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Optimal monetary policy in a Phillips-curve world

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Abstract

In this paper, we study optimal monetary policy in a model that integrates the modern theory of unemployment with a liquidity model of monetary transmission. Two policy environments are considered: period-by-period optimization (time consistency) and full commitment (Ramsey allocation). When the economy is subject to productivity shocks, the optimal policy is pro-cyclical. We also characterize the long-term properties of monetary policy and show that with commitment the optimal inflation rate is inversely related to the bargaining power of workers. Both results find empirical support in the data.
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"*Despite its disrepute within important academic and policymaking circles, the Phillips Curve persists in US data. Simple econometric procedures detect it.*" Thomas Sargent, 1998

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0. Introduction

A robust empirical feature of post-war US data is the positive correlation between inflation and employment, which is commonly referred to as the Phillips-curve relation (see [28]). This empirical feature supports the view that inflationary monetary policies have expansionary effects on the real sector of the economy, at least in the short-run. The goal of this paper is to study the optimal monetary policy in a model in which there is a direct link between these policies and employment.

We study a general equilibrium model where the real side of the economy is characterized by a search and matching framework with equilibrium unemployment. In this framework, we introduce a monetary sector in which changes in the supply of money affects the nominal interest rate by changing the supply of loanable funds (liquidity effect). The change in the interest rate, in turn, affects the financing cost of firms and impacts on the real sector of the economy. In this way the model captures the ''cost channel'' of monetary transmission that Barth and Ramey [5] find significant for the propagation of monetary shocks. This channel is also consistent with recent empirical studies that find significant liquidity effects of monetary policy shocks.[1]

We consider two policy environments. In the first environment we assume that monetary policy interventions are decided on a period-by-period basis, and the monetary authority cannot credibly commit to long-run plans (time-consistent policy). In studying the time-consistent policy, we restrict the analysis to policies that are Markov-stationary, i.e., policy rules that only depend on the current (physical) states of the economy. In the second policy environment, we assume that the monetary authority is able to commit to long-term plans (Ramsey allocation).

There are two main findings. The first finding concerns the *cyclical* properties of the optimal policy while the second concerns the *long-term* properties. Regarding the cyclical properties, we show that in both policy environments the optimal policy is pro-cyclical when business cycle fluctuations are driven by technology shocks: it increases the stock of money when employment and output are high and reduces the stock of money when they are low. Further, the optimal growth rate of money is positively correlated with employment and output. Both features—the pro-cyclicality of the monetary aggregates and the money growth—characterize the post-war history of the US economy as documented in [11].

The second finding is that there are important differences between the long-term properties of the time-consistent policy and the long-term properties of the optimal policy with commitment. We show that when the worker's share of the matching surplus is small and the employer's share high, the time-consistent policy is less inflationary than the optimal policy with commitment. This result contrasts with earlier studies of optimal monetary policy, such as [4,21].

The intuition for these results is simple. Consider first the pro-cyclicality of the optimal policy. After a positive productivity shock, the demand for loanable funds increases due to the firms' desire to expand production. The increase in the demand

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[1] See, for example [10,18,22].

for loanable funds raises the nominal interest rate and this is inefficient. To prevent the interest rate increase, the policy maker has to expand the supply of loanable funds by increasing the stock of money. Because the search and matching frictions in the labor market generate a persistent response of employment to shocks (hump-shaped response) and output grows for more than one period, the optimal growth rate of money is above its steady state level for more than one period. This implies that the growth rate of money is positively correlated with employment and output. In the absence of matching frictions, however, output will grow only in the first period and then return to the steady state. In this case, the optimal growth rate of money would be negatively correlated with employment and output: it would be below the steady state with the exception of the first period. Therefore, the search and matching frictions are key to generating the pro-cyclicality of money growth.

Consider now the long-term properties of the optimal policy. In this economy there are two possible sources of inefficiency. The first inefficiency derives from the cost of financing the current production plan for firms. On this dimension a Friedman rule of a zero nominal interest rate is optimal because a positive interest rate distorts the production decisions of firms by increasing their financing cost. The second source of inefficiency derives from the matching frictions in the labor market. As shown in [19], if the worker's share of the matching surplus is too small, there will be an excessive creation of vacancies due to the high profitability of a match for the firm. The policy maker can reduce the profitability of a match by increasing the nominal interest rates. However, the decision to create new vacancies is not affected by the current interest rate but only by future interest rates. The policy maker is able to *credibly* choose the future interest rates only if it can commit. Otherwise, after the new vacancies have been created, it no longer has the incentive to keep the high interest rate. The lack of commitment then implies that the time-consistent policy is given by a simple Friedman rule of a zero nominal interest rate while the optimal policy with commitment will set positive nominal interest rates. In the long run higher interest rates are associated with higher inflation rates (Fisher rule). As will be shown in Section 5, the importance of the worker's share of the surplus for the long-term property of the monetary policy is supported by data for a cross-section of countries.

There are several studies that are related to this paper. Shi [29] shows that with searching frictions the Friedman rule may not be efficient, although he does not conduct an explicit analysis of the optimal monetary policy. The optimal and time-consistent policy is studied in Ireland [20], but in an environment in which there are no frictions in the labor market and monetary policy affects the real sector of the economy through the rigidity of nominal prices. In Ireland's model the optimal monetary policy is also pro-cyclical when business fluctuations are driven by technology shocks. However, his results do not extend to our long-term results for which policy commitment can affect the properties of the optimal policy. Our novel results depend crucially on the consideration of search and matching frictions. A study of the differences between time-consistent policies and optimal policies with commitment in models with sticky prices and liquidity effects is conducted in [2]. In contrast to our paper, they do not find important long-term differences between the

environment with and without commitment. We reach a different conclusion because of the more complex dynamics introduced by the matching frictions that characterize the labor market.

The plan of the paper is as follows. In Section 1 we describe the model and in Section 2 we define the optimal policy in the two policy environments: absence of commitment and full commitment. Section 3 characterizes the analytical properties of the optimal policy and Section 4 examines their quantitative properties. Section 5 discusses the empirical relevance of our long-term results and provides cross-country evidence about the negative relation between the workers' share of the surplus and the inflation rate. Finally, Section 6 concludes.

1. The economy

We describe here a monetary economy that is specifically designed to generate the liquidity effect of monetary interventions, that is a reduction in the nominal lending rate after a monetary expansion. The reduction in the cost of borrowing, in turn, leads to an expansion in the real sector of the economy. By designing the economy so that inflationary policies have expansionary effects, we capture the main idea behind the Phillips-curve relation—that is, the idea that in the short run there is a trade-off between inflation and the real activity (a Phillips-curve world)—and this trade-off can be used for the design of monetary policy. The basic structure of the model is similar to the one developed in [12]. In that paper, however, we did not study the optimal policy which is the objective of the current paper.

1.1. The monetary authority and the intermediation sector

The total amount of households' nominally denominated assets is denoted by M. We interpret M as a broad monetary aggregate and will refer to it as money. Part of these assets are used for transactions and the remaining quantity is held in the form of bank deposits. The funds collected by banks are then used to make loans to firms. The monetary aggregate M is controlled by the monetary authority by making transfers to the households in the form of bank deposits. The monetary transfers are denoted by $T = gM$, where g is the growth rate of money.

For monetary interventions to have a liquidity effect—that is, a fall in the nominal interest rate after a monetary expansion—some form of rigidity has to be imposed in the households' ability to readjust their stock of deposits. We assume that the households choose the stock of nominal deposits at the end of each period after all transactions have taken place and they must wait until the end of the next period to change their portfolio. Denote by D the pre-transfer household deposits. Because the monetary transfers are in the form of bank deposits and households cannot readjust immediately these deposits, the funds available to banks to make loans are $D + gM$. Therefore, an increase in the growth rate of money increases the stock of loanable funds, which in turn induces a fall in the nominal interest rate. This is the liquidity

channel of "limited participation" models similar to Lucas [23], Fuerst [16] and Christiano and Eichenbaum [9].

1.2. Households

There is a continuum of agents of total measure 1 that maximize the expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t (c_t - \chi_t a), \tag{1}$$

where c_t is consumption of market produced goods, a is the disutility from working and χ_t is an indicator function taking the value of one if the agent is employed and zero if unemployed.

Households own three types of assets: transaction funds (cash), nominal deposits and firms' shares. Denoting by m the pre-transfers nominally denominated assets and by d the quantity of these assets kept in the form of deposits, the household's transaction funds are $m - d$. In each period, agents are subject to the following cash-in-advance and budget constraints:

$$P(c + i) \leqslant m - d, \tag{2}$$

$$P(c + i) + m' = m + gM + (d + gM)R + \chi Pw + P\pi n. \tag{3}$$

The variable P is the nominal price, i is the household's investment in the shares of new firms, n identifies the number of firms' shares that the household owns and π the real dividends paid by these firms. The real wage received by an employed worker is denoted by w and it is paid at the end of the period. The determination of the wage will be specified below. The nominal after-transfer stock of deposits is $d + gM$. These deposits earn the nominal interest rate R.

1.3. Production

The production sector is characterized by a search-matching framework similar to the labor-search model of Pissarides [27] and Mortensen and Pissarides [25] with exogenous separation. The production technology displays constant returns-to-scale with respect to the number of employees. Without loss of generality, it is convenient to assume that there is a single firm for each worker. The search for a worker involves a fixed cost κ and the probability of finding a worker depends on the matching technology $\mu V^{\alpha}(1 - N)^{1-\alpha}$, where V is the number of vacancies (number of firms searching for a worker), $1 - N$ is the number of searching workers and $\alpha \in (0, 1)$. The probability that a searching firm finds a worker is denoted by q and it is equal to $\mu V^{\alpha}(1 - N)^{1-\alpha}/V$, while the probability that an unemployed worker finds a job is denoted by h and is equal to $\mu V^{\alpha}(1 - N)^{1-\alpha}/(1 - N)$. Job separation is exogenous and occurs with probability λ. Workers can search for a new job only if unemployed and there is no cost for searching.

If the searching process is successful, the firm operates the technology $y = Ax^v$, where A is the aggregate level of technology and x is an intermediate input. Output goods and intermediate goods are perfect substitutes, and therefore, the relative price is 1. The aggregate level of technology A is subject to shocks and follows a first-order Markov process with transition density function $\Gamma(A, A')$. The purchase of the intermediate good requires liquid funds. Firms get these funds by borrowing from a financial intermediary at the nominal interest rate R.[2]

The contract signed between the firm and the worker specifies the wage $w(\mathbf{s})$ which depends on the states of the economy $\mathbf{s}$. The set of aggregate states will be specified below. The determination of the wage is such that the worker gets the share η of the matching surplus. The assumption of a constant sharing fraction of the surplus is standard in this class of models and it is motivated by assuming Nash bargaining between the firm and the worker. As we will see later, the parameter η plays a crucial role in characterizing the properties of the optimal policy.

1.3.1. Firms

Firms post vacancies and implement optimal production plans to maximize the welfare of their shareholders. Denote by $J(\mathbf{s})$ the value of a match for the firm measured in terms of current consumption. This is given by

$$J(\mathbf{s}) = \tilde{\pi}(\mathbf{s}) + \beta(1 - \lambda)EJ(\mathbf{s}'). \tag{4}$$

For notational convenience, we have defined the function $\tilde{\pi}(\mathbf{s}) = E(\beta P(\mathbf{s})/P(\mathbf{s}'))\pi(\mathbf{s})$, where $\pi(\mathbf{s})$ are the dividends paid by the firm to the shareholders at the end of the period. The function expresses the current value for the shareholder of the dividend paid by the firm. Because dividends are paid at the end of the period, the shareholder needs to wait until the next period to transform monetary payments into consumption. This implies that the real value (in terms of today's consumption) of one unit of money received at the end of the period is $\beta P(\mathbf{s})/P(\mathbf{s}')$.

The dividends paid to the shareholders are equal to the output produced by the firm minus the cost for the intermediate input, $x(1 + R)$, and the labor cost, w

$$\pi = Ax^v - x(1 + R) - w. \tag{5}$$

Notice that the cost for the intermediate input also includes the interest paid on the loan needed to finance the payment of the input.

Given $J(\mathbf{s})$ the firm's value of a match as defined above, the value of a vacancy $Q(\mathbf{s})$ is

$$Q(\mathbf{s}) = -\kappa + q(\mathbf{s})\beta EJ(\mathbf{s}') + (1 - q(\mathbf{s}))\beta EQ(\mathbf{s}') \tag{6}$$

Free entry implies that the value of a vacancy is zero in equilibrium and Eq. (6) becomes

$$\kappa = q(\mathbf{s})\beta EJ(\mathbf{s}'). \tag{7}$$

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[2] In alternative, we could assume that working hours are flexible and the intermediate input is replaced by the number of working hours. The properties of the model would not change if we assume that the part of the worker's payment that compensates the disutility from working has to be paid in advance.

Eq. (7) is the arbitrage condition for the posting of new vacancies, and accordingly, for the creation of new jobs. It simply says that the cost of posting a vacancy, κ, is in equilibrium equal to the discounted expected return from posting the vacancy.

Consider now the worker. Define $W(\mathbf{s}, \varphi)$ and $U(\mathbf{s})$ to be the values of being employed and unemployed, in terms of current consumption. They are equal to

$$W(\mathbf{s}) = \tilde{w}(\mathbf{s}) - a + (1 - \lambda)\beta EW(\mathbf{s}') + \beta\lambda EU(\mathbf{s}'), \tag{8}$$

$$U(\mathbf{s}) = h(\mathbf{s})\beta EW(\mathbf{s}') + (1 - h(\mathbf{s}))\beta EU(\mathbf{s}'), \tag{9}$$

where $\tilde{w}(\mathbf{s}) = E(\beta P(\mathbf{s})/P(\mathbf{s}'))w(\mathbf{s})$. As with dividends, the wage $w(\mathbf{s})$ is multiplied by the term $E\beta P(\mathbf{s})/P(\mathbf{s}')$ because wages are paid at the end of the period. Adding Eqs. (4)–(8) and subtracting (9) gives the total surplus generated by the match $S(\mathbf{s})$. The surplus is shared between the worker and the firm according to η, i.e., $W(\mathbf{s}) - U(\mathbf{s}) = \eta S(\mathbf{s})$ and $J(\mathbf{s}) = (1 - \eta)S(\mathbf{s})$. Using this sharing rule and Eq. (7), the surplus can be written as

$$S(\mathbf{s}) = \tilde{\pi}(\mathbf{s}) + \tilde{w}(\mathbf{s}) - a + \frac{(1 - \lambda - \eta h(\mathbf{s}))\kappa}{(1 - \eta)q(\mathbf{s})}. \tag{10}$$

Moreover, by equating $W(\mathbf{s}) - U(\mathbf{s})$ to $\eta S(\mathbf{s})$, and using (5), we derive the wage $w(\mathbf{s})$ as

$$w(\mathbf{s}) = \eta(Ax^v - x(1 + R)) + \frac{(1 - \eta)a}{E(\frac{\beta P(\mathbf{s})}{P(\mathbf{s}')})} + \frac{\eta h(\mathbf{s})\kappa}{q(\mathbf{s})E(\frac{\beta P(\mathbf{s})}{P(\mathbf{s}')})}. \tag{11}$$

The wage $w(\mathbf{s})$ as well as the surplus generated by the match depend on the intermediate input x. Because the firm and the worker split the surplus, the optimal input maximizes this surplus. The optimal input is then defined in the following proposition:

Proposition 1.1. *The optimal input x is given by*

$$x = \left(\frac{vA}{1 + R}\right)^{\frac{1}{1-v}}.$$

Proof 1.1. The differentiation of the surplus in Eq. (10), after substituting $\pi(s) + w(s) = Ax^v - x(1 + R)$, gives the result. □

According to Proposition 1.1, the intermediate input—and therefore, the firm's output—is decreasing in the nominal interest rate R. This is because the interest rate increases the marginal cost of the intermediate input. This is the direct channel through which monetary policy interventions impact on the real sector of the economy. This is in addition to the dynamic impact that will affect employment as described below.

Using Eqs. (7) and (4) we derive

$$\frac{\kappa}{q(\mathbf{s})} = \beta\tilde{\pi}(\mathbf{s}') + \beta E\left(\frac{(1 - \lambda)\kappa}{q(\mathbf{s}')}\right), \tag{12}$$

where $\tilde{\pi}(\mathbf{s})$ is the value in terms of current consumption of dividends distributed by the firm at the end of the period. Using forward substitution and the law of iterated expectations we have

$$\frac{\kappa}{q(\mathbf{s}_t)} = \beta E_t \sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1} \tilde{\pi}(\mathbf{s}_{t+j}). \tag{13}$$

Because κ is constant, an increase in the expected sum of future dividends (properly discounted) induces a reduction in the current value of q, i.e., the probability that a vacancy is filled. The fall in q requires an increase in the number of vacancies which in turn increases the next period employment. Eq. (13) provides intuition on how changes in the interest rate affect the employment rate. If a fall in the future interest rates generates an increase in the expected dividends, it will induce an increase in employment. Also notice that the future inflation rates play an important role as $\tilde{\pi}_{t+j} = \beta P_{t+j}\pi_{t+j}/P_{t+j+1}$, for $j \geqslant 1$. On the other hand, the current dividend π_t and the next period inflation rate P_{t+1}/P_t do not enter Eq. (13).

These observations are key to understanding the different properties of the optimal policies with and without commitment. These policies will be characterized in detail in later sections. Here, we would like to provide some intuition about the differences. Without commitment, the policy maker is unable to (credibly) determine the future inflation and interest rates. This implies that the policy maker is unable to affect employment. With commitment, instead, the policy maker can affect employment because it can (credibly) choose the future policies today. Consequently, if the equilibrium employment is not efficient, we would expect that the optimal policy with commitment differs from the time-consistent policy.

2. Defining the optimal monetary policy

We can now define the optimal monetary policy under the two policy regimes. We begin with the case where commitment is not possible.

2.1. Optimal and time-consistent monetary policy

In this section, we define the optimal policy when the monetary authority chooses the growth rate of money on a period-by-period basis and cannot credibly commit to the choice of future rates. We restrict the analysis to policies that are Markov stationary, i.e., policy rules that are functions of the current aggregate states of the economy. Given $\mathbf{s}$ the current states, a policy rule will be denoted by $g = \Psi(\mathbf{s})$.

The procedure we follow to derive the time-consistent policy consists of two steps. In the first step, we define a recursive equilibrium where the policy maker follows an arbitrary policy rule $\Psi(\mathbf{s})$. In the second step, we ask what the optimal growth rate of money should be today if the policy maker anticipates that from tomorrow on it will follow some arbitrary rule $\Psi(\mathbf{s})$. This allows us to derive the optimal current g as a function of the current states and the arbitrary future rule. We denote the function

that returns the optimal current policy by $g = \psi(\Psi; \mathbf{s})$. If the current policy rule ψ is equal to the policy rule that will be followed from tomorrow on, that is, $\psi(\Psi; \mathbf{s}) = \Psi(\mathbf{s})$ for all $\mathbf{s}$, then Ψ is an optimal and time consistent policy rule. We describe these two steps in detail in the next two subsections.

2.1.1. The household's problem given the policy function Ψ

Assume that the policy maker commits to the policy rule $g = \Psi(\mathbf{s})$. Then, using a recursive formulation, we describe the household's problem and define a competitive equilibrium conditional on this policy rule. In order to use a recursive formulation, we normalize all nominal variables by the pre-transfer stock of money M. The aggregate states of the economy are the aggregate level of technology, A, the normalized pre-transfer stock of nominal deposits, D, and the number of employed workers, N. Therefore, $\mathbf{s} = (A, D, N)$. The individual states are the occupational status χ, the normalized pre-transfer stock of nominally denominated assets m, the normalized pre-transfer stock of nominal deposits d, and the number of firms' shares n owned by the household. We denote the set of individual states by $\hat{\mathbf{s}} = (\chi, m, d, n)$. The household's problem is

$$\Omega(\Psi; \mathbf{s}, \hat{\mathbf{s}}) = \max_{n', d'} \{ c - \chi a + \beta E \Omega(\Psi; \mathbf{s}', \hat{\mathbf{s}}') \} \tag{14}$$

subject to

$$c \leqslant \frac{m - d}{P} - \frac{(n' - (1 - \lambda)n)\kappa}{q}, \tag{15}$$

$$m' = \frac{(d + g)(1 + R) + P(\chi w + n\pi)}{(1 + g)}, \tag{16}$$

$$\mathbf{s}' = H(\Psi; \mathbf{s}), \tag{17}$$

$$g = \Psi(\mathbf{s}). \tag{18}$$

Notice that in order to have n' shares of active firms in the next period, the household buys $(n' - (1 - \lambda)n)$ new shares. Given the matching probability for a new vacancy, q, the creation of a new firm requires the posting of $1/q$ new vacancies, each of which costs κ. Therefore, the total investment in new firm shares is $i = (n' - (1 - \lambda)n)\kappa/q$. In solving this problem, the household takes as given the policy rule Ψ and the law of motion for the aggregate states H defined in Eq. (17). To make clear that this problem is conditional on the particular policy rule Ψ, this function has been included as an extra argument in the household's value function and in the aggregate law of motion.

A solution for this problem is given by the state contingent functions $n'(\Psi; \mathbf{s}, \hat{\mathbf{s}})$ for next period firms' shares and $d'(\Psi; \mathbf{s}, \hat{\mathbf{s}})$ for bank deposits. As for the value function, we make explicit the dependence of these decisions on the policy rule Ψ.

In equilibrium, households are indifferent about the allocation of liquid funds (money) between the purchase of consumption goods and the purchase of firms' shares, independently of their employment status. This derives from the assumption

that the utility function is linear in consumption. Because the aggregate behavior of the economy is independent of the distributions of firms' shares among households, we concentrate on the *symmetric* equilibrium in which all agents make the same portfolio choices of deposits and shares of firms. This implies that differences in earned wages between employed and unemployed workers give rise to different consumption levels rather than differences in asset holdings. We then have the following definition.

Definition 2.1 (Symmetric equilibrium given Ψ). A recursive symmetric competitive equilibrium, given the policy rule Ψ, is defined as a set of functions for (i) household decisions $n'(\Psi; \mathbf{s}, \hat{\mathbf{s}})$, $d'(\Psi; \mathbf{s}, \hat{\mathbf{s}})$, and value function $\Omega(\Psi; \mathbf{s}, \hat{\mathbf{s}})$; (ii) intermediate input $x(\Psi; \mathbf{s})$; (iii) wage $w(\Psi; \mathbf{s})$; (iv) loans $L(\Psi; \mathbf{s})$; (v) interest rate $R(\Psi; \mathbf{s})$ and nominal price $P(\Psi; \mathbf{s})$; (vi) law of motion $H(\Psi; \mathbf{s})$. Such that: (i) the household's decisions are optimal solutions to the household's problem (14); (ii) the intermediate input x maximizes the surplus of the match; (iii) the wage is such that the worker obtains a fraction η of the surplus; (iv) the market for loans clears, that is $D + g = L(\Psi; \mathbf{s})$, and $R(\Psi; \mathbf{s})$ is the equilibrium interest rate; (v) the law of motion $H(\Psi; \mathbf{s})$ for the aggregate states is consistent with the individual decisions of households and firms; (vi) all agents choose the same holdings of deposits and firms shares (symmetry).

Differentiating the objective function (14) with respect n', we get:

$$\frac{\kappa}{q} = \beta E\left(\frac{\beta P' \pi'}{P''(1 + g')}\right) + \beta E\left(\frac{(1 - \lambda)\kappa}{q'}\right), \tag{19}$$

which is equivalent to (12) derived before. The first-order condition with respect to d' is

$$E\left(\frac{1}{P'}\right) = \beta E\left(\frac{1 + R'}{P''(1 + g')}\right), \tag{20}$$

which is the Euler equation in standard dynamic models with money when agents are risk neutral.

2.1.2. One-shot optimal policy and the fixed point of the policy problem

In the previous subsection, we derived the household's decision rules $n'(\Psi; \mathbf{s}, \hat{\mathbf{s}})$ and $d'(\Psi; \mathbf{s}, \hat{\mathbf{s}})$, and the value function $\Omega(\Psi; \mathbf{s}, \hat{\mathbf{s}})$ for a given policy rule Ψ. We now ask what the optimal policy would be today, if the policy maker anticipates that from tomorrow on it will follow an arbitrary policy rule Ψ. Defining the optimality of a particular policy requires the definition of a welfare objective. Our assumption is that the policy maker attributes equal weight to all households independently of their employment status.

To determine the optimal growth rate of money today, we need to derive a function that links the households' welfare to g. To derive this function, we first consider the following household's problem:

$$V(\Psi; \mathbf{s}, \hat{\mathbf{s}}, g) = \max_{n', d'}\{c - \chi a + \beta E\Omega(\Psi; \mathbf{s}', \hat{\mathbf{s}}')\} \tag{21}$$

subject to

$$c \leqslant \frac{m-d}{P} - \frac{(n'-(1-\lambda)n)\kappa}{q}, \tag{22}$$

$$m' = \frac{(d+g)(1+R) + P(\chi w + n\pi)}{(1+g)}, \tag{23}$$

$$\mathbf{s}' = \tilde{H}(\Psi; \mathbf{s}, g), \tag{24}$$

where $\Omega(\Psi; \mathbf{s}', \hat{\mathbf{s}}')$ is the next period value function conditional on the policy rule Ψ derived in the previous section. The new function $V(\Psi; \mathbf{s}, \hat{\mathbf{s}}, g)$ is the value function for the household when the current growth rate of money is g and future growth rates are determined according to the policy rule Ψ.

After solving this problem and imposing the aggregate consistency condition in the symmetric equilibrium $m = M = 1$, $d = D$, and $n = N$, the objective function of the policy maker can be written as:

$$\mathbf{V}(\Psi; \mathbf{s}, g) = N \cdot V(\Psi; \mathbf{s}, 1, M, D, N, g) + (1-N) \cdot V(\Psi; \mathbf{s}, 0, M, D, N, g), \tag{25}$$

which is simply the weighted average of the value functions for employed and unemployed households. The unemployment status is the only source of heterogeneity because we are restricting the competitive equilibrium to be symmetric in the sense that all the households choose the same level of assets (but different consumption). The policy maker chooses g to maximize the above objective, i.e.,

$$g^{\mathrm{OPT}} = \arg \max_g \mathbf{V}(\Psi; \mathbf{s}, g) = \psi(\Psi; \mathbf{s}). \tag{26}$$

We then have the following definition of an optimal and time-consistent monetary policy rule.

Definition 2.2. The optimal and time-consistent monetary policy rule $\Psi^{\mathrm{OPT}}(\mathbf{s})$ is the fixed point of the mapping $\psi(\Psi; \mathbf{s})$, i.e.,

$$\Psi^{\mathrm{OPT}}(\mathbf{s}) = \psi(\Psi^{\mathrm{OPT}}; \mathbf{s})$$

The basic idea behind this definition is that, when the agents in the economy (households, firms and the monetary authority) expect that future values of g are determined according to the policy rule Ψ^{OPT}, the optimal value of g today is the one predicted by the same policy rule Ψ^{OPT} that will determine the future values. This property assures that the policy maker will continue to use the same policy rule in the future, so it is rational to assume that future values of g will be determined by this rule.

2.2. Optimal policy with commitment

With commitment, the policy maker chooses at time zero a sequence of money growth as a function of future history realizations of the shock and the initial states.

The equilibrium allocation associated with this policy is usually referred to as the Ramsey equilibrium.

Let $\mathbf{h}^t$ be the history of shock realizations from time zero up to time t and let $\mathbf{H}^t$ be the collection of all possible histories. A monetary policy with commitment can be expressed as $g_t = g(N_0, \mathbf{h}^t)$, for all $\mathbf{h}^t \in \mathbf{H}^t$ and $t \geqslant 0$. Similarly, the realization of the interest rate induced by this policy can be expressed as a function of N_0 and $\mathbf{h}^t$, i.e., $R_t = R(N_0, \mathbf{h}^t)$. The policy maker will choose $g(N_0, \mathbf{h}^t)$ to maximize the expected discounted utility of the representative household obtained under the competitive allocation induced by the policy $g(N_0, \mathbf{h}^t)$. If we define $C(N_0, \mathbf{h}^t | g(N_0, \mathbf{h}^t))$ the aggregate consumption induced by the policy $g(N_0, \mathbf{h}^t)$ in the competitive equilibrium and $N(N_0, \mathbf{h}^t | g(N_0, \mathbf{h}^t))$ the employment rate also induced by the policy $g(N_0, \mathbf{h}^t)$ in the competitive equilibrium, the optimal policy with commitment is defined as

$$\arg \max_{\{\{g(N_0, \mathbf{h}^t)\}_{\mathbf{h}^t \in \mathbf{H}^t}\}_{t \geqslant 0}} E_0 \sum_{t=0}^{\infty} \beta^t [C(N_0, \mathbf{h}^t | g(N_0, \mathbf{h}^t)) - a N(N_0, \mathbf{h}^t | g(N_0, \mathbf{h}^t))].$$

(27)

The characterization of the optimal policy follows the primal approach and consists of choosing the optimal allocation among the set of all competitive allocations induced by a feasible policy $g(N_0, \mathbf{h}^t)$. See [8] for details about the primal approach.

3. Properties of the optimal policy

Before characterizing the properties of the optimal policy, let us observe that for a given aggregate stock of deposits, there is a simple relation between the nominal interest rate and the current growth rate of money. This is formally established in the following lemma.

Lemma 3.1. *Given the aggregate stock of deposits, the nominal interest rate is equal to* $R = \max\{\frac{v(1+g)}{D+g} - 1, 0\}$.

Proof 3.1. See appendix. □

Although we have defined the monetary policy in terms of the growth rate of money, Lemma 3.1 implies that a definition in terms of the interest rate would induce the same real allocation (employment and consumption). The next two subsections characterize the properties of the optimal monetary policy in the two policy environments.

3.1. Optimal policy without commitment

We have the following proposition.

Proposition 3.1 (Policy without commitment). *If there is not commitment, the optimal policy maintains the nominal interest rate to zero in any state of the economy.*

Proof 3.1. See appendix. □

Therefore, the Friedman rule of a zero nominal interest rate is the optimal policy when the policy maker cannot commit to future policies. This result is also obtained in [2] and depends on the fact that the current growth rate of money does not affect future employment. Future employment will be affected by future growth rates of money but without commitment the policy maker is unable to choose credibly these rates. Given the inability to affect future employment, the optimal policy will choose a current growth rate of money that leads to a zero interest rate. This is because a zero interest rate does not distort the production choice of the existing matches, i.e., the choice of the intermediate input.

Although the time-consistent policy gives a precise prediction about the nominal interest rate, a zero interest rate can be implemented with a multiplicity of growth rates of money (see Lemma 3.1). The policy indeterminacy (in terms of money growth) derives from the fact that with a zero nominal interest rate the cash-in-advance constraints of households and firms are not binding and several sequences of money growth can induce a zero interest rate. In what follows we restrict the analysis to a particular policy, i.e., the policy under which the whole quantity of money is used for transaction. The following proposition characterizes the optimal growth rate of money in the environment without policy commitment and full use of money.

Proposition 3.2 (Procyclical time-consistent policy). *Without policy commitment, the optimal growth rate of money compatible with full use of money is given by $g = \beta E_{-1}(1 + g_Y) - 1$, where $E_{-1}(1 + g_Y)$ is the expected gross growth rate of output before the observation of the shock.*

Proof 3.2. See appendix. □

Therefore, the growth rate of money depends only on the predictable part (before the shock) of the growth rate of output and current (unpredictable) productivity shocks do not affect the optimal growth rate of money. This is because in the current period the nominal interest rate is determined by the equilibrium condition $R = v(1 + g)/(D + g) - 1$ (see Lemma 3.1). Because the stock of deposits D cannot be changed, the constancy of R requires the constancy of g.

The fact that the optimal growth rate of money follows the predictable growth rate of output implies that the growth rate of money is pro-cyclical if the growth rate of output displays some persistence. In this respect the matching frictions play an important role in the model. In a limited participation model with a neoclassical production technology—and therefore, absence of matching frictions—the response of output to shocks is not hump-shaped. This implies that in this latter model the growth rate of output is positive only in the first period. Because in the first period

the increase in output is not expected, the growth rate of money does not change. After the first period the growth rate of output becomes negative because it converges back to the steady state and the growth rate of money will be negative. This implies that when output is above the steady state, the growth rate of money is negative (counter-cyclical). The matching framework, instead, is able to generate a hump-shaped response of output as shown in [3,14,24]. Consequently, output will continue to growth beyond the first period which induces an increase in the optimal growth rate of money in the first few periods. As we will see in Section 4, this generates a pro-cyclical response of the growth rate of money.

3.2. Optimal policy with commitment

We have the following proposition.

Proposition 3.3 (Policy with commitment). *If $\eta \geqslant 1 - \alpha$, the optimal policy with commitment implies $R(N_0, \mathbf{h}^t) = 0$ for all $t = 0, 1, 2, \ldots$ (Friedman rule). If $\eta < 1 - \alpha$ the optimal policy with commitment implies $R(N_0, \mathbf{h}^0) = 0$ and $R(N_0, \mathbf{h}^t) > 0$ for some $t \geqslant 1$.*

Proof 3.3. See appendix. □

According to this proposition, if the worker's share of the surplus η is too small, the optimal policy with commitment induces positive interest rates. Therefore, in contrast to the case without commitment, the Friedman rule of a zero nominal interest rate is not optimal unless the bargaining power of the worker is sufficiently large.

To understand these properties, we have to consider the two channels through which monetary policy affects the real sector of the economy: the *direct* channel and the *indirect* channel. The direct channel works through the cost of financing the intermediate input x. Given the number of employed workers, a higher interest rate increases the financing cost of the firm and reduces production. On this dimension, a zero nominal interest rate would be optimal. The indirect channel works through the incentives to create vacancies. The policy maker can increase employment by reducing the profitability of a match. This, in turn, can be obtained by increasing the inflation and interest rates. Therefore, if the employment rate is not efficient under a Friedman rule, the policy maker may deviate from this rule. More precisely, if the worker's share of the surplus η is smaller than $1 - \alpha$, the Hosios [19] conditions for the efficiency of the matching process are violated, and the high profitability of a match for the firm induces an excessive creation of vacancies. Under this condition the policy maker would like to reduce job creation. However, the decision to create new vacancies is not affected by either the current interest rate or the current inflation rate. What affects the return on a new vacancy are the future interest and inflation rates. This can be seen from Eq. (13) which for

simplicity we rewrite below:

$$\frac{\kappa}{q(\mathbf{s}_0)} = \beta E_0 \sum_{t=1}^{\infty} [\beta(1-\lambda)]^{t-1} \pi(\mathbf{s}_t)\frac{\beta P(\mathbf{s}_t)}{P(\mathbf{s}_{t+1})}. \tag{28}$$

The infinite sum on the right-hand side of this equation starts at $t = 1$. This implies that the creation of new vacancies at time zero does not depend on the current interest rate and the current and next period inflation rates (change in prices between today and tomorrow). Consequently, the optimal growth rate of money in the current period will be set such that $R_0 = 0$. Future inflation and growth rates of money, instead, will be set taking into consideration the possibility of correcting for the second source of inefficiency. Under the condition $\eta < 1 - \alpha$, this requires a higher average inflation rate which induces a higher average nominal interest rate. If $\eta > 1 - \alpha$, it would be optimal to have negative interest rates. A negative interest rate, however, is not compatible with a competitive equilibrium.[3]

With shocks the full characterization of the commitment policy is not available. In general, we would not expect that the interest rate is constant over the business cycle because the trade-off between the production efficiency (the optimal input x) and the optimal employment is affected by the shock. However, we would expect that the average interest and inflation rates are higher when $\eta < 1 - \alpha$. This will be shown numerically in Section 4. In that section, we will also show the numerical cyclical properties of the commitment policy.

3.3. Optimal policy with externality

The analysis of the previous two subsections showed that without commitment the optimal policy maintains a zero interest rate and a negative inflation rate. This would also be the optimal policy with commitment when $\eta \geqslant 1 - \alpha$. We now introduce an extra feature that allows for the optimality of a positive long-term interest rate even if there is no commitment but it does not change the basic cyclical (short-term) properties of the optimal policy. We assume that each firm generates a negative externality of the form $\xi \cdot Ax^v$, where ξ is constant. With this externality, Propositions 3.1 and 3.2 become

Proposition 3.4 (Policy without commitment). *If there is not commitment, the optimal policy maintains the nominal interest rate equal to $\xi/(1 - \xi)$ in any state of the economy and $g = \beta E_{-1}(1 + g_Y)/(1 - \xi) - 1$.*

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[3] Other models studied in the literature assume that producers are monopolistic competitors and there is underproduction. Extending our model by assuming monopolist producers should not change the basic results with some qualifications. More specifically, keeping constant the monopolistic power in the product market and assuming that this power is not too large, a positive inflation rate may still be desirable if the bargaining power of workers η is small. With monopolistic competition, however, the threshold level for η is smaller than $1 - \alpha$.

Proof 3.4. The proof follows the same steps of Propositions 3.1 and 3.2 taking into account the externality $\xi A X^v N$ in the objective of the policy maker. $\square$

The introduction of the externality is also important to differentiate the long-term properties of the optimal policy without commitment. Proposition 3.3 becomes

Proposition 3.5 (Policy with commitment). *If $\eta = 1 - \alpha$ the commitment policy implies $R(N_0, \mathbf{h}^t) = \xi/(1 - \xi)$ for all $t = 0, 1, 2, \dots$ (constant interest rate). If $\eta < 1 - \alpha$ the commitment policy implies $R(N_0, \mathbf{h}^0) = \xi/(1 - \xi)$ and $R(N_0, \mathbf{h}^t) > \xi/(1 - \xi)$ for some $t \geqslant 1$. If $\eta > 1 - \alpha$ the commitment policy implies $R(N_0, \mathbf{h}^0) = \xi/(1 - \xi)$ and $R(N_0, \mathbf{h}^t) < \xi/(1 - \xi)$ for some $t \geqslant 1$.*

Proof 3.5. The proof follows the same steps of Propositions 3.3 taking into account the externality $\xi A X^v N$ in the objective of the policy maker. $\square$

Although it is still the case that commitment may increase inflation when η is small, for large values of η the opposite may be true as we will show numerically in the next section. Therefore, our results are qualitatively similar to the results of Kydland and Prescott [21] and Barro and Gordon [4] if η is large but they differ if η is small. We will come back to this point in Section 5 when we discuss the empirical plausibility of the condition $\eta < 1 - \alpha$ and the evidence about the relationship between commitment and inflation.

4. Quantitative properties of the optimal policy

In this section, we analyze the quantitative properties of the optimal monetary policy (with and without commitment). Our analysis will be focused on the properties of the economy around the steady state. In the regime with policy commitment the steady state is the equilibrium to which the economy will converge after the initial implementation of the optimal plan in absence of shocks. The problem solved to characterize the limiting equilibrium with policy commitment is described in Appendix E.

4.1. Calibration

The model is calibrated to US data. The period is one quarter and the discount factor is $\beta = 0.99$. The parameter ξ is chosen to get a steady state quarterly interest rate equal to 0.018. This implies a steady state inflation rate of about 0.008 per quarter. The value of ξ needed to obtain an interest rate equal to 0.018 depends on the policy regime. In the regime without commitment we set $\xi = 0.017682$. As stated in Proposition 3.4 this will guarantee that the equilibrium interest rate is equal to the calibration target. In the regime with policy commitment the value of ξ depends on the whole set of parameters. In the baseline model we set $\xi = 0.0034977$.

Approximately, this implies that policy commitment increases the long-term inflation rate by about 0.014 per quarter (about 6% per year).

The production function is characterized by the parameter v and the stochastic properties of the shock. The nominally denominated assets used by households for transaction purposes (money) as a fraction of their total nominally denominated assets, is equal to $(M-D)/M(1+g)$. This can also be written as $(1-v)+RD/M(1+g)$.[4] Because $RD/M(1+g)$ is a small number, we take $1-v$ to be the approximate fraction of transaction funds used by households. A proxy for $1-v$ is then given by the stock of $M1$ used by households as a fraction of $M3$ that they own. The value chosen is $v = 0.85$.

The aggregate level of technology A follows the first-order autoregressive process $\log(A') = \rho\log(A) + \varepsilon'$, with $\varepsilon \sim N(0, \sigma^2)$. The parameter ρ is assigned the value 0.95, and σ is set such that the volatility of output generated by the model in the regime without commitment is similar to the data. The value chosen is $\sigma_z = 0.0009$. Of course, the evaluation of the model will not be based on the ability to match the volatility of output.

The workers share of the surplus is set to $\eta = 0.2$. This is about half the value that would guarantee an efficient creation of new vacancies. After fixing η, the disutility from working, a, is chosen so that the steady state capital income share is 18%. This value guarantees that the net capital income share (net of depreciation) is similar to the data.[5] To evaluate the importance of η, we will report the simulation results also for other values of this parameter.

The searching and matching section of the model is characterized by four parameters: the parameters of the matching technology, μ and α, the probability of exogenous separation, λ, and the cost of creating a new vacancy, κ. We set $\alpha = 0.6$ which is consistent with the estimate of Blanchard and Diamond [6]. Then to calibrate the parameters μ, λ and κ, we follow Andolfatto [3] and impose the following steady state targets: (a) the fraction of the population that is employed equals 0.57;[6] (b) the probability that a vacancy is successfully filled is $q = 0.9$; and (c) the transition probability from employment to non-employment is $\lambda = 0.15$.

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[4]This is obtained by using $M - D = PC = P(1-v)Y + RD$ and $PY = M(1+g)$.

[5]In standard models the gross capital income share is higher (about 35%) because it compensates for the higher depreciation of capital. In our economy, the only accumulation of capital comes from the initial cost of creating new vacancies. Consequently, the aggregate stock of capital and its replacement are smaller than in standard models.

[6]This implies that the steady state fraction of searching workers is 0.43. Here, we are adopting is broader definition of searching workers which includes not only unemployed workers but also individuals that are out of the labor force. This captures the fact that some of the new hired workers do not transit through the pool of formally defined unemployed workers. From a technical point of view, this larger fraction is important because is reduces the impact that changes in the number of employed workers have on the probability that an advertised vacancy is filled. When the fraction of steady state searchers is small, a small percentage increase in the number of employed workers corresponds to a large percentage fall in the number of searching workers, which in turn implies a large fall in the probability with which new vacancies are filled.
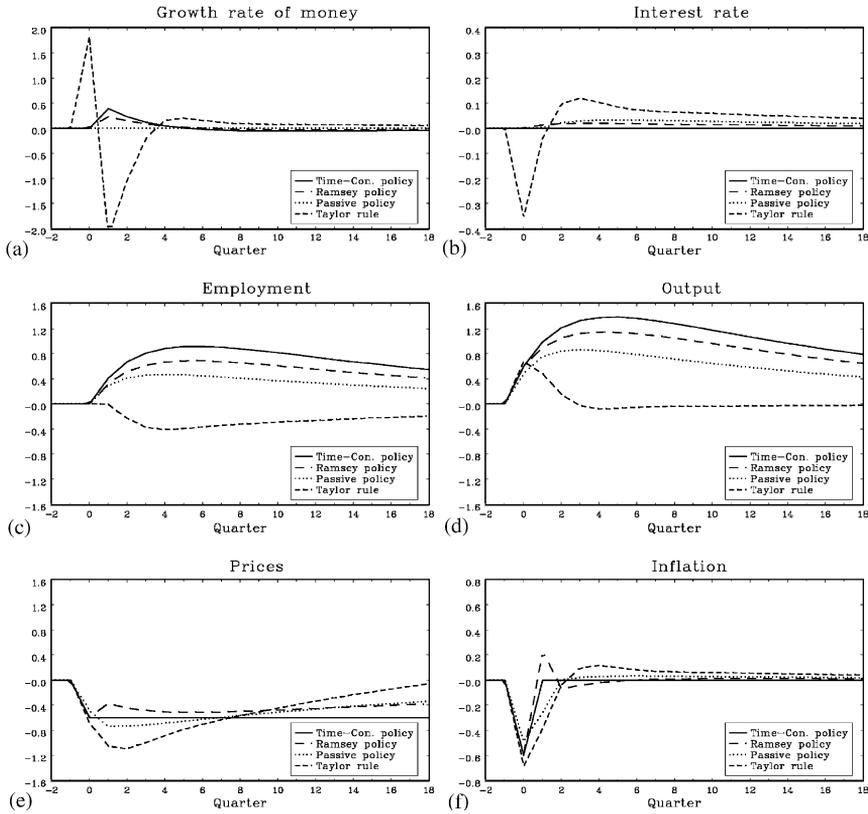
Fig. 1. Impulse responses to a positive productivity shock under alternative monetary regimes.

4.2. Cyclical properties of the calibrated economy

Fig. 1 plots the impulse responses of several variables to a positive productivity shock under the policy regimes with and without commitment. The figure also plots the impulse responses under two alternative regimes. In the first regime, the policy maker keeps the growth rate of money constant (passive policy) while in the second regime it controls the nominal interest rate according to the following specification of the Taylor rule:

$$R_t = \bar{R} + \gamma_y(Y_t - \bar{Y}) + \gamma_p(P_t - P_{t-1}), \tag{29}$$

where R_t is the nominal interest rate, Y_t is logarithm of aggregate output, P_t is the logarithm of the price level and the bar sign denotes steady state values. As suggested in [30], we set $\gamma_y = 1$ and $\gamma_p = 1.5$.

The first point to observe is that, although the Friedman rule is optimal when the policy maker cannot commit to future policy (time-consistent policy), this is not the case when the policy maker is able to commit to future policies (Ramsey allocation).

In this case the optimal interest rate is procyclical. As a result of this, the response of employment and output is smaller in the Ramsey policy regime.

The second point to note is that the optimal policies (with and without commitment) amplify the responses of employment and output relative to the passive policy. This is because the optimal policies induce a smaller or zero increase in the interest rate and allow the economy to take full advantage of the higher productivity. On the other hand, the failure to increase the growth rate of money in the passive policy induces an increase in the interest rate which dampens the response of the economy to the shock. When the policy maker follows the Taylor rule, the aggressive counter-cyclical properties of this rule goes beyond restricting the response of employment and output and generates a recession. We should point out, however, that in our specification of the Taylor rule we have assumed that potential output is constant. If we allow potential output to depend on the shock, the stabilization consequences of this rule would be smaller.

Table 1 reports some business cycle statistics computed from the simulation of the artificial economy. As expected from the impulse responses plotted in Fig. 1 (panel (c) and (d)), the volatility of employment and output is larger under the optimal policy regimes. Under these regimes the model generates volatility of money stock and money growth that are not very different from the data. It also generates positive correlations of the stock of money and the growth rate of money with employment and output. Employment is also positively correlated with the inflation rate. The correlation of inflation and output is positive although it is close to zero. Nevertheless, this is an important improvement compared to the other two policy regimes that generate a negative correlation. To explain why the optimal policy improves the performance of the model along this dimension, consider first the case of a passive policy. In this regime, when output expands prices fall and when output contracts prices increase. As shown in the first panel of Fig. 1, in the optimal policy regime the price level falls only in the first period of the shock. But in the first period the growth in output is small relative to the subsequent growth (see panel (d) of Fig. 1). Another important feature of the model is the autocorrelation of the optimal growth rate of money as reported in the lower section of the table. This autocorrelation is positive and close to the value found in the data.

It is important to emphasize that the positive correlation of employment and output with the optimal growth rate of money depends crucially on the fact that the response of output to shocks is hump-shaped. The hump-shaped response occurs because of the searching and matching frictions. Without these frictions, the response of output would not be hump-shaped and the growth rate of money would not be pro-cyclical.

To show the importance of the searching frictions, we have also considered an alternative model in which the number of employed workers is kept constant at its full employment. This model is similar to a simplified version of the standard limited participation model.[7] The cyclical properties are presented in Table 2. As anticipated

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[7] The main differences are that the endogeneity of the labor supply is replaced by the endogeneity of the intermediate input and consumption enters linearly in the utility function. These differences, however, do not affect the main properties of the model.

Table 1
Business cycle properties of the calibrated economy under alternative policy regimes

	TimeCon policy	Ramsey policy	Passive policy	Taylor rule	US economy
Standard deviation					
Employment	1.09	0.82	0.60	0.61	0.99
Output	1.63	1.38	1.22	1.01	1.67
Consumption	1.48	1.24	1.13	1.33	1.39
Price index	0.76	0.67	1.08	1.80	1.39
Inflation	0.58	0.63	0.62	0.98	0.57
Interest rate	0.00	1.36	2.44	0.50	1.29
Money stock	1.03	0.74	0.00	3.73	1.52
Money growth	0.44	0.26	0.00	3.68	0.73
Employment correlation with					
Prices	−0.61	−0.59	−0.79	0.48	−0.30
Inflation	0.35	0.28	0.34	−0.54	0.51
Interest rate	0.00	0.96	0.91	−0.60	0.40
Money stock	0.99	0.95	0.00	0.77	0.49
Money growth	0.15	0.25	0.00	−0.07	0.33
Final output correlation with					
Prices	−0.87	−0.87	−0.99	−0.41	−0.30
Inflation	0.03	0.02	−0.18	−0.97	0.51
Interest rate	0.00	0.91	0.69	−0.83	0.40
Money stock	0.93	0.92	0.00	0.63	0.49
Money growth	0.36	0.47	0.00	0.03	0.33
Autocorrelation of money growth	0.49	0.49	0.00	−0.15	0.59

Notes: Statistics for the model economy are computed from HP detrended data generated by simulating the model for 240 periods. The statistics are averages of 1000 repeated simulations. Statistics for the US economy are computed from HP detrended data 1959.1–1996.4. Consumption includes consumer expenditures in non-durable and services. The price index is the CPI index.

from the discussion above, the optimal growth rate of money is negatively correlated with output. Moreover, there is no difference between the optimal policies with and without commitment. Finally, we notice that the volatility of the economy is smaller because employment is kept constant.

Table 3 recalculates the statistics for alternative values of the bargaining parameter η. In changing η we also change three other parameters: the working disutility a, the externality parameter ξ in the regime with policy commitment, and the standard deviation of the shock σ_z. The new working disutility is such that there is no change in the steady state capital income share. The change in the externality parameter in the regime with policy commitment guarantees that the steady state interest rate does not change. The new standard deviation of the shock guarantees that the volatility of output in the regime without commitment remains the same.

A brief inspection of Table 3 shows that for $\eta < 1 - \alpha$ policy commitment reduces the volatility of employment and output relative to the regime without commitment.

Table 2
Business cycle properties of the calibrated economy without matching frictions

	TimeCon policy	Ramsey policy	Passive policy	Taylor rule	US economy
Standard deviation					
Output	0.77	0.77	0.76	1.53	1.67
Consumption	0.86	0.86	0.85	1.72	1.39
Price index	0.76	0.76	0.67	9.83	1.39
Inflation	0.58	0.58	0.61	2.02	0.57
Interest rate	0.00	0.00	1.61	1.63	1.29
Money stock	0.14	0.14	0.00	13.84	1.52
Money growth	0.01	0.01	0.00	4.67	0.73
Final output correlation with					
Prices	−0.98	−0.98	−0.98	−0.32	−0.30
Inflation	−0.88	−0.88	−0.32	−0.80	0.51
Interest rate	0.00	0.00	0.69	−0.75	0.40
Money stock	0.14	0.14	0.00	0.33	0.49
Money growth	−0.70	−0.70	0.00	−0.75	0.33

Notes: Statistics for the model economy are computed from HP detrended data generated by simulating the model for 240 periods. The statistics are averages of 1000 repeated simulations. Statistics for the US economy are computed from HP detrended data 1959.1–1996.4. Consumption includes consumer expenditures in non-durable and services. The price index is the CPI index.

However, when $\eta > 1 - \alpha$, the reverse seems to be true. This is because in this case the optimal response of the interest rate under commitment tends to be counter-cyclical (it decreases during an expansion and increases during a recession). In fact, the correlation of employment and output with the interest rate is positive for $\eta = 0.1$ but becomes negative when $\eta = 0.5$. We also notice that as we increase η, we reduce the correlation of the growth rate of money with employment. This is because the fluctuation of employment becomes less important relative to the fluctuation of output. In general, the performance of the model improves as we reduce η.

4.3. Long-term properties of the calibrated economy

Here, we show how the long-term inflation rate and interest rate are affected by the bargaining parameter η. All the other parameters are as in the baseline calibration including $\xi = 0.01768$. Table 4 reports the steady state values for the annual inflation and the interest rates. Without commitment the inflation and interest rates do not depend on η and they are equal to 3.2% and 7.4%. These would also be the equilibrium inflation and interest rates with commitment if $\eta = 1 - \alpha = 0.4$. However, when $\eta \neq 1 - \alpha$, the commitment policy differs from the time-consistent policy. More specifically, the inflation and interest rates with commitment would be higher if $\eta < 1 - \alpha$ and lower if $\eta > 1 - \alpha$. This is clearly showed in Table 4. Notice that with $\eta = 0.7$ the optimal interest rate reaches the lower bound of zero.

Table 3
Business cycle properties of the calibrated economy for alternative values of the bargaining parameter η

	$\eta = 0.1$				$\eta = 0.5$			
	TimeCon policy	Ramsey policy	Passive policy	Taylor rule	TimeCon policy	Ramsey policy	Passive policy	Taylor rule
Standard deviation								
Employment	1.18	0.68	0.59	0.50	0.69	0.71	0.46	0.90
Output	1.60	1.13	1.08	0.82	1.63	1.65	1.45	1.56
Consumption	1.47	1.03	1.00	1.08	1.15	1.52	1.40	1.98
Price index	0.62	0.56	0.95	1.44	1.14	1.06	1.29	2.99
Inflation	0.47	0.51	0.53	0.78	0.86	0.91	0.88	1.56
Interest rate	0.00	1.86	2.21	0.40	0.00	0.32	2.71	0.82
Money stock	1.13	0.53	0.00	2.97	0.62	0.77	0.00	6.16
Money growth	0.48	0.19	0.00	2.97	0.25	0.28	0.00	5.65
Employment correlation with								
Prices	−0.61	−0.57	−0.82	0.50	−0.61	−0.66	−0.67	0.39
Inflation	0.35	0.32	−0.16	−0.54	0.35	0.26	0.39	−0.58
Interest rate	0.00	0.97	0.89	−0.58	0.00	−0.70	0.99	−0.66
Money stock	0.99	0.98	0.00	0.77	0.95	0.82	0.00	0.77
Money growth	0.17	0.34	0.00	−0.07	0.06	0.03	0.00	−40.11
Final output correlation with								
Prices	−0.84	−0.87	−0.98	−0.41	−0.96	−0.94	−0.99	−0.44
Inflation	0.09	0.04	−0.15	−0.98	−0.16	−0.17	−0.24	−0.95
Interest rate	0.00	0.92	0.69	−0.84	0.00	−0.79	0.71	−0.80
Money stock	0.95	0.91	0.00	0.64	0.85	0.88	0.00	0.58
Money growth	0.34	0.53	0.00	0.03	0.38	0.36	0.00	−0.01

Notes: Statistics for the model economy are computed from HP detrended data generated by simulating the model for 240 periods. The statistics are averages of 1000 repeated simulations.

Table 4
Steady state annual inflation and interest rates in the commitment regime

	$\eta = 0.1$	$\eta = 0.3$	$\eta = 0.4$	$\eta = 0.5$	0.7%
Inflation rate (%)	8.94	5.50	3.16	0.86	−1.00
Interest rate (%)	13.41	9.82	7.40	4.99	0.00

5. Some empirical evidence

The theoretical model has important predictions about the *long-term* properties of the data. There are two main implications: (i) the lack of policy commitment has a negative impact on inflation if the bargaining power of workers (the parameter η) is small and potentially a positive impact if it is large; (ii) conditional on commitment, the bargaining power of workers has a negative impact on inflation. We will discuss each of these two implications below.

5.1. Policy commitment and inflation

The major problem in testing the first implication derives from the measurement of the bargaining parameter η and the degree of policy commitment. Several papers have estimated structural search models with wage bargaining. Due to well-known identification problems, the estimation of the bargaining parameter is not very precise and it is very sensitive to the specification of the model. With this in mind, we report here the results of two studies that conduct this estimation using US data.

Flinn [15] estimates a search model with bargaining to study the welfare implications of the minimum wage. Without restrictions on the feasible values of the bargaining parameter, the estimates for η are all smaller than 0.2. Another recent study is Moscarini [26]. This paper jointly estimates the bargaining parameter η and the matching parameter α. The estimated value of η is much smaller than $1 - \alpha$ with a difference of 0.3.[8] Although the models used by these two papers differ in several respects from our model, they show that the condition $\eta < 1 - \alpha$ cannot be considered empirically irrelevant.

The second problem in testing the importance of policy commitment is to find a proxy for commitment. One possibility is to assume that countries in which the monetary authority has greater independence are also countries with a greater ability to commit to future monetary policies. Several studies have investigated the importance of Central Bank Independence for explaining inflation but they reach contrasting results depending on the set of countries used in the empirical investigation. For the restricted group of high-income countries, Central Bank Independence seems to reduce inflation (see [17]), but for developing countries the impact is estimated to be positive, although not always statistically significant (see [7,13]).

These results are consistent with the predictions of our theoretical model in the following sense. According to our proxy variable for the parameter η (which we will describe below), the bargaining power of workers is on average greater in high income countries than in countries with middle and low levels of income. This is clearly shown in Appendix F. Therefore, the condition $\eta > 1 - \alpha$ is more likely to be satisfied in high income countries while the condition $\eta < 1 - \alpha$ is more likely to be satisfied in middle and low income countries.[9] Under this interpretation our theory predicts that the inflation impact of Central Bank Independence is negative in high income countries and positive in middle- and low income countries. This seems consistent with the findings of the previous empirical literature as discussed above.

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[8] The estimates are $\eta = 0.4$ and $\alpha = 0.3$. If we re-parameterize the model using these specific values of η and α, we would obtain similar results to our baseline parameterization.

[9] The evidence of a small value of η for the United States is not inconsistent with this view. In fact, the bargaining power of workers in the Unites States is considered to be one of the lowest among industrialized countries. Consequently, the condition $\eta > 1 - \alpha$ may still dominate for the whole group of industrialized countries even if it does not hold for the United States. From table in Appendix F, we can verify that the proxy variable for the bargaining power of employers in the United States is one of the highest among the high income countries.

Although this is not a rigorous testing of our theory, the empirical evidence is consistent with it.

We should also point out that several authors find the positive correlation between Central Bank Independence and inflation in low income countries surprising because it is contrary to the the existing theory. Cukierman et al. [13] then discuss alternative measures of central bank independence which could revert the sign of the correlation. Of course, the fact that some authors prefer the use of alternative measures of central bank independence simply because the previous indices are not consistent with the existing theory, is not a valid argument to discard these indices. After all, our paper shows that the previous empirical results are not inconsistent with the (new) theory.

5.2. Bargaining power and inflation

If we think that policy makers are subject to some form of commitment, then countries in which employers have greater bargaining power should experience higher inflation. In this section we provide some evidence in support of this result. Before discussing the empirical evidence, however, we should justify the assumption of commitment.

Although the previous analysis focused on the extreme cases of full discretion and full commitment, the result that the bargaining power of workers affects the equilibrium inflation rate requires only a weak form of commitment. More specifically, this result also holds if the growth rate of money is chosen two periods in advance (two-period commitment). In fact, according to the analysis of Section 3, the profitability of a new job does not depend on the current inflation rate but on the inflation rate two periods from now (see Eq. (28)). This pre-commitment of policy could be the result of a lag between the moment in which the policy maker chooses the policy instruments, and the moment in which these instruments affect the real economy. When we adopt this interpretation of policy commitment, the assumption that countries "commit" to future policies is not unreasonable. Notice that the same result would be reached if the policy maker chooses the nominal interest rate instead of the growth rate of money. In this case, a one-period commitment would be sufficient.

To capture the cross-country differences in the bargaining power of employers and workers, we use the ratio of the value added generated in the manufacturing sector to the wages paid in that sector. The idea underlying the use of this proxy variable is that greater bargaining power of employers should allow them to appropriate a larger share of the surplus, where the surplus is approximated with value added. Higher values of this ratio are then interpreted as higher bargaining power of employers (and lower bargaining power of workers). We use data published by the United Nation Industrial Development Organization (UNIDO) for the years 1990–2000 for a large set of countries. The set of countries and years available for each country are reported in Appendix F.

Fig. 2 relates our proxy for the bargaining power of employers to the inflation rate. Each point in the graph corresponds to one country observation in a particular
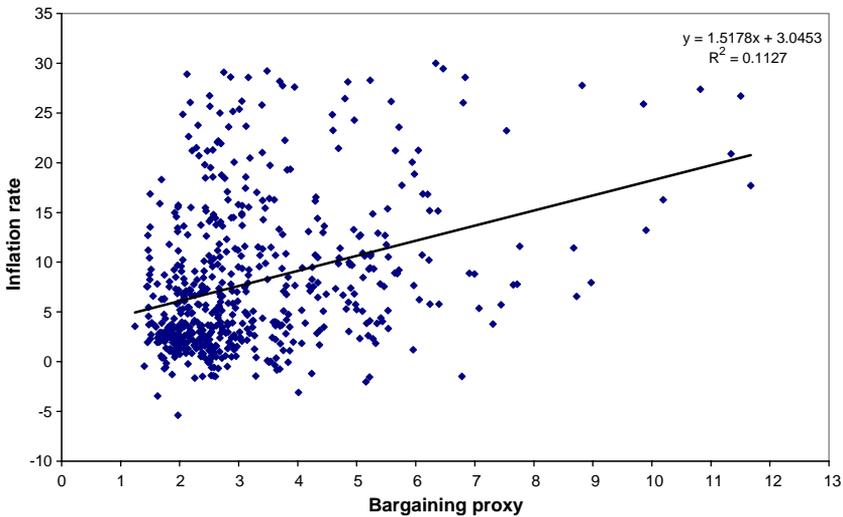
Fig. 2. Cross-country correlation of inflation with the bargaining proxy.

year. For example, if for a certain country there is data from 1990 to 1995, then the figure plots 6 distinct points for this country. If only 2 years of data is available, then only two points are plotted. Inflation rates are based on the GDP deflator published by the World Bank in the World Development Indicators. The inflation rates are for the same year of the proxy variable.

Fig. 2 shows that there is a positive association between the bargaining power proxy and the inflation rate. The correlation coefficient is 0.34. A similar correlation coefficient is obtained if we average the variables for each country over the years of available data (so that countries with a smaller number of observations do not get lower weights). In constructing the graph we have excluded countries for which the inflation rates are above 30%. The reason is that situations of hyperinflation cannot be rationalized as the outcome of optimal monetary policies. However, the sign of the correlation is not affected by this upper bound.

The top panel of Table 5 reports the estimates of the regression of inflation on the bargaining proxy for different cut off point of inflation. Even if we include only observations for which the inflation rate is smaller than 10%, the relationship remains statistically significant. Also notice that if we consider the whole sample, the coefficient remains statistically significant but the proxy variable explains a smaller fraction of inflation variability. In this case, the sample includes countries with inflation rates that are above 1000% as in the case of some Latin American countries in the early 1990s.

The positive correlation between the inflation rate and our bargaining proxy may be the result of the fact that developing countries have lower fractions of value added paid in the form of wages and these countries experience on average higher inflation rates. To capture the impact of the country development, we extend our regression

Table 5
Linear regression of inflation on the bargaining proxy

	Whole sample	Inflation <50%	Inflation <30%	Inflation <10%
Intercept	−6.05	4.57*	4.15*	2.44*
	(21.8)	(0.75)	(0.57)	(0.34)
Bargaining proxy	14.69*	1.50*	1.15*	0.51*
	(5.08)	(0.19)	(0.15)	(0.10)
# Obs.	672	606	573	403
R-square	0.012	0.093	0.096	0.056
Intercept	−6.41	9.52*	8.28*	4.40*
	(27.55)	(0.88)	(0.66)	(0.40)
Bargaining proxy	14.71*	1.22*	0.91*	0.34*
	(5.12)	(0.18)	(0.14)	(0.10)
Productivity/10 000	0.13	−1.60*	−1.30*	−0.48*
	(6.00)	(0.17)	(0.13)	(0.06)
# Obs.	672	606	573	403
R-square	0.012	0.206	0.236	0.185

*Note: Significant at 1% level.

equation by including a proxy for the level of development. This variable is given by the average productivity in the manufacturing sector computed by dividing the value added by the number of workers employed in this sector. To avoid endogeneity problems, the productivity variable is for the year preceding the first year of available data for inflation. The bottom panel of Table 5 shows that the bargaining proxy remains statistically significant. This result is robust to the use of per-capita GDP as a proxy for the level of development instead of manufacturing productivity.

Before closing we should acknowledge that there could be alternative interpretations about the positive correlation between the share of value added going to employers and inflation. For example, in an environment in which wages are pre-set in nominal terms, an unanticipated increase in inflation increases the fraction of value added going to employers. This alternative mechanism relies on the effects of *unanticipated* inflation and should be neutral to *anticipated* inflation. Our results, however, are robust to the use of averages of data over time, which are better proxies for anticipated inflation.

6. Conclusion

In this paper we have analyzed the properties of the optimal monetary policy in a world where inflationary monetary interventions have expansionary effects in the economy through the liquidity channel. We find that if technology shocks are the main driving force of business cycle fluctuations, then the optimal monetary policy is pro-cyclical and amplifies the response of the economy to these shocks. The

optimality of a pro-cyclical policy derives from the assumption that technology shocks are the main source of business cycle fluctuations. Different conclusions may be reached if we consider alternative sources of fluctuations.

We have also analyzed the long-run properties of the optimal and time-consistent policy and compared it to the long-term properties of the optimal policy under commitment. The main finding is that the ability to commit could lead to higher inflation if certain conditions pertaining to the structure of the labor market are satisfied. More specifically, higher inflation would be optimal if the employers' share of the surplus is too large. In this case higher inflation and interest rates are optimal because they reduce the surplus generated by a match, and therefore, the excessive creation of jobs. This result requires only a weak version of commitment. For example, this would be the outcome if the policy maker chooses the policy instruments two periods in advance (two-period commitment). This weak form of commitment could derive from formal and informal institutional restrictions to the discretion of the policy maker, as well as from the lags through which the policy instruments impact on the real sector of the economy. In this sense, the assumption of commitment is not unreasonable and there is some cross-country evidence supporting our result. In particular, countries in which the fraction of value added paid in the form of wages (proxing for the bargaining power of workers) is lower, are also the countries that tend to experience higher inflation.

We conclude by pointing out that there could be other mechanisms that make the Friedman rule suboptimal. One of this mechanism derives from the distributional effects of inflation which are ignored in this paper. This mechanism is studied in Albanesi [1].

Appendix A. Proof of Lemma 3.1

If the cash-in-advance constraint is binding, the aggregate version of the budget constraint (16) can be written as $(1 + g) = (D + g)(1 + R) + (1 - v)PNAX^v$ where the last term is simply the aggregate value of wages and profits paid by firms. Remember that the wage and profit paid by an individual firm is $(1 - v)AX^v$.[10] Combining the budget constraint with the equilibrium in the loans market $D + g =$

---

[10] All variables are denoted in capital letters because they are aggregate variables.

PXN and rearranging, we get $R = v(1+g)/(D+g) - 1$. This relation is valid as long as $v(1+g)/(D+g) - 1 \geqslant 0$. Otherwise the cash-in-advance constraint is not binding and the interest rate cannot be smaller than zero. $\square$

Appendix B. Proof of Proposition 3.1

Consider the following planner's problem in the choice of the input X and the number of vacancies V:

$$\Omega(A, N) = \max_{X, V}\{C - aN + \beta E\,\Omega(A', N')\} \tag{B.1}$$

subject to

$$C = N(AX^v - X) - V\kappa, \tag{B.2}$$

$$N' = (1 - \lambda)N + m(V, 1 - N). \tag{B.3}$$

Eq. (B.2) defines consumption from the aggregate resource constraint and (B.3) is the law of motion for the next period employment. The first-order conditions are

$$-\kappa + \beta m_1 E\left[A'(X')^v - X' - a + \frac{\kappa}{m_1'}(1 - \lambda - m_2')\right] = 0, \tag{B.4}$$

$$X = (vA)^{\frac{1}{1-v}}, \tag{B.5}$$

where m_1 and m_2 are the derivatives of the matching function with respect to the first and second arguments.

Now consider the household problem specified in (14). The first-order conditions are

$$\frac{\kappa}{q} = \beta E\left(\frac{\beta P'\pi'}{P''(1+g')}\right) + \beta E\left(\frac{(1-\lambda)\kappa}{q'}\right), \tag{B.6}$$

$$-E\left(\frac{1}{P'}\right) + \beta E\left(\frac{(1+R')}{P''(1+g')}\right) = 0. \tag{B.7}$$

These conditions must be satisfied in the competitive equilibrium. Using Eq. (11) to eliminate π', condition (B.6) can be written as

$$-\kappa + \beta(1-\eta)qE\left[(A'(X')^v - X'(1+R'))E\left(\frac{\beta P'}{P''(1+g')}\right)\right.$$
$$\left. -a + \frac{\kappa}{(1-\eta)q'}(1 - \lambda - \eta h')\right] = 0. \tag{B.8}$$

Because in the competitive equilibrium $X = (vA/(1+R))^{\frac{1}{1-v}}$ (see Proposition 1.1), Eq. (B.5) implies that the optimal interest rate for the planner is zero. With $R = 0$, Eq. (B.8) is not necessarily equal to (B.4), that is, the optimal number of new vacancies may not be optimal. However, Eq. (B.8) does not depend on the current interest rate, but only on the future interest rate. The planner will be able to affect

the rate of vacancy creation only if it can credibly commit to R' today. Without commitment the value of R' chosen today will not be optimal tomorrow. Consequently, the zero interest rate policy is the optimal and time-consistent policy. However, the optimal policy in terms of the growth rate of money is not necessarily unique because at $R = 0$ the cash-in-advance constraint is not binding. To show that the zero interest rate policy is unique, it is enough to show that the policy maker will always deviate from a policy rule that do not implement $R = 0$. Let Ψ be the policy rule that determined the future growth rates of money and let g be the current growth rate of money determined by this policy, that is, $g = \Psi(\mathbf{s})$. Given (g, Ψ), the equilibrium condition implies either $R = 0$ or $R > 0$ (the interest rate cannot be negative). In the first case the planner does not need to change g to obtain $R = 0$. If instead $R > 0$, then it will change g to obtain $R = 0$. This is because a change in the current growth rate of money will not affect the next period states (in particular employment). Therefore, the planner will deviate from any policy that allows for a positive nominal interest rate. $\square$

Appendix C. Proof of Proposition 3.2

Using the equilibrium condition in the loans market $D + g = PXN$ to eliminate the term $D + g$ in the aggregate budget constraint, we get $1 + g = PN[X(1 + R) + (1 - v)AX^v]$. Because $X(1 + R) + (1 - v)AX^v = AX^v$, this equation can be written as $1 + g = PY$ where $Y = NAX^v$ is aggregate gross output. This condition must be satisfied in each period. Therefore, it must also be satisfied in the next two periods, that is, $P'Y' = 1 + g'$ and $P''Y'' = 1 + g''$. Using these two conditions we can derive:

$$E\left(\frac{Y''}{Y'(1 + g'')}\frac{1}{P'}\right) = E\left(\frac{1}{P''(1 + g')}\right). \tag{C.1}$$

Assume that the optimal policy is $g_t = \beta E_{t-1} Y_t / Y_{t-1} - 1$. To verify that this policy implements a zero nominal interest rate, substitute this policy in Eq. (C.1). This gives

$$E\left(\frac{1}{P'}\right) = \beta E\left(\frac{1}{P''(1 + g')}\right). \tag{C.2}$$

From the household's first-order conditions, Eq. (B.7), we observe that this condition implies a zero nominal interest rate. Therefore, the policy $g_t = \beta E_{t-1} Y_t / Y_{t-1} - 1$ is optimal and time-consistent. We want to show now that this policy is unique. Notice that any policy that implements a zero nominal interest rate must satisfies condition (C.2). Then given a policy that satisfies this condition, the next period stock of deposits is determined and it cannot be changed in the next period. If the optimal policy rule is different from $g_t = \beta E_{t-1} Y_t / Y_{t-1} - 1$, it must have some stochastic component that is unpredictable so that condition (C.2) is still satisfied. But then ex-post, this stochastic component may induce an interest rate different from zero and the policy maker will deviate from this policy. Therefore, this stochastic component cannot be part of the optimal policy. Moreover, because the

growth rate of final output $N(AX^v - X)$ is equal to the growth rate of gross output $Y = NAx^v$ when $R = 0$, then $g = \beta E_{-1}(1 + g_Y) - 1$. $\quad \square$

Appendix D. Proof of Proposition 3.3

Consider the planner problem (B.1) with first-order conditions (B.4) and (B.5). The first-order conditions for the household in the competitive equilibrium are given by (B.7) and (B.8). If $\eta = 1 - \alpha$, $R = 0$ and $E'[\beta P'/(P''(1 + g'))] = 1$, it can be verified that (B.8) is exactly equal to (B.4). This is because $m_1 = (1 - \eta)q$ and $m_2 = \eta h$. This policy can be implemented by setting the growth rate of money as follows:

$$1 + g_{t+1} = \frac{\beta P_t}{1 + g_t} E_t Y_{t+1}, \tag{D.1}$$

where $Y_{t+1} = A_{t+1} X_{t+1}^v$ is gross output under the assumption that $R_t = 0$ and $E_t[\beta P_t/(P_{t+1}(1 + g_t))] = 1$ for all $t = 0, 1, \dots$. Notice that the growth rate of money does not depend on the current shock but only on the shocks up to the previous period.

We can verify that policy (D.1) implements $R_t = 0$ and $E[\beta P_t/(P_{t+1}(1 + g_t))] = 1$. Because $1 + g_{t+1} = P_{t+1} Y_{t+1}$ (see proof of Proposition 3.2), after rearranging and taking expectations we get $E_t(1 + g_{t+1}/P_{t+1}) = E_t Y_{t+1}$. Using this equation to eliminate EY_{t+1} in (D.1) we obtain $E_t[\beta P_t/(P_{t+1}(1 + g_t))] = 1$. This condition also holds one period forward. Therefore, $1/P_{t+1} = E_{t+1}[\beta 1/(P_{t+2}(1 + g_{t+1}))]$. Taking the expectation at time t we have

$$E_t\left(\frac{1}{P_{t+1}}\right) = \beta E_t\left(\frac{1}{P_{t+2}(1 + g_{t+1})}\right). \tag{D.2}$$

Comparing this equation with condition (B.7) we verify that policy (D.1) delivers a zero nominal interest rate and reaches the first best.

Now consider the case $\eta < 1 - \alpha$ and assume that the policy maker follows the same policy (D.1). As we have seen above this policy delivers a zero interest rate. Comparing (B.4) with (B.8), we can see that this policy induces a number of vacancies which is smaller than the social optimum. To see this, let us first observe that under policy (D.1) Eq. (B.8) can be written as

$$-\kappa + \beta q_t E\left[(1 - \eta)(A_{t+1}(X_{t+1})^v - X_{t+1} - a) + \frac{\kappa}{q_{t+1}}(1 - \lambda - \eta h_{t+1})\right] = 0. \tag{D.3}$$

This must always be satisfied. We want to show now that when $\eta > 1 - \alpha$, the left-hand side of condition (B.4) is greater than zero. Because $m_{1t} = \alpha q_t$ and $m_{2t} = (1 - \alpha)h_t$, substituting these functions in Eq. (B.4) we get

$$-\kappa + \beta q_t E\left[\alpha(A_{t+1}(X_{t+1})^v - X_{t+1} - a) + \frac{\kappa}{q_{t+1}}(1 - \lambda - (1 - \alpha)h_{t+1})\right] > 0. \tag{D.4}$$

The inequality sign can be verified by comparing this expression with condition (D.3). This implies that q_t is too large or equivalently that the number of new vacancies V_t is too small. To increase employment the policy maker would need to

decrease the future interest rates. However, from Lemma 3.1 we know that the interest rate cannot be negative. Therefore, the Friedman rule of a zero interest rate is still optimal.

In the case $\eta < 1 - \alpha$, the zero interest rate policy cannot be optimal. Let us consider again condition (D.4) which is now satisfied with the inverted inequality sign. Therefore, the value of q_t is too small and the value of vacancies V_t too large. This implies that the allocation could be improved by decreasing V_t. This, in turn, can be obtained by increasing the interest rate R_{t+1} as can be verified from Eq. (B.8). Therefore, the Friedman rule of a zero nominal interest rate is no longer optimal.

Appendix E. Optimal policy problem with commitment

Before setting the whole problem let us define the following functions:

$$
\begin{aligned}
&U(N(\mathbf{h}^{t-1}), R(\mathbf{h}^t), V(\mathbf{h}^t), A_t) \\
&\quad = N(\mathbf{h}^{t-1})[A_{t+1}X(\mathbf{h}^{t+1})^\nu - X(\mathbf{h}^{t+1}) - a] - \kappa V(\mathbf{h}^t),
\end{aligned}
\tag{E.1}
$$

$$
\begin{aligned}
&G(N(\mathbf{h}^{t-1}), V(\mathbf{h}^t), N(\mathbf{h}^t)) \\
&\quad = (1-\lambda)N(\mathbf{h}^{t-1}) + \mu V(\mathbf{h}^{t-1})^\alpha (1 - N(\mathbf{h}^{t-1}))^{1-\alpha} - N(\mathbf{h}^t),
\end{aligned}
\tag{E.2}
$$

$$
\begin{aligned}
&Q(N(\mathbf{h}^{t-1}), V(\mathbf{h}^t), N(\mathbf{h}^t), V(\mathbf{h}^{t+1}), N(\mathbf{h}^{t+1})) \\
&\quad = -\kappa + \beta(1-\eta)q(\mathbf{h}^t) + \sum_{z_{t+1}} \left[\frac{A_{t+1}X(\mathbf{h}^{t+1})^\nu - X(\mathbf{h}^{t+1})(1 + R(\mathbf{h}^{t+1}))}{1 + R(\mathbf{h}^{t+1})} \right. \\
&\quad\quad \left. -a + \frac{\kappa(1 - \lambda - \eta h(\mathbf{h}^{t+1}))}{(1-\eta)q(\mathbf{h}^{t+1})} \right].
\end{aligned}
\tag{E.3}
$$

Eq. (E.1) defines the period utility for the policy maker. It is per-capita consumption minus the per-capita disutility from working. Eq. (E.2) is the low of motion for employment. Eq. (E.3) is the first-order conditions of the household's problem which we derived in (B.8). In rewriting this constraint we have assumed that $E_{t+1}[\beta P(\mathbf{h}^{t+1})/(P(\mathbf{h}^{t+2})(1 + g(\mathbf{h}^{t+1})))] = 1/(1 + R(\mathbf{h}^{t+1}))$. It can be shown that this condition holds in the optimal policy.[11] This implies that the first-order condition (B.7) in the household's problem is satisfied. Using these functions the Ramsey problem can be written as

$$
\max_{\{N(\mathbf{h}^t), R(\mathbf{h}^t), V(\mathbf{h}^t)\}_{t=0}^\infty} \sum_{t=0}^\infty \beta^t \sum_{\mathbf{h}^t \in \mathbf{H}^t} p(\mathbf{h}^t)[U(N(\mathbf{h}^{t-1}), R(\mathbf{h}^t), V(\mathbf{h}^t), A_t)]
\tag{E.4}
$$

---

[11] To see this result, it is sufficient to show that the interest rate at time t depends only on the history up to time $t - 1$ (it does not depend on the current shock). This can be seen by looking at condition (B.8). Once we have established that the interest rate does not depend on the current shock (but only on the previous shocks), we can show that $E_{t+1}[\beta P(\mathbf{h}^{t+1})/(P(\mathbf{h}^{t+2})(1 + g(\mathbf{h}^{t+1})))] = 1/(1 + R(\mathbf{h}^{t+1}))$ following the same argument of the proof of Proposition 3.3.

subject to

$$G(N(\mathbf{h}^{t-1}), V(\mathbf{h}^t), N(\mathbf{h}^t)) = 0, \tag{E.5}$$

$$Q(N(\mathbf{h}^{t-1}), V(\mathbf{h}^t), N(\mathbf{h}^t), V(\mathbf{h}^{t+1}), N(\mathbf{h}^{t+1})) = 0 \tag{E.6}$$

with $N(\mathbf{h}^{-1})$ given. In this problem the policy maker chooses the sequence of interest rates, vacancies and employment. The numerical solution of this problem is obtained by linearizing the first-order conditions around the steady state.

Appendix F. Data set for the analysis of Section 5

Country	Bargaining proxy	Inflation rate
Albania (1990)	2.8	−0.5
Algeria (1990–1996)	2.6	28.7
Argentina (1990, 1993–1996)	2.9	416.2
Australia (1990–1992)	2.5	2.3
Austria (1990–2000)	1.9	2.2
Bahamas (1990–1991)	2.1	2.2
Bangladesh (1990–1992, 1995, 1997)	3.5	5.5
Barbados (1990–1997)	2.6	3.0
Belgium (1990–1992)	2.4	3.1
Bolivia (1990–1997)	9.2	11.3
Botswana (1990–1995)	2.3	8.9
Brazil (1990,1992–1995)	5.0	1558.3
Bulgaria (1991–1996)	3.0	99.0
Burundi (1990–1991)	5.0	5.0
Cameroon (1990–1999)	2.8	4.2
Canada (1990–2000)	2.4	1.7
Central African Republic (1990–1993)	2.0	−0.1
Chile (1990–2000)	5.4	9.3
Colombia (1990–2000)	6.0	21.5
Costa Rica (1990–1999)	2.9	20.8
Croatia (1990–1992)	4.2	264.7
Cyprus (1990–2000)	2.1	3.7
Denmark (1990–1991)	1.6	3.2
Ecuador (1990–1999)	6.9	39.1
El Salvador (1993–1998)	3.2	8.0
Ethiopia (1990–1996)	4.8	7.9
Fiji (1990–1992)	2.1	5.5
Finland (1990–1999)	2.2	2.2
France (1990–1992)	1.7	2.6
Gabon (1991–1995)	1.7	7.3
Gambia (1993)	5.5	5.1
Ghana (1993–1995)	6.1	35.0
Greece (1990–2000)	2.6	10.6
Honduras (1990–1995)	2.1	20.6

Country	Bargaining proxy	Inflation rate
Hong Kong (1990–1999)	1.9	5.1
Hungary (1990–2000)	2.4	19.9
Iceland (1990–1996)	1.5	5.3
India (1990–2000)	3.0	8.2
Indonesia (1990–1999)	6.6	16.4
Iran (1990–1993)	2.8	26.3
Ireland (1990–1999)	4.8	3.0
Israel (1990–1996)	1.5	13.0
Italy (1990–2000)	2.0	4.3
Cote d'Ivoire (1994–1995, 1997)	2.7	19.1
Jamaica (1990–1996)	2.9	36.4
Japan (1990–2000)	3.1	0.5
Jordan (1990–1997)	4.2	4.9
Kenya (1990–1999)	2.8	13.6
Korea (1990–2000)	4.3	5.4
Luxembourg (1990–1994)	1.8	3.1
Macau (1990–1997)	1.8	7.2
Malawi (1990–1998)	3.3	29.1
Malaysia (1990–2000)	3.7	3.8
Malta (1990–1996)	2.0	3.1
Mauritius (1990–1997)	2.3	6.9
Mexico (1990–2000)	5.1	19.2
Mongolia (1990–1995)	4.3	107.8
Moldova (1993–1995)	6.8	391.9
Morocco (1990–1999)	2.7	3.3
Namibia (1994)	3.0	15.7
Nepal (1990–1994, 1996)	4.6	9.1
Netherlands (1990–2000)	2.0	2.1
New Zealand (1990–1996)	1.9	1.8
Nigeria (1991–1996)	17.1	46.2
Norway (1990–1999)	1.7	2.4
Pakistan (1990–1991, 1996)	4.5	9.3
Panama (1990–1999)	2.6	2.0
Paraguay (1991)	4.6	24.8
Peru (1990–1996)	6.0	1222.6
Philippines (1990–1997)	5.0	9.46
Poland (1990–2000)	3.2	28.0
Romania (1990–1993)	2.7	159.0
Russia (1993–1998)	3.2	238.9
Senegal (1990–1997)	2.8	4.6
Sierra Leone (1993)	11.5	26.7
Singapore (1990–2000)	3.3	1.6
Slovakia (1993–1994)	2.6	17.9
Slovenia (1990–1999)	1.8	71.7
South Africa (1990–2000)	2.0	10.4
Spain (1990–1995, 1990–2000)	2.2	5.1

Country	Bargaining proxy	Inflation rate
Sri Lanka (1990–1998)	5.3	10.9
Suriname (1990–1993)	3.7	43.9
Swaziland (1990–1995)	3.5	13.3
Sweden (1990–2000)	2.3	2.8
Syria (1990–1998)	4.5	8.3
Thailand (1990–1994)	7.7	5.0
Trinidad and Tobago (1990–1995)	2.7	8.1
Tunisia (1993–1999)	2.8	4.3
Turkey (1990–2000)	4.7	71.3
Egypt (1990–1997)	3.0	11.7
United Kingdom (1990–1995)	2.2	4.2
Unites States (1990–1995)	2.9	2.8
Uruguay (1990–1999)	4.2	45.4
Venezuela (1990–1998)	9.1	45.8
Zimbabwe (1990–1999)	3.5	25.3

Source: UNIDO Industrial Statistics and World Bank Development Indicators.

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