Carnegie-Rochester Conference Series on Public Policy 45 (1996) 89-121
North-Holland

Will social security survive the baby boom?*

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Abstract

In this paper we consider the design and implementation of a pay-as-you-go social-insurance system as a problem in political economy. We consider whether a society of forward-looking rational economic agents would implement a system in which the level of benefits depends on the relative shares of different age groups in the population. We calibrate a model economy to match long-run features of the U.S. economy and then look at the nature of the social-security system that results. We show that such a system would collapse given realizations of the population growth rate that the U.S. has experienced since World War II. If the benefits of the current retired generation are viewed as an obligation that must be paid, the system would survive the baby boom.

1 Introduction

The social-security system in the United States has been in existence since the late 1930s and has had fairly broad coverage since the early 1950s. In

*We thank Jeremy Greenwood, Per Krusell, and David Austin-Smith for helpful suggestions. This work has been supported in part by the Bradley Policy Research Center, Simon School of Business, University of Rochester. Soares gratefully acknowledges support from JNICT, "Praxis XXI," fellowship.

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PII 0167-2231(96)00019-X

the years since its inception both the coverage and the levels of benefits have changed. Coverage became quite broad beginning in the 1950s and the benefits have become ever more generous through the 1980s. The system operates on a pay-as-you-go basis, with current benefits to retirees paid by taxes on current working generations. This arrangement has always been controversial because it is different from the intent of the initial framers of the social-insurance system who envisioned a funded retirement savings system. Recently, the viability of the existing system has come into serious question in public policy debates. The increasing burden of the higher payroll taxes combined with the aging of the baby boomers has led many to predict the collapse or bankruptcy of the system early in the next century.[1]

When the social-security system was created in the 1930s about 6% of the population was over age 65. By 1995, the combined effects of increased life expectancy and the pattern of birthrates had more than doubled that share. As the baby-boom generation (born in the late 1940s and early 1950s) ages, the share of the population over age 65 will increase even more dramatically. These demographic forces, combined with a trend toward earlier retirement, mean that, by early in the next century, there will be far fewer workers for every retiree collecting social-security benefits than there are now. he demographic forces that are at the heart of this problem have been in place since the late 1940s. This is something of a puzzle: why would rational forward-looking economic agents who have a democratic choice over fiscal policy choose to keep this system in place given that it has a demographic time bomb built into it?

In an earlier paper (Cooley and Soares (1995)) we took a step toward understanding the existence of social-security systems by showing that a society would choose to put in place a pay-as-you-go social insurance system as the outcome of a majority-rule voting process. We studied the quantitative properties of such a system, detailed how it treats different generations, and showed why it would be sustainable in that environment. The model economy in that paper is incapable of addressing the primary issues of concern for the survival of a pay-as-you-go social-insurance system.[2] It assumes that population growth is constant and the only choice voters have each period is to continue with the existing constant level of social-security benefits.

In this paper we describe an economic environment in which an initial population is offered the opportunity to choose a *rule* for a pay-as-you go social-insurance system. The rule relates the level of benefits for retirees to the state of the economy. In each subsequent period the generations alive at

[1] See, for example, the report of The Bipartisan Commission on Entitlement and Tax Reform, January 1995.

[2] Two recent related papers that do address this issue in different contexts are Boldrin and Rustichini (1995), and Galasso (1996).

that time get to vote to continue with the rule in place or abandon social insurance altogether. In this environment, realizations of the state variables altogether can lead to abandonment of the social-insurance system even though it was sustainable when put in place. We consider the quantitative dimensions of this issue by calibrating a version of this environment to long-run features of the U.S. economy. We then consider a particular equilibrium – one characterized by median voter decisions – in which the initial population of voters chose a rule that relates the level of social-insurance benefits to the shares of each generation in the population. The shares of the generations in the population follow a random walk. We trace out the evolution of this economy given the realizations of the shares of generations since the 1950s. In this model economy the social-insurance system would collapse. However, if the working generations are obligated to pay off the claims of the current generation of retirees before privatizing the system, then the system will be preserved. We illustrate the impact of social security on the economy by showing the quantitative effect it has on the capital/output ratio and the return on capital. Finally, we consider some policies that allow for a smooth transition (as opposed to sudden collapse) from a pay-as-you-go to a private social-security system. None of the policies we have considered so far are implementable.

2 The economic environment

We study an economy where a large number of identical agents are born each period and have a lifetime of 4 periods. Population growth is stochastic, implying that the relative size of each cohort is a random variable. The share of age-i individuals in the population, given by the measure $\mu_{i,t}, i = 1,2,3,4$ with $\Sigma_{i=1}^{4}\mu_{i,t} = 1$, will change over time according to realizations of a stochastic process to be described later.

The agents in each generation maximize their discounted lifetime utility. The "momentary" utility function is assumed to take the constant relative risk aversion form

$$U(c_{i,t+i-1}) = \frac{c_{i,t+i-1}^{1-\rho} - 1}{1-\rho}, \tag{1}$$

where ρ is the inverse of the intertemporal elasticity of substitution.

We differentiate risk-sensitive behavior from intertemporal substitution by assuming that agents face a nonstationary recursive "risk-sensitive" discounted dynamic programming problem as defined by Hansen and Sargent

(1994).[3] For an agent born in period t this is given by

$$V_{1,t} = U(c_{1,t}) + \beta\Gamma_t(V_{2,t+1}), \tag{2}$$

where β is the subjective discount factor, $c_{i,t+i-1}$ is consumption of an age-i individual at period $t_i - 1$, and

$$\Gamma_t(V_{2,t+1}) = \frac{2}{\sigma} logE_t[exp(\frac{\sigma V_{2,t+1}}{2})].$$

σ is the risk-sensitivity parameter. By attributing different weights to different realizations of the next period value function, Γ allows us to incorporate some risk sensitivity in the discounting of the future. For example, if $\sigma > 0, \Gamma(\cdot)$ is concave indicating a preference for risk.

Agents in this economy accumulate claims on real capital, used in production by firms, to help smooth consumption across time. The budget constraint facing an individual of age i at time t can be written as

$$a_{i+1,t+1} = (1 + r_t)a_{i,t} + y_{i,t} - c_{i,t}, \tag{3}$$

where $y_{i,t}$ is the real net labor income plus transfers (in units of the consumption good) of an age-i individual at $t, a_{i,t}$ denotes the beginning-of-period asset holdings of an age-i individual at time t, and r_t denotes the rate of return on these assets. Agents will not accumulate assets in the last period of life and we assume there are no intergenerational transfers so that

$$a_{5,t} = 0, \forall t. \tag{4}$$

We choose the construct of four period lived agents because, in the U.S., current life expectancies and work-life expectancies imply that individuals spend somewhere between three and four years working for every year of retirement. We assume that agents may work the first three periods of their lives, but must be retired in the fourth period. In our specification of preferences leisure is not valued, so labor is inelastically supplied. We do try to incorporate some aspects of labor supply behavior by assuming that, before their mandatory retirement, age-i workers supply h_i hours of labor inelastically and earn $w_t h_i \epsilon_i$. The term h_i, varies to reflect the allocation of hours to market work over the life cycle, w_t is the real hourly wage rate (in terms of the consumption good) in period t, and ϵ_i is an efficiency index representing the productivity of an hour of work supplied by an agent of age i. After retirement, the net labor income plus transfers of a retiree is equal to his

[3] This specification will allow us to work with a LEQA (Linear/Exponential/Quadratic Approximation) method to study overlapping generations models in a stochastic environment. The linear decision rules that we obtain when we apply LEQA will incorporate "risk-sensitive" behavior.

social-security benefits, b_t. The level of benefits is computed by applying a replacement rate, θ_t, to a base income that we take to be a function of the income of the agents currently employed:

$$b_t = \theta_t w_t \overline{h\varepsilon}, \tag{5}$$

where $\overline{h\varepsilon}$ is the weighted average earnings of the working generations. Under these assumptions the net labor income plus transfers of an individual is given by

$$y_{i,t} = \begin{cases} (1-\tau_{t,s})w_t h_i \varepsilon_i, & \text{for } i = 1,2,3, \\ b_t, & \text{for } i = 4. \end{cases} \tag{6}$$

The production technology of the economy is described by a constant-returns-to-scale function,

$$Y_t = F(K_t, L_t) = \Psi K_t^{1-\alpha} L_t^{\alpha}, \tag{7}$$

where $\Psi \geq 0, \alpha \in (0,1)$ is the labor share of output, and Y_t, K_t, and L_t are the levels of output and of the capital and labor inputs, respectively. The capital stock is equal to the aggregate asset holdings of the agents in the economy. It depreciates at a constant rate δ and evolves according to the law of motion,

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{8}$$

There is a government in this economy whose only role is to implement the pay-as-you-go social-insurance system chosen by the agents through voting. The government must implement the required payroll tax rates so that its budget is balanced each period.

3 Equilibrium

Individuals have two roles in this model; they are economic agents who buy goods, accumulate assets, and supply labor, and they are participants in the political process through which policies are determined. We begin by describing their economic decisions, taking as given a sequence of political decisions or policies. We then describe how the policies are determined. Finally, we define an equilibrium for this economy.

3.1 *Economic decisions*

The economic problem of an age i individual at time t is to choose a sequence of consumption and asset holdings, given a sequence of policies for social insurance, that maximize the expected discounted value of lifetime utility subject to budget constraints. We write this as:

$$V_{i,t}(a_{i,t}, A_t, \mu_t; S_t) =$$

$$max_{c_{i,t}, a_{i+1,t+1}} \{U(c_{i,t}) + \beta\Gamma_t(V_{i+1,t+1}(a_{i+1,t+1}, A_{t+1}, \mu_{t+1}; S_{t+1}))\}$$

$$s.t. \;\; a_{i+1,t+1} = (1+r_t)a_{i,t} + y_{i,t} - c_{i,t}, \tag{9}$$

$$y_{i,t} = \begin{cases} (1-\tau_{t,s})w_t h_i \varepsilon_i, & \text{for } i = 1,2,3, \\ \theta_t w_t \overline{h\varepsilon_t}, & \text{for } i = 4. \end{cases}$$

$$A_{t+1} = H(A_t, \mu_t; S_t),$$

$$\text{given } \; S_t = \{\theta_l\}_{l=t}^{\infty}$$

where A_t represents the distribution of capital across agents and μ_t represents the distribution of agents across generations. $H_t(A_t, \mu_t; S_t)$ is the law of motion of the distribution of capital. S_t is a given sequence that determines the level of the social-security benefits in each period. Hence, we are assuming that, for each period, the level of the social-security replacement rate, θ_t is given.

A set of decision functions $C_{i,t}(a_{i,t}, A_t, \mu_t; S_t), H_{i,t}(a_{i,t}, A_t, \mu_t; S_t)$, laws of motion $H_t(A_t, \mu_t; S_t)$, and value functions $V_{i,t}(a_{i,t}, A_t, \mu_t; S_t)$ are obtained for the current state of the economy (A_t, μ_t).

The correct formulation of this problem would have to take into account the probability of the system collapsing or being sustained in the future. For the standard dynamic programming problem, this would be represented in the following way,

$$V_i(.|S) = \max_{c_i, a_i'}\{U(c_i) + \beta([1 - P(.)]e\{V_{i+1}'(.|S')\} + P(.)E\{V_{i+1}'(.|0)\})\} \tag{10}$$

where $P(.)$ is the probability that the system will collapse.

The probability of collapse is a function of the state of the economy and also depends on the stochastic process for the random variables in the model economy. Fully rational behavior involves taking into account both the effect of uncertainty on the value function for a given social-security system, and the effect on the probability of the system collapsing. To compute these probabilities for each period, we would need to compute the path for the economy for each possible sequence of the shocks. This makes it impossible to derive even a numerical approximation of the equilibrium path.

Accordingly, for pragmatic reasons, we assume that the agents do not take into account how the unexpected shocks might affect the future sustainability of the system. That is, when evaluating the possibility of the social-security system collapsing, the agents only look at the path of the economy corresponding to the expected sequence of the random variables. For the expected population shares, the system will then collapse with probability one or not collapse at all.

In our model, competitive firms maximize profits, which are equal to $Y_t - w_t L_t - r_t K_t$, taking the wage rate and interest rate as given. The first-order conditions for the firm's problem determine the following functions for the net real return to capital and the real wage rate.

$$\begin{aligned} R_t &= (1-\alpha)\Psi(\frac{K_t}{L_t})^{-\alpha} - \delta, \\ W_t &= \alpha\Psi(\frac{K_t}{L_t})^{1-\alpha}. \end{aligned} \tag{11}$$

3.2 *Political decisions*

The most general framework that would accommodate policies that are sensitive to the evolution of the economy would allow agents to choose a sequence of policy functions $\{\Phi_t\}_{t=0}^{\infty}$ that will determine in each period the level of the social-security replacement rate as a function of the state of the economy, $\theta_t = \Phi_t(A_t, \mu_t)$.

To keep this problem tractable we restrict the set of possible sequences of policy functions to be a sequence of constant policy functions. Agents in the initial generation vote for a linear policy function described by the vector of parameters P. In particular we assume it will be a linear function:

$$\theta_t = P\begin{pmatrix}1\\ \mu_t\end{pmatrix}. \tag{12}$$

The vector of parameters of the function P will be chosen by the agents through a democratic voting process. This function will determine the level of social-security benefits in each period as a function of the realized shares of each generation in the population. Once a social-security system is implemented, agents in subsequent periods will only vote to continue with this rule – represented by the chosen vector P^* – or abandon it in favor of no social insurance at all.

3.2.1 *Truthful voting:*

In this economy each agent has measure zero. We assume they will act as "measure zero voters" and behave as though their decisions do not influence the aggregate political outcome.[4] In addition, we make the assumption of sincere voting; everyone votes for the most preferred alternative at every stage

[4]While this assumption might seem perfectly reasonable for economists, used to models where the agents take the aggregate variables as given, in a political environment it is not so obvious. In fact, the best strategy for the agents, if they have some positive probability of being the pivotal voter, is to act as if they were the pivotal voter.

of the game. Although the game is sequential, this assumption seems perfectly reasonable because voters have no strategic gains from misrepresenting preferences.

3.2.2 *The voting decision:*

In Cooley and Soares (1995), social insurance is time inconsistent because forward-looking rational agents will not believe it is sustainable. To show how social security could be implemented and sustained by rational forward-looking agents, we introduced a reputational mechanism. Because social security is a dynamic game that involves repeated interactions between generations, reputational considerations can be used to deal with the time inconsistency problem.[5]

The reputation mechanism is represented by a trigger-strategy where the equilibrium of the one-shot game serves as a credible threat to induce more "cooperative" behavior from agents. If the workers today vote against paying social-security benefits, then agents next period lose confidence in the sustainability of the system. This loss of credibility means the cost of defecting today involves the collapse of the system tomorrow.

Let S_t^* be a rule that specifies the social-security system. The assumed expectations mechanism is,

$$S_{t+1}^e = \begin{cases} S_{t+1}^*, & \text{if } S_t = S_t^* \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

If the social-security benefits this period are the ones expected, agents trust the "majority" to perform according to the specified rule for period $t+1$. The social-security benefits are "given" to the retirees as a reward for not having deviated from the equilibrium. If the current generations of workers fail to go along, they will not be rewarded in the future.

3.3 *Equilibrium*

The solution to the agents' political problem involves evaluating the utility obtained under alternative values for the policy parameters. This requires that the agents predict the future path of the economy under the alternative current policies, which in turn requires them to predict the corresponding future policies. An equilibrium for this economy requires that agents consider the outcomes of policies that will never be realized, and rationality needs to be maintained in the subgames which would occur in the case of any deviations from the equilibrium path.

[5]Other discussions of the support of equilibria with social security may be found in Boldrin and Rustichini (1995), Browning (1975), Hanson and Stuart (1989), Jungenfelt (1991), Kotlikoff, Persson, and Svensson (1988), Tabellini (1991), and Verbon (1987).

In the initial period, agents will choose an equilibrium policy sequence with an associated expectations mechanism like the one described by (13). Only policy sequences that are sustainable along the equilibrium path need to be considered. This is because rational forward-looking workers would not support the current costs of a social-security system if they did not expect to benefit from it in their own retirement. We assume that, once a social-security system is implemented, agents in the following periods can only vote to retain the rule corresponding to the chosen P^*, or abandon it.

The existence of an equilibrium (or equilibria) where the agents form expectations according to (13) has to be confirmed. To determine whether P^* is a trigger-strategy equilibrium level of the social-security system under the reputational mechanism described in (13), one needs to consider whether the agents have any incentive to deviate and vote for $P_t \neq P^*(P_t = 0)$ at any point in time.

The effects of a policy change can be analyzed by finding the future law of motion of the state variable implied by the responses of agents to the new policy. Because of the reputation mechanism, beliefs about the sustainability of a social-security system depend in every period on the political outcome of the preceding period. This means the realization of P in each period will affect the decisions of the agents in the subsequent periods, and it has to be included as a state variable, along with the distribution of capital.

The aggregate policy for the next period will depend on the political outcome of the current period.

$$P_{t+j} = \begin{cases} P^* & \text{if } P_{t+j-1} = P^*, \\ & \\ 0 & \text{otherwise} \end{cases} \qquad \text{for } j \geq 1 \tag{14}$$

With the foregoing assumption we can define the function I describing the political outcome as

$$I(P_{-1}, P^*) = \begin{cases} 1 & \text{if } P_{-1} = P^*, \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

and we can now formulate the problem of the age-i agent in the following way:

$$\tilde{V}_i(a_i, A, \mu, P; P^*) = \max_{c, a'_{i+1}} \{U(c) + \beta\Gamma(V_{i+1}(a'_{i+1}, A', \mu'; I(P_{-1}, P^*)P^*))\} \tag{16}$$

$$s.t. \qquad a'_{i+1} = (1+r)a_i + y_i - c$$

$$y_i = \begin{cases} (1-\tau_s)w_t h_i \varepsilon_i, & \text{for } i = 1, 2, 3, \\ \theta w \overline{h\varepsilon}, & \text{for } i = 4, \end{cases}$$

$$\theta = P.\left(\frac{1}{\mu}\right)$$

$$A' = H(A, \mu, P; P^*)$$

$$P \in \{0, P^*\}.$$

A set of decision functions $\tilde{C}_i(a_i, A, \mu, P; P^*)$, $\tilde{H}_i(a_i, A, \mu, P; P^*)$, laws of motion $\tilde{H}_i(A, \mu, P; P^*)$, and value functions $\tilde{V}_i(a_i, A, \mu, P; P^*)$ are obtained for the current state of the economy (A, P) where P is the chosen vector of parameters for the social-security policy of the current period. Next period's decisions are given by the functions $H(A, \mu; I(P_{-1}, P^*), C_i(a_i, A, \mu; I(P_{-1}, P^*)P^*)$ from the individual's economic problem (9).

Note that, after the initial period, once the social-security rule, P^*, is chosen, age-i agents compare

$$\tilde{V}_i(a_i, A, 0; P^*) = \max_{c, a'_{i+1}}\{U(c) + \beta\Gamma(V_{i+1}(a'_{i+1}, A', \mu'; 0))\}$$

to

$$\tilde{V}_i(a_i, A, \mu, P^*; P^*) = \max_{c, a'_{i+1}}\{U(c) + \beta\Gamma(V_{i+1}(a'_{i+1}, A', \mu'; P^*))\}$$

and decide to vote for or against the implemented rule. Because there are only two possible choices, the majority-rule aggregator is easily applicable.

Lemma 1 *The optimal political outcome for an agent of generation i with an asset stock a_i when the aggregate state is A given the expectation mechanism (13) with $P_t^* = P^*, \forall t$ is*

$$\Pi_i(a_i, A, \mu; P^*) = arg \max_{P \in \{0, P^*\}} \tilde{V}_i(a_i, A, \mu, P; P^*) \tag{17}$$

Lemma 2 *The political outcome for the state of the economy A and given the expectation mechanism (13) with $P_t^* = P^*, \forall t$ is*

$$\Pi_{ag}(A, \mu, P^*) = arg \max_{P \in \{0, P^*\}} \Sigma_{\{i: P_i(a_i, A, \mu; P^*) = P\}} \mu_i = \tag{18}$$

$$P^* M\left(\frac{\Sigma_{i=1}^I M(P_i(a_i, A, \mu; P^*) = P^*)}{I} > .5\right)$$

where $M(.)$ is simply a majority-rule function that delivers the value 1 if the argument is true and 0 otherwise.

Definition: Given the expectation mechanism (13) with $P_t^* = P^* \forall t$, if the policy vector, P^*, is a political outcome in every period along the equilibrium path, then it is *sustainable.*[6]

Definition: An *equilibrium* is a set of value functions, $V_i(a_{i,t}, A_t, \mu_t; P^*)$, $\tilde{V}_i(a_{i,t}, A_t, \mu_t, P_t; P^*)$, decision rules for consumption and asset holding, $C_i(a_{i,t}, A_t, \mu_t; P^*)$, $\tilde{C}_i(a_{i,t}, A_t, \mu_t, P_t; P^*)$, $H_i(a_{i,t}, A_t, \mu_t; P^*)$, $\tilde{H}_i(a_{i,t}, A_t, \mu_t, P_t; P^*)$, for $i = 1, ..., 4$, laws of motion for the distribution of capital $H((A_t, \mu_t; P^*)$, $\tilde{H}(A_t, P_t, \mu_t; P^*)$, a pair of relative factor price functions $\{W(A_t, \mu_t), R(A_t, \mu_t)\}$, a function for the level of capital per capita $\hat{K}(A_t, \mu_t)$, [7] and a vector describing the social-security system P^* such that these functions satisfy:

1. The individual's dynamic programs (9) and (16).
2. The first-order conditions of the firm's problem (11).
3. Factor markets clear:

$$\hat{K}_t(A_t, \mu_t) = \Sigma_{i=1}^4 \mu_{i,t} a_{i,t}, \tag{19}$$

$$\hat{L}_t = \Sigma_{i=1}^3 \mu_{i,t} h_i \varepsilon_i. \tag{20}$$

4. The commodity market clears:

$$\Sigma_i \mu_i [C_i(a_{i,t}, A_t, \mu_t; P^*) + H_i(a_{i,t}, A_t, \mu_t; P^*)] = \hat{F}(\hat{K}_t, \hat{L}_t) + (1 - \delta) \underbrace{\Sigma_i \mu_{i,t} a_{i,t}}. \tag{21}$$

$$\Sigma_i \mu_i [\tilde{C}_i(a_{i,t}, A_t, \mu_t, P_t; P^*) + \tilde{H}_i(a_{i,t}, A_t, \mu_t, P_t; P^*)] = \hat{F}(\hat{K}_t, \hat{L}_t) + (1 - \delta) \underbrace{\Sigma_i \mu_{i,t} a_{i,t}}. \tag{22}$$

5. The laws of motion for the distribution of capital are generated by the decision rules of the agents:

$$H(A_t; P^*) = [H_i(a_{i,t}, A_t, \mu_t; P^*)]_{i=1,...,3}. \tag{23}$$

$$\tilde{H}(A_t, P_t; P^*) = [\tilde{H}_i(a_{i,t}, A_t, \mu_t, P_t; P^*)]_{i=1,...,3}. \tag{24}$$

6. The social-security system is self-financing:

$$\tau_t = \frac{\mu_{i,t} b_t}{\Sigma_{i=1}^3 \mu_{i,t} w_t h_i \varepsilon_i} = \frac{\theta_t \mu_{i,4} \overline{h\varepsilon}}{\Sigma_{i=1}^3 \mu_{i,t} h_i \varepsilon_i}. \tag{25}$$

[6] As only policy functions that are sustainable along the equilibrium path need to be considered, each level for the replacement rate can be presumed sustainable and then tested for any deviations along the path.

[7] A variable with a hat indicates that the variable is expressed in per capita terms.

7. The consumer problems (9) and (16) are consistent (for all i):[8]

$$H(A_t, \mu_t; P^*) = \tilde{(H)}(A_t, \mu_t, P^*; P^*),$$

$$V_i(a_{i,t}, A_t, \mu_t; P^*) = \tilde{(V)}_i(a_{i,t}, A_t, \mu_t, P^*; P^*),$$

$$C_i(a_{i,t}, A_t, \mu_t : P^*) = \tilde{(C)}_i(a_{i,t}, A_t, \mu_t, P^*; P^*) \text{ and}$$

$$H_i(a_{i,t}, A_t, \mu_t; P^*) = \tilde{H}_t(a_{i,t}, A_t, \mu_t P^*; P^*)$$

8. The replacement rate rule, P^*, is the political outcome:

$$P^* = \Pi_{ag}(A_t, \mu_t, P^*) \tag{26}$$

3.4 *Implementation of P^*:*

Once the set of the sustainable sequences of policy functions is determined, we can consider the problem facing the agents alive in the first period when a pay-as-you-go social-insurance system is proposed. In the initial period, agents choose the equilibrium sequence of policy functions that will be implemented. Agents (workers) will only vote for sustainable policy functions.

The problem of the age-i agent in the period when social security is first proposed is;

$$\overline{V}_i(a_i, A, \mu; P^*) = \max_{c, a'_{i+1}} \{U(c) + \beta\Gamma(V_{i+1}(a'_{i+1}, A', \mu'; P^*))\}$$

$$s.t. \quad a'_{i+1} = (1+r)a_i + y_i - c \tag{27}$$

$$y_i = \begin{cases} (1-\tau_s)wh_i\varepsilon_i, & \text{for } i = 1,2,3, \\ \theta w\overline{h\varepsilon}, & \text{for } i = 1. \end{cases}$$

$$\theta = P.(\tfrac{1}{\mu})$$

$$P^* \in \Omega(A, \mu),$$

$$4A' = \tilde{H}(A, \mu; P^*),$$

where $\Omega(A, \mu)$ is the set of sustainable replacement ratios for the current state of the economy. A set of decision functions $\tilde{C}_i(a_i, A, \mu; P^*)$, $\tilde{H}_i(a_i, A, \mu; P^*)$,

[8] That is, the sequence of policy functions P^* is currently generated by the preferences of the agents when the agents take P^* as the expected equilibrium political outcome for the next period, if there are no current deviations from the equilibrium. If the agents believe that the social-security system described by the sequence P^* will be sustained in the case where the punishment mechanism is not triggered, then the political outcome for the present period will be the level P^*. Their decision functions will therefore be the same as the ones obtained when the agents take the whole sequence of policy decisions as given (see problem 9).

laws of motion $\tilde{H}(A, \mu; P^*)$, and value functions $\overline{V}_i(a_i, A, \mu; P^*)$ are obtained for this problem.

The problem of a generation i agent in the initial period can also be described by (27) with the following objective function

$$\overline{V}_i(a_i, A, \mu, P^*) = max_{c,a'_{i+1}}\{U(c) + \beta V_{i+1}(a'_{i+1}, A', \mu', \Pi_{ag}(A', P^*))\} \quad (28)$$

where the use of the aggregator $\Pi_{ag}(A', \mu', P^*)$ replaces the sustainability constraint. For any nonsustainable P^* next period's political outcome will be $\Pi_{ag}(A', \mu', P^*) = 0$ and therefore current workers will end up paying for a system that would never benefit them. For sustainable P^*'s, $\Pi_{ag}(A', \mu', P^*) = P^*$ and they will get the benefit level corresponding to P^*.

3.5 *Political environment*

In the first period, when agents vote for P^*, there is a continuum of possible transfer levels from which to choose. In this context, where there are more than two possible choices, the majority rule is not a well-defined aggregator. Furthermore, work in formal political theory has demonstrated that, when dealing with a multidimensional choice set, voting systems rarely possess an equilibrium, especially majority-rule system (see e.g., Kramer (1973), Grandmont (1978)). To guarantee existence of equilibrium, we will assume issue-by-issue (sequential) voting – in our case the issues will be the elements of the vector P – where the agenda is set competitively (see for example Shepsle (1979) and Denzau and Mackay (1981)).

The first assumption implies that the political process consists of a sequence of elections in which levels for each of the parameters are voted on. An element of P is chosen in each one of these elections. The assumption of competitive agenda setting allows any proposal of the policy parameters to be considered. Because we assumed sincere voting, strategic collusion between the agents is ruled out. Further, because the choice of social-security policy parameters is restricted to the implementation period, multi-election proposals would not be credible. Decisions made in the i^{th} election are implemented before $i+1st$ election decisions. Once they are implemented, the decisions for the next election will be revised to take this into account. Therefore, when making the i^{th} election decision, the agents will incorporate this effect into their political choice. That is, if the political behavior is to be subgame perfect, voters will be unable to vote for certain levels of the parameters in future elections.

In this setting, if the preferences over each one of the possible parameters are single-peaked, there exists a policy function, defined by the choices of the median voter in each election, that is "invulnerable" to every set of proposals to change and thus constitutes a voting equilibrium.[9] This is the equilibrium we will study.

Lemma 3 *Let m be the generation where the median voter is located, then the aggregate choice will be determined according to:*

$$P_a(A) = \Pi_m(a_m, A, \mu) = arg\max_P \overline{V}_m(a_m, A, \mu, P). \tag{29}$$

3.6 *Simulation and transition policies*

Once we have chosen a policy function for a given initial realization of the state variables (A, μ), we simulate the model for a sequence of realizations of the exogenous stochastic state of the economy μ. For each realization of the shares, we compute the expected path of the economy and check for sustainability of the social-security system by solving problem 16.

We also examine some possible smoother transitions to an economy without a social-security system. We assume that in each period after the implementation, the agents, besides voting for the simple abandonment of the social-security system, are given the option to move to a policy path where the social-security system will be smoothly phased away. Along this path, the social-security benefits will be decreased by a given amount in order to fully "depreciate" in a certain number of periods, while depreciating the system will be financed by a predetermined type of taxation, labor, and/or capital taxation. For now, we rule out financing this transition with debt.

In order to focus solely on possible transition policies, we will abstain from considering sustainability problems along the path if the transition policy is ever chosen. But to restrict the possible choices of this policy, we will limit them to the ones where either the young agents (that benefit most from the abandonment of the system) or the median voter generation will be the first generation to pay contributions without ever getting direct benefits.

A proposed transition policy can be described by a sequence $\{DP^*\}$. For instance, if we want the social-security system to fully depreciate in T periods starting now, we can define the t-th element of this sequence as,

$$DP_t^* = max(DP_{t-1}^* - \frac{P^*}{T}, 0).$$

[9]Single-peakedness implies the existence of a majority winner. We will show numerically that the preferences are single-peaked with respect to each of the parameters. In this environment, for the equilibrium to be unique requires strict quasi-concavity with respect to the parameters.

We can formulate the problem of the age-i agent in the following way:[10]

$$\hat{V}_i(a_i, A, \mu; SP; P^*) = max_{SP}\{max_{c,a'_{i+1}}[U(c)+\beta\Gamma(\hat{V}_{i+1}(a'_{i+1}, A', \mu'; \{SP\}; P^*))],$$

$$max_{c,a'_{i+1}}[U(c) + \beta\Gamma(CV_{i+1}(a'_{i+1}, A', \mu'; \{DP^*\}))]\}$$

$$s.t. \quad a'_{i+1} = (1 + r)a_i + y_i - c \tag{30}$$

$$y_i = \begin{cases} (1 - \tau_s)w_t h_i \varepsilon_i, & \text{for } i = 1, 2, 3, \\ \theta w \overline{h\varepsilon}, & \text{for } i = 4. \end{cases}$$

$$\theta = SP.(^1_\mu)$$

$$A' = \tilde{H}(A, \mu, P; P^*)$$

$$SP \in \{P^*, \{DP^*\}\}. \tag{31}$$

As in the sustainability problem (16), age-i agents compare the expected utility level from sustaining the social-security system.

$$\hat{V}_i(a_i, A, \mu; P^*; P^*) = max_{c,a'_{i+1}}\{U(c) + \beta\Gamma(\hat{V}_{i+1}(a'_{i+1}, A', \mu'; SP; P^*))\}$$

to the expected utility level from a transition policy according to which the social-security system depreciates in the way described in

$$\{DP^*\}\hat{V}_i(a_i, A, \{DP^*\}; P^*) = max_{c,a'_{i+1}}\{U(c)+\beta\Gamma(V_{i+1}(a'_{i+1}, A', \mu'; \{DP^*\}))\}$$

and they choose the best option.[11]

4 Calibration

We assign values to the parameters of preferences and technology in this economy based on long-run features of the U.S. economy. We calibrate the model assuming that the model period is 60/4 years long. Agents in our model are assumed to be born as workers at age 21 living 60 years to a real-life age of 80. Therefore, a period in the model will correspond to 15 years.

The driving mechanism for change in this economy is population dynamics. Our framework requires calibrating the process for innovations to shares of each cohort in the population. We calibrate the relative size of

[10] We will suppose that the transition policy will be sustained in each period after it is implemented. The objective of this section is to find a transition scheme that will be chosen by the agents as an option to the social-security system.

[11] Note that if the agents decide to sustain the system in the current period, the agents living in the next period will face the same problem. This is incorporated in their value functions.

each generation in the population based on the average shares for the period 1946–1959, a period chosen because social security began to have fairly broad coverage in the early 1950s. The shares corresponding to each cohort follow unit root processes with standard deviations that are given below:[12] also presented are the realized shocks to these processes for three subsequent periods in the model. Figure 1 shows the evolution of the population shares as a consequence of these shocks.

Table 4.1

Shares - No Intercept				
	μ_0	μ_1	μ_2	μ_3
Initial Level	.273402	.248356	.225046	.158708
St. Dev.	.00419	.004313	.003257	.000881
Shocks				
1960-1974	.043333	-.0283	-.02704	-.00019
1975-1989	-.05936	.057453	-.01057	-.00517
1990-2009	-.0294	-.01355	-.043942	-.01304

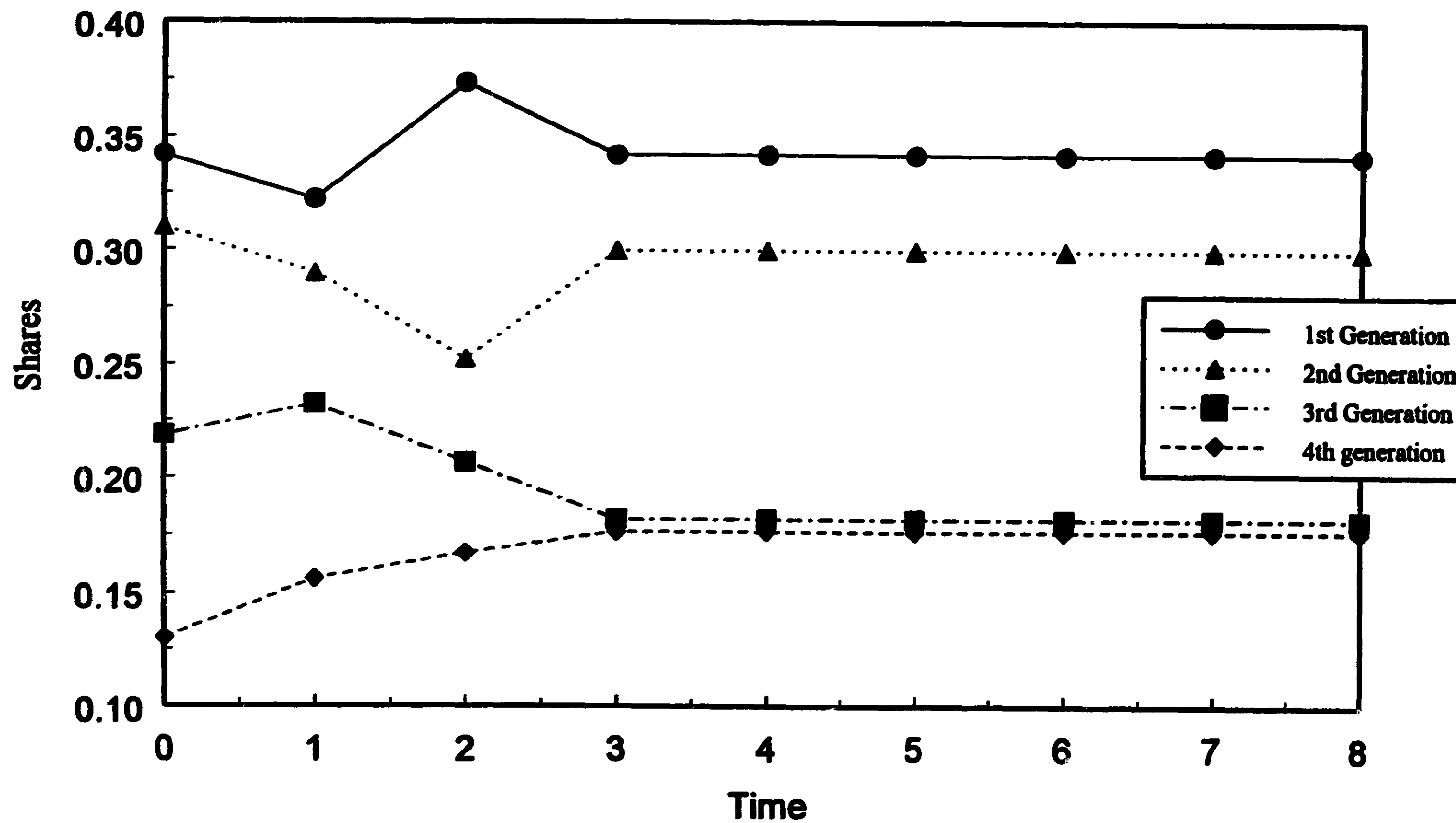
4.1 *Preferences*

In our benchmark model we set the intertemporal elasticity of substitution to 1. We choose a value less than one for the effective discount factor which has the effect of not biasing the results towards the existence of a social-security system. We set the discount rate β to be the equivalent in the four-generations model of the value (.975) in a 60-generations model. The risk sensitivity parameter is set to 1, a small positive value that is necessary to choose a random replacement rate.

We also solved the model under several different assumptions on the risk sensitivity. We assumed $\sigma = 0$ and $\sigma = -1$ but the behavior of the aggregate variables changes very little. As σ increases the accumulation of assets drops very little. The implications for the collapse of the social-security system are the same for all values of σ. We set $\sigma = 1$ so that the agents will choose a replacement rate that is a function of the population shares. Unless $\sigma > 0$ the voters will prefer a replacement rate that will not change with the aggregate shocks. $\sigma > 0$ implies that $\Gamma(V)$ is concave in $E[V]$ and therefore the agents prefer a late resolution of uncertainty. This means that in this set-up, a social-security system depending on the stochastic variables of the economy

[12]The shares and the respective processes were computed from Citibase data for the U.S. population. As a proxy for the average shares from 1990 to 2009, we used the average shares for the period 1990–1995.

Figure 1: Relative Size of the Different Generations



will not be an insurance against the shocks but a game that only optimistic agents will be willing to play.

4.2 *Technology*

The share of labor in the production function is set to be .6 following Cooley and Prescott (1995). The parameter Ψ is chosen so that for a capital-output ratio of 3 and for the given initial aggregate labor input, annual output is normalized to one, as in Imrohoroglu et al. (1994).[13] We calibrate the hours of work supplied by each agent in our model to be the average hours worked by the agents in the corresponding age groups in the Current Population Survey (CPS) March demographic files for 1989-91. The age-specific endowments of efficiency units are constructed to provide a realistic age distribution of earnings using the CPS data. We compute these indexes as the ratio between the average hourly wage for each age group and the average hourly wage for all the age groups.

Table 4.2

Labor Supply			
Generations	1	2	3
Hours of work	.2569	.2691	.1681
Efficiency index	.9043	1.1828	1.1873

The supply of labor of each generation is then given relative to the total lifetime supply. For the retirees we use $\overline{h\varepsilon} = \frac{\Sigma_{i=1}^{3}\mu_{i,t}h_i\varepsilon_i}{\Sigma_{i=1}^{3}\mu_{i,t}}$. Lastly, we set the depreciation rate to be 4.8% on an annual basis. The parameter choices are summarized in the following table:

Table 4.3

Benchmark Calibration - 4 Generations								
β^*	π	σ	α	δ	Ψ	$h_1\varepsilon_2$	$h_2\varepsilon_2$	$h_3\varepsilon_3$
.684	1	1	.6	.5219	3.957	.3097	.4243	.266

5 Findings

For the benchmark calibration, the utility levels of the agents alive in the initial period when social security is first proposed are single peaked over

[13]Total factor productivity will be such that $1 = \Psi \cdot 3^{.3} \cdot (\Sigma_{i=1}^{3}\mu_i h_i)^{.6} \cdot 4/60$.

the policy parameters, P. The utility of the youngest agents alive at that time is strictly decreasing over positive values of the parameters, and the utility of retirees is strictly increasing. The utilities of the middle-aged agents (generations 2 and 3) have interior maxima. The third generation agents always prefer higher levels of benefits than the second generation, but the median voter belongs to the second generation. Having established single-peakedness for the benchmark economy, and having located the median voter, it is possible to determine the equilibrium level of the replacement rate for the initial population.

As noted previously, the equilibrium level of P must be sustainable. To avoid the burden of determining all the sustainable values for this parameter, it is more efficient to check first whether the unconstrained level of P for the median voter is sustainable. The levels of the policy parameters that maximize the utility level of the first-period median voter are

$$P^* = (3.5532 - 2.6531 - 3.495 - 3.7105 - 1.9774).$$

This policy vector is sustainable for the expected path of the parameters, so it will be a political equilibrium.

For the initial population distribution this policy implies $\theta = .8116$. This corresponds to a replacement rate of .6493, which is computed by dividing the level of social-security benefits by the **peak** labor income of the agents.[14]

The time paths for the replacement rate and for the tax rate on labor income implied by this social-security rule are shown in Figures 2 and 3, respectively. These paths are calculated assuming that the social-security system would survive. The population dynamics are such that the replacement rate would rise from its initial level, then fall and then rise steadily. The tax rate on labor incomes implied by these dynamics rises steadily.

The equilibrium social-security system is sensitive to the assumed generational structure and the location of the median voter. The median voter in this economy is located in the second generation and this finding is robust: it holds for all the realizations of the population distribution.

Table 5.1 shows the equilibrium ratios of assets to output held by the working generations, $(a_{i,ss})$, the capital/output ratio k_{xx}/y_{ss}, and the interest rate r_{ss} in the absence of a social-security system ($P = 0$) and for the equilibrium policy vector ($P = P^*$), given the initial age distribution of the population in this economy. Here we see the reason that social security has very important general equilibrium effects on the economy: the equilibrium levels of the assets for each generation and the aggregate stock of capital per capita are strictly decreasing with the level of the social-security benefits.

[14]We use the peak as the base because the labor supply and lifecycle earnings profile of agents imply a fairly sharp decline in the effective earnings of third-generation agents.

Figure 2: Level of the Replacement Rate Along the Simulation Path

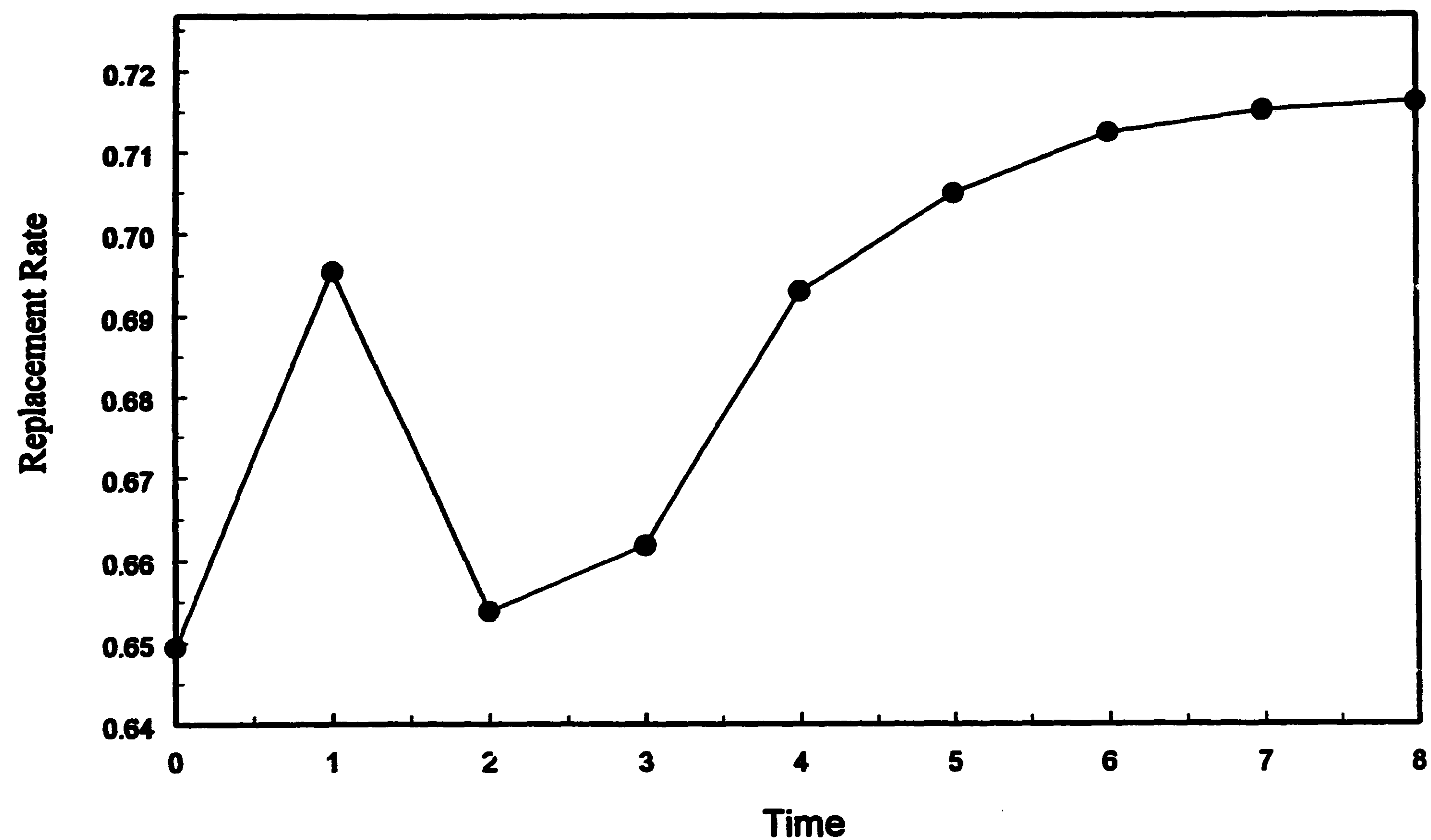


Figure 3: Level of the Social Security Tax Rate Along the Simulation Path

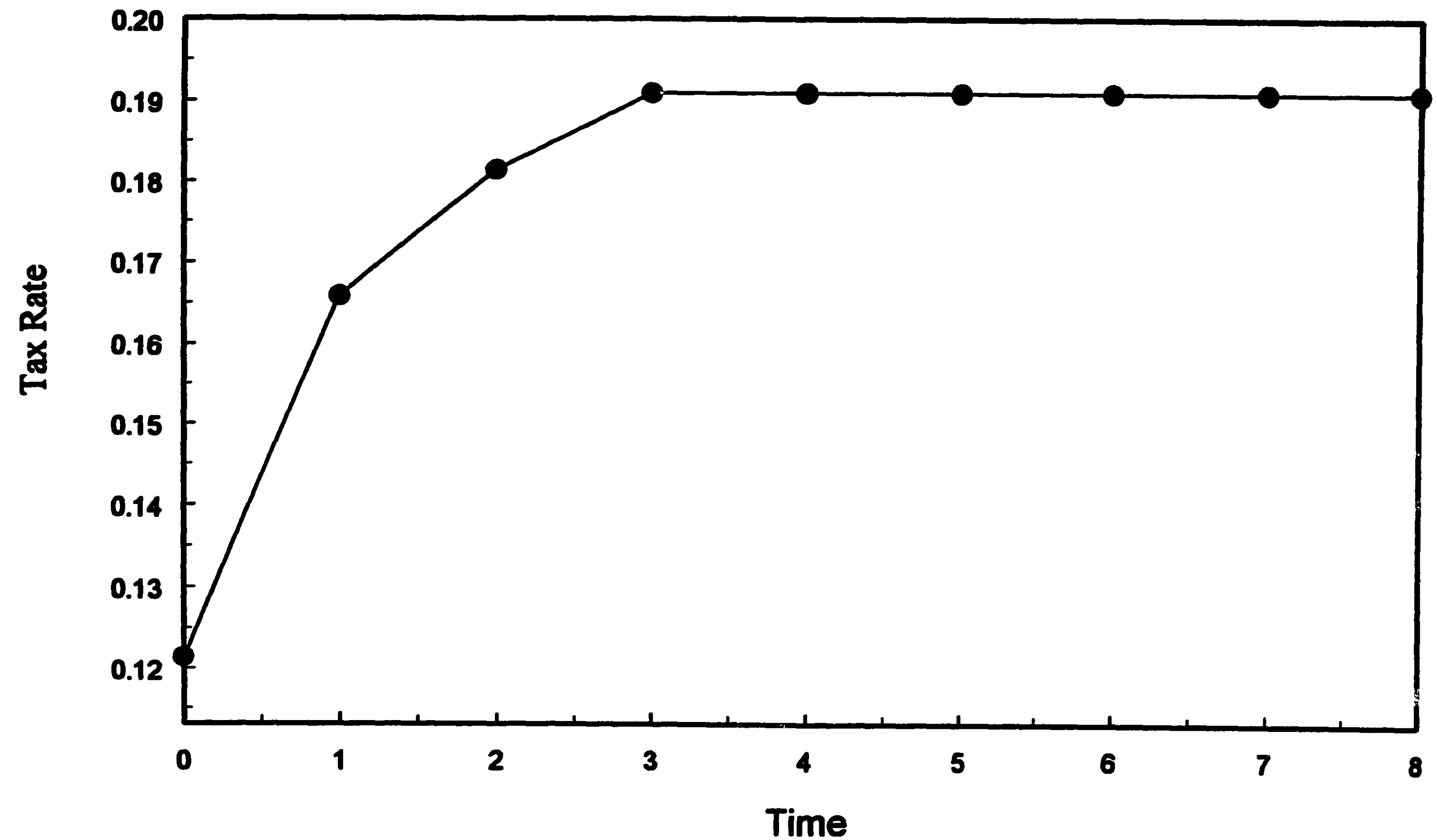


Table 5.1

Steady States					
	$a_{2,ss}/y_{ss}$	$3, ss/y_{ss}$	$a_{4,ss}/y_{ss}$	k_{ss}/y_{ss}	$r_{ss(annual)}$
$P = 0$	.7336	1.6977	1.2869	3.7182	.0504
$P = P^*$	.6947	1.4975	.965	3.1572	.0595

Figure 4 shows the evolution of the capital stock along the transition path from a steady state without social security to a steady state with $P = P^*$ and a different population distribution. Although the transition appears fairly rapid, it is important to recall that the periods here are generations - i.e., 15 years. Figure 5 shows the corresponding evolution of the real interest rate, while Figure 6 shows the evolution of the wage rate (measured per efficiency unit of the labor input).

5.1 *Value of the social-security system*

Social security affects social welfare by altering aggregate consumption levels and lifetime consumption profiles. To assess how different generations far under a social-security system, we need a measure of welfare that takes into account the important general equilibrium effects of the system.

We construct a measure of the welfare costs based on the Hicks compensation principle. We compute the aggregate extra amount of income required to make all the agents able to attain the same expected level of lifetime utility in the economy with the social-security system as in an economy without social security. The compensation scheme that results is one that, being optimally allocated between consumption and savings, maintains the reference level of expected utility in each period.

The compensation to be given to an age-i agent endowed with a level of assets $a_{i,t}$ in an economy with a social-security system described by S and an aggregate state described by A_t, is $x_{i,t}$ such that $V(a_{i,t} + x_{i,t}, A_t, \mu_t; S) = V(a_{i,t}, A_t; 0)$, where $V(a_{i,t}, A_t; 0)$ would be the lifetime utility level of that agent an economy without social security.

The values of the compensations as a fraction of per capita output are shown in Figure 7. These are shown along the transition path from a steady state without social security to a steady state with $P = P^*$. An important attribute of the equilibrium is that the second and older generations would require a significant increment to income to be as well off without social security once their contributions have been made.

Figure 4: Level of the Capital-Output Ratio Along the Simulation Path

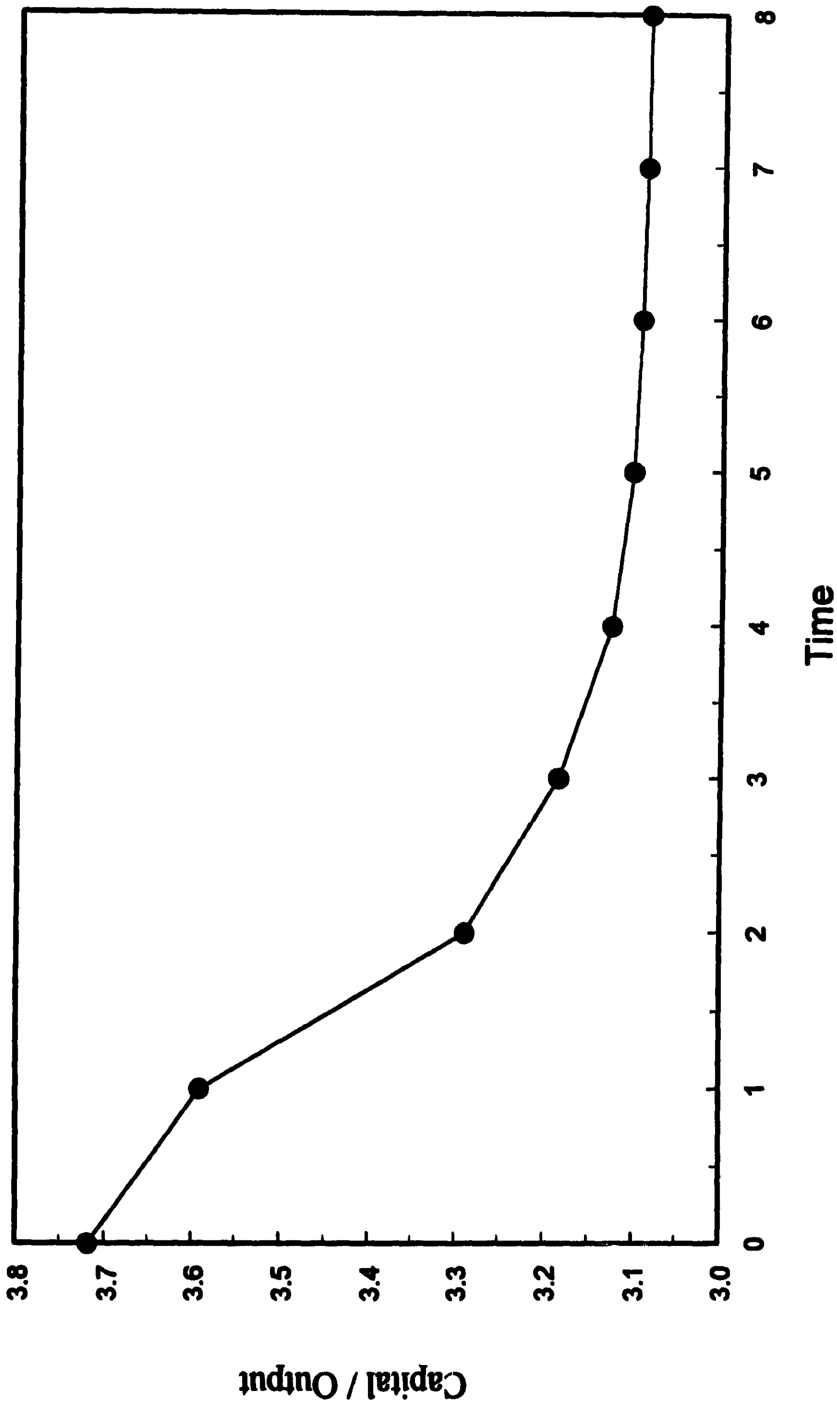


Figure 5: Level of the Real Interest Rate Along the Simulation Path

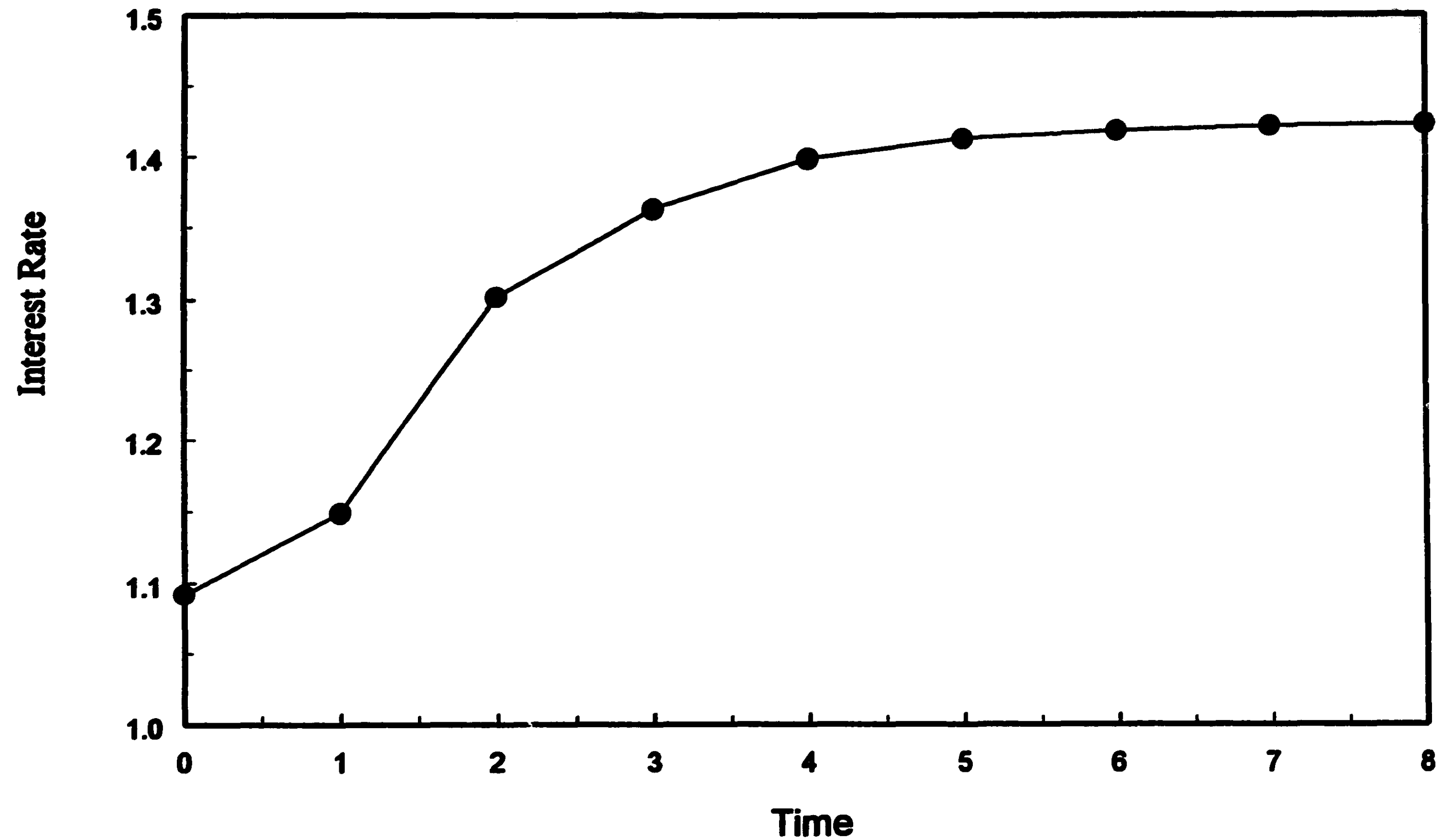


Figure 6: Level of Wage per Efficiency Units Along the Simulation Path

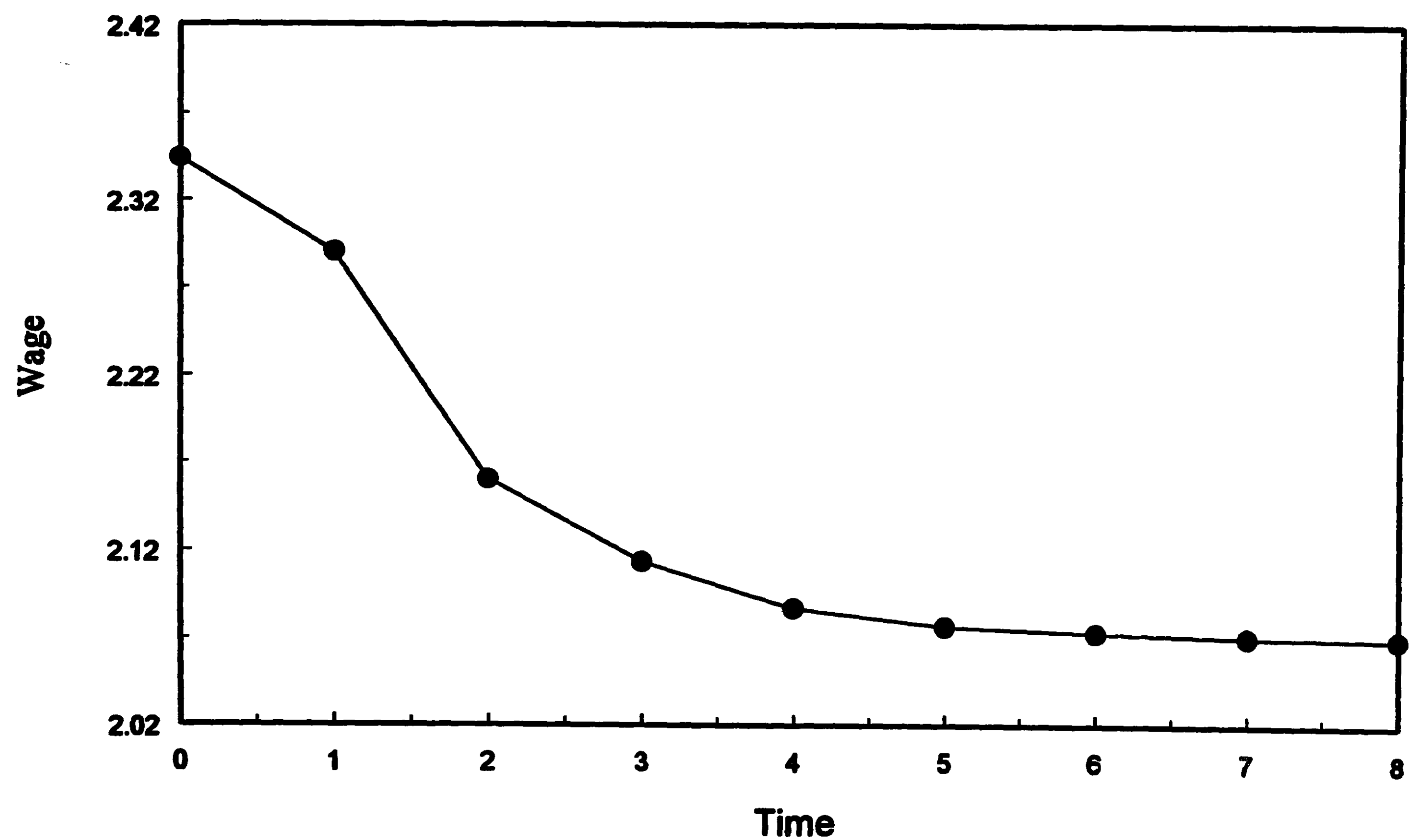
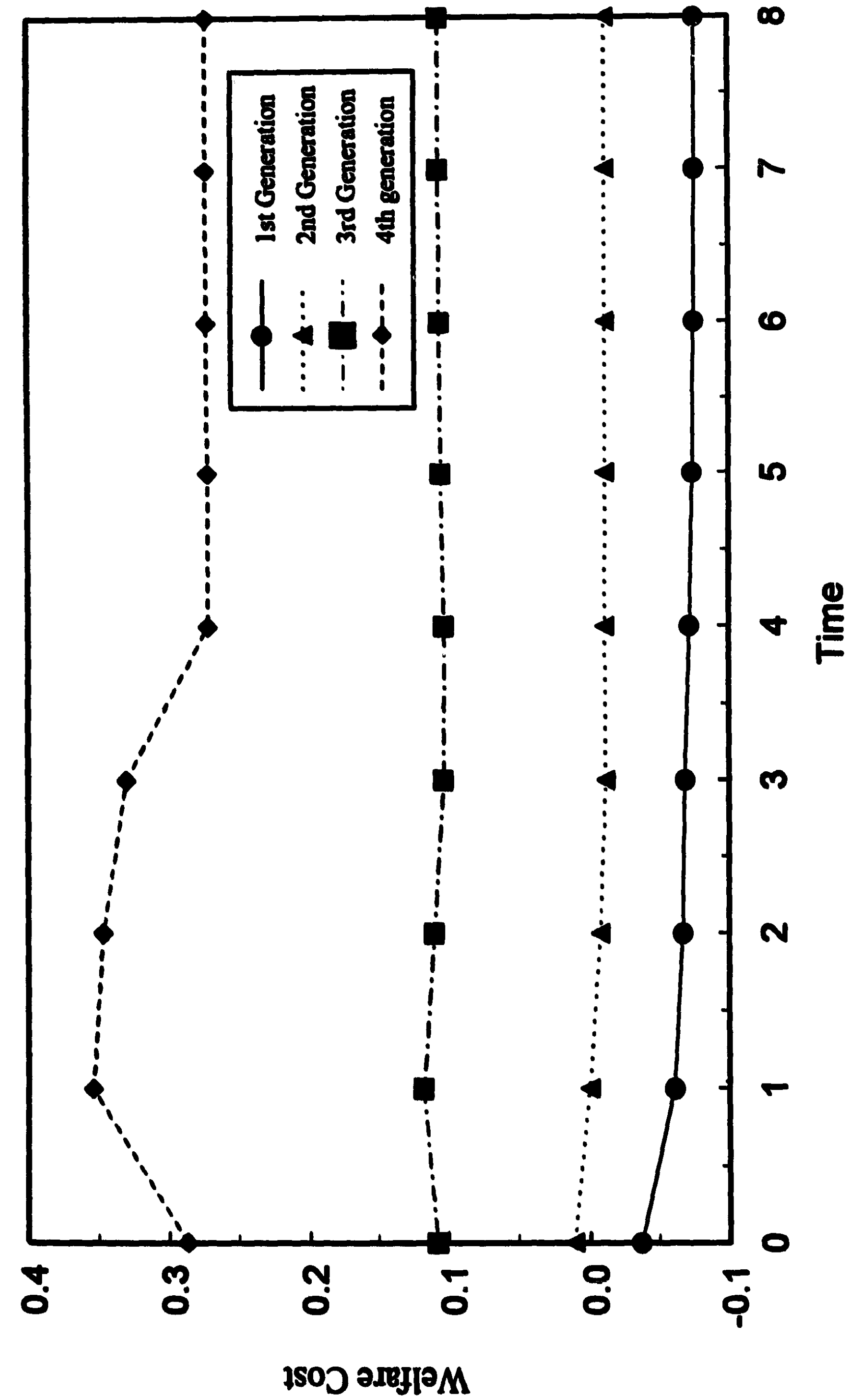


Figure 7: Level of the Welfare Costs per Output ALong the Path



In our economy, the realized dynamics of the post war baby boom would cause the implemented social-security system to collapse after only two periods. Even though it is sustained after the first big unexpected change in the age distribution of the population, they would choose to privatize the system in the third period. The prolonged increase in the size of the older generation is the fundamental cause of the abandonment of social security. If the system were sustained in the third period, it would have collapsed in the next period where the shares of the youngest generations return to levels close to the initial ones.

Figure 8 shows the impact that privatizing the social-security system would have on the capital/output ratio along the path as the economy returns to a steady state without social security. Figure 9 shows the corresponding path of the real interest rate. The capital/output ratio recovers to presocial-security levels within two generations and eventually reaches a higher level because of the change in the relative shares of the generations. The return on capital falls sharply with the abandonment of the social-security system.

The conclusions just described are based on the assumption that voters could simply abandon the social-security system in favor of a fully private system at any point in time. This would involve an abandonment of their obligations to the current generation of retirees. Since social security as we have described it is simply a tax and transfer system, there is nothing to preclude this. If, however, the claims of the current generation of retirees are viewed as an entitlement that must be honored, the conclusion is quite different; the system would not collapse. Figure 10 shows the present value of social security for each generation alive when the consequences of the baby boom become known. It also shows welfare calculation assuming the existing obligations must be honored and assuming an instant collapse of the system. It is obvious that the prospect of having to pay for one more generation of retirees makes the consequences for the (2nd generation) median voters very different.

We considered several smoother transition policies that involve gradual shrinking of the social-security system. We allowed for the possibility that these transitions could be financed by taxes on labor income or capital income and that the social-security system be phased out over two, three, or four periods. None of the transition policies we have considered would have been implementable – that is, they would not have been chosen by the median voters. The voters always prefer to abandon social security abruptly.

Figure 8: Level of the Capital-Output Ratio Along the Simulation Path With Abandonment of the Social Security System

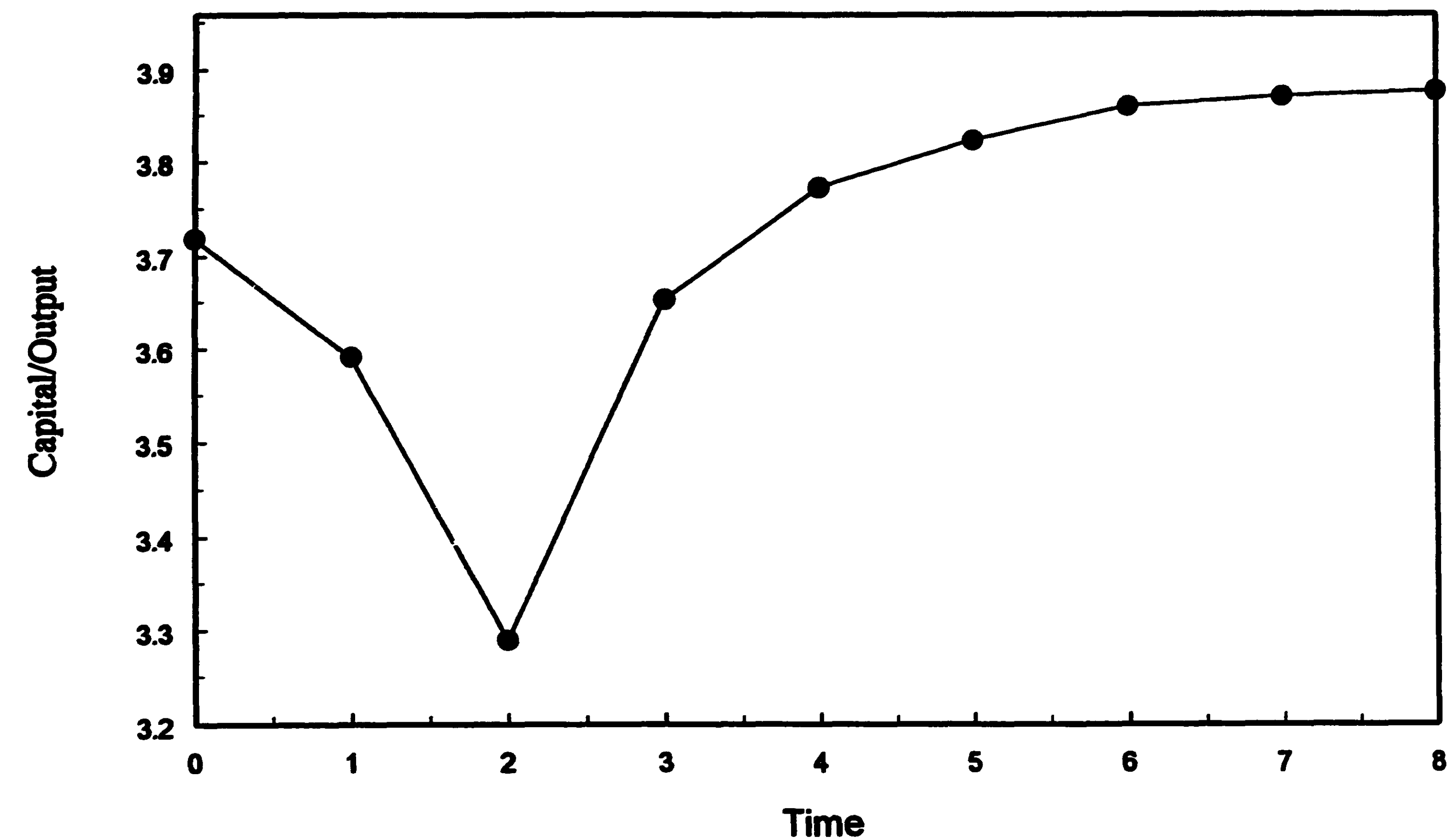


Figure 9: Level of the Real Interest Rate Along the Simulation Path With Abandonment of the Social Security System

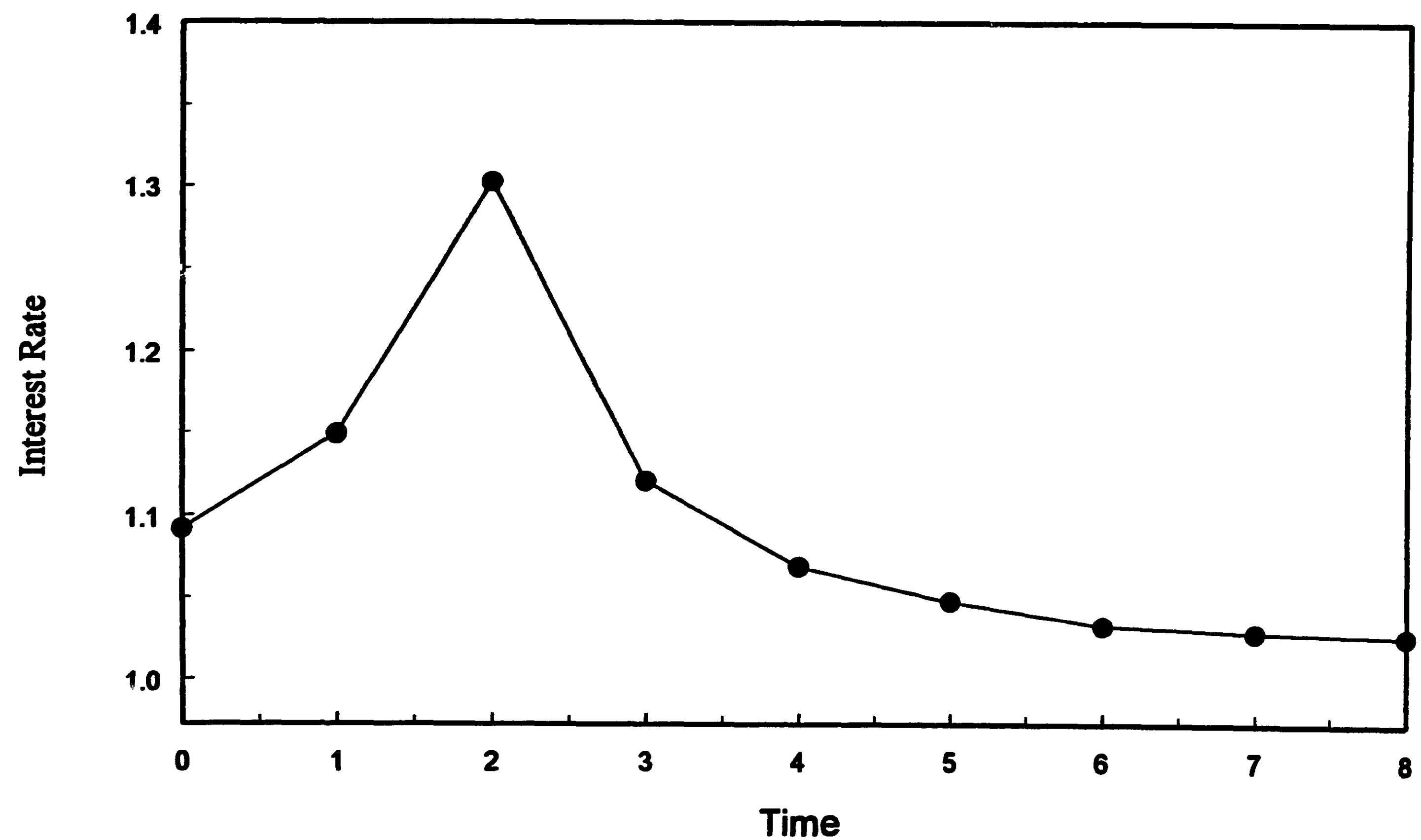
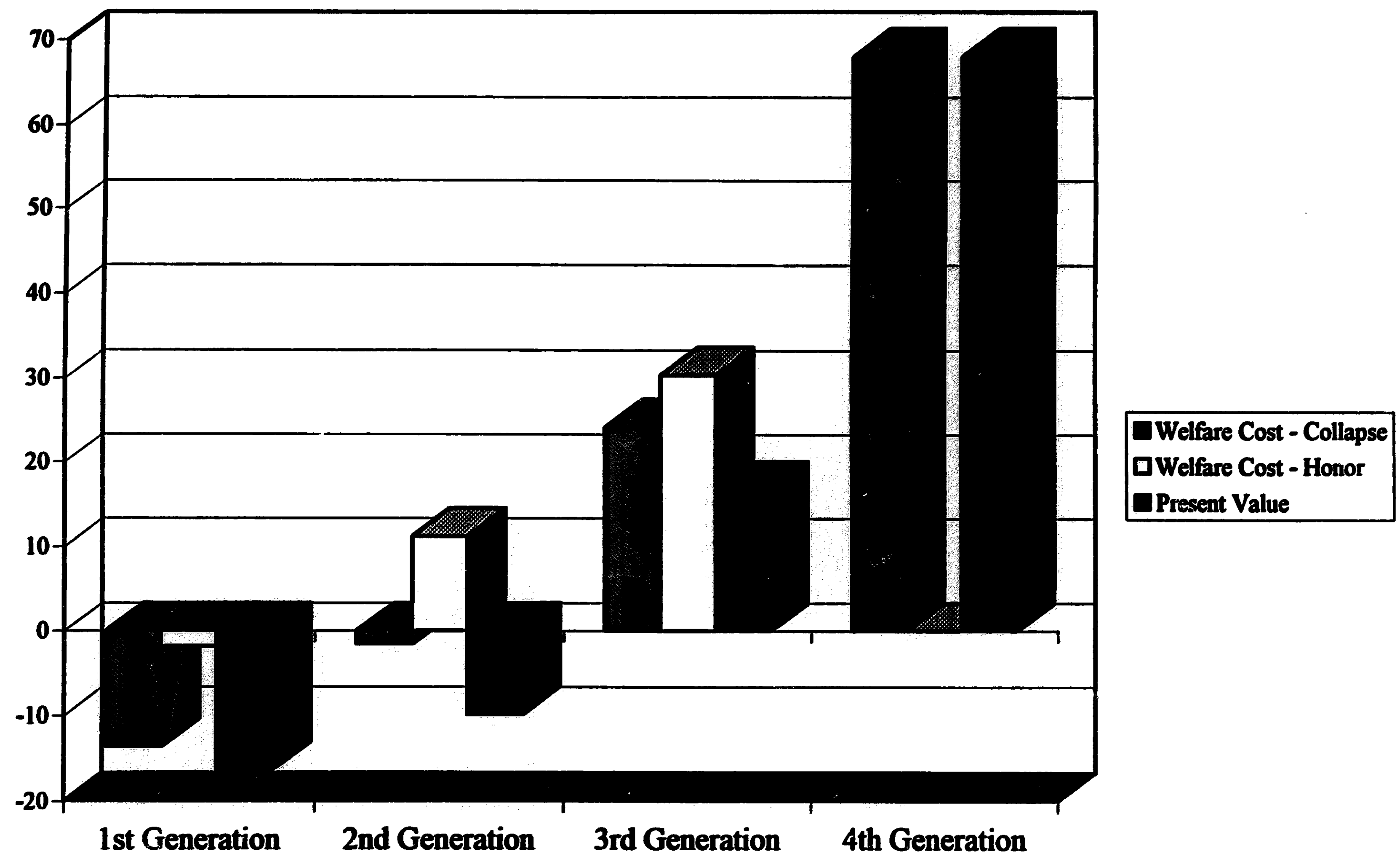


Figure 10: Welfare Cost



6 Concluding remarks

A pay-as-you-go social-security system is an intergenerational social contract where generations of workers are rewarded for supporting the retired generation with the implicit promise that the system will also support them on retirement. We have addressed the issue of the sustainability of such a system in a world with stochastic population growth. Our results suggest that such a system is not sustainable given the population dynamics in the United States. Our model predicts that majority-rule voters would already have abandoned social security. If social security is viewed as an entitlement for those who have already contributed over their working lives, the system will not be abandoned.

The model economy we studied abstracts from many important issues that affect the viability of social insurance. In particular we do not address endogenous labor supply or retirement decisions, nor do we allow for bequests by the oldest generation. All of these will affect equilibrium outcomes in important ways. Instead, we focus on the endogenous *choices* that agents make through the democratic process. We do this to understand what kinds of choices rational forward-looking voters would make and whether they would choose to sustain the system given the dynamics of population growth. This seems important to us as a background against which to evaluate proposals to reform the social-security system. Many critics of the current system cite the reforms carried out in Chile as an example to be followed. The Chilean reforms, while successful, were not the result of a democratic process. In evaluating reforms for the U.S., it seems important to understand whether they would be both implementable and sustainable in a democratic system.

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