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Indivisible Assets, Equilibrium, and the Value of Intermediation*

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This paper considers a standard monetary economy with indivisible primary assets and transaction costs. When assets are indivisible, if a steady-state equilibrium with positive savings exists, there necessarily exists a very large set of equilibria. The intermediation of indivisible assets substantially reduces the set of competitive equilibria, and enhances the "flexibility" of prices. We state sufficient conditions for intermediaries to form and hold all primary assets directly. We define and analyze various measures of the consumer surplus created by intermediaries. We show that conventional measures of intermediary output bear no obvious relation to the consumer surplus created by intermediation. *Journal of Economic Literature* Classification Numbers: E40, G20. © 1995 Academic Press, Inc.

INTRODUCTION

Many assets, including U.S. Treasury liabilities and corporate debt, are issued only in relatively large minimum denominations. This has also

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been true historically for both metallic and paper currencies.[1] When assets exist only in large denominations and when there are transactions costs associated with trading in those assets, intermediaries may arise to provide "divisibility" of the assets. The size and importance of money market and bond funds and the securitized mortgage industry attest to the contemporary importance of this kind of intermediation. Moreover, Carothers (1930) and Rolnick and Weber (1988) indicate that, historically, many banks were primarily in the business of intermediating large-denomination assets. Should such intermediaries be regulated? This is a topic of long-standing debate. Many have argued that banking must be regulated because "free banks" proliferate equilibria and thereby contribute to instability. To understand how these arguments apply to the intermediation of indivisibilities, we must first understand the economic consequences of having indivisible assets. In this paper we describe the properties of an economy with indivisible assets. We then explore the conditions under which intermediaries are likely to form and describe the consequences of allowing that to occur. As we will demonstrate, even rudimentary intermediaries can alter the attainable competitive equilibria. In addition, the analysis yields observations about how to measure the social value of intermediary activity. By focusing on intermediaries whose only role is to provide conveniently denominated liabilities, we can examine intermediation in a way which represents a minimal departure from standard models of money.

Given the practical importance of the limited divisibility of currency and other assets, it is surprising that relatively little attention has been given to its consequences. Some of the consequences have been explored by Klein (1973), Bryant and Wallace (1979, 1984), Marimon and Wallace (1987), Smith (1989), and Cooley and Smith (1993). However, a general assessment of the consequences of asset indivisibility for the uniqueness and stability of competitive equilibria or for the welfare properties of those equilibria in standard monetary economies is lacking.[2]

Financial intermediaries can reduce transactions costs and allow agents to share large denomination assets. Moreover, if asset indivisibility limits trade, then the unfettered formation of intermediaries may have implica-

[1] Carothers (1930), Timberlake (1974), Hanson (1979, 1980), Rolnick and Weber (1986), Cipolla (1987), and Glassman and Redish (1988) document a number of historical episodes in which currency was issued only in relatively large denominations and could be divided only to a limited extent. Moreover, in a variety of these cases, shortages of small-denomination currency caused economic disruption. It bears emphasis that "small change shortages" have been observed in economies with paper currencies and have been observed in relatively modern, developed economies. On this point see Kemmerer (1910), Ross (1922), or Rich (1989).

[2] Reasons why (outside) assets might be issued in a form with limited divisibility are discussed by Bryant and Wallace (1984), Villamil (1988), and Cooley and Smith (1993).

tions for the existence, multiplicity, and welfare properties of competitive equilibria in a monetary economy. Formal investigation of the intermediation of large-denomination assets seems limited to Bryant and Wallace (1979), who do not examine these kinds of issues. A related question is whether bank liabilities and outside money can coexist in a world without restrictions on intermediation and where money is not assigned any special role in transactions. This question has troubled several authors [see White (1987) for an example], but appears not to have received formal consideration elsewhere.

Finally, in an economy where intermediaries emerge, how should we measure their social value? We treat this question as being closely related to the problem of measuring the "output" of financial intermediaries, an issue on which there is a large literature.[3] Most of this literature measures intermediary output based on some combination of cost data (value of inputs) and the quantity of intermediary assets or liabilities, which are identified as outputs. However, we show that the consumer surplus created by intermediaries will, under several definitions, be *inversely* related both to intermediary costs and to the quantity of intermediary assets or liabilities. Thus the intermediation of more assets does not imply the creation of more services.[4]

We use a two-period overlapping generations model, in which each generation contains a continuum of identical agents. There is a single nonstorable good and a single primary (outside) asset. In addition, as in Williamson (1986), there is a fixed cost of participating in the market for this asset. We then consider three situations: (i) all participants in primary asset markets bear a fixed cost, there is no intermediation, and assets are perfectly divisible; (ii) the same circumstances apply except that the asset is indivisible; (iii) intermediaries exist that purchase the primary asset and issue (at a constant marginal cost) perfectly divisible secondary securities. These secondary securities can be thought of as mutual fund shares or bank notes/bank deposits.

Our findings are as follows. When assets are perfectly divisible but there is a fixed cost of participating in asset markets, only stationary equilibria exist. In contrast, the same model with a zero fixed cost would allow for a continuum of nonstationary equilibria, as in Gale (1973) or Sargent (1987). Thus the effect of a (nonconvex) transaction cost is to

[3] See Berger and Humphrey (1990), Fixler and Zieschang (1990), or Hornstein and Prescott (1990) for examples.

[4] According to Kuznets (1953, p. 193), "It is not only permissible but necessary to view national income measures as approximations to economic welfare, since they are, by definition, appraisals of the yield of the country's economy from the standpoint of the wants of its ultimate consumers." Thus a deviation between methods of measuring output and the creation of consumer surplus indicates a substantial problem with traditional output measures.

reduce the set of competitive equilibria. With a fixed market participation cost and indivisible outside assets, there will also be a continuum of nonstationary equilibria, if a stationary equilibrium with positive savings exists. Moreover, even if the economy starts at a steady-state equilibrium with positive savings (that is, even if there is a given initial condition stating the value of outside assets), there is a continuum of nonstationary equilibria. We refer to equilibria with this property as indeterminate. We also show that if a steady-state equilibrium with positive savings exists, then either (a) there is a continuum of Pareto-ranked stationary equilibria with positive savings or (b), starting from the steady-state debt level, there is a continuum of nonstationary equilibrium paths that can be Pareto ranked. Thus, asset indivisibilities are a source of indeterminacies. Interestingly, however, all competitive equilibrium paths (nonstationary as well as stationary) achieve a constant inflation rate/rate of return in finite time. In this sense asset indivisibilities create a kind of price stickiness.

We also examine the role of intermediaries as holders of primary assets. Intermediaries allow the (finite) fixed cost of participating in primary asset markets to be borne by a large number of agents and also allow agents to pool funds and "share" primary assets (in large denominations). If an equilibrium with intermediated assets exists, the set of equilibrium paths qualitatively resembles that for standard overlapping generations models with homogeneous agents. However, for some initial debt levels—levels that would be consistent with equilibrium in the absence of intermediation—there will be no equilibrium if intermediaries are allowed to form. In particular, if the initial debt level is too high, and if intermediaries raise market returns (as is often argued in the development literature), debt service may "explode" if intermediaries form. Consequently, governments with large initial debt levels may wish to inhibit the formation of intermediaries. That such repression is common in developing countries has been argued by McKinnon (1973) and Shaw (1973) among others.

The free formation of unregulated intermediaries also precludes the possibility that any equilibrium is indeterminate. Thus laissez-faire intermediation *reduces* the potential for multiple equilibria, in contrast to what has often been asserted in criticisms of the real-bills doctrine.[5] Finally, when assets are intermediated at a positive cost, primary assets will bear a higher rate of return than intermediary liabilities. Thus the model explains, for instance, how bank notes that are perfectly safe claims to specie could bear a lower rate of return than specie, a situation that has often been viewed as inconsistent with the class of models at hand [see White (1987).] These results are summarized in Table I.

The remainder of the paper proceeds as follows: Section I considers the economy with divisible and indivisible primary assets and a fixed cost of

[5] See Mints (1945) for an example of such criticisms.

TABLE I

	Divisible assets (no fixed costs)	Divisible assets (fixed costs)	Indivisible assets (fixed costs; no intermediation)	Intermediation
Existence of nonstationary equilibrium	Yes	No	Yes	Yes
Noninitial conditions, indeterminacy	No	No	Yes	No
Sticky inflation rate	No	Yes	Yes	No
Rate of return dominance of bank liabilities	—	—	—	Yes

market participation in the absence of intermediation. Section II considers the set of equilibria with intermediaries and discusses conditions under which intermediaries will be active. Section III develops some measures of the social value of intermediation and compares them to conventional measures of intermediary activity. Section IV concludes with comments on some possible extensions.

I. THE MODEL: NO INTERMEDIATION

The economy consists of an infinite sequence of two-period-lived overlapping generations and an initial old generation. Each generation contains a continuum of identical agents of measure one. Time is obviously discrete and indexed by $t = 1, 2, \ldots$.

At each date there is a single nonstorable consumption good. Young agents are endowed with a quantity $w_1 > 0$ of the good when young and with $w_2 > 0$ when old. Letting c_j denote age j consumption ($j = 1, 2$), these agents have preferences described by the additively separate utility function $u(c_1) + v(c_2)$. The functions u and v are assumed to be strictly increasing, concave, and twice-continuously differentiable. In addition, we assume that $\forall c \in R_+$ (the consumption set)

$$0 \geq cv''(c)/v'(c) \geq -1, \tag{A1}$$

so that young and old consumption are gross substitutes, and we also assume that v is strictly concave.

There is a single outside asset that agents in this economy can hold. We

assume that the asset comes in indivisible units with a real value of x, so agents can only hold integer multiples of x.[6] Also, we assume that $x < w_1$, so that it is feasible for an agent to hold at least one unit of the asset. The indivisibility of the asset has various possible interpretations. One is that it is a U.S. Treasury liability issued in a minimum denomination. Such a liability might be either indexed, in which case it is natural to take its minimum denomination as fixed in real terms, or unindexed. If government debt is not indexed, then clearly we must assume that the minimum (nominal) denomination in which this debt is issued is adjusted as the price level changes over time. [The latter formulation mimics that of Bryant and Wallace (1984).] Alternatively, we could interpret the asset as a specie currency (coins), but where specie is in fixed total supply and has no alternative uses.[7] Finally, we assume that there is a fixed transaction cost $\phi > 0$ associated with acquiring the asset. This cost could be interpreted as the cost of participating in a T-bill market if the asset is a treasury liability or the cost of a scale (to verify that coins are of full weight and not counterfeited) if the asset is specie. The cost, ϕ, is assumed to satisfy

$$\phi < w_1 - x$$
$$u'(w_1 - \phi) < v'(w_2). \tag{A2}$$

This inequality, (A2), implies that this is a "Samuelson-case economy," in Gale's (1973) terminology.

A. *Divisible Assets*

As a benchmark, we begin by considering the case where assets are held directly and are perfectly divisible, but we retain the assumption of a positive fixed transaction cost.[8] Here the asset can be interpreted as fiat money, but young agents bear a cost of bringing goods to market to be

[6] Of course below we allow intermediaries to form. Then the primary asset must be purchased in integer multiples of x, but intermediary liabilities will be assumed to be perfectly divisible.

[7] See Marimon and Wallace (1987), who also interpret an asset in fixed supply (paying no dividends and with no alternative uses) and that is divisible only at cost as a specie currency. However, in our analysis, it is also possible to allow specie to have an alternative use so long as the specie stock remains constant over time. Sargent and Wallace (1983) provide an example of how this could be accomplished. If specie has an alternative use, if the metallic content of the smallest coin is legally fixed, and if coins pass at their intrinsic value, then each coin will clearly have a real value of x. See Sargent and Smith (1994) for an example of an economy with these properties.

[8] There is assumed to be no intermediary activity owing to a government prohibition or because the cost of intermediation is prohibitive. On the latter point, see Section III.

exchanged for currency. (Of course, the other interpretations of the asset mentioned above can be retained as well.)

Let r_t denote the gross real return on the asset between t and $t+1$; let B_t be the real per capita quantity of the asset outstanding at t; and let

$$f(r_t) = \arg\max[u(w_1 - \phi - s) + v(w_2 + r_t s)].$$

Conditional on agents entering the asset market, f describes their savings behavior. Assumption (A1) implies that $f'(r_t) > 0$, and (A2) implies that $f(1) > 0$. Furthermore, define $H(w_1 - \phi, w_2, r_t)$ by

$$H(w_1 - \phi, w_2, r_t) \equiv u[w_1 - \phi - f(r_t)] + v(w_2 + r_t f(r_t)],$$

so that H is a standard indirect utility function. We assume that

$$H(w_1 - \phi, w_2, 1) > u(w_1) + v(w_2). \tag{A3}$$

Also, define $\bar{r}$ by

$$H(w_1 - \phi, w_2, \bar{r}) \equiv u(w_1) + v(w_2).$$

Then clearly

$$1 > \bar{r} > f^{-1}(0) = u'(w_1 - \phi)/v'(w_2).$$

It follows that agents are willing to participate in asset markets at any rate of return greater than or equal to unity. Note that if agents do participate in asset markets, they behave as if their endowment stream is given by $(w_1 - \phi, w_2)$.

We now define a competitive equilibrium for an economy with divisible primary assets and a fixed transaction cost.

DEFINITION. An equilibrium is a pair of nonnegative sequences $\{r_t\}$ and $\{B_t\}$ such that

$$B_t = f(r_t) \qquad \text{(market clearing)} \tag{1}$$

$$B_{t+1} = r_t B_t \qquad \forall t, \tag{2}$$

and $r_t \geq \bar{r}$ if $B_t > 0$.

Equation (2) can be interpreted as the government budget constraint,[9] and $r_t \geq \bar{r}$ must hold for agents to voluntarily exchange goods for assets.

[9] If the asset is currency issued in a fixed per capita nominal amount M, then $B_t = M/p_t$, where p_t is the time t price level, and $r_t = p_t/p_{t+1}$.

This economy has two stationary equilibria:

(i) $B_t = f(1)$, $r_t = 1$ $\forall t$, and

(ii) $B_t = 0$; $r_t \in [f^{-1}(0), \bar{r}]$ $\forall t$.

In contrast to standard overlapping generations models, this economy has no nonstationary equilibria. This is because any such equilibria have $B_1 \in (0, f(1))$, and hence have $\lim B_t = 0$. But then there exists a finite data T such that $f^{-1}(B_T) < \bar{r}$, which contradicts the condition that agents are willing to participate in asset markets. (This argument is depicted in Fig. 1.) Thus a fixed transaction cost affects the equilibria of this economy. However, this conclusion requires that the asset is perfectly divisible, as we now demonstrate.

B. Indivisible Assets

We now require assets to be purchased in integer multiples of $x > 0$. Throughout we let $n = 0, 1, \ldots,$ denote an integer. Also, in order to preclude problems with the existence of an equilibrium with nonzero savings (due to the presence of nonconvexities), we assume that $f^{-1}(x)$ exists and that

$$\bar{r} \leq f^{-1}(x). \tag{A4}$$

Thus there is an interest rate at which agents willingly hold one unit of the asset, and at this interest rate agents are willing to participate in asset markets.

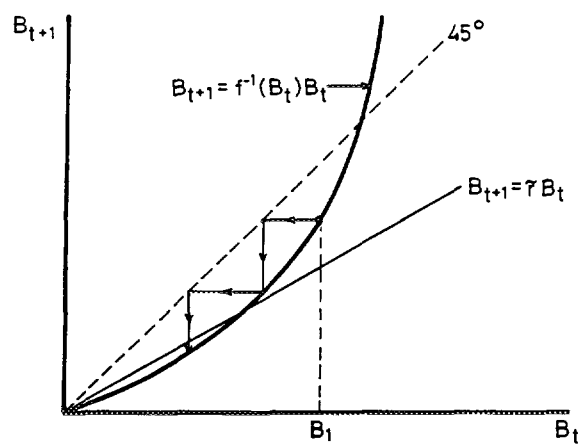

FIG. 1. Nonexistence of nonstationary equilibria.

Savings behavior. We next describe the optimal savings behavior of a young agent facing the interest rate r_t at t and a fixed transaction cost of ϕ, and who is constrained to hold integer multiples of the asset. Let n^* be the smaller integer such that $(n^* + 1)x > w_1 - \phi$, let $I(n)$ be an indicator function satisfying

$$I(n) = \begin{cases} 0; & n = 0 \\ 1; & n > 0, \end{cases}$$

and define the correspondence $M(r_t)$ by

$$M(r_t) \equiv \{n: n = 0, 1, \ldots, n^*; \\ n \text{ maximizes } u[w_1 - I(n)\phi - nx] + v(w_2 + r_t nx)\}.$$

Then $M(r_t)$ gives the set of optimal savings levels, expressed as integer multiples of x, at each interest rate. Apparently $M(r_t)$ is nonempty, $\forall r_t > 0$.

We now wish to characterize the correspondence $M(r_t)$. To do so, define the function $r(n)$ by

$$u(w_1 - \phi - nx) + v[w_2 + r(n)nx] \\ \equiv u[w_1 - \phi - (n-1)x] + v[w_2 + r(n)(n-1)x], \tag{3}$$

for $n > 1$, and by

$$u(w_1 - \phi - x) + v[w_2 + r(1)x] \equiv u(w_1) + v(w_2), \tag{4}$$

for $n = 1$. Clearly $r(1)$ exists [by (A4)], is unique, and satisfies $r(1) \geq \bar{r}$. For $n > 1$, $r(n)$ may or may not exist. However, whenever $r(n)$ exists, it is the interest rate that leaves young agents indifferent between saving $(n-1)x$ and nx.

The properties of the function $r(n)$ are useful in describing the properties of the correspondence $M(r)$. The properties of $r(n)$ are summarized in

PROPOSITION 1. (a) *If $r(n)$ exists, it is unique.* (b) *If $f^{-1}(nx)$ exists, then $r(n)$ exists and satisfies $r(n) \in (f^{-1}[(n-1)x], f^{-1}(nx))$.* (c) *If $r(n)$ exists, then $f^{-1}[(n-1)x]$ exists and satisfies $r(n) > f^{-1}[(n-1)x]$.* (d) *If $r(n)$ exists then $r(n-1)$ exists and satisfies $r(n-1) < r(n)$.*

See the Appendix for proof.

We are now prepared to describe the properties of the correspondence $M(r)$. These are summarized in

PROPOSITION 2. *If $r(n)$ exists, then $\{n-1, n\} = M[r(n)]$. In addition, $\{n-1\} = M(r)\ \forall r \in (r(n-1), r(n))$, while $\{0\} = M(r)\ \forall r < r(1)$. Finally, if $r(n)$ does not exist, then $n \notin M(r)$ for any r.*

See the Appendix for proof.

Equilibrium. Let $\mu_t(n)$ be the fraction of the young population saving nx at t. Evidently

$$\begin{aligned} &\mu_t(n) = 0; && n \notin M(r_t) \\ &\mu_t(n) \in [0, 1]; && n \in M(r_t). \end{aligned} \tag{5}$$

Then the asset market clears at t if

$$\sum_{n=0}^{n^*} \mu_t(n) nx = B_t. \tag{6}$$

Define $n(B_1)$ to be the smallest integer satisfying $n(B_t)x \geq B_t > [n(B_t) - 1]x$. If $n(B_t)x = B_t$, then (6) and Proposition 2 require that $\mu_t[n(B_t)] = 1$. By Proposition 2, this condition will hold iff $r_t \in [r[n(B_t)], r[n(B_t) + 1]]$. If, on the other hand, $n(B_t)x > B_t$, Proposition (2) and Eq. (6) imply that $r_t = r[n(B_t)]$ must hold and that

$$\mu_t[n(B_t)]n(B_t)x + \{1 - \mu_t[n(B_t)]\}[n(B_t) - 1]x = B_t. \tag{7}$$

We are now prepared to define an equilibrium for an economy with indivisible assets and a fixed transaction cost.

DEFINITION. An equilibrium is a set of nonnegative sequences $\{r_t\}$, $\{B_t\}$, and $\{\mu_t[n(B_t)]\}$ such that Eq. (7),

$$\begin{aligned} &r_t = r[n(B_t)] && \text{if } n(B_t)x > B_t, \\ &r_t \in [r[n(B_t)], r[n(B_t) + 1]] && \text{if } n(B_t)x = B_t, \end{aligned} \tag{8}$$

$$B_{t+1} = r_t B_t, \tag{9}$$

hold $\forall t$.

We now characterize the set of sequences $\{B_t\}$ which are consistent with the existence of a perfect foresight equilibrium. To do so we define the correspondence $\Psi(B_t)$ by

$$\Psi(B_t) \equiv \begin{cases} \{r[n(B_t)]B_t\}; & n(B_t)x > B_t \\ [r[n(B_t)]B_t, r[n(B_t) + 1]B_t]; & n(B_t)x = B_t. \end{cases}$$

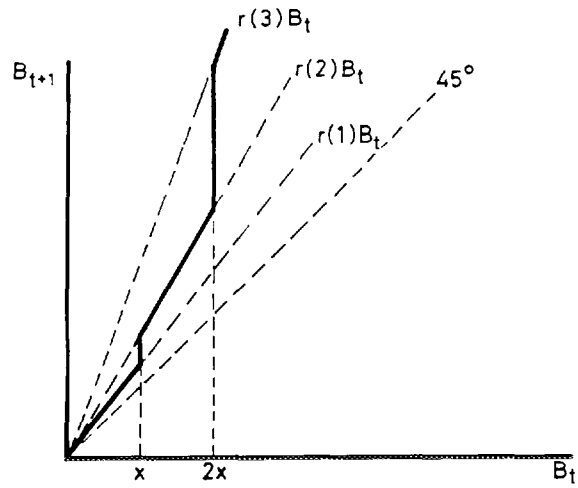

FIG. 2. $r(1) > 1$.

Then $\{B_t\}$ is an equilibrium sequence of debt levels if $B_t \in [0, w_1 - \phi]$ $\forall t$ and if B_t satisfies

$$B_{t+1} \in \Psi(B_t); \qquad \forall t \geq 0.$$

There are three possible configurations of the correspondence $\Psi(B_t)$; in each case, Ψ is convex-valued and upper hemi-continuous.

Case 1. $r(1) > 1$. Then, by Proposition 1, $r(n) > 1$ $\forall n$ such that $r(n)$ exists. This situation is depicted in Fig. 2. Clearly the only equilibrium has $B_t = 0$ $\forall t$.

Case 2. $r(1) < 1$; $r(n) \neq 1$ $\forall n$ such that $r(n)$ exists. In this case, depicted in Fig. 3, there is a stationary equilibrium with $B_t = 0$ $\forall t$ and one with $r_t = 1$ $\forall t$. In addition, there is a continuum of nonstationary equilibria with $\lim B_t = 0$. This is possible since, once $r_t = r(1) < 1$, $B_t \to 0$ because $\mu_t[n(B_t)] = \mu_t(1) \to 0$. Thus, the indivisible asset economy behaves substantially differently from the divisible asset economy.

Note that, in Fig. 3, $B_t = B^*$ $\forall t$ constitutes a steady-state equilibrium. However, there also exist equilibrium sequences in Fig. 3 that have $B_0 = B^*$ and $B_t \to 0$. This situation cannot arise (under our assumptions) without asset indivisibilities, and we will refer to it as indeterminacy.

DEFINITION. An equilibrium sequence $\{B_t\}$ is *determinate* if there is no other equilibrium sequence $\{B_t'\}$ with $B_0' = B_0$ and $B_t' \neq B_t$ for some t.

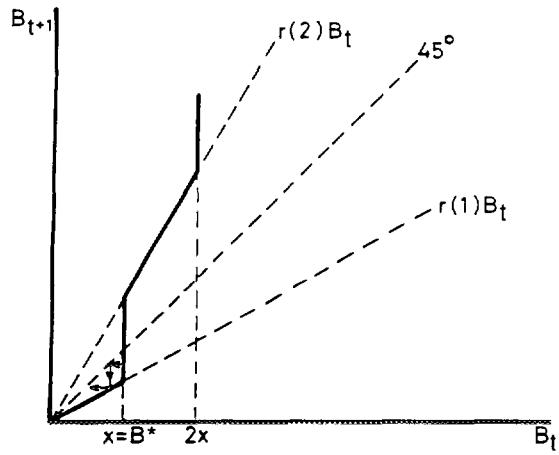

FIG. 3. $r(1) < 1$.

In other words, an equilibrium is determinate if the initial condition determines the time path of B_t. An equilibrium which is not determinate is *indeterminate*.[10]

When $r(n) \neq 1$ for any n, the steady-state equilibrium with positive savings is indeterminate. The result is that there is a very large set of equilibrium paths. Not only can we choose any $B_0 \in (0, B^*)$ and construct a nonstationary equilibrium, but we can also proceed as follows. Set $B_t = B^*$ for $t = 0, 1, \ldots, T-1$, and set $B_T \in [r[n(B^*)]B^*, B^*)$. Then $\{B_t\}_{t=T}^{\infty}$ will be a decreasing sequence with $B_t \to 0$. Since this can be done for any $T = 1, 2, \ldots$, and since for each choice of T there is a continuum of such equilibria, the equilibrium set is much larger than that in corresponding overlapping generations models with divisible assets [see Gale (1973)]. Thus, asset indivisibilities dramatically increase the set of equilibria in this case.

Finally, we note that nonstationary equilibria with indivisible assets will necessarily display long periods of stable inflation. This is the case because, whenever $B_t \in ((n-1)x, nx)$, the equilibrium rate of return is constant. Moreover, $B_t \in (0, x)$ will hold for large enough t, so that the rate of return on the outside asset will (in finite time) become constant at $r(1)$. This represents a form of price stickiness and is in contrast to nonstationary equilibria in divisible asset overlapping generations models

[10] Note that this type of indeterminacy bears no relationship to the standard indeterminacies that arise in overlapping generations models of money due to the lack of an initial condition. For a discussion of these "initial conditions indeterminacies" see Gale (1973) or Sargent (1987).

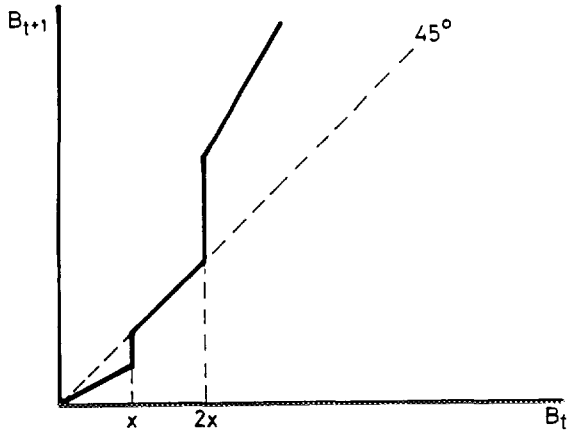

FIG. 4. $r(n) = 1$.

(with $f' > 0$), where inflation necessarily accelerates over time. However, when $B_{t+1} < nx < B_t$ holds, there will be a discrete, one-time jump in the rate of inflation. This pattern of inflation, where long periods of price stability are interrupted by discrete jumps, has often been observed historically in economies with commodity monies (and where the denomination structure of the coinage was a matter of significance). For instance, Cipolla (1982) discusses some examples of this phenomenon; our analysis indicates that they can probably be accounted for by the fact that indivisibilities of the coinage were important in such economies.

This pattern of stable inflation, punctuated by discrete increases in the inflation rate, has also been observed in modern economies. For example, Columbia had a fairly stable inflation rate in the range of 9.7% per year in 1970. After a discrete increase, the Colombian rate of inflation averaged 22.1% per year over the period 1973–1976, 23.1% over the period 1978–1989, and was 25.4% in 1992. A similar pattern of abrupt jumps in the inflation rate has been observed in Brazil. Dornbusch and Fischer (1993) describe these events, but do not provide enough information to determine whether the method of issuing government liabilities can account for the behavior of inflation in these cases.

A continuum of Pareto-ranked stationary equilibria. We discusss the third case separately. Marimon and Wallace (1987) have shown that costly asset divisibility can result in multiple Pareto-ranked stationary equilibria with positive savings.[11] Here suppose $r(n) = 1$ for some n. This case is depicted in Fig. 4. Clearly there is a continuum of stationary

[11] Under regularity conditions normal overlapping generations models with divisible assets have at most one stationary equilibrium with positive savings, as in Gale (1973).

equilibria with $r_t = r(n) = 1$. All such equilibria yield young agents utility equal to $u(w_1 - \phi - nx) + v[w_2 + r(n)nx]$, which is independnet of the stationary level of B_t. However, equilibria with higher B_t yield the initial old higher utility and hence are Pareto superior. Thus, a continuum of Pareto-ranked stationary equilibria with positive savings is possible.

Hence, if a steady-state equilibrium with positive savings exists, there is either a continuum of such equilibria or the steady-state equilibrium is indeterminate. In the latter case, there is a continuum of nonstationary equilibria, even if the initial debt level coincides with the steady-state debt level. These equilibria can also be Pareto ranked.

Young welfare. Under assumptions implying increasing savings functions, standard overlapping generations models have the feature that increases in B_t (that are consistent with equilibrium) raise r_t and hence the welfare of young savers. Here an analogous result obtains. For $n \geq 1$ define

$$G(n) = u(w_1 - \phi - nx) + v[w_2 + r(n)nx]$$

whenever $r(n)$ exists, and define $G(0) \equiv u(w_1 + v(w_2)$. Then for a given (equilibrium) value of B_t, $G[n(B_t)]$ is young welfare at t if $B_t > n(B_t)x$. If $B_t = n(B_t)x$, then young welfare at t lies in the interval $[G[n(B_t)], G[n(B_t) + 1]]$. Clearly, if $\{B_t\}$ and $\{B_t'\}$ are two equilibrium sequences of debt levels, if $n(B_t) = n(B_t')$, and $n(B_t)x \neq B_t n(B_t')x \neq B_t'$ holds, then young agents have the same utility at B_t and B_t'. However, we now show

PROPOSITION 3. *$G(n + 1) \geq G(n)$ for all n such that $r(n + 1)$ exists. $G(n + 1) > G(n)$ for all such $n > 0$.*

Proof. For $n = 0$, this follows from the definition of $r(1)$. For $n > 0$, by definition we have

$$\begin{aligned} G(n + 1) &= u[w_1 - \phi - (n + 1)x + v[w_2 + r(n + 1)(n + 1)x] \\ &\equiv u(w_1 - \phi - nx) + v[w_2 + r(n + 1)nx] \\ &> u(w_1 - \phi - nx) + v[w_2 + r(n)nx] \equiv G(n), \end{aligned}$$

where the inequality follows from Lemma 3 in the Appendix.

EXAMPLE. Let $u(c_1) + v(c_2) = c_1 + \ln c_2$. Then from (3), $r(n)$, $n > 1$, is defined by

$$\begin{aligned} &w_1 - \phi - nx + ln[w_2 + r(n)nx] \\ &\quad \equiv w_1 - \phi - (n - 1)x + \ln[w_2 + r(n)(n - 1)x]. \end{aligned} \tag{10}$$

Solving (10) for $r(n)$ gives

$$r(n) = \frac{w_2(e^x - 1)}{nx - (n - 1)xe^x}, \tag{11}$$

and $r(n)$ exists (is positive) $\forall n$ satisfying $n/(n - 1) > e^x$. Also, $r(1)$ is given by

$$w_1 - \phi - x + \ln[w_2 + r(1)x] \equiv w_1 + \ln w_2.$$

Solving for $r(1)$ yields

$$r(1) = (e^{\phi + x} - 1)w_2/x.$$

In addition, $f(r) = 1 - (w_2/r)$, so that $f^{-1}(x) = w_2/(1 - x)$. Then $f^{-1}(x) \geq \tilde{r}$ iff $1 \geq (1 - x)e^{\phi + x}$. Of course (A4) could be replaced with the weaker assumption that $r(1) \leq r(2)$, which would hold here iff $1 \geq e^{\phi + x}(2 - e^x)$. Finally, it is apparent when $r(1) < 1$ will hold.

II. THE MODEL: INTERMEDIATION

Intermediaries have two roles in this economy. First, if the fixed cost is a cost of one agent participating in a T-bill market, buying a scale, or taking goods to market, one agent could bear this cost while representing others. The fixed cost could then be borne by a large number of agents, thereby rendering it negligible. Second, agents could pool their resources and, in effect, share assets of limited divisibility. This allows agents to achieve convex combinations of integer multiples of x. Consequently, if intermediaries could form at zero cost, they clearly would.

We now suppose that intermediaries can form, but at a cost. The intermediary bears the fixed cost ϕ (but divides this cost among an arbitrarily large number of agents) and can intermediate z units of assets at a cost of γz, where $\gamma > 0$. We assume that intermediaries have no assets of their own (capital). Rather, the intermediary sells shares in itself in amount q_t per customer. (Alternatively, q_t could be deposits or the real value of bank notes issued.) The gross return on these shares is R_t with certainty. Then if the entire per capita debt, B_t, is intermediated, $q_t = (1 + \gamma)B_t$ shares must be sold to purchase the debt and cover costs (the per capita fixed cost is zero if the intermediary serves a large number of clients).[12]

[12] Note that we have implicitly assumed a linear pricing scheme by the intermediary; intermediary clients pay a fee of $r_t - R_t$ per unit intermediated for the service. Another possibility would be that intermediaries also charge a fixed fee. It is straightforward to show, however, that if agents can buy as many shares as they want at the going rate of return and intermediaries are Nash competitors, in equilibrium the fixed fee charged will be zero; that is, the pricing scheme described in the text is the equilibrium pricing scheme.

With free entry, zero profits requires that $R_t q_t = r_t B_t$. Therefore

$$R_t = r_t/(1 + \gamma). \tag{12}$$

A. Equilibrium Conditions

Behavior of agents. Given the fixed cost associated with direct individual participation in the market for the primary asset, any agent will either hold only primary assets or hold only intermediary liabilities. Of course for an individual, saving through an intermediary involves no fixed costs. Therefore, an agent who holds intermediary shares will have the savings function $g(R_t)$, where g is defined by

$$g(R_t) \equiv \arg\max[u(w_1 - s) + v(w_2 + R_t s)].$$

Clearly, g is continuous, and assumptions (A1) and (A2) imply that it satisfies $g(1) > 0$ and $g'(R_t) > 0$.

Given the prevailing interest rate on the primary asset, r_t, the utility of an agent who holds the primary asset directly (and hence bears the fixed cost ϕ) cannot exceed $H(w_1 - \phi, w_2, r_t)$. On the other hand, agents who save through intermediaries will achieve the utility level $u[w_1 - g(R_t)] + v[w_2 + R_t g(R_t)] \equiv H(w_1, w_2, R_t)$, where $R_t = r_t/(1 + \gamma)$. Thus, whenever $B_t > 0$, all agents will wish to save through intermediaries if

$$H[w_1, w_2, r_t/(1 + \gamma)] > H(w_1 - \phi, w_2, r_t) \tag{13}$$

holds, for all possible equilibrium values of r_t. We now provisionally assume that (13) holds for all such r_t and derive the conditions that an equilibrium must satisfy. We then describe conditions implying that (13) holds for any r_t that can satisfy the appropriate equilibrium conditions.

Equilibrium. We now define a competitive equilibrium for an economy where the primary asset is intermediated.

DEFINITION. An equilibrium for the intermediated economy is a set of sequences $\{r_t\}$, $\{R_t\}$, and $\{B_t\}$ satisfying (12),

$$B_{t+1} = r_t B_t \tag{14}$$

and

$$(1 + \gamma)B_t = g(R_t). \tag{15}$$

Equation (15) requires that agents purchase enough intermediary shares for the intermediary to acquire the debt and cover its costs. Substituting

(12) and (15) into (14) gives the equilibrium law of motion for B_t:

$$B_{t+1} = (1 + \gamma)B_t g^{-1}[(1 + \gamma)B_t]. \tag{16}$$

If $(1 + \gamma)g^{-1}(0) \geq 1$, a unique equilibrium obtains with $B_t = 0$ $\forall t$. If $(1 + \gamma)g^{-1}(0) < 1$, there are two stationary equilibria and a continuum of nonstationary equilibria, as shown in Fig. 5.

In general, the relation between the law of motion for the intermediated and unintermediated economy can be almost anything. For instance, consider the example of Section I, with the additional condition $w_1 > 1$. Then, $r(1) > 1$ holds iff $(e^{\phi+x} - 1)w_2 > x$ holds, while $r(1) < 1$ in the opposite case, and $g^{-1}(0)(1 + \gamma) > (<) 1$ iff $(1 + \gamma)w_2 < (>) 1$. Clearly, all configurations are possible. Similarly, the steady-state debt level with intermediation, which exists if $(1 + \gamma)w_2 < 1$, is $(1 + \gamma)^{-1} - w_2$. For the intermediated economy $B_t = x$ can be selected as the steady-state equilibrium, with $x \gtrless (1 + \gamma)^{-1} - w_2$ by appropriate choice of parameters. Thus intermediation can raise, lower, or leave unchanged the steady-state equilibrium savings level.

Note that, under our assumptions, all intermediated equilibria are determinate. Thus intermediation reduces the scope for multiplicity of equilibria. This is in contrast to arguments often given (see Mints (1945), for example) that free and unregulated entry into intermediation might add scope for indeterminacies. However, as we describe below, there is still a rationale for regulating entry into intermediation in this model.

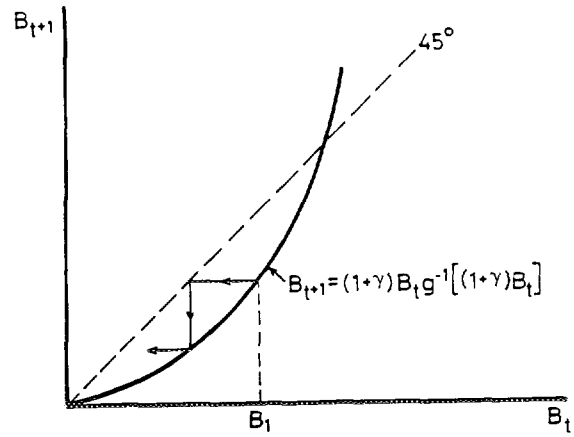

FIG. 5. An intermediated economy.

B. The Role for Intermediaries

Section I analyzes equilibria without intermediation whereas Section II.A describes equilibrium conditions that obtain with active intermediaries. Agents will, of course, prefer to save through intermediaries when (13) holds for all possible equilibrium values of r_t. We now describe what this entails.

Consider first the situation when intermediaries are initially absent. Then if the equilibrium asset supply at t is $B_t > 0$, $r_t = r[n(B_t)]$ if $n(B_t)x > B_t$, and $r_t \in [r[n(B_t)], r[n(B_t) + 1]]$ otherwise. In either event, an intermediary could form, divide the fixed cost among a large number of agents, and allow agents to share the primary asset. At the prevailing market interest rate r_t (which agents take as given when deciding to form an intermediary), this will result in a utility gain for savers if (13) holds. Moreover, since $B_t > 0$, $r_t \in [r(1), 1]$. Then if (13) holds $\forall r_t \in [r(1), 1]$, there is always an incentive for the formation of an intermediary. Note that under the same condition, agents will view it as optimal to hold only intermediary shares. Finally, since $H_3 > 0$, for any $\phi > 0$ there always exists a positive value $\tilde{\gamma}$ below which (13) will hold $\forall r_t \in [r(1), 1]$.

Now suppose that an economy is on an intermediated equilibrium path, as in Section II.A. We ask whether, at the prevailing market rate r_t, agents have any incentives to hold indivisible assets directly. If they do so they bear the fixed cost ϕ and have utility not exceeding $H(w_1 - \phi, w_2, r_t)$. As holders of (only) intermediary shares, their utility is $H[w_1, w_2, r_t/(1 + \gamma)]$. Therefore if (13) holds for all relevant r_t, no agent will have an incentive to hold the indivisible asset directly. It only remains, then, to describe "relevant" values of r_t.

Clearly if $r_t \leq r(1)$, an agent cannot obtain utility exceeding $u(w_1) + v(w_2)$ by holding the indivisible asset directly. Therefore, if (13) holds $\forall r_t \in [r(1), 1]$, no agent will have an incentive to purchase unintermediated primary assets. Of course $r_t \leq r(1)$ can hold only if $(1 + \gamma)g^{-1}(0) \leq r(1)$. Then the relevant values of r_t lie in the interval $(\max[r(1), (1 + \gamma)g^{-1}(0)], 1]$.

This discussion raises the question of whether (13) might be satisfied for some relevant values of r_t and be violated for others. In this instance intermediaries might exist during certain time intervals (depending on B_1) and not others. There might also be existence issues if for a given r_t, (13) held, say giving intermediaries an incentive to form, but if their formation resulting in an equilibrium value r_t' that violated (13). At this point we merely raise these as questions and henceforth assume that (13) holds for all relevant r_t.[13]

[13] Of course (13) is not a necessary condition for intermediation, so violation of (13) need not indicate that there is no incentive for intermediation.

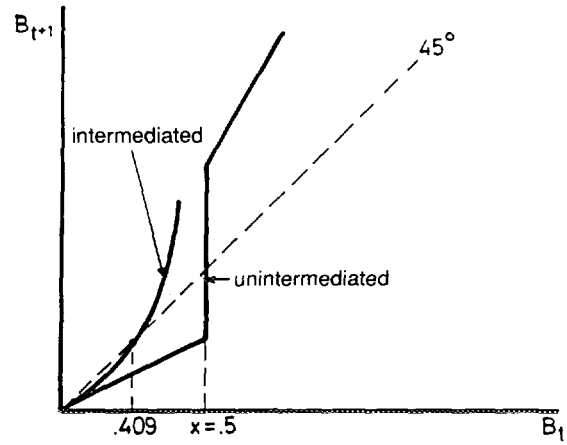

FIG. 6. The case of financial repression.

The formation of intermediaries and the necessity of legal restrictions. As indicated in the Introduction, governments (especially in developing countries) often discourage the formation of intermediaries.[14] The foregoing discussion suggests a reason why this might be the case. Figure 6 depicts the equilibrium laws of motion for B_t for the intermediated and unintermediated economies of the example in Section I, under the assumptions that $x = w_2 = 1/2$, $\gamma = 0.1$, $\phi = \ln(1.9) - x$, and $w_1 = 2$. As is clear, if $B_1 \in (0.409, 0.5]$, the unintermediated economy has an equilibrium. However, for this initial debt level, the intermediated economy would have to have $r_t > 1\ \forall t$, so that interest obligations on the debt would "blow up." In this case, as the development literature often argues, intermediation raises equilibrium returns for a given debt level. Here, returns are raised sufficiently for debt service to become infeasible. Thus, a government with a sufficiently large initial debt will need to prevent intermediaries from forming until $B_t \leq 0.409$. Note in particular that governments with large debt levels would be the ones that are motivated to represss intermediaries for this reason.[15]

[14] See, e.g., McKinnon (1973) or Shaw (1973) for a discussion of "financial repression."

[15] In a coinage economy, the argument just given could be formulated as follows: the government must prevent intermediation until there is a "currency shortage" (i.e., the value of real balances is sufficiently small). Such an argument would appear to account for a number of historical instances. It is also the case that financial repression often occurs for considerations of deficit finance. See Bencivenga and Smith (1992) for a discussion of this type of financial repression.

EXAMPLE. For the example economy of Section I, (13) is

$$\phi > \ln(1 + \gamma) - (\gamma w_2/r_t).$$

This condition is satisfied for all relevant r_t by the numerical example just given.

III. THE VALUE OF INTERMEDIATION

We define the social value of a firm or industry to be the amount agents are willing to pay for the goods or services it produces. Here the service offered by intermediaries is asset divisibility. Accordingly we ask how much agents are willing to pay to avoid holding the primary asset directly [assuming that (13) holds]. We develop three measures of the value of intermediation: one uses prices that prevail with absent intermediaries, one uses prices that prevail with intermediaries, and one allows for the fact that intermediaries affect market rates of return. In each case we show that typical methods for measuring the output of intermediaries cannot accurately reflect the social value of intermediation over the entire range of values for equilibrium debt levels.

A. *The "No-Intermediary" Benchmark*

Suppose at date t there is an unintermediated equilibrium with $B_t > 0$. Then the utility of young agents at t in this equilibrium is $u[w_1 - \phi - n(B_t)x] + v[w_2 + r_t n(B_t)x]$, where r_t is the date t equilibrium interest rate. We define the date t value of intermediation (per customer), y_t, to be the amount agents would be willing to pay to avoid holding the primary assets directly. Then y_t is given by

$$H[w_1 - y_t, w_2, r_t/(1 + \gamma)] \equiv u[w_1 - \phi - n(B_t)x] + v[w_2 + r_t n(B_t)x], \tag{17}$$

which gives y_t as a function of B_t whenever r_t is a function of B_t. (This occurs whenever $B_t \neq n(B_t)x$.) We henceforth ignore the (finite) set of values B_t such that B_t is an integer multiple of x. For the remaining values B_t, (17) becomes

$$H[w_1 - y(B_t), w_2, r[n(B_t)]/(1 + \gamma)] \equiv G[n(B_t)]. \tag{17'}$$

Apparently, if $n(B_t) = n(B_t')$, then $y(B_t) = y(B_t')$. Thus, intermediary services provided are constant $\forall B_t \in ([n(B_t) - 1]x, n(B_t)x)$, that is, are independent of the quantity of assets intermediated. Moreover, in prac-

tice intermediary output is often measured using costs of operation. Here per capita operating costs are γB_t, which are increasing in B_t. Thus, over time intermediary costs can change while intermediary service provision remains constant.

It is also common to measure intermediary output per unit of assets or liabilities. Measured in this way intermediary output is $y(B_t)/B_t$, which is decreasing in B_t $\forall B_t \in ([n(B_t) - 1]x, n(B_t)x)$. Costs per unit intermediated are, of course, γ, so again costs will not accurately reflect the social value of intermediary output movements over time.

B. The Intermediation Benchmark

Now suppose that an intermediated equilibrium exists with $B_t > 0$ and with equilibirum return $r_t = (1 + \gamma)g^{-1}[(1 + \gamma)B_t]$ at t. We ask how much agents would be willing to pay, at this interest rate, to avoid holding the indivisible assets directly. To answer this question, note that if $r_t \in [r(n), r(n + 1)]$, agents holding the indivisible asset directly would optimally purchase nx units. Then agents would be willing to pay $y_t = y(B_t)$ to avoid holding the indivisible asset directly, where $y(B_t)$ is defined by

$$H(w_1 - y(B_t), w_2, g^{-1}[(1 + \gamma)B_t]) = u(w_1 - \phi - nx) + v(w_2 + (1 + \gamma)g^{-1}[(1 + \gamma)B_t]nx) \quad (18)$$

$\forall B_t$ such that $(1 + \gamma)g^{-1}[(1 + \gamma)B_t] \in [r(n), r(n + 1)]$. Note that $y'(B_t)$ is well defined whenever $(1 + \gamma)g^{-1}[(1 + \gamma)B_t] \neq r(n)$ for some integer n.

In general the sign of $y'(B_t)$ (when y' exists) is ambiguous. However, $y'(B_t) < 0$ is possible, in which case intermediary services are inversely related (over some interval) to both the quantity of assets intermediated and intermediary costs (γB_t). A sufficient condition for $y'(B_t) < 0$ is now given.

PROPOSITION 4. *Let* $(1 + \gamma)g^{-1}[(1 + \gamma)B_t] \in (r(n), r(n + 1))$ *for some* n. (a) *Then* $y'(B_t) < 0$ *iff* $n \geq n(B_t)$ [*with strict inequality if* $B_t = n(B_t)x$], *and* (b) $n \geq n(B_t)$ *holds if* $r_t = (1 + \gamma)g^{-1}[(1 + \gamma)B_t] \geq r[n(B_t)]$. *Thus* $(1 + \gamma)g^{-1}[(1 + \gamma)B_t] \geq r[n(B_t)]$ *and* $B_t \neq n(B_t)x$ *is sufficient for* $y'(B_t) < 0$.

Proof. For B_t as stated, $y'(B_t)$ exists and satisfies

$$-H_1(-)y'(B_t) + H_3(-)(1 + \gamma)(g^{-1})' = (1 + \gamma)^2 v'(w_2 + (1 + \gamma)g^{-1}[(1 + \gamma)B_t]nx)nx(g^{-1})',$$

where

$$H_3(-) = v'\left[w_2 + \left(\frac{r_t}{1 + \gamma}\right) g\left(\frac{r_t}{1 + \gamma}\right)\right] g\left(\frac{r_t}{1 + \gamma}\right).$$

Then, since $H_1 > 0$, $y'(B_t) < 0$ iff

$$v'(w_2 + r_t nx)(1 + \gamma)nx > v'\left[w_2 + \left(\frac{r_t}{1 + \gamma}\right) g\left(\frac{r_t}{1 + \gamma}\right)\right] g\left(\frac{r_t}{1 + \gamma}\right). \tag{19}$$

But, (A1) implies that $v'(w_2 + z)z$ is increasing in z, so that (19) is equivalent to $(1 + \gamma)nx > g[r_t/(1 + \gamma)]$ or to

$$(1 + \gamma)g^{-1}[(1 + \gamma)nx] > r_t = (1 + \gamma)g^{-1}[(1 + \gamma)B_t]. \tag{19'}$$

Expression (19') reduces to $nx > B_t$, or equivalently $n \geq n(B_t)$ (with strict inequality if $B_t = n(B_t)x$).

Furthermore, suppose that $r_t \geq r[n(B_t)]$. Then $n \geq n(B_t)$, which implies that (19') holds if $B_t \neq n(B_t)x$.

Parenthetically, the economy depicted in Fig. 6 satisfies $r_t = (1 + \gamma)g^{-1}[(1 + \gamma)B_t] > r[n(B_t)]$ $\forall r_t \leq 1$. Thus economies satisfying this condition for all relevant r_t are easily constructed. All such economies will have a social value of intermediary output, as defined by (18), that is inversely related almost everywhere to the quantity of assets intermediated and to intermediary operating costs.

C. *A General Equilibrium Measure*

In this section we propose a measure of the value of intermediary services that takes account of the fact that intermediation has general equilibrium consequences for rates of return. In particular, per capita intermediary output at t, y_t, is now defined to be the amount that agents are willing to pay to avoid holding indivisible assets directly, with any changes in equilibrium rates of return taken into account. Thus, if the time t asset supply per capita is B_t, $y_t = y(B_t)$, with $y(B_t)$ defined by

$$H[w_1 - y(B_t), w_2, g^{-1}[(1 + \gamma)B_t]] \equiv G[n(B_t)]. \tag{20}$$

It is now demonstrated that $y(B_t)$ cannot be monotone in B_t. In particular, for $B_t \in ((n - 1)x, nx)$, $y'(B_t) > 0$ since over this interval $G[n(B_t)]$ is constant. We now show that if $B_t + \varepsilon = n(B_t) = B_t' - \varepsilon$, then $y(B_t') < y(B_t)$ for $\varepsilon > 0$ and sufficiently small. To see this, observe that by definition (for ε sufficiently small)

$$H[w_1 - y(B_t'), w_2, g^{-1}[(1 + \gamma)B_t']] - H[w_1 - y(B_t), w_2, g^{-1}[(1 + \gamma)B_t]] \equiv G[n(B_t) + 1] - G[n(B_t)]. \tag{21}$$

As $\varepsilon \to 0$ the right-hand side of (21) is a positive constant (by Proposition 3). If $y(B_t') \geq y(B_t)\ \forall B_t' > B_t$, then as $\varepsilon \to 0$ the left-hand side of (21) approaches a number bounded above by zero. But this is a contradiction. Thus $y(B_t)$ must be decreasing at certain values of B_t.

Since intermediary costs per capita (γB_t) are monotone in B_t, in certain neighborhoods measured costs will move in the opposite direction from $y(B_t)$. Costs then do not reliably reflect movements in intermediary services for any of our output measures. Furthermore, for at least two of our intermediary output measures, $y(B_t') < y(B_t)$ for some B_t, B_t' with $B_t' > B_t$. Intermediary output can decline while intermediary assets/liabilities increase in real terms. Consequently, changes in intermediary assets or liabilities also need not give any indication of directions (much less magnitudes) of movement in the provision of intermediation services.

Discussion. We have assumed that direct participation in the market for the primary (indivisible) asset involves a fixed cost (ϕ), whereas intermediation is a constant marginal cost (γ) activity. We have previously described how ϕ might be interpreted, while γ simply represents intermediary costs (costs of record keeping and service provision), which are probably reasonably represented as proportional to the level of intermediary transactions. Indeed, our assumptions have a substantial empirical basis, as it is widely believed that there are decreasing costs to financial market activity at small levels of activity, whereas average costs appear to be well approximated by a constant at the levels at which most financial intermediaries operate.[16]

We have assumed that the costs of direct participation in the market for primary assets is of this fixed form for a simple reason. In the absence of a fixed component of transaction costs, when $B_t = n(B_t)x$, agents are not inconvenienced by the indivisibility of the asset, and hence in the absence of a transactions cost advantage there would be no role for intermediaries (at such a debt level). In equilibrium, intermediaries would then be active at some debt levels and not at others. It would be straightforward to let both direct asset market participation and intermediary activity have a fixed and a variable cost component (so long as intermediaries retain a cost advantage at all values of B_t). However, we avoid this because it complicates notation without materially affecting the analysis.

[16] See Clark (1988). Some empirical studies find evidence of decreasing costs of intermediation globally, while others find evidence of increasing costs. The evidence against constant average costs of intermediation—at least at reasonable scales of operation—does not appear to be strong on either side.

IV. CONCLUSIONS

Indivisibility of assets is an important institutional feature of asset markets. The preceding sections present a framework for analyzing economies with indivisible assets and for analyzing the incentives for and equilibrium consequences of the intermediation of such assets.

Not surprisingly, asset indivisibilities create considerable scope for indeterminacy.[17] In fact, for a variety of economies there will be either a continuum of stationary equilibria or indeterminacies. We also demonstrated that nonstationary equilibria with indivisible assets must eventually display long periods of stable inflation. This is the case even though the indivisibility potentially creates substantial deadweight losses.

We have also shown that, for appropriate costs associated with asset trading and intermediation, there is always a role for intermediaries. Intermediated equilibria are qualitatively similar to standard equilibria in homogeneous agent, overlapping generations models but have the feature that the primary asset bears a higher return than intermediary liabilities. It is also possible that, over some range of values for the initial debt, intermediation raises the rates of return on debt sufficiently that the formation of intermediaries will cause debt service to explode. In this case the existence of equilibria may depend on the government inhibiting the formation of intermediaries. Free and unregulated entry into intermediation does not increase the problem of multiplicity of equilibria, as is often asserted.

The fact that the primary asset will dominate intermediary liabilities in rate of return also is of interest in economies with a specie currency. For instance, during periods with free banking intermediaries issued bank notes that were claims to specie.[18] To the extent that there were defaults or note-holder losses, bank notes may appear to have been dominated in rate of return by other assets. White (1987) has made this point and argued that since free banks were largely unregulated, observations of this type constitute evidence against "legal restrictions theories" of money as articulated by Wallace (1983). However, if we interpret our indivisible asset as a metallic currency, banks will emerge that issue notes that are completely backed by specie and yet bear lower rates of return to specie.[19]

[17] This point is also a theme of Marimon and Wallace (1987) and Smith (1989).

[18] See Rolnick and Weber (1988) for an interpretation of free banks as mutual funds—which is, of course, what our intermediaries are.

[19] Two points deserve mention. One is that while small change often existed in earlier eras, it was often not in circulation. This point is discussed by Rolnick and Weber (1986) and Glassman and Redish (1988). Second, many banks were primarily in the business of intermediating specie. According to Carothers (1930, p. 79–80), after the War of 1812, the per capita

This situation is consistent with the point of view put forth by Wallace (1983). Finally, we have observed that intermediary output—the consumer surplus created by intermediaries—can be negatively related both to costs and to the quantity of assets intermediated.

We now comment on some issues that have not been addressed. First, we have not discussed why assets might be issued in indivisible forms. For historical coinage economies the answer is probably technological.[20] For treasury liabilities or other securities the answer is less obvious, although issuing these in indivisible forms may reduce the costs of issue, as argued by Klein (1973), or permit nonlinear pricing, as in Guesnerie and Seade (1982) or, more explicitly, in Bryant and Wallace (1984), or Villamil (1988). This possibility is further considered by Cooley and Smith (1993), who demonstrate the existence of a unique minimum denomination which maximizes steady-state welfare subject to a constraint that the relevant steady-state equilibrium be determinate.

Second, we have not considered the possibility of altering the denomination structure. Cooley and Smith (1993) allow a one-time choice of the denomination structure. However, in historical coinage economies, changes in the money supply often required recoinage, so that there were repeated alterations of the minimum denomination. The possibility of recoinage, with consequent seigniorage income and changes in denomination structure, would be an interesting topic for further investigation.

Another topic meriting further investigation would be an analysis of a world with indivisible assets and heterogeneous agents. With a finite set of different agent types we conjecture that the qualitative nature of our results would survive. However, the form of the analysis would be far more complicated than that presented here.

APPENDIX

Propositions 1 and 2 are immediate implications of the following four lemmas.

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supply of outside money in denominations of less than 50 cents was less than 25 cents. Carothers argues that many banks existed largely to intermediate specie, issuing notes in denominations of less than 50 cents. Parenthetically, at the time 50 cents was in the neighborhood of a day's per capita income. Finally, while free banks apparently did not intermediate specie, by 1830 many states had prohibited bank note issues in denominations of less than $1. Indeed, White (1990) documents that a surprisingly large number of states prohibited the issuance of notes in denominations less that $5 and that these restrictions carried over into the free banking era. (Free banking in the United States began in 1837.) These issues are discussed in some detail by Carothers (1930).

[20] See Hanson (1979, 1980), Rolnick and Weber (1986), and Glassman and Redish (1988) on problems of dividing coins or that inhibited the creation of small change.

LEMMA 1. *Suppose at the interest rate r_t', agents (weakly) prefer saving n_2x to n_1x, where n_1 and n_2 are integers. If $n^* > n_2 > n_1$, then n_2x is (strictly) preferred to n_1x $\forall r_t > r_t'$. Similarly, if $n_2 < n_1 < n^*$, n_2x is preferred to n_1x $\forall r_t < r_t'$.*

Proof. By assumption, if $n_2, n_1 \geq 1$, then

$$u(w_1 - \phi - n_2x) + v(w_2 + r_t'n_2x) \geq u(w_1 - \phi - n_1x) + v(w_2 + r_t'n_1x).$$

Now define $\Theta(n_1, n_2, r)$ by

$$\Theta(n_1, n_2, r) = u(w_1 - \phi - n_2x) + v(w_2 + rn_2x) - u(w_1 - \phi - n_1x) - v(w_2 + rn_1x).$$

Then

$$\Theta_3 = v'(w_2 + rn_2x)n_2x - v'(w_2 + rn_1x)n_1x$$

and $(n_2 - n_1)\Theta_3 > 0$, where the inequality follows from (A1). (In particular $v'(w_2 + z)z$ is an increasing function of z.) This establishes the result for $n_2, n_1 \geq 1$. If $n_2 > n_1 = 0$, then

$$u(w_1 - \phi - n_2x) + v(w_2 + r_t'n_2x) \geq u(w_1) + v(w_2),$$

and the result is obvious.

LEMMA 2. *Suppose that $f^{-1}(nx)$ exists, $n > 1$. Then $r(n)$ exists and satisfies $f^{-1}[(n-1)x] < r(n) < f^{-1}(nx)$.*

Proof. By definition,

$$u(w_1 - \phi - nx) + v[w_2 + f^{-1}(nx)nx] > u[w_1 - \phi - (n-1)x] + v[w_2 + f^{-1}(nx)(n-1)x].$$

Similarly,

$$u(w_1 - \phi - nx) + v[w_2 + f^{-1}[(n-1)x]nx] < u[w_1 - \phi - (n-1)x] + v[w_2 + f^{-1}[(n-1)x](n-1)x].$$

Then by the intermediate value theorem there exists a value $r(n) \in (f^{-1}[(n-1)x], f^{-1}(nx))$ satisfying (3).

LEMMA 3. *If $r(n)$ exists, then $r(n) > r(n-1)$.*

Proof. Cooley and Smith (1993) demonstrate that if $r(n)$ exists then $f^{-1}[(n-1)x]$ also exists. If $n = 2$, then by (A4) and the definition of $r(1)$,

$r(1) \leq f^{-1}(x)$. By Lemmas 1 and 2, $r(2) > f^{-1}(x)$. Thus $r(2) > r(1)$. For $n > 2$ we have

$$r(n-1) < f^{-1}[(n-1)x] < r(n),$$

by Lemmas 1 and 2. This establishes the result.

LEMMA 4. *If $r_t = r(n)$, $(n-1)x$ and nx are preferred to any other positive asset choice.*

Proof. (Case 1: $n > 1$) Suppose that $\hat{n}x$, with $\hat{n} \neq (n-1)$, n, is a (weakly) preferred choice, and suppose $\hat{n} > n$. Then $u(w_1 - \phi - \hat{n}x) + v[w_2 + r(n)\hat{n}x] \geq u[w_1 - \phi - (n-1)x] + v[w_2 + r(n)(n-1)x]$. Moreover, there exists a value $\lambda \in (0, 1)$ such that $\lambda\hat{n} + (1-\lambda)(n-1) = n$. Therefore $u[w_1 - \phi - nx] + v[w_2 + r(n)nx] > u[w_1 - \phi - (n-1)x] + v[w_2 + r(n)(n-1)x]$ (since v is strictly concave). But this contradicts the definition of $r(n)$. A similar contradiction is derived if $0 < \hat{n} < n - 1$.

(Case 2: $n = 1$) Now suppose $\hat{n}x$ is a preferred asset choice. If $\hat{n} > 2$, then $2x$ is also a preferred asset choice, since $2 = \lambda\hat{n} + (1-\lambda)$ for some $\lambda \in (0, 1)$. Thus

$$u(w_1 - \phi - 2x) + v[w_2 + r(1)2x] \geq u(w_1 - \phi - x) + v[w_2 + r(1)x].$$

But this implies that $r(2) \leq r(1)$, contradicting Lemma 3.

Note that the lemmas imply that, if $r_t = r(n)$, then nx and $(n-1)x$ are the only optimal asset choices (including the choice of zero assets).

REFERENCES

BENCIVENGA, V. R., AND SMITH, B. D. (1992). Deficits, inflation, and the banking system in developing countries: The optimal degree of financial repression, *Oxford Econ. Pap.* **44,** 767–790.

BERGER, A. N., AND HUMPHREY, D. B. (1990). "Measurement and Efficiency Issues in Banking," manuscript.

BRYANT, J., AND WALLACE, N. (1979). The inefficiency of interest-bearing national debt, *J. Polit. Econ.* **87,** 365–381.

BRYANT, J., AND WALLACE, N. (1984). A price discrimination analysis of monetary policy, *Rev. Econ. Stud.* **51,** 279–288.

CAROTHERS, N. (1930). "Fractional Money." Wiley, New York.

CIPOLLA, C. M. (1982). "The Monetary Policy of Fourteenth Century Florence." Univ. of California Press, Berkeley.

CIPOLLA, C. M. (1987). "The Monetary Policy of Sixteenth Century Florence." Univ. of California Press, Berkeley.

CLARK, J. A. (1988). Economies of scale and scope at depository institutions: A review of the literature, *Fed. Res. Bank Kansas City Econ. Rev.* **73,** 16–33.

COOLEY, T. F., AND SMITH, B. D. (1993). A theory of optimal denominations for government liabilities, *Econ. Theory* **3,** 597–623.

DORNBUSCH, R., AND FISCHER, S. (1993). Moderate inflation, *World Bank Econ. Rev.* **7,** 1–44.

FIXLER, D. J., AND ZIESCHANG, K. D. (1990). "User Costs, Shadow Prices, and the Real Output of Banks," manuscript.

GALE, D. (1973). Pure exchange equilibrium of dynamic economic models, *J. Econ. Theory* **6,** 12–36.

GLASSMAN, D., AND REDISH, A. (1988). Currency depreciation in early modern England and France, *Exploration Econ. Hist.* **25,** 75–97.

GREENWOOD, J., AND HUFFMAN, G. (1987). A dynamic equilibrium model of inflation and unemployment, *J. Monet. Econ.* **19,** 203–228.

GUESNERIE, R., AND SEADE, J. (1982). Nonlinear pricing in a finite economy, *J. Public Econ.* **17,** 157–179.

HANSON, J. R., II (1979). Money in the colonial American economy: An extension, *Econ. Inquiry* **17,** 281–286.

HANSON, J. R., II (1980). Small notes in the American colonies, *Exploration Econ. Hist.* **17,** 411–420.

HORNSTEIN, A., AND PRESCOTT, E. C. (1990). "Measures of the Insurance Sector Output," manuscript.

KEMMERER, E. W. (1910). "Seasonal variations in the relative demand for money and capital in the United States." National Monetary Commission, Washington.

KLEIN, M. A. (1973). The economics of security divisibility and financial intermediation, *J. Finance* **28,** 923–931.

KUZNETS, S. (1953). National income and economic welfare, *in* "Economic Change." Norton, New York.

MARIMON, R., AND WALLACE, N. (1987). Trade using assets divisible at a cost, *J. Econ. Theory* **43,** 223–251.

MCKINNON, R. I. (1973). "Money and Capital in Economic Development." Brookings Institute, Washington, DC.

MINTS, L. W. (1945). "A History of Banking Theory in Great Britain and the United States." Univ. of Chicago Press, Chicago.

RICH, G. (1989). Canadian banks, gold, and the crisis of 1907, *Exploration Econ. Hist.* **26,** 135–160.

ROGERSON, R., AND WRIGHT, R. (1988). Involuntary unemployment in economies with efficient risk sharing, *J. Monet. Econ.* **22,** 501–516.

ROLNICK, A. J., AND WEBER, W. E. (1986). Gresham's law or Gresham's fallacy, *J. Polit. Econ.* **94,** 185–199.

ROLNICK, A. J., AND WEBER, W. E. (1988). Explaining the demand for free bank notes, *J. Monet. Econ.* **21,** 47–71.

ROSS, V. (1922). "A History of the Canadian Bank of Commerce," Vol. 2. Oxford Univ. Press, Toronto.

SARGENT, T. J. (1987). "Dynamic Macroeconomics." Harvard Univ. Press, Cambridge.

SARGENT, T. J., AND SMITH, B. D. (1994). "Gresham Laws and Their Obverses," manuscript, Cornell University.

SARGENT, T. J., AND WALLACE, N. (1983). A model of commodity money, *J. Monet. Econ.* **12,** 163–187.

SHAW, E. S. (1973). "Financial Deepening in Economic Development." Oxford Univ. Press, London.

SMITH, B. D. (1989). Legal restrictions, sunspots, and cycles, *J. Econ. Theory* **47,** 369–392.

TIMBERLAKE, R. J., JR. (1974). Denominational factors in nineteenth century currency experience, *J. Econ. Hist.* **34,** 835–850.

VILLAMIL, A. P. (1988). Price discriminating monetary policy: A non-uniform pricing approach, *J. Public Econ.* **51,** 378–386.

WALLACE, N. (1983). A legal restrictions theory of the demand for money and the role of monetary policy, *Fed Res. Bank Minneapolis Quart. Rev.*, 1–7.

WHITE, E. N. (1990). "Free banking, denominational restrictions and liability insurance," manuscript, Rutgers University, April.

WHITE, L. H. (1987). Accounting for non-interest-bearing currency: A critique of the legal restrictions theory of money, *J. Money Credit Banking* **19,** 448–455.

WILLIAMSON, S. D. (1986). Increasing returns to scale in financial intermediation and the non-neutrality of government policy, *Rev. Econ. Studies* **53,** 863–876.