

Employment and hours over the business cycle*

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Approximately one quarter of the adjustment in total hours of employment over the business cycle represents adjustments in hours, while the remainder is explained by changes in employment. Real business cycle models characterize agents as either continuously adjusting their hours or making only labor force participation decisions about jobs with indivisible hours. In this paper we extend the representative agent framework to allow for decisions on both participation and hours. We calibrate and simulate a dynamic version of the model and show that it is better able to mimic some features of the aggregate data.

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1. Introduction

One of the striking features of U.S. data is that as much as three quarters of the variation in total hours of employment takes the form of movements in and out of the labor force rather than adjustments in average hours of work. Modern business cycle models have not addressed this feature of the data. Real business cycle theories based on representative agent models have abstracted from these facts by characterizing agents as either continuously adjusting their hours or

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making only labor force participation decision about jobs with indivisible hours. One consequence of this simplification is that these models have not been entirely successful at explaining the fluctuations in hours worked relative to productivity. These models also imply labor supply elasticities that are inconsistent with the available microeconomic evidence. It is a cornerstone of modern empirical research on labor supply that the participation decision and the hours of work decision are distinct and that it is important to model them both. For example, the econometric techniques pioneered by Heckman and applied by many [e.g., Cogan (1981)] involve estimating a participation equation as a prelude to obtaining unbiased estimates of labor supply. In this paper we extend the representative agent framework in a way that is precisely in the spirit of the empirical labor supply literature; workers decide on both participation and hours. In addition, there are fixed costs associated with the decision to participate in employment. We study the implications of this model for aggregate labor supply elasticities and for the volatility of hours, employment, and productivity in a simple model economy.

One of the major challenges to equilibrium real business cycle theories has been the claim that they assume a degree of intertemporal substitution in labor supply that seems inconsistent with the available empirical evidence. It is difficult to reconcile the large fluctuations in aggregate hours of work and the fluctuations in hours relative to productivity with existing estimates of the elasticity of labor supply. Kydland and Prescott (1982) presented a model with time-to-build technology and non-time-separable preferences that implies substantial intertemporal substitution in labor supply: when wages are temporarily high workers increased their hours. This highly elastic labor supply behavior is viewed as inconsistent with both microeconomic evidence based on panel studies [Ashenfelter (1984)] and macroeconomic evidence [see Altonji (1982), Mankiw, Rotemberg, and Summers (1985)]. These empirical studies reveal insufficient intertemporal substitution to explain the observed fluctuations in hours worked. Moreover, the evidence indicates that much of the fluctuation in aggregate hours of work over the business cycle takes the form of fluctuations in employment, the extensive margin, rather than changes in hours by employed workers, the intensive margin, as is assumed in the model economy studied by Kydland and Prescott.

Rogerson (1984) constructed a model economy in which labor supply is indivisible, that is, individuals either work a given number of hours or not at all. In this setting, all fluctuations in aggregate hours of work are due to fluctuations in employment. Hansen (1985) extended Rogerson's model to a growth setting and then calibrated it using the methods of Kydland and Prescott. His results demonstrated that such a model was capable of explaining the high variability in total hours worked even though individuals do not substitute across time. Such a model could thus reconcile low measurements of the intertemporal substitution elasticity with observed fluctuations in aggregate hours. It has the unfortunate

feature that *all* fluctuations in aggregate hours are due to fluctuations along the extensive margin. Moreover, it implies a ratio of fluctuations in aggregate hours to productivity nearly twice that found in U.S. data.

In this paper we extend the representative agent business cycle framework in a way that permits workers to adjust their labor supply along both the intensive and extensive margins. We then compare the implications of this more realistic specification to models that display only adjustment on one or the other margin. We show that ignoring either margin of adjustment can lead to significant bias in the implied labor supply elasticity.

A model with adjustment along both margins has been developed by Cho and Rogerson (1988). They achieve this feature by introducing heterogeneity in the opportunity sets of household decision makers.¹ In this paper, we assume a continuum of agents with identical preferences and opportunity sets. The special feature of our model is that agents are assumed to have a fixed cost associated with labor supply that depends on the fraction of days in a period that they will be employed. In equilibrium the cost of participating in the labor force turns out to be an increasing function of the employment rate. We interpret this feature as reflecting the costs of replacing home production. Other ways of displaying adjustment on both margins are also possible. In a recent paper Kydland and Prescott (1991) achieve essentially the same thing by assuming that there is a cost associated with moving people between the household sector and the production sector. Card (1989) describes a model in which both margins of adjustment arise from the demand side because of features of the technology.² Whatever abstraction is used to capture this features of the labor market, our results suggest that it is important to represent both margins.

In the next section of the paper we describe a static version of our economy and discuss the decision problem facing the representative worker. Section 3 describes the equilibrium and the fourth section presents some examples that illustrate how what labor economists call the labor supply elasticity depends on both margins of adjustment. These examples illustrate the dramatic differences in aggregate labor supply elasticities implied by different model economies. Section 5 extends the model to a dynamic setting and discusses calibration and simulation. We present the results from three examples. In the first example the parameters are calibrated to match the observation in aggregate data that three quarters of fluctuations in the total hours of works are due to fluctuations in

¹Another alternative would be to introduce heterogeneity in preferences. One problem with this approach is that, to collapse the model into a representative agent framework requires weighting individual utilities. Rogerson (1987) reports one interesting case of weight determination.

²Card looks at the breakdown of aggregate hours into average hours and movements in and out of employment using a different data set. His results suggest a slightly different breakdown than the one quarter/three quarter we use.

employment while one quarter is due to fluctuations in average hours. Our results show that this model is able to replicate almost exactly the variability of hours relative to productivity that is found in the U.S. data. In the second example we calibrate the model using observations from the Panel Study on Income Dynamics. The results from this model economy display considerably less variation in aggregate hours than do the data from the U.S. economy or from the previous example. In the third example, we use a log linear specification of preferences like that used in Hansen (1985) but add a fixed cost term. This allows us to calculate the welfare costs of moving workers from the household to the market sector that are consistent with the observed U.S. output fluctuations.

2. The economy

In this section we describe a model economy with a continuum of agents (or households) uniformly distributed on the unit interval $[0, 1]$. Each agent has identical preferences and the same opportunity set. There are three goods: labor, capital, and output. We first describe a static single-period model which we later extend to a dynamic setting. Capital and labor are inputs to the production function:

$$f(K, N): R_+ \times R_+ \rightarrow R_+, \quad (1)$$

where K and N are the aggregate capital stock and labor input. We will use uppercase letters to denote aggregate per capita variables and lowercase letters to denote individual variables. The production function is continuous and strictly monotonic in K and N , and concave in K and N separately. In addition, it is assumed to be homogeneous of degree one and $f(0, 0) = 0$. Anticipating the dynamic version, we introduce a multiplicative productivity shock, λ , and write the production function as $\lambda f(K, N)$. For the time being we will assume λ is fixed.

Each agent is endowed with one unit of time and one unit of capital. Time is completely divisible, so there is no indivisibility in labor supply. The utility function is assumed to be separable between consumption and leisure:

$$U(c, l) = u(c) - v(1 - l), \quad (2)$$

where c and l are consumption and leisure, respectively, and $n = 1 - l$ is labor supplied to the market. We further assume that:

- (i) u and v are twice continuously differentiable and increasing.
- (ii) u is strictly concave, v is strictly convex, and $v(0) = 0$.
- (iii) $\lim_{c \rightarrow 0} u'(c) = \infty$, $\lim_{c \rightarrow \infty} u'(c) = 0$.
- (iv) $\lim_{n \rightarrow 0} v'(n) = 0$, $\lim_{n \rightarrow 1} v'(n) = \infty$.
- (v) $u'(c + y) + u''(c + y)c \geq 0$ for all $c > 0$ and $y \geq 0$.

These are all standard conditions except (v) which is imposed to guarantee that the labor supply curve is not backward-bending.

In order to capture the features that we are interested in, that people choose on both margins, we need to introduce some additional assumptions. Each time period, say a quarter, is divided up into a large number of days. Individuals in this economy are assumed to make two kinds of choices; they must choose the number of days in each period on which to work and they must choose the number of hours to work on each of the days that they do work. We will let n denote the number of hours they work on any given day and we let e denote the fraction of days they work in each quarter. On any day that the agent does work the utility will be

$$U(c, l) = u(c) - v(n).$$

The *average* daily utility for an individual will be

$$U(c, l) = u(c) - v(n)e, \quad (3)$$

where e represents the fraction of days in the period that the agent works.

In addition to the above noted standard features we assume that there is a fixed cost associated with each day the agent chooses to work. The idea that such costs exist and may be important has been discussed by many authors; Hall (1987), Kydland and Prescott (1989), and Hansen (1985) are just a few examples. Typically in these discussions the fixed cost is associated with the costs of commuting, getting ready for work, and so on. Here we relate these costs to household production. For each day that the agent participates in the labor force we assume that there is some household production activity that must be replaced. One may think of this as the replacement of child care, household services, or whatever. In the appendix we indicate how the household production can be represented as mapping into agents preferences. The cost associated with participation in the labor market will depend on the fraction of days that an agent participates, since the larger the fraction, the more household production that must be replaced.³ We denote this fixed cost $\psi(e)$. We can now represent the average daily utility function as

$$U(c, l; e) = u(c) - v(n)e - \psi(e)e. \quad (4)$$

Now, e is essentially the agents' *employment rate*, and since we have adopted a representative agent construct this will be the aggregate employment rate in

³There is nothing about this specification that is inconsistent with commuting costs kind of story because there too the fixed costs will vary with the number of days worked.

equilibrium. The function ψ is thus increasing in e and we assume further that it is twice differentiable.

The key element in this specification is that the utility function is written as a nonlinear function of the *two* labor supply variables e and n . There are arguments other than the one we present above to justify this specification, but this seems the most direct and simple.⁴

3. Equilibrium

A competitive equilibrium can be defined in a purely standard way. Let X be a consumption set, $X = \{(c, n, e, k) \in \mathbb{R}^4 : c \geq 0, 0 \leq n \leq 1, e = 0 \text{ or } 1, 0 \leq k \leq 1\}$.

Definition. An allocation for the economy is a list $\{c(t), n(t), k(t), e(t), K, N, E, w, r\}$, where for each $t \in [0, 1]$, $(c(t), n(t), e(t), k(t)) \in X$, and $K, N \geq 0$.

Definition. A competitive equilibrium for the economy is a list $\{c(t), n(t), k(t), e(t), K, N, w, r\}$ such that:

(i) for each $t \in [0, 1]$, $(c(t), n(t), e(t), k(t))$ is a solution to the consumer's problem

$$\max [u(c) - v(n)e - \psi(e)],$$

subject to

$$c \leq wne + rk,$$

$$c \geq 0, \quad 0 \leq n \leq 1, \quad 0 \leq k \leq 1, \quad 0 \leq e \leq 1.$$

(ii) N, K are a solution to the firm's problem

$$\max [\lambda f(K, N) - rK - wN],$$

subject to

$$K \geq 0, \quad N \geq 0.$$

(iii)

$$K = \int_0^1 k(t) dt, \quad \lambda f(K, N) = \int_0^1 c(t) dt, \quad N = \int_0^{e(t)} n(t) dt, \quad E = \int_0^1 e(t) dt.$$

⁴In an earlier version of this paper we presented an argument in terms of employment lotteries that could be used to arrive at this specification.

The competitive equilibrium defined above is standard except for the feature that there is a fixed cost associated with labor supply. Assuming $0 < e < 1$, the first-order conditions for the consumers problem are

$$wu'(wne + r) = v'(n), \tag{5}$$

$$nv'(n) = v(n) + \psi(e) + \psi'(e)e. \tag{6}$$

These are the marginal conditions that must hold simultaneously when both margins of adjustment are operating.

4. Examples

In this section we consider several examples based on different specifications of preferences. As we noted in the introduction, equilibrium business cycle models have been criticized because they assume a responsiveness of hours of work to wages that is inconsistent with microeconomic estimates of labor supply equations. Since the model considered in this paper is an equilibrium model, it is somewhat misleading to talk about labor supply. Nevertheless, the purpose of this exercise is to provide a basis for comparison with estimated labor supply elasticities. These examples illustrate the implications for aggregate labor supply, employment, and welfare of assuming that workers adjust only on one margin or the other. The examples are all based on versions of the following specification of preferences and technology:

$$u(c) = (1/\sigma)c^\sigma, \tag{7}$$

$$v(n) = (a/(\gamma + 1))n^{\gamma + 1}, \tag{8}$$

$$\psi(e) = (b/(\tau + 1))e^\tau, \tag{9}$$

where it is assumed that $0 < \sigma < 1$, $\gamma > 0$, and $\tau > 0$. The technology is given by

$$\lambda f(N) = \lambda N^\alpha. \tag{10}$$

(Here we abstract from capital stock to simplify the exposition.)⁵

The first example is simply the model described above in which individuals choose both hours of work and the number of days to work or the fraction of time to be working. The second example considers the pure indivisible labor

⁵To keep our examples simple we assume that the firm is owned by an agent whose only role is to dispose of the profits associated with this decreasing returns production function.

model that has been studied by Rogerson (1984) and Hansen (1985). In that model workers choose only whether to work or not – employment decisions are made on the extensive margin – but do not vary the hours of work. The third and final example considers the case where workers adjust only along the hours of work margin – the intensive margin – as is assumed by Kydland and Prescott.

Example 1

When both margins of adjustments are considered as described in the previous section, the cost of labor supply depends on the fraction of days worked by the representative agent which, in equilibrium, will equal the aggregate employment rate. The representative agent has to solve the maximization problem:

$$\max [(1/\sigma) c^\sigma - (a/(\gamma + 1))n^{\gamma+1}e - (b/(\tau + 1))e^{\tau+1}], \quad (11)$$

subject to

$$c \leq wne,$$

$$c \geq 0, \quad 0 \leq n \leq 1, \quad 0 \leq e \leq 1.$$

The first-order conditions for (11) are

$$w(wne)^{\sigma-1} - an^\gamma = 0, \quad (12)$$

$$w(wne)^{\sigma-1}n - (a/(\gamma + 1))n^{\gamma+1} - be^\tau = 0. \quad (13)$$

These equations can be solved simultaneously for n^* and e^* , the equilibrium employment and hours supplied by the representative agent.

Plugging $w(wne)^{\sigma-1} = an^\gamma$ in (13), we have

$$e = Hn^{(\gamma+1)/\tau}, \quad (14)$$

where $H = [a\gamma/b(\gamma + 1)]^{1/\tau}$. If we substitute (14) into (12), we get the supply of hours of work,

$$n^* = Jw^{\sigma/R}, \quad (15)$$

where $R = \gamma + (\tau + \gamma + 1)(1 - \sigma)/\tau$ and $J = (H^{\sigma-1}/a)^{1/R}$. The employment rate is determined as

$$e^* = Lw^{\sigma(\gamma+1)/\tau R}, \quad (16)$$

where $L = HJ^{(\gamma+1)/\tau}$. Aggregate labor supply is simply the product of the employment rate and the hours of work, $N^s = n^*e^*$.

The demand for labor can be obtained from the firm's maximization problem

$$\max [\lambda N^\alpha - wN], \tag{17}$$

subject to

$$N \geq 0.$$

The first-order condition for (17) is

$$\lambda \alpha N^{\alpha-1} - w = 0, \tag{18}$$

and the demand for aggregate labor is

$$N^d = Gw^{1/(\alpha-1)}, \tag{19}$$

where $G = (\lambda \alpha)^{1/(1-\alpha)}$. Equating the aggregate demand and supply of labor, we can solve for the equilibrium wage rate by equating (19) and N^s obtained above. If we substitute the equilibrium wage rate into (14) and (15), we obtain the equilibrium hours of work and the employment rate. Note that the elasticity of aggregate labor supply is the sum of the elasticity of hours of work and the elasticity of employment.

To get a concrete idea of the implications of this model for aggregate labor supply elasticities we assume the parameter values $a = 7.5$, $b = 0.8$, $\sigma = 0.8$, $\gamma = 2$, $\tau = 0.8$, $\lambda = 1$, and $\alpha = 0.64$. These parameters imply the following labor supply and demand functions:

$$n^* = 0.43w^{0.27}, \tag{20}$$

$$e^* = 0.43w^{1.02}, \tag{21}$$

$$N^s = 0.18w^{1.29}, \tag{22}$$

$$w^* = 1.12. \tag{23}$$

The equilibrium allocation in this example is $(c^*, n^*, e^*, N^*, w^*) = (0.24, 0.46, 0.48, 0.21, 1.12)$ and the utility of the representative agent is 0.17. The elasticity of hours with respect to the wage is 0.27, that of employment is 1.02, and the aggregate is the sum of these two, 1.29. This number is well above the range of empirical estimates that are typically found for males [Pencavel (1986)], but our main purpose is to compare it to the values for alternative specifications of preferences that ignore one of the margins of adjustment.

Example 2

For comparison purposes, we consider an economy with a simple fixed utility cost that does not depend on the aggregate employment rate. This turns out to correspond to the indivisible labor economy considered by Hansen and Rogerson and a comparison of the implied elasticities is instructive.

The economy with fixed utility cost of labor supply has

$$\psi(e) = b, \quad (24)$$

for all e , without changing other features of the previous examples. The form of the problem considered by Hansen and Rogerson assumes that agents face an employment lottery that determines their probability of working. With the lottery, the agent's problem is

$$\max [(1/\sigma)c^\sigma - (a/(\gamma + 1))n^{\gamma+1}e - be], \quad (25)$$

subject to

$$c \leq wne,$$

$$c \geq 0, \quad 0 \leq n \leq 1, \quad 0 \leq e \leq 1.$$

The first-order conditions are

$$w(wne)^{\sigma-1} - an^\gamma = 0, \quad (26)$$

$$w(wne)^{\sigma-1}n - [a/(\gamma + 1)]n^{\gamma+1} - b = 0. \quad (27)$$

Using (26), we can rewrite (27) as

$$[a\gamma/(\gamma + 1)]n^{\gamma+1} = b,$$

which implies that the hours of work do not depend on the wage rate:

$$n^* = [b(\gamma + 1)/a\gamma]^{1/(\gamma+1)} = \bar{n}(\text{fixed}). \quad (28)$$

This latter feature is discussed in Grilli and Rogerson (1988). With separable preferences and employment lotteries, fixed time cost and fixed utility cost imply an indivisibility as in (28). Using the hours the employment rate can be obtained as

$$e^* = (K/\bar{n}^\sigma)^{1/(\sigma-1)}w^{\sigma/(1-\sigma)}, \quad (29)$$

where $K = b + (a/(\gamma + 1))\bar{n}^{\gamma+1}$. The aggregate labor supply is obtained as $N^s = n^*e^*$. The firm's problem is the same as in Example 1.

With the parameter values specified in Example 1 and with $b = 0.33$, we obtain the functions:

$$n^* = 0.40(\text{fixed}), \tag{30}$$

$$e^* = 0.90w^{4.00}, \tag{31}$$

$$N^s = 0.36w^{4.00}, \tag{32}$$

$$w^* = 0.97. \tag{33}$$

The equilibrium in this example is $(c^*, n^*, e^*, N^*, w^*) = (0.31, 0.40, 0.79, 0.32, 0.97)$. The elasticity of the aggregate labor supply is 4.00, dramatically larger than in the previous examples. In this economy the labor market adjusts only along the extensive margin, so the elasticity of the aggregate labor supply is necessarily greater than in the previous examples.

Example 3

Finally, we consider the case where there is no fixed cost of labor supply. For this case $b = 0$, and the representative agent has to solve the problem

$$\max [(1/\sigma)c^\sigma - (a/(\gamma + 1))n^{\gamma+1}], \tag{34}$$

subject to

$$c \leq wn,$$

$$c \geq 0, \quad 0 \leq n \leq 1.$$

The first-order condition is

$$(wn)^{\sigma-1}w - an^\gamma = 0, \tag{35}$$

and the equilibrium labor supply is

$$N^s = n^s = (1/a)^{1/(\gamma+1-\sigma)}w^{\sigma/(\gamma+1-\sigma)}. \tag{36}$$

Once again the firm's problem is the same. We can solve for the equilibrium wage rate and plug it into (37) to obtain the hours of work.

Table 1
Elasticities in the examples.^a

Case	Hours	Employment	Aggregate
Both margins	σ/R (0.27)	$\sigma(1+\gamma)/\tau R$ (1.02)	$\sigma(1+\gamma+\tau)/\tau R$ (1.29)
Pure fixed cost ($\tau=0$) (extensive margin)	0	$\sigma/(1-\sigma)$ (4.00)	$\sigma/(1-\sigma)$ (4.00)
Convex environment ($b=0$) (intensive margin)	$\sigma/(1+\gamma-\sigma)$ (0.40)	0	$\sigma/(1+\gamma-\sigma)$ (0.40)

^a(1) The assumed utility function is $U(c, l; e) = 1/\sigma c^\sigma - a/(1+\gamma)n^{1+\gamma} - b/(1+\tau)e^\tau I(n > 0)$. (2) $R = \gamma + (\tau + \gamma + 1)(1 - \sigma)/\tau$. (3) The numbers in parentheses are elasticities when $\sigma = 0.8$, $\gamma = 2$, and $\tau = 0.8$.

With the parameter values specified in Example 1, we get the labor supply function:

$$N^s = n^* = 0.40w^{0.36}, \quad (38)$$

$$w^* = 0.90. \quad (39)$$

The equilibrium in this example is $(c^*, n^*, N^*, w^*) = (0.35, 0.39, 0.39, 0.90)$. The elasticity of aggregate labor supply is 0.36, much smaller than in the previous examples. This is entirely due to the fact that labor market adjustment takes place only along the intensive margin. Table 1 summarizes the labor supply elasticity in the three examples. The pure fixed cost economy shows the greatest elasticity, while the economy without nonconvexities shows the smallest. The model economy studied in Example 1 shows an elasticity between these extremes. These results underscore the importance of considering both margins of adjustment. Simply modeling the intensive margin is likely to bias downward estimates of the labor supply elasticity, while simply modeling the extensive margin is likely to bias the elasticity upwards. This is true whether one is engaged in an econometric estimation exercise or a calibration and simulation exercise. Since this point has been made in a very different form in the econometric literature on labor supply, in the next section we consider its implications in a simulation exercise.

5. Simulation of a dynamic model economy

One of the primary motivations for introducing the indivisible labor model into the business cycle literature was to have a model economy that was capable of generating the kind of fluctuations in aggregate hours that are observed in U.S. aggregate data. The Kydland and Prescott model which considered hours variation only on the intensive margin fell short in that respect. In this section

we consider whether modelling agents as adjusting on both margins – a specification that is less of an abstraction than either of the other two – is capable of replicating features of the aggregate data.

The static model characterized in section 3 can be extended into a dynamic setting by incorporating capital accumulation and an information structure. Suppose there is a continuum of agents uniformly distributed over the closed interval $[0, 1]$ as was assumed in section 2. Each individual is initially endowed with one unit of time and one unit of capital, and lives forever. There is one firm with technology which can be represented with the production function

$$Y_t = \lambda_t f(K_t, N_t), \quad (40)$$

where K_t , N_t , and Y_t are aggregate capital, aggregate labor, and aggregate output in period t , respectively. We will abstract from population and technological growth. λ_t is a random shock which is assumed to be a realization of the AR(1) process:

$$\lambda_{t+1} = \eta \lambda_t + \varepsilon_{t+1}, \quad (41)$$

where the ε_t are assumed to be independently and identically distributed with distribution function F . It is assumed that the distribution has a positive support to guarantee that output is positive. Since we abstract from growth, λ_t will have an unconditional mean of 1 by assuming the mean of the distribution F to be $1 - \eta$. Individuals are assumed to observe λ_t at the beginning of the period t .

Output can be either consumed or invested implying that the following constraint has to be satisfied in the aggregate:

$$C_t + I_t \leq \lambda_t f(K_t, N_t), \quad (42)$$

where C_t and I_t are aggregate consumption and investment in period t . The law of motion for the aggregate capital stock is given by

$$K_{t+1} = (1 - \delta)K_t + I_t, \quad (43)$$

where δ is the rate of capital depreciation and $0 \leq \delta \leq 1$. The stock of capital is assumed to be owned by the individuals who sell capital services to the firm. Thus, the aggregate law of motion for the capital stock, (43), arises from individual optimizing behavior. In this model, all agents are identical and are treated equally. From now on, uppercase letters will denote aggregate variables, while lowercase letters will denote per capita variables. Anticipating the equilibrium, we use these interchangeably.

The representative agent will maximize the expected value of the discounted sum of temporal utilities,

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - v(1 - l_t)e_t - \psi(e_t)e_t], \quad (44)$$

subject to the constraints (40)–(43). The programming problem to be solved can be stated as follows:⁶

$$\max E_0 \left[\sum_{t=0}^{\infty} \beta^t (u(c_t) - v(n_t)e_t - \psi(e_t)e_t) \right], \quad (45)$$

subject to

$$\begin{aligned} c_t + i_t &\leq \lambda_t f(k_t, N_t), \quad N_t = e_t n_t, \\ k_{t+1} &= (1 - \delta)k_t + i_t, \quad \lambda_{t+1} = \eta \lambda_t + \varepsilon_{t+1}, \\ c_t &\geq 0, \quad 0 \leq e_t \leq 1, \quad 0 \leq n_t \leq 1, \quad k_t \geq 0. \end{aligned}$$

To explore the quantitative implications of this model we adopt the following functional forms for technology and preferences:

$$Y_t = \lambda_t k_t^\alpha N_t^{1-\alpha}, \quad (46)$$

$$u(c_t, l_t, e_t) = \log(c_t) - a/(1 + \gamma)(1 - l_t)^{1+\gamma} - b/(1 + \tau)e_t^{\tau+1}. \quad (47)$$

Following Kydland and Prescott (1982), we approximate the model economy with a quadratic objective and linear constraints. The details of the approximation method are described in Kydland and Prescott (1982).

The steady state of the model is described by the following conditions:

$$i = \delta k, \quad (48)$$

$$\lambda(1 - \alpha)k^\alpha N^{-\alpha} = an^\gamma c, \quad (49)$$

$$\lambda(1 - \alpha)k^\alpha N^{-\alpha} n = [a/(1 + \gamma)n^{1+\gamma} + be^\tau]c, \quad (50)$$

$$\lambda \alpha k^{\alpha-1} N^{1-\alpha} = (\delta + \rho), \quad (51)$$

$$c + I = \lambda k^\alpha N^{1-\alpha}, \quad (52)$$

⁶Since there is no distortion, we don't need to distinguish aggregate variables from their individual counterparts.

where the steady state of a variable is denoted by the variable's symbol without any script, $n = 1 - l$, $N = ne$, and $\rho = (1/\beta) - 1$. Condition (48) is a standard one for a steady state. (49) and (50) equate the marginal benefits from adjustments along the intensive and extensive margins to the marginal costs of those adjustments, respectively. Condition (51) requires the rental rate of capital to be equal to the marginal productivity of the capital stock, and (52) is the budget constraint. We can solve these conditions for n , e , k , i , and c .

In order to simulate the model we must first assign values to the parameters. We borrow most of the parameter values from Kydland and Prescott (1982), Hansen (1985), and Prescott (1986). The values used for calibration are: $\alpha = 0.36$, $\beta = 0.99$, $\delta = 0.025$, $\lambda = 1$, and $\eta = 0.95$. The details of the justification for these parameter values, except those for utility, can be found in Prescott (1986). Calibrating the utility parameters a , τ , b , and γ presents a more difficult problem which we approach in two different ways.

In the first approach we arbitrarily fix $\tau = 0.62$ and choose the remaining three parameters to fit three facts observed in the U.S. economy. *First*, the model does not make a distinction between people who are in or out of the labor force. Consequently, the data for the U.S. that corresponds to the employment rate in the model economy are formed by the product of the employment rate and the participation rate. For the U.S. economy the value of this product is about 65%. *Second*, about one-third of the time endowment is spent in labor market activity. This value may overestimate the true fraction, but it is not a bad estimate if we take into account the portion of time spent on commuting and preparing for work. *Third*, one of the features of the business cycle that we have stressed earlier is that 75% of the aggregate labor fluctuation is due to the fluctuation in employment and the remaining 25% is due to the fluctuation in hours of work per person. This ratio of fluctuation in hours per person relative to that in employment has been fixed at one third. We determine the values of the utility parameters (a , b , τ , γ) to hit these three numbers.⁷

In the second approach we draw on microeconomic observations analyzed by Bils and Cho (1991) to calibrate these preference parameters. From the first-order conditions for the household we obtain the following relation between employment and hours:

$$\ln(e_t) = \frac{1}{\tau} \cdot \ln\left(\frac{a_1\gamma}{b_2(1 + \gamma)}\right) + \frac{1 + \gamma}{\tau} \cdot \ln(n_t).$$

This relationship between employment and hours can be estimated. Bils and Cho used observations from the Panel Study on Income Dynamics (hereafter PSID) to estimate the linear relationship between weeks of employment and

⁷See the note to table 2.

Table 2
Calibration results, first parameterization.^a

Series	U.S.		Model	
	Std. dev.	Corr. with output	Std. dev.	Corr. with output
Output	1.76	1.00	1.76 (0.17)	1.00 (0.00)
Consumption	1.29	0.85	0.53 (0.06)	0.88 (2.49)
Investment	8.60	0.92	5.63 (0.57)	0.98 (0.40)
Capital stock	0.63	0.04	0.47 (0.08)	0.07 (6.73)
Aggregate hours	1.74	0.77	1.06 (0.12)	0.98 (0.56)
Hours	0.46	0.76	0.25 (0.02)	0.98 (1.24)
Employment	1.50	0.81	0.81 (0.08)	0.98 (1.04)
Productivity	1.18	0.35	0.75 (0.08)	0.96 (0.81)
Agg. hrs/Productivity				
in physical units		1.47		1.42
in efficiency units		1.42		1.42

^a(1) The data used are quarterly time series from the third quarter of 1955 to first quarter of 1984. Before the statistics were calculated, the data were logged and detrended by the Hodrick–Prescott filter. Standard deviations are in percentage terms. The statistics are means of 100 simulations. The numbers in parentheses are standard deviations of the 100 simulations in percentage term. (2) The values of the parameters assumed in the model are $a = 6.0$, $b = 0.87$, $\gamma = 1.0$, and $\tau = 0.62$.

hours of employment.⁸ That relationship pins down two of the preference parameters. The remaining two are estimated using the steady state fractions of total hours of work and the employment population ratio alluded to above. This gives us four relationships in four unknowns which yields values of $a = 13.5$, $b = 1.75$, $\gamma = 2$, and $\tau = 1.2$.

Using the value function for the economy, we solve for the equilibrium decision rules as functions of the state variables, the technology shock and the capital stock. With the equilibrium decision rules, we generate time series for the model economy. One-hundred time series were generated and each of the time series was logged and detrended using the Hodrick–Prescott filter. Second moments were calculated from each of the time series and means of the one-hundred simulations were calculated. The results are reported in table 2 for the first parameterization and in table 3 for the second. The statistics for the model economy in table 2 are computed with the standard deviation of the technology shock equal to 0.00825. This number, which lies in a range suggested by Prescott (1986), was chosen because it implies the mean of the standard deviation in output from the one-hundred simulations equal to the standard deviation in actual U.S. output. In the table 3 results the standard deviation of the technology shock had to be set equal to 0.0102 to match the volatility of output.

⁸The sample includes prime-aged (25–55) male heads of households, who worked and reported weeks and hours.

Table 3
 Calibration results, second parameterization.^a

	Std. dev.	Corr. with output
Output	1.74 (0.30)	1.00 (0.00)
Consumption	0.55 (0.10)	0.87 (0.10)
Investment	5.42 (0.87)	0.98 (0.10)
Capital	0.46 (0.11)	0.06 (0.06)
Aggregate hours	0.74 (0.12)	0.97 (0.10)
Hours	0.27 (0.04)	0.97 (0.10)
Employment	0.48 (0.08)	0.97 (0.10)
Productivity	1.03 (0.17)	0.98 (0.10)

^aThe statistics are the means of 100 simulations. Each simulated time series is detrended using Hodrick–Prescott filter before the statistics are calculated. The size of the technology shock is $\sigma = 0.0102$ and the preference parameters are

$$u_t = \ln(c_t) - \frac{a}{1 + \gamma} \cdot n_t^{1+\gamma} \cdot e_t - \frac{b}{1 + \tau} \cdot e_t^{1+\tau},$$

where $a = 13.5$, $b = 1.75$, $\gamma = 2$, $\tau = 5/3$.

The summary statistics generated by the model economy reported in table 2 resemble the statistics from the U.S. economy with a few notable exceptions. The most important, from the standpoint of our objectives, is that the model economy shows less fluctuation than the U.S. economy judged by the standard deviations in table 2. The correlations of the simulated variables with output from the model economy are very close to those from the U.S. economy except that the hours, employment, and productivity are more highly correlated in the model economy. This is due in part to the fact that the time series in the model economy were created by a single shock. To create time series having correlations more like those from the U.S. economy we would need to introduce either more shocks or measurement errors. This stochastic singularity problem is common to real business cycle models. In spite of the above-mentioned discrepancies the results show that the model economy captures some important features of the U.S. economy.

The results of our simulations reveal that this specification produces a ratio of aggregate hour variability relative to productivity variability in the model economy of about 1.4, which is quite close to the ratio implied by the U.S. data. For the U.S. economy, the ratio is about 1.47 in physical units but 1.42 in efficiency units [see Hansen (1985)]. That this ratio is so high has been a problem for other business cycle models. Kydland and Prescott (1982), Hansen (1985), Prescott (1986), and Bencivenga (1987) all focus attention on this key ratio. For the model economy studied by Kydland and Prescott this ratio turns

out to be 1.17, while it is 2.70 for the indivisible labor economy studied by Hansen (1985). Employment, average hours of work, aggregate hours of work, and productivity all fluctuate less in the model economy than in the data. This model economy is not as successful as capturing the volatility of aggregate hours as the pure indivisible labor model considered by Hansen. In his model economy the standard deviation of aggregate hours was 1.35.

The results reported in table 3, based on a model economy calibrated to microeconomic observations, are not as encouraging. The fluctuations in aggregate hours are much lower than observed in the aggregate data and the ratio of hours to productivity is much smaller. If we place more confidence in the preferences implied by these microeconomic observations, it suggests that there is some important feature of the economy that is missing. Finally, we consider an experiment that will enable us to relate our results more directly to those in Hansen (1985) and Cooley and Hansen (1989) who use the indivisible labor specification. They consider preferences that are logarithmic in consumption and linear in leisure. Here we use exactly that specification augmented by a fixed cost of labor supply so that preferences are assumed to be

$$u = \ln(c_t) + a \cdot e_t \cdot \ln(1 - n_t) - \frac{b}{1 + \tau} \cdot e_t^{1+\tau}. \quad (53)$$

Table 4 contains simulation results from the model with a log linear specification for consumption and leisure where the preference parameters are assumed to be $a = 1.5$, $b = 0.45$. These imply that steady state aggregate hours are about one third and the employment population ratio is about 65 percent when we fix the cost of labor supply parameter to be $\tau = 0.6$. We can now study the effects of changes in costs of labor supply on the volatility of output and relative volatility of employment to hours by varying the value of τ . Table 4 shows a strong pattern of response to increases in the value of τ . The volatility of aggregate output decreases, the elasticity of aggregate hours decreases, and the ratio of the elasticity of employment relative to per capita hours decreases with increasing τ . One interesting finding is that the ratio of the standard deviations of employment and hours is exactly the same as the elasticity ratio.

The final column of table 4 contains estimates of the welfare costs of moving labor from the household sector to the market sector as a fraction of output. We compute these costs by first measuring aggregate utility without the fixed cost term and then measuring it with the fixed costs. We then determine what increment to consumption would be necessary to make aggregate utility in the latter case equal to aggregate utility in the economy without fixed costs of labor supply. The last column shows that increment as a fraction of output. The table shows that the cost of labor supply as a fraction of output is decreasing as τ increases and that the costs of labor supply necessary to explain the labor

Table 4
Log linear specification.^a

τ	Std(Y)	$\varepsilon(Q)$	$\varepsilon(A)$	$\frac{\text{Std}(E)}{\text{Std}(H)}$	$\frac{\text{Std}(Q)}{\text{Std}(Py)}$	$\frac{\text{Cost}}{\text{Output}}$
0.6	1.96	1.13	5.14	5.14	1.77	5.5
1.0	1.84	1.03	3.02	3.02	1.55	4.3
2.0	1.76	0.90	1.47	1.47	1.28	2.7
3.0	1.74	0.83	0.96	0.96	1.16	2.0

^a(1) The preference is assumed to be

$$u = \ln(c_t) + a \cdot e_t \cdot \ln(1 - n_t) - \frac{b}{1 + \tau} \cdot e_t^{1 + \tau}.$$

The parameter values are $a = 1.5$, $b = 0.45$, and the value of τ is indicated in the table. (2) Std(Y) = standard deviation of output, $\varepsilon(Q)$ = elasticity of aggregate hours with respect to the technology shock, $\varepsilon(A)$ = ratio of elasticity of employment with respect to the technology shock relative to that of per capita hours, Std(E)/Std(H) = ratio of standard deviations of employment and hours, Std(Q)/Std(Py) = ratio of standard deviations of aggregate hours and productivity, and Cost/Output = ratio of costs of labor supply in a steady state relative to output in the steady state. (3) The standard deviation of the technology shock is assumed to be 0.009.

market fluctuations are less than 6 percent of GNP. We can compare these estimates to similar costs estimated by Cogan (1981). Using a data set of working women, Cogan estimated the costs of labor supply to be 28.3 percent of the average earnings of working women. Clearly the costs will be much smaller in the cases of men and singles, so Cogan's estimate cannot be representing the economy-wide costs of labor supply. Nevertheless, our estimates of this cost are considerably smaller. Kydland and Prescott (1991) examine a model economy that employs a similar construct, the cost of moving workers between the household and market sector. They experiment with different values of such a parameter but don't directly compute the welfare costs associated with their preferred specification.

It is encouraging that a model economy with adjustment along both the extensive and intensive margin can reproduce some features of the observed data. This combined with the evidence that neglecting one or the other margin can lead to substantial biases in estimated elasticities suggests that the fixed cost model developed here may be a useful extension to the usual specification of preferences adopted in representative agent studies. Any such model is a serious abstraction from reality and at best can serve as a paradigm for what might go on the economy. The results of this paper suggest that representative agent models that abstract from the fact that there are two margins of adjustment in the labor market may miss features of the economy that are important for some purposes.

Appendix

The specification of the utility function employed in the paper can be motivated by considering an explicit model of household production in the spirit of Becker (1965). Again, assume that each agent is endowed with one unit of time and one unit of capital. Time is completely divisible and each agent consumes the final output from home production. The production function is

$$y = g(c, 1 - n + n'), \quad (\text{A.1})$$

where c represents goods purchased in the market as an input in home production (consumption in the usual sense), n is labor supplied to the market, and n' is labor input purchased from other agents. We assume that $n = n'$, i.e., that household work must be replaced one for one as must be true for things like child care and domestic services. As a result it must be the case that those who work in the household production sector (who are not counted as employed in this model economy) take care of the home production of more than one household. We further assume that when an agent chooses labor force participation and replaces his home production with that of others, then he faces some output loss that depends on the fraction of days worked. For example, if child care is an important form of household production, then 'output' (nurturing) will be diminished as the scale of the childcare enterprise increases, which it must as everybody's fraction of work increases. Suppose the output loss function can be expressed as

$$q[m/(1 - e)], \quad (\text{A.2})$$

where m is the household's home production that must be taken care of by a given agent, e is the employment rate (or fraction of days worked) and also the aggregate employment rate. For simplicity we assume m to be fixed. Combining yields the home production function:

$$y = g(c, 1) - q[m/(1 - e)]. \quad (\text{A.3})$$

We assume

- (i) $g(\cdot)$ is increasing, twice differentiable, and strictly concave in c .
- (ii) $\lim_{c \rightarrow 0} g_c(c, 1) = \infty$ and $\lim_{c \rightarrow \infty} g_c(c, 1) = 0$.
- (iii) $g_c(c + y, 1) + g_{cc}(c + y, 1)c \geq 0$ for all $c \geq 0$ and $y \geq 0$.
- (iv) q is continuously differentiable and increasing in both arguments.
- (v) $q(m) = 0$ if $e = 0$.

Assumption (iii) guarantees that labor supply is not backward-bending, while assumption (iv) says that the function $q(\cdot)$ is an increasing function of the employment rate.

The utility function is assumed to be separable between the output of home production and market activity:

$$U(y, l) = y - v(n), \tag{A.4}$$

where y represents home production. We assume that the disutility of labor supply is characterized by:

(vi) $v(\cdot)$ is increasing, strictly convex, and twice differentiable.

(vii) $\lim_{n \geq 0} v'(n) = 0$ and $\lim_{n \geq 1} v'(n) = \infty$.

Now, if we combine utility functions, we have

$$U(c, l) = u(c) - v(n) - \psi(e), \tag{A.5}$$

where

$$l = 1 - n, \quad u(c) = g(c, 1) \quad \psi(e) = q[e, m/(1 - e)].$$

If we assume that an agent who works in the home production sector incurs no disutility associated with work and if we fix the wage rate in home production at some level, say w_h , then we can characterize equilibrium in the home production sector exactly as in the text. The only additional constraint in this case is that the employment rate, e , can never be equal to 1 because then the output loss associated with participation would become infinite.

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