The role of monetary shocks in equilibrium business cycle theory: Three examples

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Abstract

We study three equilibrium business cycle models that differ according to the mechanism through which monetary growth shocks affect the economy. These include models with inflation tax effects, with staggered nominal wage contracts, and with unanticipated inflation effects. We review some monetary features of business cycles in postwar US data and compare these with the same properties computed for the artificial economies. Our goal is to identify the features of the business cycle that these mechanisms help to explain, the features that remain puzzling, and how the form of the mechanism matters. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

To what extent and by what mechanism does monetary policy affect real economic activity? Robert E. Lucas Jr. wrote in his Nobel lecture: 'So much

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thought has been devoted to this question and so much evidence is available that one might reasonably assume that it had been solved long ago. But this is not the case . . .' (Lucas, 1996, p. 661). For decades economists have explored the ways in which changes in money can influence real activity. The mechanisms by which this can happen are, by now, familiar to every economist. These include that prices are slow to adjust ('sticky prices'), that wages are set in nominal terms ('wage contracting'), that monetary changes cause confusion making it hard to differentiate relative price changes from average price level changes ('monetary misperceptions'), or that households and firms change their portfolios at different frequencies ('limited participation').

Each of these is a potentially important feature of the real world. In addition, it is relatively straightforward to incorporate these features in a standard neo-classical growth model and explore the implications for economic activity, as the recent equilibrium business cycle literature attests.[1] It is common in this literature to proceed by writing down a model with one of the features described above and then claim success if this feature helps to explain more features of the data than the standard real business cycle model. Alternatively, models are commonly dismissed if they fail to explain some key features of the data.

Unfortunately, studying the properties of these models with the idea that there is some unique mechanism by which monetary shocks get transmitted seems unlikely to be successful. If there were a unique mechanism we would probably know what it is by now. All of the features cited above may interact in important ways. It is also possible that different factors are more important in different periods of time. This makes the ruling out of a particular transmission mechanism fairly precarious.

There are many features of the data that we would like a model to match and this makes it difficult to decide what is a legitimate basis for a claim of success or failure. Some of the existing literature passes judgment on models based on their ability to match the shape of impulse response functions from structural vector autoregressions. These are features of the data that are dependent on theory in the form of (often unstated) identifying restrictions. To accept judgments based on this one must also accept the identifying restrictions. We take a different approach to assess models and the importance of alternative transmission mechanisms.

In the next section we describe three model economies in which money affects real economic activity, but technology shocks are the primary cause of aggregate fluctuations. The first is a simple cash-in-advance economy and the other two incorporate more powerful monetary transmission mechanisms – nominal wage

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[1] In addition to papers cited in the context of the three examples presented in this paper, this literature includes Kydland (1989), Christiano and Eichenbaum (1995), King (1991), Chari et al. (1996), Christiano et al. (1997), Ohanian et al. (1995), Farmer (1997), and Cole and Ohanian (1997).

contracting and incomplete information. In Section 3 we compare the business cycle properties computed from simulations of these economies with the same properties computed from actual data. The empirical facts we focus on are simple correlations computed from H–P filtered US and artificial data. Our goal is not to pick a single best model, but instead to determine the features of the business cycle that these mechanisms help to explain, the features that remain puzzling, and how the mechanisms differ.

2. Model economies

The model economy studied in this paper is a basic real business cycle model with money introduced by imposing a cash-in-advance constraint (see Cooley and Hansen, 1995). This economy consists of a continuum of infinitely lived households with identical preferences and initial endowments. Agents choose consumption and work effort to maximize expected discounted lifetime utility

$$E \sum_{t=0}^{\infty} \beta^t [\alpha \log c_{1t} + (1-\alpha) \log c_{2t} - \gamma h_t], \qquad 0 < \beta < 1 \quad \text{and} \quad 0 < \alpha < 1. \tag{1}$$

Utility depends on consumption of a 'cash good' (c_1), a 'credit good' (c_2), and hours worked, h. Households are endowed with one unit of time that can be allocated to work or leisure.[2] The two consumption goods, which are produced using the same technology, differ in that previously accumulated cash balances are needed to purchase c_1 but not c_2.

A typical household begins period t with m_t units of cash and k_t units of capital carried over from the previous period. Purchases of c_{1t} are subject to the cash-in-advance constraint:

$$P_t c_{1t} \leq m_t + T_t, \tag{2}$$

where P_t is the price level and T_t is a lump-sum transfer of new cash issued by the government.

Credit goods $(c_{2t}$ and $i_t)$ can be financed with income earned in period t. Hence, the period budget constraint faced by a representative household is the following:

$$P_t(c_{1t} + c_{2t} + i_t) + m_{t+1} \leq P_t(w_t h_t + r_t k_t) + m_t + T_t. \tag{3}$$

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[2] The fact that hours worked enters linearly in the utility function follows from the assumptions that labor is indivisible, utility is separable in consumption and leisure, and agents trade employment lotteries (see Hansen, 1985; Rogerson, 1988).

Output, Y_t, is produced by a firm with access to the following technology, where K_t and H_t are the per capita stock of capital and hours worked, respectively:[3]

$$Y_t = e^{z_t} K_t^\theta H_t^{1-\theta}, \quad 0 < \theta < 1. \tag{4}$$

Given that labor and capital markets are competitive, the equilibrium real-wage rate and rental rate will equal the marginal product of labor and capital, respectively.

The law of motion for the capital stock is

$$K_{t+1} = (1 - \delta)K_t + I_t, \quad 0 < \delta < 1, \tag{5}$$

where I_t is per capita investment. The variable z_t is a shock to technology that is observed at the beginning of period t and evolves over time as follows:

$$z_{t+1} = \rho_1 z_t + \varepsilon_{t+1}^1, \quad 0 < \rho_1 < 1. \tag{6}$$

The random variable ε^1 is i.i.d. normal with mean zero and standard deviation σ_1.

The economy wide money supply, M_t, grows at the rate $\mu + g_t$ and new money is introduced at the beginning of period t (g_t is observed at the beginning of t), i.e.

$$M_{t+1} = e^{\mu + g_t} M_t, \tag{7}$$

where

$$g_{t+1} = \rho_2 g_t + \varepsilon_{t+1}^2, \quad 0 < \rho_2 < 1. \tag{8}$$

The random variable ε^2 is i.i.d. with mean zero and standard deviation σ_2.

The fiscal policy of the government determines how the seignorage obtained from money creation is divided between government spending and lump-sum transfers. We assume that there is no government debt. Hence, the government budget constraint is

$$G_t + T_t = (e^{\mu + g_t} - 1)M_t. \tag{9}$$

2.1. Example 1: the inflation tax

This example, apart from some parameter values, is the same as the model studied in Section 4 of Cooley and Hansen (1995). Here we assume that $G_t = 0$ for all t. As in our 1995 paper, values for β, γ, θ and δ are chosen so that the

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[3] We follow the convention of using capital letters, e.g. K and H, to denote *per capita* values of endogenous variables and lower case letters to denote values under the control of a particular household.

steady-state capital–output ratio, investment–output ratio, labor income share, and time spent participating in the labor market are equal to the average of these values computed from US data. We also employ the same value for α as in our previous paper, which is based on information from a survey of consumer transactions administered by the Federal Reserve Board in 1984 and 1986.

The parameters of the money supply process (μ and ρ_2 in Eqs. (7) and (8)) were assigned values from an estimated first-order autoregression of the growth rate of M1. Similarly, the autoregressive parameter of the technology shock process (ρ_1) was chosen to match the stochastic properties of the Solow residual. The standard deviations, σ_1 and σ_2, were chosen so that the standard deviation of the cyclical component of the Solow residual and the M1 growth rate, respectively, equal the same statistic computed from simulations of the artificial economy.[4]

For details on the numerical methods used to solve for a recursive competitive equilibrium, see Hansen and Prescott (1995). The output of these numerical methods is a set of linear equations determining the price level, hours worked, and next period's capital stock:

$$\log P_t = a_0 + a_1 z_t + a_2 g_t + a_3 \log K_t + \log M_{t+1},$$
$$\log H_t = b_0 + b_1 z_t + b_2 g_t + b_3 \log K_t, \tag{10}$$
$$\log K_{t+1} = c_0 + c_1 z_t + c_2 g_t + c_3 \log K_t.$$

2.2. Example 2: nominal wage contracts

In this example, we incorporate nominal wage contracts of the sort studied by Cho and Cooley (1995). In particular, we assume that a fraction ϕ of the firms in our economy set nominal wages four periods (quarters) in advance. The remaining $(1 - \phi)$ firms trade labor on a competitive spot market as in Example 1. For those firms that set nominal wages in advance, $\frac{1}{4}$ of them revise their wage each quarter. All other aspects of this example, including parameter values and the fiscal policy employed, are the same as in Example 1.

The contract wage is assumed to equal the expected value of the market-clearing nominal wage that would prevail in an economy without contracts.[5] The log of the nominal wage in a market-clearing economy can be written as

$$\log W_t = \log(1 - \theta) + z_t + \theta(\log K_t - \log H_t) + \log P_t, \tag{11}$$

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[4] The parameter values that follow from this procedure and used in this paper are, $\beta = 0.989$, $\alpha = 0.84$, $\gamma = 2.556$, $\theta = 0.4$, $\delta = 0.019$, $\rho_1 = 0.95$, $\sigma_1 = 0.0071$, $\rho_2 = 0.49$, $\sigma_2 = 0.0084$ and $\mu = 0.015$.

[5] This follows much of the previous literature on wage contracting, including Gray (1976), Fischer (1977) and Blinder and Mankiw (1984).

where the evolution of K, H and P over time is governed by Eqs. (5)–(10). Under this arrangement, the contract wage for time t set j periods prior, $\tilde{W}_t^{t-j}$, is given by

$$\log \tilde{W}_t^{t-j} = E[\log W_t | \Omega_{t-j}], \tag{12}$$

where Ω_{t-j} is the information available to agents in period $t-j$.

Taking as given the nominal wage that a particular firm is obligated to pay, hours worked are chosen unilaterally by the firm in time t in order to maximize profits. Hence, firms will choose hours so that the marginal product of labor is equal to the realized real wage. For a firm whose nominal wage was set j periods in the past, this means

$$\log \tilde{H}_t^{t-j} = \frac{1}{\theta} [\log(1-\theta) + z_t + \log P_t - \log \tilde{W}_t^{t-j}] + \log K_t. \tag{13}$$

To preserve identical households, we assume they consist of large 'families' that supply labor simultaneously to all of the firms. Hence, equilibrium hours worked by a household in period t is equal to the average hours worked in the economy as a whole, i.e.

$$\log \tilde{H}_t = (1-\phi) \log H_t + \frac{\phi}{4} \sum_{j=1}^{4} \log \tilde{H}_t^{t-j}. \tag{14}$$

Eqs. (10) and (11) are used to evaluate the conditional expectation in Eq. (12), and the result is combined with Eqs. (13) and (14) to obtain an expression for hours worked, $\tilde{H}_t$. This expression, which is a function of current and past values of the state variables, is used in place of the first-order condition for hours worked when solving for a recursive competitive equilibrium using the methods of Hansen and Prescott (1995).

This example adds one new parameter to those already calibrated. This parameter ϕ, the fraction of firms that set wages in advance, is calibrated so that in an economy with both technology shocks and money shocks, technology shocks alone account for 78% of the variance of output.[6] This implies a value for ϕ equal to 0.42.

2.3. Example 3: unanticipated changes in the money supply with incomplete information

This example is drawn without modification from Cooley and Hansen (1997), and is based on modeling features introduced by Lucas (1972, 1975). In this

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[6] Aiyagari (1994) estimates the lower bound of the fraction of the total variance of output explained by technology shocks to be 78%.

environment, agents live on a large number of spatially separated markets, or islands, and newly printed money is distributed unequally across these islands. Production is carried out on a household's home island, but purchases of goods are made elsewhere.[7] The key feature of the model is that households do not observe, and cannot infer, the average price level in the economy as a whole. They are only able to observe the price level in their own market and in the markets in which they shop. As a result, households will mistakenly respond to changes in the average price level as though they were changes in market specific relative prices.

In contrast with the previous examples, the fiscal policy in this example sets $T_t = 0$ for all t. Government spending, which still must satisfy Eq. (9), will differ across islands depending on the realization of a market specific shock, s_t, i.e. $G_t = \int G_t(s)\,d\varphi(s)$, where $\varphi(s)$ is the distribution function of the random variable s across markets and $G_t(s)$ is nominal government spending on island s. In addition

$$G_t(s_t) = e^{s_t}M_{t+1} - M_t. \tag{15}$$

The shock s_t evolves according to the autoregressive process

$$s_{t+1} = \rho_3 s_t + \varepsilon_{t+1}^3, \quad 0 < \rho_3 < 1, \tag{16}$$

where ε^3 is i.i.d. normal with mean 0 and standard deviation σ_3. These features imply that that the demand for output will differ across islands.[8]

The price of output in a market depends on the aggregate money growth rate, g_t, and the market specific shock, s_t. If a price increase is due to a change in g_t, it is the result of economy-wide inflation and agents will respond to it as a change in the implicit tax rate on money holdings (the 'inflation tax'). If a price increase is due to a change in s_t, then there has been a real increase in the demand for this island's output. Residents of markets characterized by higher than average government demand $(s_t > 0)$ will expect, on average, to purchase goods at a lower price than they sell their own output. In addition, if this high level of government spending is expected to persist, residents will expect a relatively high return on investment in new capital.

Our informational assumptions imply that households are not able to observe the two shocks, g_t and s_t, separately. Hence, agents may confuse changes in these two shocks and respond to a change in the economy-wide price level (resulting from a change in g_t) as though it were at least partly a change in the relative price of output in the home market (a change in s_t).

Values need to be assigned to the additional parameters, ρ_3 and σ_3. We set the autoregressive parameter equal to 0.99, as in Cooley and Hansen (1997), so that

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[7] The assignment of households to a market for shopping is random and independent over time.
[8] See Cooley and Hansen (1997) for details.

misperceptions persist as long as possible. The parameter σ_3 is set according to the same criteria used to set ϕ in Example 2, i.e. σ_3 is set equal to 0.0034 so that in an economy with both technology and money growth shocks, technology shocks alone explain 78% of the variance of output.

3. Business cycle facts and theoretical findings

In this section we describe and compare statistics summarizing the cyclical properties of US data from 1954 : 3 to 1994 : 4 and our three model economies. We follow the common practice in the real business cycle literature of reporting the moments of data that have been detrended using the Hodrick–Prescott filter.[9] We are also concerned with the robustness of the nominal features of the business cycle, so we also report statistics for two sub-samples during which it is commonly agreed that the Federal Reserve was using a stable operating procedure.

From Table 1, which contains the percent standard deviations of the cyclical component of various time series, it is clear that the relatively strong monetary non-neutralities present in Examples 2 and 3 can contribute significantly to the volatility of real variables. Output, hours, productivity and investment are all significantly more volatile in these two examples than they are in the inflation tax model. In the unanticipated money model consumption becomes far more volatile for reasons that are discussed at length in Cooley and Hansen (1997). Introducing monetary non-neutralities also increases the volatility of interest rates and velocity, which is more in keeping with what is observed in the data.

Table 2 shows the contemporaneous correlation of various money-related series with output. We see here that a negative correlation of the cyclical component of output and the price level is a robust feature of the data. This feature is consistent with the behavior of prices and output in the inflation tax model, but is inconsistent with the two models where stronger non-neutralities have been incorporated. In contrast, a positive correlation of output and inflation (as measured by the CPI) is also a robust feature of the data. However, in this case, the models with the stronger non-neutralities are consistent with this property of the data while the inflation tax model is not. None of these examples is consistent with both features of the data.

Our models are, however, roughly consistent with the procyclical nature of short-term nominal interest rates (as measured by the federal funds rate or the 90 day T-bill rate) and velocity. Either of the stronger transmission mechanisms

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[9] Statistics that summarize the US business cycle have been presented in many places (for example, Kydland and Prescott, 1990). The facts presented here differ slightly from other treatments in that we take as our measure of output US Gross Domestic Product (GDP) rather than GNP.

Table 1
Standard deviation of various series

	US data			Model economies		
	1954:3–1994:4	1954:3–1979:3	1982:4–1994:4	Inflation tax	Wage contracts	Unanticipated money
M1 growth rate	0.87	0.57	1.20	0.87	0.87	0.87
Real GDP	1.60	1.50	1.13	1.72	1.95	2.02
Hours	1.63	1.61	1.02	1.37	2.05	2.19
Nondurables and services	0.81	0.75	0.59	0.55	0.54	1.64
Fixed investment	5.52	5.19	4.58	5.94	7.31	9.30
Productivity	0.84	0.94	0.47	0.42	0.76	0.79
CPI	1.39	1.14	0.78	1.85	1.76	2.20
GDP deflator	0.85	0.70	0.60			
Inflation (CPI)	0.56	0.50	0.47	1.24	1.21	1.35
Federal funds rate	1.71	1.44	1.22			
Three month T-bill rate	1.33	0.98	1.14	0.58	0.71	0.88
10 Y bond rate	0.82	0.43	1.01			
Velocity (GDP/M1)	2.26	1.12	3.66	1.41	1.86	2.19

Note: The statistics reported in this table have been computed from Hodrick–Prescott filtered data. Series not expressed in percentage terms have been logged first. Results for the model economies are averages of 100 simulations, where each simulation has been filtered in the same manner as the US data.

Table 2
Correlation of money-related series with output

	US data			Model economies		
	1954:3–1994:4	1954:3–1979:3	1982:4–1994:4	Inflation tax	Wage contracts	Unanticipated money
M1 growth rate	−0.19	−0.03	−0.56	−0.03	0.41	0.34
CPI	−0.55	−0.64	−0.16	−0.21	0.06	0.30
GDP deflator	−0.50	−0.42	−0.27			
Inflation (CPI)	0.34	0.22	0.24	−0.16	0.20	0.17
Federal funds rate	0.41	0.47	0.69			
Three month T-bill rate	0.41	0.45	0.67	0.01	0.31	0.34
10 Y bond rate	0.07	0.04	0.51			
Velocity (GDP/M1)	0.29	0.56	0.26	0.95	0.95	0.93

Note: The statistics reported in this table have been computed from Hodrick–Prescott filtered data. Series not expressed in percentage terms have been logged first. Results for the model economies are averages of 100 simulations, where each simulation has been filtered in the same manner as the US data.

helps considerably in explaining the procyclical nature of short-term nominal interest rates. All of our models exhibit a similar high correlation of velocity and output, but the correlation in the data, although positive, is weaker.

Given our focus on the monetary aspects of the business cycle, Fig. 1 shows the correlation between the cyclical component of the money growth rate and leads and lags of various real and nominal variables. We find only a very weak association between M1 growth and GDP in the US sample (see also the first line of Table 2). The contemporaneous correlation is negative while the correlation of money growth with future output is positive but small and insignificant.[10] Similar results hold for hours and investment. These results, however, are not robust across the two sub-periods. In the early period, the contemporaneous correlation between money growth and GDP is essentially zero, but the correlation with future output is positive and significant and peaks at about three quarters. In the later period the correlation between money and output is strongly negative contemporaneously and stays negative for three quarters.

The correlations between money growth and real variables in the model economies differ significantly from those in the data. In particular, the model economies with strong transmission mechanisms exhibit relatively large positive contemporaneous correlations not found in the data. As one would expect, these correlations are much smaller in the inflation tax example.

Turning now to the monetary variables, there is a fairly strong negative correlation between money growth and interest rates in the US data. All three models, however, exhibit a strong *positive* correlation between these variables.[11] In addition, the price level correlations and inflation correlations are quite different in the model economies than in the data. The choice of monetary transmission mechanism appears to have very little effect on these correlations and, in particular, does not help to explain the correlation pattern observed in the data.

4. Conclusions

We find from our experiments that economies with labor contracting and misperception of monetary shocks have similar cyclical properties. In addition, the cyclical properties of our model economies and the US economy differ in important respects. In particular, our theory does not capture the (weak) phase shift found in the correlation of money growth with real variables. Also, the

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[10] This feature appears to be related to the hump-shaped response of output to a money growth shock focused on by various authors, including King (1991) and Chari et al. (1996).

[11] Others have found that 'sticky price' or 'limited participation' models are consistent with this feature of the data. See, for example, Christiano et al. (1997).
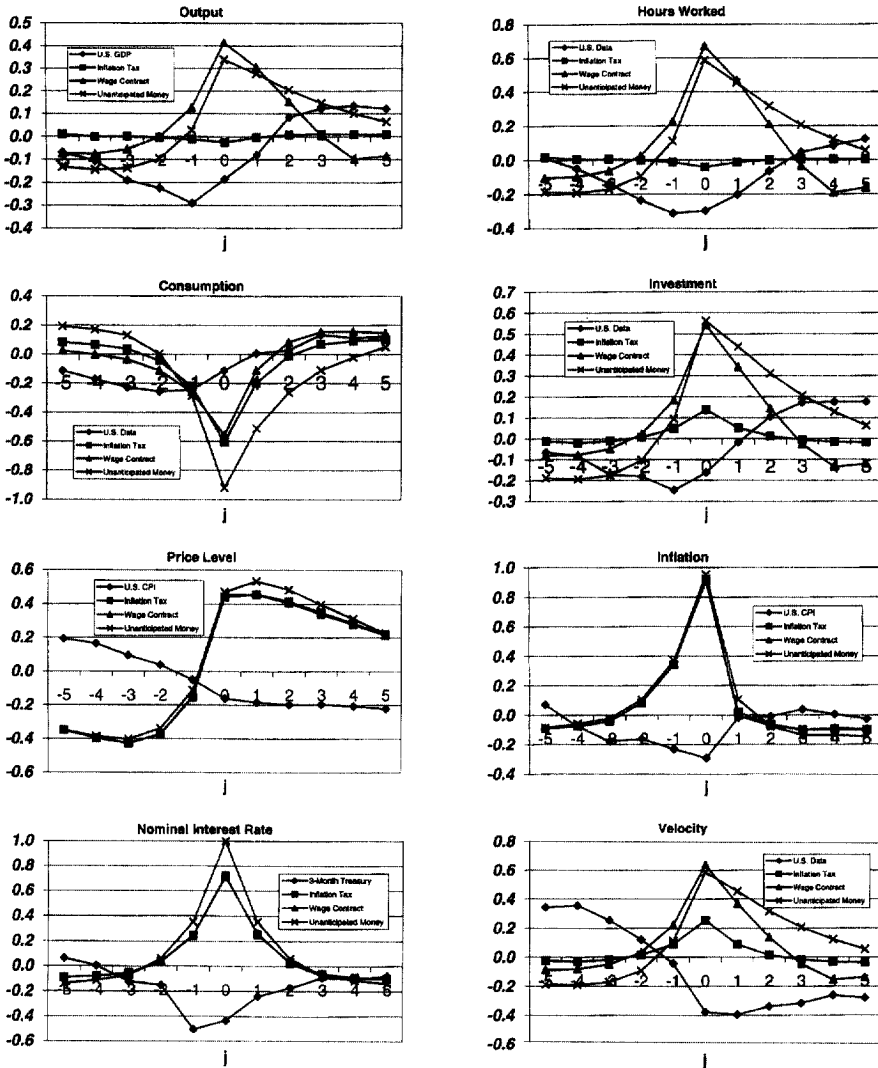
Fig. 1. Correlation with money growth, correlation ($\Delta \ln M(t)$, variable $(t + j)$).

weak correlation between the cyclical component of money growth and prices (or inflation) is puzzling, as is the negative correlation between money growth and nominal interest rates. Finally, none of these calibrated models can simultaneously account for the observed counter-cyclical price level and the procyclical rate of inflation. In ongoing work we plan to explore whether the addition of transmission mechanisms not considered here help illuminate these issues.

References

Aiyagari, S.R., 1994. On the contribution of technology shocks to business cycles. Federal Reserve Bank of Minneapolis Quarterly Review 18, 22–34.

Blinder, A.S., Mankiw, N.G., 1984. Aggregation and stabilization policy in a multi-contract economy. Journal of Monetary Economics 13, 67–86.

Chari, V.V., Kehoe, P.J., McGrattan, E.R., 1996. Sticky price models of the business cycle: can the contract multiplier solve the persistence problem? Staff Report 217, Federal Reserve Bank of Minneapolis.

Cho, J.-O., Cooley, T.F., 1995. The business cycle with nominal contracts. Economic Theory 6 (1), 13–33.

Christiano, L.J., Eichenbaum, M., 1995. Liquidity effects, monetary policy, and the business cycle. Journal of Money Credit and Banking 27 (4), 1113–1136.

Christiano, L.J., Eichenbaum, M., Evans, C.L., 1997. Sticky price and limited participation models of money: a comparison. European Economic Review 41, 1201–1249.

Cole, H.L., Ohanian, L.E., 1997. Shrinking money and monetary business cycles. Working Paper 579, Federal Reserve Bank of Minneapolis.

Cooley, T.F., Hansen, G.D., 1995. Money and the business cycle. In: Cooley, T.F. (Ed.), Frontiers of Business Cycle Research. Princeton University Press, Princeton, pp. 175–216.

Cooley, T.J., Hansen, G.D., 1997. Unanticipated money growth and the business cycle reconsidered. Journal of Money Credit and Banking 29 (4).

Farmer, R.E.A., 1997. Money in a real business cycle model. Journal of Money Credit and Banking 29 (4).

Fischer, S., 1977. Long-term contracts, rational expectations, and the optimal money supply rule. Journal of Political Economy 85, 191–206.

Gray, J.A., 1976. Wage indexation: a macroeconomic approach. Journal of Monetary Economics 2, 221–235.

Hansen, G.D., 1985. Indivisible labor and the business cycle. Journal of Monetary Economics 16, 309–328.

Hansen, G.D., Prescott, E.C., 1995. Recursive methods for computing equilibria of business cycle models. In: Cooley, T.F. (Ed.), Frontiers of Business Cycle Research. Princeton University Press, Princeton, pp. 39–64.

King, R.G., 1991. Money and business cycles. Working Paper, University of Virginia.

Kydland, F.E., 1989. The role of money in a business cycle model. Institute for Empirical Macroeconomics Discussion Paper 23, Federal Reserve Bank of Minneapolis.

Kydland, F.E., Prescott, E.C., 1990. Business cycles: real facts and a monetary myth. Federal Reserve Bank of Minneapolis Quarterly Review 14, 3–18.

Lucas, R.E. Jr., 1972. Expectations and the neutrality of money. Journal of Economic Theory 4, 103–123.

Lucas, R.E. Jr., 1975. An equilibrium model of the business cycle. Journal of Political Economy 83, 1113–1144.

Lucas, R.E. Jr., 1996. Nobel lecture: monetary neutrality. Journal of Political Economy 104 (4), 661–682.

Ohanian, L.E., Stockman, A.C., Kilian, L., 1995. The effects of real and monetary shocks in a business cycle model with some sticky prices. Journal of Money Credit and Baking 27 (4), 1209–1234.

Rogerson, R., 1988. Indivisible labor, lotteries and equilibrium. Journal of Monetary Economics 21, 3–16.