end{bmatrix}
\begin{bmatrix}
D_1 \\ D_2 \\ D_3 \\ D_4
\end{bmatrix}
=
\begin{bmatrix}
1{,}000 \\ 980 \\ 960 \\ 940
\end{bmatrix},
$$

$$\mathbf{CF} \qquad\cdot\quad \mathbf{d} \;=\; \mathbf{P};$$

$$
\begin{bmatrix}
0.000909 & 0 & 0 & 0 \\
-0.000083 & 0.000909 & 0 & 0 \\
-0.000075 & -0.000083 & 0.000909 & 0 \\
-0.000068 & -0.000075 & -0.000083 & 0.000909
\end{bmatrix}
\begin{bmatrix}
1{,}000 \\ 980 \\ 960 \\ 940
\end{bmatrix}
=
\begin{bmatrix}
D_1 \\ D_2 \\ D_3 \\ D_4
\end{bmatrix},
$$

$$\mathbf{CF}^{-1} \qquad\qquad =\; \mathbf{P} \;=\; \mathbf{d}.$$

Solve first for $D_1 = 0.90909$, then $D_2 = 0.80826$, then $D_3 = 0.71660$, and finally $D_4 = 0.63328$. Then, $y_{0,1} = 0.10$, $y_{0,2} = 0.1123$, $y_{0,3} = 0.1175$, and $y_{0,4} = 0.1210$.

(b) $y_{1,3} = [(1 + y_{0,3})^3 \div (1 + y_{0,1})]^{0.5} - 1 = [(1.1175)^3 \div (1.1)]^{0.5} - 1 = 0.1264$.

7.12. (a) First, solve the following system for the discount functions $\mathbf{d}$:

$$
\begin{bmatrix}
50 & 50 & 1{,}050 \\
80 & 80 & 1{,}080 \\
110 & 1{,}110 & 0
\end{bmatrix}
\begin{bmatrix}
D_1 \\ D_2 \\ D_3
\end{bmatrix}
=
\begin{bmatrix}
878.9172 \\ 955.4787 \\ 1{,}055.4190
\end{bmatrix},
$$

$$\mathbf{CF} \qquad\cdot\quad \mathbf{d} \;=\; \mathbf{P}_0.$$

We find that $D_1 = 0.943396$, $D_2 = 0.857338$, and $D_3 = 0.751314$. The spot rates are obtained as follows:

$$\frac{1}{D_1} - 1 = \frac{1}{0.943396} - 1 = 0.06,$$

$$\frac{1}{D_2^{1/2}} - 1 = \frac{1}{0.857338^{1/2}} - 1 = 0.08,$$

$$\frac{1}{D_3^{1/3}} - 1 = \frac{1}{0.751314^{1/3}} - 1 = 0.10.$$

(b) The weights are found by solving for $\mathbf{w}$ as follows:

$$
\begin{bmatrix}
50 & 80 & 110 \\
50 & 80 & 1{,}110 \\
1{,}050 & 1{,}080 & 0
\end{bmatrix}
\begin{bmatrix}
w_X \\ w_Y \\ w_Z
\end{bmatrix}
=
\begin{bmatrix}
150 \\ 150 \\ 1{,}150
\end{bmatrix},
$$

$$\mathbf{CF} \qquad\cdot\quad \mathbf{w} \;=\; \mathbf{P}_0;$$

$$
\begin{bmatrix}
-0.03996 & 0.00396 & 0.002666 \\
0.03885 & -0.00385 & -0.001667 \\
-0.00100 & 0.00100 & 0
\end{bmatrix}
\begin{bmatrix}
150 \\ 150 \\ 1{,}150
\end{bmatrix}
=
\begin{bmatrix}
w_X \\ w_Y \\ w_Z
\end{bmatrix},
$$

$$\mathbf{CF}^{-1} \qquad\qquad\cdot\quad \mathbf{P}_0 \;=\; \mathbf{w}.$$

We find that $w_X = -2.3341$, $w_Y = 3.33295$, and that $w_Z = 0$. This means that bond Q is replicated by a portfolio with a short position in 2.3341 bonds X and a long position in 3.33295 bonds Y.

7.13. The following system may be solved for **b** to determine exactly how many of each of the bonds are required to satisfy the fund's cash flow requirements:

$$\begin{bmatrix} 1{,}100 & 100 & 110 & 120 \\ 0 & 1{,}100 & 110 & 120 \\ 0 & 0 & 1{,}110 & 120 \\ 0 & 0 & 0 & 1{,}120 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 80{,}000{,}000 \\ 100{,}000{,}000 \\ 120{,}000{,}000 \\ 140{,}000{,}000 \end{bmatrix},$$

$$\mathbf{CF} \cdot \mathbf{b} = \mathbf{P}_0.$$

First, we invert matrix **CF** to obtain $\mathbf{CF}^{-1}$:

$$\begin{bmatrix} 0.000909 & -0.000083 & -0.00008 & -0.000079 \\ 0 & 0.000909 & -0.00009 & -0.000087 \\ 0 & 0 & 0.00090 & -0.000096 \\ 0 & 0 & 0 & 0.000892 \end{bmatrix},$$

$$\mathbf{CF}^{-1}.$$

Thus, by inverting matrix **CF** to obtain $\mathbf{CF}^{-1}$, and premultiplying vector $\mathbf{P}_0$ by $\mathbf{CF}^{-1}$ to obtain solutions vector **b**, we find that the purchase of 43,767.2 bonds 1, 67,929 bonds 2, 95,650.9 bonds 3, and 124,880 bonds 4 satisfy the fund's exact matching requirements.

7.14. (a) $120D_1 + 1{,}120D_2 = 957.9920$, $\quad D_1 = 0.925925$,
$\quad\quad 50D_1 + 1{,}050D_2 = 840.2471$, $\quad D_2 = 0.756143$,
$\quad\quad y_{0,1} = 1/D_1 - 1 = 0.08$;

(b) $y_{0,2} = (1/D_2)^{0.5} - 1 = 0.15$;

(c) $1{,}000 \div (1 + y_{0,2})^2 = 756.1437$;

(d) $120w_A + 50w_B = 0$, $\quad w_A = -0.714285$,
$\quad\quad 1{,}120w_A + 1{,}050w_B = 1{,}000$, $\quad w_B = 1.714285$;

(e) $15{,}000 = 120\#A + 50\#B$,
$\quad\quad 12{,}000 = 1{,}120\#A + 1{,}050\#B$,
$\quad\quad \#A = 216.42857$; $\#B = -219.42857$.

Thus, sell 219.42857 B bonds for $184,374.22 and pay $207,336.83 for 216.42857 A bonds.

7.15. Since the riskless return rate is 0.125, the current value of a security guaranteed to pay $1 in one year would be $1/1.125 = 0.8888889$. The security payoff vectors are as follows:

$$\mathbf{bu} = \begin{bmatrix} 20 \\ 32 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 4 \\ 16 \end{bmatrix}.$$

Portfolio holdings are determined as follows:

$$\begin{bmatrix} 20 & 1 \\ 32 & 1 \end{bmatrix}\begin{bmatrix} \#bu \\ \#b \end{bmatrix}=\begin{bmatrix} 4 \\ 16 \end{bmatrix}.$$

The following includes the inverse matrix:

$$\begin{bmatrix} -0.0833333 & 0.083333 \\ 2.666667 & -1.666667 \end{bmatrix}\begin{bmatrix} 4 \\ 16 \end{bmatrix}=\begin{bmatrix} 1 \\ -16 \end{bmatrix}.$$

We find that $\#bu = 1$ and $\#b = -16$. This implies that the payoff structure of a single call can be replicated with a portfolio comprising 1 share of stock for a total of $24 and short-selling 16 T-bills for a total of $14.2222222. This portfolio requires a net investment of $9.7777778. Since the call has the same payoff structure as this portfolio, its current value must be $9.7777778.

7.16. (a) First, we define the following payoff vectors:

$$\begin{bmatrix} 15 \\ 25 \end{bmatrix}\quad\begin{bmatrix} 0 \\ 7 \end{bmatrix},$$

Stock $X = 18$

We have a set of two payoff vectors in a two-outcome economy. The set is linearly independent. Hence, this set forms the basis for the two-outcome space. Since we have market prices for these two securities, we can price all other securities in this economy. First, we solve for the value of the $22-exercise price call as follows:

$$\begin{bmatrix} 15 & 0 \\ 25 & 7 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 3 \end{bmatrix}=\begin{bmatrix} \#S \\ \#C_{X=18} \end{bmatrix},$$

$$\begin{bmatrix} 0.066667 & 0 \\ -0.238095 & 0.142857 \end{bmatrix}\begin{bmatrix} 0 \\ 3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0.42857 \end{bmatrix}.$$

Thus, the call with an exercise price equal to $22 can be replicated with 0.42857 calls with an exercise price equal to $18.

(b) The riskless return rate is determined as follows:

$$\begin{bmatrix} 0.066667 & 0 \\ -0.238095 & 0.142857 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}=\begin{bmatrix} 0.066667 \\ -0.095238 \end{bmatrix}.$$

Since the riskless asset is replicated with 0.066667 shares of stock and short positions in 0.095238 calls, the value of the riskless asset is 0.666674, implying a riskless return rate equal to $1/0.666674 = 0.50$.

(c) Solve for the value of the put as follows:

$$\begin{bmatrix} 0.066667 & 0 \\ -0.238095 & 0.142857 \end{bmatrix}\begin{bmatrix} 25 \\ 15 \end{bmatrix}=\begin{bmatrix} 1.666667 \\ -3.90952 \end{bmatrix},$$

implying that its value is $1.6666667 \cdot \$20 - 3.90952 \cdot 7 = 5.9667$. Note that this put value is lower than either of the two potential cash flows that it may generate. This is due to the particularly high riskless return rate.
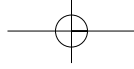
CHAPTER 8

8.1. Solve as follows:

$$y' = \frac{7x^2 + 2 \cdot 7x \cdot h + 7h^2 - 7x^2}{h}$$

$$= \lim_{h \to 0} \frac{14x \cdot h + 7h^2}{h} = \lim_{h \to 0} [14x + 7h] = 14x.$$

8.2. The derivatives are found by using the power rule (polynomial rule) as follows:

(a) $dy/dx = 5 \cdot 0 \cdot x^{0-1} = 0$;
(b) $dy/dx = 3 \cdot 7 \cdot x^{3-1} = 21x^2$;
(c) $dy/dx = 4 \cdot 2 \cdot x^{4-1} + 3 \cdot 5 \cdot x^{3-1} = 8x^3 + 15x^2$;
(d) $dy/dx = 0.5 \cdot 10 \cdot x^{0.5-1} - 3 \cdot 11 \cdot x^{3-1} = 5x^4 - 33x^2$;
(e) $dy/dx = (1/5) \cdot 5 \cdot x^{1/5-1} = (5/5)x^{-4/5} = \sqrt[5]{x^4}$;
(f) $dy/dx = -2 \cdot 2 \cdot x^{-2-1} + 1/2 \cdot 2/3 \cdot x^{1/2-1} + 1/5 \cdot 3 \cdot x^{-1/5-1} - 1 \cdot 1 \cdot x^{-1-1}$;
     $= -4x + (1/3)x^{-1/2} + 0.6x^{-0.8} - 1 = -4x + (1/3)/\sqrt{x} + 0.6/x^{0.8} - 1$.

8.3. Second derivatives are found as follows:

(a) $dy/dx = 0$;
(b) $dy/dx = 42x$;
(c) $dy/dx = 24x^2 + 30x$;
(d) $dy/dx = 20x^3 - 66x$;
(e) $dy/dx = -0.8x^{-1.8}$;
(f) $dy/dx = -4 - (1/6)/\sqrt{x^3} - 0.48/x^{-1.8}$.

8.4. Find first derivatives, set them equal to zero, and solve for x. Then check second derivatives to ensure that they are negative:

(a) $dy/dx = 30x$; $d^2y/dx^2 = 30$; there is no finite maximum.
(b) $dy/dx = 6$; $d^2y/dx^2 = 0$; there is no finite maximum.
(c) $dy/dx = -6x + 6$; $d^2y/dx^2 = -6$; $x_{min} = 1$.
(d) $dy/dx = 3x^2 + 6x + 2 = 0$; $d^2y/dx^2 = 6x + 6$; $x_{max} = -3.803848$, $x_{max} = -14.19615$.
(e) $dy/dx = 36x^2$; $d^2y/dx^2 = 72x$; there is no finite maximum; the first derivative is zero when $x = 0$, but when $x = 0$, d^2y/dx^2 is not negative.
(f) $dy/dx = 2x + 10 = 0$; $d^2y/dx^2 = 2$; the minimum occurs when $x = -5$.

8.5. Find first derivatives, set them equal to zero, and solve for x. Then check second derivatives to ensure that they are positive:

(a) $dy/dx = 30x$; $d^2y/dx^2 = 30$; $x_{min} = 0$.
(b) $dy/dx = 20$; $d^2y/dx^2 = 0$; there is no finite minimum.
(c) $dy/dx = 6x + 6$; $d^2y/dx^2 = 6$; $x_{min} = -1$.
(d) $dy/dx = 6x^2 - 12x + 1$; $d^2y/dx^2 = 6x - 12$; using the quadratic formula, we find that $x = 3.1366$ and 68.86335. The second derivative is positive in both cases. This function has two minima.
(e) $dy/dx = 36x^2$; $d^2y/dx^2 = 72x$; there is no finite minimum; the first derivative is zero when $x = 0$, but when $x = 0$, d^2y/dx^2 is not positive.
(f) $dy/dx = 2x + 10 = 0$; $d^2y/dx^2 = 2$; the minimum occurs when $x = -5$.
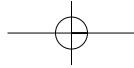
8.6.   (a)   (i)   First, find the yield to maturity (*ytm*) of the bond:

$$0 = NPV = \sum_{t=1}^{n} \frac{CF_t}{(1+ytm)^t} - P_0, \qquad \text{yield to maturity} = ytm;$$

$$0 = NPV = \frac{1,000}{(1+ytm)^1} - 900, \qquad \text{solve for } ytm;$$

$$ytm = 0.111.$$

(ii)   Use *ytm* from part (i) in the duration formula:

$$Dur = \frac{\sum_{t=1}^{n} t \cdot \frac{CF_t}{(1+ytm)^t}}{P_0}$$

(note that negative signs are omitted);

$$Dur = \frac{1 \cdot \frac{1,000}{1.111}}{900} = Dur = 1 \text{ year.}$$

(b)   (i)
$$0 = NPV = \sum_{t=1}^{n} t \frac{CF_t}{(1+ytm)^t} - P_0 = \frac{1,000}{(1+ytm)^t} - 800;$$

$$ytm = 0.118.$$

(ii)
$$Dur = \frac{\sum_{t=1}^{n} t \cdot \frac{CF_t}{(1+ytm)^t}}{P_0} = \frac{2 \cdot \frac{1,000}{1.118^2}}{800} = 2.$$

(c)   *ytm* = 0.126:

$$Dur = \frac{3 \cdot \frac{2,000}{(1.126)^3}}{1.400} = 3.$$

(d)   There are several ways to work this problem. First, consider the cash flows of the portfolio:

$$P_0 = 900 + 800 + 1,400 = 3,100;$$

$$CF_1 = 1,000, \qquad CF_2 = 1,000, \qquad CF_3 = 2,000;$$

$$0 = NPV = \frac{1,000}{(1+ytm)^1} + \frac{1,000}{(1+ytm)^2} + \frac{2,000}{(1+ytm)^3} - 3,100, \qquad ytm = 0.122;$$

$$Dur = \frac{\sum_{t=1}^{n} t \cdot \frac{CF_t}{(1+ytm)^t}}{P_0} = \frac{\frac{1 \cdot 1,000}{1.122} + \frac{2 \cdot 1,000}{(1.122)^2} + \frac{3 \cdot 2,000}{(1.122)^3}}{3,100}; \qquad Dur = 2.161 \text{ years.}$$

Second, notice that the portfolio duration is a weighted average of the bond durations: $(900/3,100) \cdot 1 + (800/3,100) \cdot 2 + (1,400/3,100) \cdot 3 = 2.161$.

(e)   An equal dollar sum is invested into each portfolio. Thus, this portfolio's duration is simply an average of the three bonds' durations. The portfolio duration equals 2.

8.7.   (a)   (i)   First find the bond's *ytm*:

$$0 = NPV = \frac{70}{(1+ytm)^1} + \frac{70}{(1+ytm)^2} + \frac{70+1,000}{(1+ytm)^3} - 950, \qquad ytm = 0.0897.$$
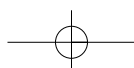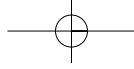
(ii)   Now, use *ytm* to find Duration:

$$Dur = \frac{\dfrac{1 \cdot 70}{1.0897} + \dfrac{2 \cdot 70}{(1.0897)^2} + \dfrac{3 \cdot 1070}{(1.0897)^3}}{950} = 2.8027.$$

(b) $$0 = NPV = \frac{120}{(1 + ytm)^1} + \frac{120}{(1 + ytm)^2} + \frac{1{,}120}{(1 + ytm)^3} - 1{,}040,$$

$$ytm = 0.1038;$$

$$Dur = \frac{\dfrac{1 \cdot 120}{1.1038} + \dfrac{2 \cdot 120}{(1.1038)^2} + \dfrac{3 \cdot 1120}{(1.1038)^3}}{900} = 2.696.$$

(c) $$0 = NPV = \frac{100}{(1 + ytm)^1} + \frac{100}{(1 + ytm)^2} + \frac{100}{(1 + ytm)^3} + \frac{1{,}100}{(1 + ytm)^4} - 900,$$

$$ytm = 0.134;$$

$$Dur = \frac{\dfrac{1 \cdot 100}{1.134} + \dfrac{2 \cdot 100}{(1.134)^2} + \dfrac{3 \cdot 100}{(1.134)^3} + \dfrac{4 \cdot 1{,}100}{(1.134)^4}}{900} = 3.456.$$

(d) $$0 = NPV = \frac{100}{(1 + ytm)^1} + \frac{100}{(1 + ytm)^2} + \frac{1{,}100}{(1 + ytm)^3} - 800,$$

$$ytm = 0.194;$$

$$Dur = \frac{\dfrac{1 \cdot 100}{1.194} + \dfrac{2 \cdot 100}{(1.194)^2} + \dfrac{3 \cdot 1{,}100}{(1.194)^3}}{800} = 2.703.$$

8.8.   (a) $P_1 = P_0 + Dur \cdot \Delta(1 + r)/(1 + r) \cdot P_0$, $P_1 = 950 - 2.8027 \cdot 0.01/1.0897 \cdot 950 = 925.57$;
       (b) $P_1 = 1{,}040 - 2.696 \cdot 0.01/1.1038 \cdot 1{,}040 = 1{,}014.60$;
       (c) $P_1 = 900 - 3.457 \cdot 0.01/1.1339 \cdot 900 = 872.56$;
   (d) $P_1 = 800 - 2.703 \cdot 0.01/1.194 \cdot 800 = 781.89$.

8.9. $Dur = 2.8027 \cdot \dfrac{950}{3{,}690} + 2.696 \cdot \dfrac{1{,}040}{3{,}690} + 3.456 \cdot \dfrac{900}{3{,}690} + 2.703 \cdot \dfrac{800}{3{,}690} = 2.910.$

8.10.   Partial derivatives are found as follows:

(i)   (a) $\partial y/\partial x = 5$;   (d) $\partial y/\partial x = 15x^2 + 7z$;
      (b) $\partial y/\partial x = 5x$;   (e) $\partial y/\partial x = 36x^2 z^5 + 3z^2$;
      (c) $\partial y/\partial x = 14x^6$;   (f) $\partial y/\partial x = \sum_{i=1}^{n} nix^{i-1}z^2$.

(ii) (a) $\partial y/\partial z = 0$;   (d) $\partial y/\partial z = 20z + 7x$;
      (b) $\partial y/\partial z = 10$;   (e) $\partial y/\partial z = 60x^3 z^4 + 6xz$;
      (c) $\partial y/\partial z = 0$;   (f) $\partial y/\partial z = 2z\sum_{i=1}^{n} nx^i$.

8.11.   Derivatives are found as follows:

(a) $dy/dx = 12(4x + 2)^2$;

(b) $dy/dx = 3x/\sqrt{(3x^2 + 8)}$;

(c) $dy/dx = 6x(12x^2 + 10x) + 6(4x^3 + 5x^2 + 3) = 96x^3 + 90x^2 + 18$;

(d) $dy/dx = 4.5(1.5x - 4)(2.5x - 3.5)^3 + 10(1.5x - 4)(2.5x - 3.5)^3$;
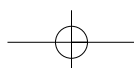
(e) $dy/dx = -50/x^3$;

(f) $dy/dx = (60x - 84) - (60x - 160)/(10x - 14)^2 = 76/(10x - 14)^2$.

8.12. First, we solve the following linear system for $z(1)$ and $z(2)$:

$$0.25z(1) + 0.05z(2) = 0.15 - 0.05;$$

$$0.05z(1) + 0.16z(2) = 0.08 - 0.05.$$

$$z(1) = 0.38666667, \qquad z(2) = 0.06666667.$$

(a) Thus, $w(1) = 0.852941$ and $w(2) = 0.147049$;

(b) $E[R(m)] = 0.139706$, $\sigma_m^2 = 0.197881$, $\sigma_m = 0.444838$.

8.12. First, we solve the following linear system for $z(1)$ and $z(2)$:

$$0.25z(1) + 0.05z(2) + 0.04z(3) = 0.15 - 0.05;$$

$$0.05z(1) + 0.16z(2) + 0.03z(3) = 0.08 - 0.05;$$

$$0.04z(1) + 0.03z(2) + 0.09z(3) = 0.06 - 0.05.$$

We multiply the following to solve for z-values:

$$\begin{bmatrix} 4.479098 & -1.09489 & -1.62575 \\ -1.09489 & 6.934307 & -1.82482 \\ -1.62575 & -1.82482 & 12.44194 \end{bmatrix} \begin{bmatrix} 0.10 \\ 0.03 \\ 0.01 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}.$$

$$z(1) = 0.398806, \qquad z(2) = 0.080292, \qquad z(3) = -0.0929.$$

(a) Thus, $w(1) = 1.032646$, $w(2) = 0.207904$, and $w(3) = -0.24055$;

(b) $E[R(m)] = 0.157096$, $\sigma_m^2 = 0.277309$, $\sigma_m = 0.526602$.

8.13. (a) $\sigma_p = \sqrt{w_f^2 \cdot \sigma_f^2 + w_m^2 \cdot \sigma_m^2 + 2w_f \cdot w_m \cdot \sigma_{f.m}}$,

$$\sigma_p = \sqrt{w_f^2 \cdot 0 + w_m^2 \cdot \sigma_m^2 + 2w_f \cdot w_m \cdot 0} = w_m \cdot \sigma_m;$$

(b) $E[R_p] = w_f E[r_f] + w_m E[R_m]$, $\qquad w_m = \dfrac{\sigma_p}{\sigma_m}$, $\qquad w_f = 1 - w_m$;

$$E[R_p] = \left(\left(1 - \frac{\sigma_p}{\sigma_m} \right) r_f + \frac{\sigma_p}{\sigma_m} E[R_m] \right) = r_f + \frac{\sigma_p}{\sigma_m} (E[R_m] - r_f]).$$

8.14. This problem is complicated by having different borrowing and lending rates. This essentially means that there will be two "Capital Market Lines," one for lending and one for borrowing. Notice that the investor's 18% required return exceeds the return of any of the three securities. This means that the the investor will probably need to leverage up her portfolio by borrowing in order to meet her requirement for expected return. Risky asset portfolio characteristics are found from the following:

$$0.02 = 0.09z_1 + 0z_2, \qquad z_1 = 0.2222;$$

$$0.06 = 0z_1 + 0.36z_2, \qquad z_2 = 0.1667, \qquad w_1 = 0.571, \qquad w_2 = 0.429;$$

$$E[R_m] = 0.097, \qquad w_f = (1 - w_m), \qquad E[R_p] = 0.18 = (1 - w_m) \cdot 0.06 + w_m \cdot 0.097.$$

Now, the allocations of funds to the stock portfolio and to the bonds are made:

$$0.18 = 0.06 + w_m \cdot 0.037, \qquad w_m = 3.243, \qquad 1 - w_m = w_f = -2.243.$$

Borrow $1,121,500.

Invest $1,621,500 in the market: $925,876.5 in security 1 and $695,623.5 in security 2.

8.15. (a) $dy/dx = 0.05e^{0.05x}$;
 (b) $dy/dx = e^x(x-1)/x^2$;
 (c) $dy/dx = 5/x$;
 (d) $dy/dx = e^x(1/x + \ln(x))$ (using the product rule).

8.16. Durations and convexities are as follows:

 (a) $ytm = 0.052612$
  $Dur = [57.00064 + 108.3024 + 2,726.544]/1,020 = -2.8351446$;
  $Con = [102.8885 + 293.2354 + 9,843.047/1,020 = 10.0384$;
 (b) $ytm = 0.0610701$;
  $Dur = [84.82003 + 159.8764 + 226.012 + 3,439.618]/1,100 = -3.554842$;
  $Con = [150.6747 + 426.0077 + 802.9775 + 15,275.38]/1,100 = 15.1409455$.

8.17. (a) $P_{1A} = 944.78$, $P_{1B} = 1,030.24$;
 (b) $P_{1A} = 948.62$, $P_{1B} = 1,033.22$;
 (c) $P_{1A} = 948.56$, $P_{1B} = 1,033.12$.

 The new bond values given in 8.17(c) are precise (subject to rounding). Note how much better the bond convexity model in 8.17(b) estimates revised bond prices than the duration model in 8.17(a). The duration model will tend to underestimate bond prices; the convexity model will tend to overestimate bond prices. However, the convexity model is normally closer to the true value.

8.18. First, set up the Lagrange function:

$$L = 50x^2 - 10x + \lambda(100 - 0.1x).$$

Next, find the first order conditions:

$$\partial y/\partial x = 100x - 0.1\lambda = 10;$$
$$\partial y/\partial \lambda = -0.1x \qquad = -100.$$

We find that $x = 1,000$ and $\lambda = 999,900$.

8.19. First set up the Lagrange function:

$$L = (25 + 3x + 10x^2) + \lambda(100 - 5x).$$
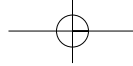
Next, find the first order conditions:

$$\frac{\partial L}{\partial x} = 3 + 20x - 5\lambda = 0, \qquad 20x - 5\lambda = -3;$$

$$\frac{\partial L}{\partial z} = 100 - 5x = 0, \qquad -5x - 0\lambda = -100.$$

(3) Next, solve the above system of equations for x and λ:

$$\begin{bmatrix} 20 & -5 \\ -5 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ \lambda \end{bmatrix} = \begin{bmatrix} -3 \\ -100 \end{bmatrix},$$

$$\mathbf{C} \qquad \cdot \ \mathbf{x} \ = \quad \mathbf{S};$$

$$x = 20, \qquad \lambda = 80\tfrac{3}{5}.$$

8.20. (a) (i) Our problem is defined as:

$$min: \sigma_p^2 = 0.04w_A^2 + 0.16w_B^2 + 0.08w_Aw_B \qquad \text{(objective function)}$$

$$s.t.: 0.15 = 0.1w_A + 0.2w_B \qquad \text{(constraint 1)},$$

$$1 = 1w_A + 1w_B \qquad \text{(constraint 2)}.$$

First, set up the Lagrange function:

$$L = (0.04w_A^2 + 0.16w_B^2 + 0.08w_Aw_B) - \lambda_1(0.15 - 0.1w_A - 0.2w_B) - \lambda_2(1 - 1w_A - 1w_B).$$

Find first order conditions:

$$\partial L/\partial w_A = 0.08w_A + 0.08w_B + 0.1\lambda_1 + 1\lambda_2 = 0;$$

$$\partial L/\partial w_B = 0.32w_B + 0.08w_A + 0.2\lambda_1 + 1\lambda_2 = 0;$$

$$\partial L/\partial \lambda_1 = 0.15 - 0.1w_A - 0.2w_B = 0;$$

$$\partial L/\partial \lambda_2 = 1 - 1w_A - 1w_B = 0.$$

Now, we have four equations with four unknowns, which can be arranged as follows:

$$0.08w_A + 0.08w_B + 0.1\lambda_1 + 1\lambda_2 = 0;$$

$$0.08w_A + 0.32w_B + 0.2\lambda_1 + 1\lambda_2 = 0;$$

$$0.1w_A + 0.2w_B + 0\lambda_1 + 0\lambda_2 = 0.15;$$

$$1w_A + 1w_B + 0\lambda_1 + 0\lambda_2 = 1.$$

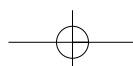We structure matrices to solve this system as follows:

$$\begin{bmatrix} 0.08 & 0.08 & 0.1 & 1 \\ 0.08 & 0.32 & 0.2 & 1 \\ 0.1 & 0.2 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_A \\ w_B \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0.15 \\ 1 \end{bmatrix},$$

$$\mathbf{C} \qquad \cdot \quad \mathbf{x} \ = \quad \mathbf{S}.$$

Step-by-step, we invert the coefficients matrix (augmented by the identity matrix) as follows:

$$\begin{array}{l} \text{row 1} \\ \text{row 2} \\ \text{row 3} \\ \text{row 4} \end{array} \begin{bmatrix} 0.08 & 0.08 & 0.1 & 1 & \vdots & 1 & 0 & 0 & 0 \\ 0.08 & 0.32 & 0.2 & 1 & \vdots & 0 & 1 & 0 & 0 \\ 0.1 & 0.2 & 0 & 0 & \vdots & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & \vdots & 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{original system}$$

$$\begin{array}{l} \text{1a} \\ \text{2a} \\ \text{3a} \\ \text{4a} \end{array} \begin{bmatrix} 1 & 1 & 1.25 & 12.5 & \vdots & 12.5 & 0 & 0 & 0 \\ 0 & 3 & 1.25 & 0 & \vdots & -12.5 & 12.5 & 0 & 0 \\ 0 & 1 & -1.25 & -12.5 & \vdots & -12.5 & 0 & 10 & 0 \\ 0 & 0 & -1.25 & -12.5 & \vdots & -12.5 & 0 & 0 & 1 \end{bmatrix} \begin{array}{l} (\text{row 1}) \cdot 12.5 \\ (\text{row 2}) \cdot 12.5 - (1a) \\ (\text{row 3}) \cdot 10(1a) \\ (\text{row 4}) \cdot 1 - (1a) \end{array}$$

$$
\begin{array}{l}
1b \\
2b \\
3b \\
4b
\end{array}
\left[
\begin{array}{cccc:cccc}
1 & 0 & 0.8\overline{3} & 12.5 & 16.\overline{6} & -4.1\overline{6} & 0 & 0 \\
0 & 1 & 0.41\overline{6} & 0 & -4.1\overline{6} & 4.16 & 0 & 0 \\
0 & 0 & -1.\overline{6} & -12.5 & -8.\overline{3} & -4.1\overline{6} & 10 & 0 \\
0 & 0 & -1.25 & -12.5 & -12.5 & 0 & 0 & 1
\end{array}
\right]
\begin{array}{l}
(1a)-(2b) \\
(2a)\cdot 1/3 \\
(3a)-(2b) \\
(4a)
\end{array}
$$

$$
\begin{array}{l}
1c \\
2c \\
3c \\
4c
\end{array}
\left[
\begin{array}{cccc:cccc}
1 & 0 & 0 & 6.25 & 12.5 & -6.25 & 5 & 0 \\
0 & 1 & 0 & -3.125 & -6.25 & 3.125 & 2.5 & 0 \\
0 & 0 & 1 & 7.5 & 5 & 2.5 & -6 & 0 \\
0 & 0 & 0 & -3.125 & -6.25 & 3.125 & -7.5 & 1
\end{array}
\right]
\begin{array}{l}
(1b)-(3c)\cdot 0.83 \\
(2b)-(3c)\cdot 0.416 \\
(3b)\cdot[-1/(1.6)] \\
(4b)-(3c)\cdot(-1.25)
\end{array}
$$

$$
\begin{array}{l}
1d \\
2d \\
3d \\
4d
\end{array}
\left[
\begin{array}{cccc:cccc}
1 & 0 & 0 & 0 & 0 & 0 & -10 & 2 \\
0 & 1 & 0 & 0 & 0 & 0 & 10 & -1 \\
0 & 0 & 1 & 0 & -10 & 10 & -24 & 2.4 \\
0 & 0 & 0 & 1 & 2 & -1 & 2.4 & -0.32
\end{array}
\right]
\begin{array}{l}
(1c)-(4d)\cdot 6.2 \\
(2c)-(4d)\cdot -3.125 \\
(3c)-(4d)\cdot 7.5 \\
(4b)\cdot[-1/(3.125)]
\end{array}
$$

$$
\qquad \mathbf{I} \qquad\qquad\qquad \mathbf{C}^{-1}
$$

$$
\begin{bmatrix}
0 & 0 & -10 & 2 \\
0 & 0 & 10 & -1 \\
-10 & 10 & -24 & 2.4 \\
2 & -1 & 2.4 & -0.32
\end{bmatrix}
\begin{bmatrix}
0 \\ 0 \\ 0.15 \\ 1
\end{bmatrix}
=
\begin{bmatrix}
w_A \\ w_B \\ \lambda_1 \\ \lambda_2
\end{bmatrix}.
$$

$$
\qquad \mathbf{C}^{-1} \qquad\qquad \cdot\ \mathbf{s}\ =\ \mathbf{x}
$$

We multiply to find the following:

$$
w_A = (0\cdot 0)+(0\cdot 0)+(-10\cdot 0.15)+(2\cdot 1)=0.5;
$$

$$
w_B = (0\cdot 0)+(0\cdot 0)+(10\cdot 0.15)+(-1\cdot 1)=0.5;
$$

$$
\lambda_1 = (-10\cdot 0)+(10\cdot 0)+(-24\cdot 0.15)+(2.4\cdot 1)=-1.2;
$$

$$
\lambda_2 = (2\cdot 0)+(-1\cdot 0)+(2.4\cdot 0.15)+(-0.32\cdot 1)=0.04.
$$

Notice that since only two securities will be included in the portfolio, we can infer immediately from the expected return constraint that w_A must equal w_B, which must equal 0.5. This simple algorithm will not work when the number of securities in the portfolio exceeds 2.
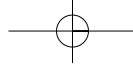
(ii)
$$
min:\ \sigma_p^2 = 0.04w_A^2 + 0.16w_B^2 + 0.08w_Aw_B
$$

$$
s.t.:\ 0.12 = 0.1w_A + 0.2w_B,
$$

$$
1 = 1w_A + 1w_B.
$$

$$
L = 0.04w_A^2 + 16w_B^2 + 0.08w_Aw_B - \lambda_1(0.12 - 0.1w_A - 0.2w_B) \\
- \lambda_2(1 - 1w_A - 1w_B).
$$

First order conditions are:

$$
\partial L/\partial w_A = 0.08w_A + 0.08w_B + 0.1\lambda_1 + 1\lambda_2 = 0;
$$

$$
\partial L/\partial w_B = 0.08w_A + 0.32w_B + 0.2\lambda_1 + 1\lambda_2 = 0;
$$

$$\partial L/\partial \lambda_1 = 0.1w_A + 0.2w_B + 0\lambda_1 + 0\lambda_2 = 0.12;$$

$$\partial L/\partial \lambda_2 = 1w_A + 1w_B + 0\lambda_1 + 0\lambda_2 = 1.$$

Thus, our system is:

$$\begin{bmatrix} 0.08 & 0.08 & 0.1 & 1 \\ 0.08 & 0.32 & 0.2 & 1 \\ 0.1 & 0.2 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_A \\ w_B \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0.12 \\ 1 \end{bmatrix}.$$

$$\mathbf{C} \qquad \cdot \ \mathbf{x} \ = \ \mathbf{S}$$

Notice that the coefficients matrix ($\mathbf{C}$) is identical to that in 21.a.i above. Thus, its inverse is identical to $\mathbf{C}^{-1}$ in part 8.19(a,i). Notice also that only element three in the solutions vector has changed. Thus, we determine our weights as follows:

$$\begin{bmatrix} 0 & 0 & -10 & 2 \\ 0 & 0 & 10 & -1 \\ -10 & 10 & -24 & 2.4 \\ 2 & -1 & 2.4 & -0.32 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0.12 \\ 1 \end{bmatrix} = \begin{bmatrix} w_A \\ w_B \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 0.2 \\ -0.48 \\ -0.032 \end{bmatrix},$$

$$\mathbf{C}^{-1} \qquad \cdot \quad \mathbf{S} \ = \ \mathbf{x} \qquad w_A = 0.8,$$
$$w_B = 0.2.$$

(iii) Notice that only the third element in $\mathbf{x}$ has changed. Thus:

$$\begin{bmatrix} w_A \\ w_B \\ w_C \\ w_D \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.8 \\ 1.92 \\ 0.112 \end{bmatrix},$$
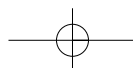
$$\mathbf{x} \qquad w_A = 0.2,$$
$$w_B = 0.8.$$

(b) Our asset returns and standard deviations are as follows:

Asset	E(R)	σ	
A	0.10	0.20	$\sigma_{A,B} = 0.04$
B	0.20	0.40	$\sigma_{A,rf} = 0$
r_f	0.09	0	$\sigma_{B,rf} = 0$

(i) The Lagrange Function is now:

$$L = (0.04w_A^2 + 0.16w_B^2 + 0.08w_A w_B) - \lambda_1(0.15 - 0.1w_A - 0.2w_B - 0.09w_{rf})$$
$$- \lambda_2(1 - 1w_A - 1w_B - 1w_{rf}).$$

Because $\sigma_{rf} = \sigma_{B,rf} = \sigma_{A,rf} = 0$, w_{rf} terms are dropped from the first set of parentheses. The first order conditions are:

$$\partial L/\partial w_A = 0.08w_A + 0.08w_B + 0.1\lambda_1 + 1\lambda_2 = 0;$$

$$\partial L/\partial w_A = 0.32w_B + 0.08w_A + 0.2\lambda_1 + 1\lambda_2 = 0;$$

$$\partial L/\partial w_{rf} = -0.09\lambda_1 - 1\lambda_2 = 0;$$

$$\partial L/\partial \lambda_1 = 0.15 - 0.1w_A - 0.2w_B - 0.09w_{rf} = 0;$$

$$\partial L/\partial \lambda_2 = 1 - 1w_A - 1w_B - 1w_{rf} = 0.$$

Now we have a (5×5) coefficient matrix:

$$\begin{bmatrix} 0.08 & 0.08 & 0 & 0.1 & 1 \\ 0.08 & 0.32 & 0 & 0.2 & 1 \\ 0 & 0 & 0 & 0.09 & 1 \\ 0.1 & 0.2 & 0.09 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_A \\ w_B \\ w_{rf} \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.15 \\ 1 \end{bmatrix},$$

$$\mathbf{C} \qquad\qquad \cdot \quad \mathbf{x} \quad = \quad \mathbf{s}.$$

Simply solve this system, to obtain:

$$\begin{bmatrix} 14.68447 & -1.33495 & -13.3495 & -6.79612 & 0.61165 \\ -1.33495 & 0.121359 & 1.213592 & 9.708738 & -0.87379 \\ -13.3495 & 1.213592 & 12.13592 & -2.91262 & 1.262136 \\ -6.79612 & 9.708738 & -2.91262 & -23.301 & 2.097087 \\ 0.61165 & -0.87379 & 1.262136 & 2.097087 & -0.18874 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.15 \\ 1 \end{bmatrix} = \begin{bmatrix} w_A \\ w_B \\ w_{rf} \\ \lambda_1 \\ \lambda_2 \end{bmatrix},$$

$$\mathbf{C^{-1}} \qquad\qquad\qquad \cdot \quad \mathbf{s} \quad = \quad \mathbf{x}.$$

Thus, we have

$$w_A = -0.40777,$$

$$w_B = 0.582524,$$

$$w_{rf} = 0.825243.$$

(ii)   If the return constraint were to decrease to 0.12, only the fourth element in the solutions vector would change:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.12 \\ 1 \end{bmatrix}, \quad \text{which results in the following weights:} \quad \begin{aligned} w_A &= -0.20388, \\ w_B &= 0.291262, \\ w_{rf} &= 0.912621. \end{aligned}$$

(iii)   Increasing the return constraint to 0.18 results in the following weights:

$$w_A = -0.61165, \qquad w_B = 0.873786, \qquad w_{rf} = 0.737864.$$

8.21. $L = 0.16w_2^2 + 0.64w_3^2 + \lambda_1(0.20 - 0.05w_1 - 0.15w_2 - 0.25w_3) + \lambda_2(1 - w_1 - w_2 - w_3);$

$\partial L/\partial w_1 = -0.05\lambda_1 - 1\lambda_2 = 0,$

$\partial L/\partial w_2 = 1.00w_2 - 0.07\lambda_1 - 1\lambda_2 = 0,$

$\partial L/\partial w_3 = 1.62w_3 - 0.11\lambda_1 - 1\lambda_2 = 0,$

$\partial L/\partial \lambda_1 = -0.05w_1 - 0.07w_2 - 0.11w_3 = -0.10,$

$\partial L/\partial \lambda_2 = -w_1 - w_2 - w_3 = -1;$

$w_1 = -0.27451, \qquad w_2 = 0.661765, \qquad w_3 = 0.612745.$

Chapter 9

9.1.   (a) $F(x) = k;$
      (b) $F(x) = 5x + k;$
      (c) $F(x) = 5x^3 + k;$
      (d) $F(x) = 5x^3 + 5x + k;$
      (e) $F(x) = e^x + k;$
      (f) $F(x) = e^{0.5x} + k;$
      (g) $F(x) = 5^x + k;$
      (h) $F(x) = \ln(x) + k.$

9.2.   (a) $\displaystyle\int_0^1 x\,dx = \tfrac{1}{2}x^2 \Big|_0^1 = (\tfrac{1}{2} + k) - (0 + k) = \tfrac{1}{2};$

      (b) $\displaystyle\int_2^4 (x + 5)\,dx = [\tfrac{1}{2}x^2 + 5x]\Big|_2^4 = (\tfrac{1}{2}\cdot 16 + 5\cdot 4 + k) - (\tfrac{1}{2}\cdot 4 + 5\cdot 2 + k) = 16;$

      (c) $\displaystyle\int_0^{20} 100{,}000e^{0.10t}\,dt = 100{,}000\left[\frac{e^{0.10t}}{0.10}\right]\Bigg|_0^{20}$

      $= 1{,}000{,}000e^2 - 1{,}000{,}000e^0 = 1{,}000{,}000(e^2 - 1) = 6{,}389{,}056.$

9.3.   (a) $(0.2 \cdot 0.2) + (0.2 \cdot 0.4) + (0.2 \cdot 0.6) + (0.2 \cdot 0.8) + (0.2 \cdot 1) = 0.04 + 0.08 + 0.12 + 0.16 + 0.20 = 0.6;$
      (b) $2.96 + 3.12 + 3.28 + 3.44 + 3.6 = 16.4;$
      (c) $596{,}730 + 890{,}216 + 1{,}328{,}047 + 1{,}981{,}213 + 2{,}955{,}622 = 7{,}751{,}828.$

9.4.   (a) $P(x) = \int p(x)\,dx = 0.5x^3 + 0.5x^2.$
      (b)   The distribution function for x will be $\int f(x)p(x)\,dx$, which, since $f(x) = x$, equals $\int (x \cdot 1.5x^2 + x \cdot x)\,dx = \int (1.5x^3 + x^2)\,dx = 0.375x^4 + 0.3333x^3$. The probability that x will be in the range 0.2–1.0 equals $P[0.2 < x < 1] = (0.5x^3 + 0.5x^2)|_{0.2}^1 = [(0.5 \cdot 1^3 + 0.5 \cdot 1^2) - (0.5 \cdot 0.2^3 + 0.5 \cdot 0.2^2)][1 - 0.024] = 0.976$. The expected value of x given that it falls within this range is a conditional distribution determined by:

      $E[x|0.2 < x < 1] = \{\int_{0.2}^1 xp(x)\}/\{P[0.2 < x < 1]\}$

      $= [(0.375 \cdot 1^4 + 0.3333 \cdot 1^3) - (0.375 \cdot 0.2^4 + 0.3333 \cdot 0.2^3)]/0.976$

      $= 0.7050666/0.976 = 0.7224043.$

      (c)   The probability that x will be in the range 0–0.2 equals $(0.5x^3 + 0.5x^2)|_0^{0.2} = [(0.5 \cdot 0.2^3 + 0.5 \cdot 0.2^2) - 0] = 0.024$. The expected value of x given that it falls within this range is a conditional distribution determined by:
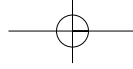
$$E[x|0 < x < 0.2] = \{\int_0^{0.2} xp(x)\}/\{P[0 < x < 0.2]\}$$
$$= (0.375 \cdot 0.2^4 + 0.3333 \cdot 0.2^3) - (0.375 \cdot 0^4 + 0.3333 \cdot 0^3)]/0.024$$
$$= 0.0032667/0.024 = 0.136111.$$

(d) $E[x] = \int_0^1 xp(x) = [(0.375 \cdot 1^4 + 0.3333 \cdot 1^3) - (0.375 \cdot 0^4 + 0.3333 \cdot 0^3) = 0.7083333.$

(e) $\sigma^2 = \int_0^1 [f(x)]^2 p(x)dx - (\int_0^1 f(x)p(x)dx)^2$
$$= \int_0^1 x^2 p(x)dx - (\int_0^1 xp(x)dx)^2 = \int_0^1 x^2 \cdot (1.5x^2 + x)dx - [\int_0^1 x \cdot (1.5x^2 + x)dx]^2.$$

And since the term to the right of the minus sign is the expected value squared:

$$\sigma^2 = \int_0^1 (1.5x^4 + x^3)dx - 0.708333^2$$
$$= (0.3x^5 + 0.25x^4)|_0^1 - 0.7083333^2$$
$$= 0.55 - 0 - 0.5017 = 0.0483.$$

9.5. (a) $P(x) = \int p(x)dx = x.$

(b) $E[S|50 \le S \le 100] = \{\int_{0.5}^1 100xp(x)dx\}/\{P[50 \le S \le 100]\} = \{\int_{0.5}^1 100x \cdot 1 \, dx\}$
$/\{\int_{0.5}^1 p(x)dx\} = \{50 \cdot 1^2 - 50 \cdot 0.5^2\}/0.5 = 75.$

(c) $E[S|0 \le S \le 50] = \{\int_0^{0.5} 100xp(x)dx\}/\{P[0 \le S \le 50]\} = \{\int_0^{0.5} 100x \cdot 1 \, dx\}/\{\int_0^{0.5} p(x)dx\}$
$= \{50 \cdot 0.50^2 - 50 \cdot 0^2\}/0.5 = 12.5.$

(d) $E[S] = \int_0^1 f(x)p(x)dx = \int_0^1 100x \, dx = 50x^2|_0^1 = 50.$

(e) $\sigma^2 = \int_0^1 [f(x)]^2 p(x)dx - (\int_0^1 f(x)p(x)dx)^2 = \int_0^1 10,000x^2 p(x)dx - (\int_0^1 100xp(x)dx)^2$
$= \int_0^1 10,000x^2 dx - (\int_0^1 100x \, dx)^2.$

Since the term to the right of the minus sign is the expected value:

$$\sigma^2 = \int_0^1 10,000x^2 dx - 50^2 = (3333.33)x^3|_0^1 - 50^2$$
$$= (3,333.33) \cdot 1 - 0 - 2,500 = 833.33.$$

(f) $E[S - 50|50 \le S \le 100] = 25.$

(g) $E[CF_t] = \{0.5 \cdot 0\} + \{0.5 \cdot E[S - 50|50 \le S \le 100]\} = 12.5.$

(h) $PV(E[CF]) = 12.5e^{-0.1 \cdot 1} = 11.31.$

9.6. We integrate the density functions as follows:

$$P_f(x) = \int 4x^3 = x^4 \qquad \text{for } 0 \le x \le 1,$$
$$P_g(x) = \int (3x^4 + 0.08x) = \tfrac{3}{5}x^5 + \tfrac{2}{5}x^2 + k \qquad \text{for } 0 \le x \le 1.$$

9.7. The amount of dividend payment to be received at any infinitesimal time interval dt equals $f(t)dt = 3,000,000dt$. The present value of this sum equals $f(t)e^{-kt}dt = 3,000,000e^{-0.06t}$. To find the present value of a sum received over a finite interval beginning with $t = 0$, one may apply the definite integral as follows:

$$PV[0,T] = \int_0^T f(t)e^{-kt}dt,$$

$$PV[0,10] = \int_0^{10} 3,000,000e^{-0.06t}dt = 3,000,000\left[-\frac{e^{-0.06t}}{0.06}\right]\Bigg|_0^{10}$$

$$= \frac{3,000,000}{0.06}(1 - e^{-0.6}) = \$22,559,418.$$

9.8. The dividend stream is evaluated as follows:

$$PV[0,2] = \frac{10,000}{0.05 - 0.03}(1 - e^{(0.03-0.05) \cdot 2}) = 19,605.28.$$

9.9. (a) The differential equation for this problem can be created in two steps. First, the account should generate interest continuously (including interest on any payments deducted from the account) as follows:

$$\frac{\mathrm{d}FV_t}{\mathrm{d}t} = iFV_0.$$

However, deductions will be made continuously from the account. The interest that these deductions would have accumulated must be deducted from the total interest given above. These deductions will reduce the amount of interest that the account will draw in the future. Assume $T - t$ relevant years in the future. The payments deducted from the account continuously are $PMT\mathrm{d}t$. These payments plus $T - t$ years of interest that would otherwise have accumulated on those payments are as follows:

$$PMT e^{i(T-t)}\mathrm{d}t.$$

The difference between these is the differential equation representing the evolution of the account:

$$\mathrm{d}FV_t = [iFV_0 - PMT e^{i(T-t)}]\,\mathrm{d}t.$$

(b) To find the state of the account (system) at any time T, integrate over t the final equation from part 9.9(a) as follows:

$$FV_T = \int_0^T [iFV_0 - PMT e^{i(T-t)}]\mathrm{d}t,$$

$$FV_T = FV_0 e^{iT} - \int_0^T [PMT e^{i(T-t)}]\mathrm{d}t,$$

where $FV_0 = \$1,000,000$. This integral is solved as follows:

$$FV_T = FV_0 e^{iT} - \frac{1}{-i}[PMT e^{i(T-t)}]\Big|_0^T,$$

$$FV_T = FV_0 e^{iT} + \frac{PMT}{i}[e^{i(T-T)}] - \frac{PMT}{i}[e^{i(T-0)}],$$

$$FV_T = FV_0 e^{iT} + \frac{PMT}{i}[1 - e^{iT}].$$

(c) Substitute numbers in the solution to part 9.9(b) as follows:

$$FV_{10} = \$1,000,000 e^{0.06 \cdot 10} + \frac{\$75,000}{0.06}[1 - e^{0.06 \cdot 10}]$$

$$= \$1,822,118.8 - \$1,027,648.5 = \$794,470.3.$$
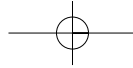
(d) Solve the following for T, to find that the retiree runs out of money in approximately 26.83 years:

$$FV_T = 0 = \$1,000,000 e^{0.06 \cdot T} + \frac{\$75,000}{0.06}[1 - e^{0.06 \cdot 10}]$$

$$= -\$250,000 e^{0.06 \cdot T} + \$1,250,000;$$

$$\ln(1,250,000) = \ln(250,000) + 0.06T, \qquad T = 26.82397.$$

9.10. (a) First, the account should generate interest continuously (including interest on any payments deducted from the account) as follows:

$$\frac{\mathrm{d}FV_t}{\mathrm{d}t} = iFV_0.$$

However, deductions will be made continuously from the account. The interest that these deductions would have accumulated must be deducted from the total interest given above. These deductions will reduce the amount of interest that the account will draw in the future. Assume $T - t$ relevant years in the future. The payments deducted from the account continuously are $PMT_0 e^{gt} dt$. These payments plus $T - t$ years of interest that would otherwise have accumulated on those payments are as follows:

$$PMT_0 e^{gt} e^{i(T-t)} dt.$$

The difference between these is the differential equation representing the evolution of the account:

$$dFV_t = [iFV_0 - PMT_0 e^{gt} e^{i(T-t)}] dt.$$

(b) To find the state of the account (system) at any time T, integrate over t the final equation from part 9.10(a) as follows:

$$FV_T = \int_0^T [iFV_0 - PMT_0 e^{gt} e^{i(T-t)}] dt,$$

$$FV_T = FV_0 e^{iT} - \int_0^T [PMT_0 e^{gt} e^{i(T-t)}] dt,$$

where $FV_0 = \$1,000,000$. This integral is solved as follows:

$$FV_T = FV_0 e^{iT} - \frac{1}{g-i} [PMT_0 e^{gt+i(T-t)}]|_0^T,$$

$$FV_T = FV_0 e^{iT} + \frac{PMT_0}{i-g} [e^{gT+i(T-T)}] - \frac{PMT_0}{i-g} [e^{g \cdot 0 + i(T-0)}],$$

$$FV_T = FV_0 e^{iT} + \frac{PMT_0}{i-g} [e^{gT} - e^{iT}].$$

(c) Substitute numbers in the solution to part 9.10(b) as follows:

$$FV_{10} = \$1,000,000 e^{0.06 \cdot 10} + \frac{\$75,000}{0.06 - 0.02} [e^{0.02 \cdot 10} - e^{0.06 \cdot 10}]$$

$$= \$1,822,118.8 - \$1,126,342.6 = \$695,776.22.$$

(d) Solve the following for T, to find that the retiree runs out of money in approximately 19.05 years:

$$FV_T = 0 = \$1,000,000 e^{0.06 \cdot T} + \frac{\$75,000}{0.06 - 0.02} [e^{0.02T} - e^{0.06 \cdot T}]$$

$$= -\$875,000 e^{0.06 \cdot T} + \$1,875,000 e^{0.02T};$$

$$\ln(1,875,000) = \ln(875,000) + 0.04T, \qquad T = 19.0535.$$

CHAPTER 10

10.1. (a) $c_T = \text{MAX}[0, S_T - X]$; $c_T = \$0$ or $\$15$.

(b) One can construct a one period hedge for a call option by shorting α shares of stock per option contract such that $c_u - \alpha u S_0 = c_d - \alpha d S_0$. We solve for the hedge ratio α as follows:
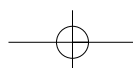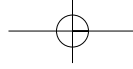
$$\alpha = \frac{C_u - C_d}{S_0(u - d)},$$

$$\alpha = \frac{15 - 0}{50(1.4 - 0.6)} = 0.375.$$

(c) Since shorting α shares of stock ensures that the portfolio is perfectly hedged, the hedged portfolio must earn the riskless rate of return $c_u - \alpha u S_0 = c_d - \alpha d S_0 = (c_0 - \alpha S_0)e^{-r_f T}$. Thus, the current value of the call can be solved for as follows:

$$C_0 = \frac{1 + r_f \alpha S_0 + C_d - \alpha d S_0}{1 + r_f},$$

$$C_0 = \frac{(1.1111) \cdot 0.375 \cdot 50 + 0 - 0.375 \cdot 0.6 \cdot 50}{1.1111} = 8.68.$$

10.2. The hedge ratio for the call equals 1. Since the riskless return rate is 0.125, the call's current value must be \$4.8888889.

10.3. The following are call and put values:

| n	c_0	p_0
2	4.62	29.96
3	3.91	29.25
8	3.87	29.21

10.4. $c_0 = \$5.10$; $p_0 = \$11.61$.

10.5. (a) $d_1 = 0.6172$; $d_2 = 0.1178$; $N(d_1) = 0.7314$; $N(d_2) = 0.5469$; $c_0 = 11.05$; with put–call parity, $p_0 = 4.34$.

(b) Use $X = 30$; $d_1 = 0.925$; $d_2 = 0.4245$; $N(d_2) = 0.6644$; $1 - N(d_2) = 0.3356$.

10.6. First, value the calls using the Black–Scholes model, then use put–call parity to value the puts:

$$p_0 = Xe^{-rT} + c_0 - S_0.$$

Thus, we will first compute d_1, d_2, $N(d_1)$, and $N(d_2)$ for each of the calls; then we will compute each call's value. Finally, we will use put–call parity to value each of the puts.

	Option 1	Option 2	Option 3	Option 4
d_1	0.957739	−0.163836	0.061699	0.131632
d_2	0.657739	−0.463836	−0.438301	−0.292626
$N(d_1)$	0.830903	0.434930	0.524599	0.552365
$N(d_2)$	0.744647	0.321383	0.330584	0.384904
Call	7.395	2.455	4.841	4.623
Put	0.939	5.416	7.803	5.665

10.7. First, value the calls using the Black–Scholes model, then use put–call parity to value the puts. Thus, we will first compute d_1, d_2, $N(d_1)$, and $N(d_2)$ for each of the calls; then
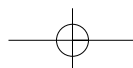
we will compute each call's value. Finally, we will use put–call parity to value each of the puts.

First, find for each of the 15 call values for d_1:

X	March	April	May
50	1.412245	1.147191	1.020949
55	0.733636	0.625012	0.578796
60	0.114115	0.148301	0.175142
65	−0.455789	−0.290231	−0.196184
70	−0.983438	−0.696249	−0.539978

Next, find for each of the 15 calls values for d_2:

X	March	April	May
50	1.271796	0.964667	0.805390
55	0.593187	0.442488	0.363237
60	−0.026334	−0.034223	−0.040417
65	−0.596239	−0.472755	−0.411743
70	−1.123888	−0.878773	−0.755537

Now, find $N(d_1)$ for each of the 15 calls:

X	March	April	May
50	0.921061	0.874349	0.846361
55	0.768415	0.734019	0.718637
60	0.545427	0.558947	0.569516
65	0.324271	0.385820	0.422233
70	0.162696	0.243137	0.294606

Next, determine $N(d_2)$ for each of the 15 calls:

X	March	April	May
50	0.898277	0.832644	0.789703
55	0.723472	0.670932	0.641786
60	0.489495	0.486350	0.483880
65	0.275508	0.318194	0.340264
70	0.130530	0.189762	0.224963

Finally, use $N(d_1)$ and $N(d_2)$ to value the calls and use put–call parity to value the puts:

Calls

X	March	April	May
50	10.626	<u>11.260</u>	11.866
55	<u>6.558</u>	7.522	8.329
60	3.536	<u>4.658</u>	5.557
65	1.658	2.681	3.536
70	<u>0.681</u>	<u>1.442</u>	2.156

Puts

X	March	April	May
50	0.319	0.742	<u>1.145</u>
55	1.220	1.952	2.536
60	3.168	<u>4.037</u>	4.692
65	<u>6.259</u>	7.008	7.599
70	<u>10.251</u>	10.717	11.147

The options whose values are underlined are overvalued by the market; they should be sold. Other options are undervalued by the market; they should be purchased.

10.8.   Implied volatilities are given as follows:

(a) $X = 40$, $\sigma = 0.2579$;
(b) $X = 45$, $\sigma = 0.3312$;
(c) $X = 50$, $\sigma = 0.2851$;
(d) $X = 55$, $\sigma = 0.2715$;
(e) $X = 60$, $\sigma = 0.2704$.