NYU STERN
NEW YORK UNIVERSITY LEONARD N. STERN SCHOOL OF BUSINESS

Department of Economics

Econometric Analysis of Panel Data

Assignment 6
Nonlinear Models for Panel Data

Part I. A Concentrated Log Likelihood

Consider an exponential regression model with fixed effects, The density is

$f(y_{it}|\mathbf{x}_{it}) = [1/\theta_{it}] \exp(-y_{it} / \theta_{it})$, $y_{it} \geq 0$, where $\theta_{it} = \exp(\alpha_i + \mathbf{x}_{it}'\boldsymbol{\beta})$, $i = 1,\ldots,n$; $t = 1,\ldots,T$.

It will prove convenient to let $\gamma_i = \exp(\alpha_i)$ so $\theta_{it} = \gamma_i \exp(\mathbf{x}_{it}'\boldsymbol{\beta}) = \gamma_i \lambda_{it}$.
The log likelihood for this exponential regression model with fixed effects is

$$\log L = \sum_{i=1}^{n} \sum_{t=1}^{T} (-\log \theta_{it} - y_{it} / \theta_{it})$$

(a) Obtain the first order condition for maximizing $\log L$ with respect to γ_i. Note, there is one of these for each i, so you need only differentiate

$$\log L_i = \sum_{t=1}^{T} (-\log \theta_{it} - y_{it} / \theta_{it})$$

with respect to γ_i and equate it to zero. You will gain some convenience by defining $a_{it} = y_{it}/\lambda_{it}$.

(b) Now, treating $\boldsymbol{\beta}$ as if it were known, show that the implicit solution of this likelihood equation for γ_i in terms of $\boldsymbol{\beta}$ is

$$\gamma_i = \left(\frac{\sum_{t=1}^{T} \left[y_{it} / \exp(\mathbf{x}_{it}'\boldsymbol{\beta}) \right]}{T} \right) = \frac{\sum_{t=1}^{T} a_{it}}{T} = \bar{a}_i = \gamma_i(\boldsymbol{\beta})$$

(c) It follows that at the solution for the MLE, it will be true that $\gamma_i(\boldsymbol{\beta}) = \bar{a}_i$ where $\bar{a}_i$ is the sample mean of a_{it}. Denote $\theta_{it}^c = \bar{a}_i \lambda_{it}$. Insert this solution back into the log likelihood function, to obtain the *concentrated log likelihood function*

$$\log L^c = \sum_{i=1}^{n} \sum_{t=1}^{T} (-\log \theta_{it}^c - y_{it} / \theta_{it}^c).$$

Note that this is a function of $\boldsymbol{\beta}$ but not of γ_i. To obtain the maximum likelihood estimator of $\boldsymbol{\beta}$, we can now maximize this function with respect to $\boldsymbol{\beta}$. This is equivalent to maximizing the whole log likelihood function, while considering only the solutions for γ_i that satisfy $\gamma_i = \gamma_i(\boldsymbol{\beta})$ as shown above. When we find $\boldsymbol{\beta}$, we can then compute γ_i. (No assignment for this part.)

(d) With this in hand, it is now possible to maximize the function with respect to $\boldsymbol{\beta}$. Show that the likelihood equation will be

$$\frac{\partial \log L^c}{\partial \boldsymbol{\beta}} = \sum_{i=1}^{n} \sum_{t=1}^{T} \left(-\frac{1}{\theta_{it}^c} + \frac{y_{it}}{(\theta_{it}^c)^2} \right) \frac{\partial \theta_{it}^c}{\partial \boldsymbol{\beta}} = \sum_{i=1}^{n} \sum_{t=1}^{T} \frac{1}{\theta_{it}^c} \left(\frac{y_{it}}{\theta_{it}^c} - 1 \right) \frac{\partial \theta_{it}^c}{\partial \boldsymbol{\beta}}.$$

You now need the derivative, $\partial \theta_{it}{}^c / \partial \boldsymbol{\beta}$. Continuing, show that

$$\partial \theta_{it}{}^c / \partial \boldsymbol{\beta} = \bar{a}_i \lambda_{it} \mathbf{x}_{it} - \frac{1}{T} \sum_{t=1}^{T} a_{it} \lambda_{it} \mathbf{x}_{it}$$ Hint: $\partial \lambda_{it} / \partial \boldsymbol{\beta} = \lambda_{it} \mathbf{x}_{it}$. Insert your result in the log likelihood equation to obtain the implicit solution for $\boldsymbol{\beta}$,

$$\frac{\partial \log L^c}{\partial \boldsymbol{\beta}} = \sum_{i=1}^{n} \sum_{t=1}^{T} \frac{1}{\theta_{it}^c} \left(\frac{y_{it}}{\theta_{it}^c} - 1 \right) \left[\lambda_{it} \bar{a}_i \mathbf{x}_{it} - \frac{1}{T} \Sigma_{t=1}^{T} \lambda_{it} a_{it} \mathbf{x}_{it} \right] = \mathbf{0}.$$

Part II. Solving for FE in Panel Probit

For the binary fixed effects panel probit model,

$\text{Prob}(y_{it} = 1 \mid \mathbf{x}_{it}) = \Phi(\alpha_i + \mathbf{x}_{it}'\boldsymbol{\beta}),$
$\text{Prob}(y_{it} = 0 \mid \mathbf{x}_{it}) = 1 - \text{Prob}(y_{it} = 1 \mid \mathbf{x}_{it}) = \Phi(-\alpha_i - \mathbf{x}_{it}'\boldsymbol{\beta}).$

a. Write out the full log likelihood function.

b. Write out the first order condition for maximizng the function with respect to α_i, taking $\boldsymbol{\beta}$ as known. Take this derivation as far as possible – you will ultimately find that unlike the exponential model we examined earlier, in this model, there is no explicit solution for α_i in terms of $\boldsymbol{\beta}$ and the data.

c. Show that regardless of the finding in b, there is no solution for α_i when y_{it} is always 1 or always 0 within a given group (i).

Part III. The Incidental Parameters Problem.

This is a purely empirical exercise. It will involve some computations using the German health care data.

As we discussed in class, for the binary logit model, there are two estimators for the fixed effects model

$$\text{Prob}(y_{it} = 1 \mid x_{it}) = \Lambda(\alpha_i + \beta'x_{it}), i = 1,...,n, t = 1,...,T.$$

The 'brute force,' unconditional approach maximizes the whole log likelihood for $\alpha_i, i=1,...,n$ and β. This estimator is known to suffer from the 'incidental parameters problem;' when T is small, the estimator is biased away from zero. The best known result is that when T = 2, there is a 100% bias. The other approach is the Rasch/Chamberlain method, which computes a conditional MLE using the probabilities conditioned on the sum of the y_{it}s for each group. This estimator is known to be consistent. For this exercise, we will see if the effect is visible in a sample, using precisely the estimators described.

a. We first see if we can observe Hsiao/Abrevaya's finding when T = 2. The following commands compute the estimates of the logit model both ways. Estimate the equations, and report your results. Do the empirical results seem to conform to the theory?

```
Setpanel ; Group = id ; Pds = ti $
Namelist ; x = hhninc,age,married,working $
Proc = FELogit $
Logit ; if[ti = T] ; Lhs = doctor ; Rhs = x ; Panel ; Fixed
      ; Table = Uncond $
Logit ; if[ti = T] ; Lhs = doctor ; Rhs = x ; Panel
      ; Table = Cond $
MakeTable ; Uncond,Cond $
EndProc $
Exec  ; Proc=FELogit ; T=2 $
```

b. A second result that seems intuitively reasonable is that the IP bias diminishes as T increases. Is this the case? Change the 2 in the Exec command above to 3 and redo the experiment. What do you find? Now, change the 2 to 7 and repeat the experiment. In each case, report your findings and your conclusions. (Tip: You could use **Exec;Proc=FELogit;T=2,7$** to do all 6 experiments with one instruction. The syntax T=2,7 means T=2,3,4,5,6,7, not T = 2 and 7.)

c. What do you conclude about the fixed effects estimators?

Part IV. A Common Effects Probit Model

In this exercise, you will fit a probit model with common effects, and develop the appropriate model based on your findings. The probit model we will use is

$$\text{Prob}(y_{it} = 1 \mid x_{it}) = \Phi(c_i + \beta'x_{it})$$
$y_{it} = \text{Public}_{it} = $ whether or not the individual chose public health insurance in that year.
$$x_{it} = \textbf{one,age,educ,hhninc,handper,working,hsat}$$

1. Suppose, for the moment, we ignore the heterogeneity, c_i and just pool the data and fit a simple probit model. Is the estimator consistent? What assumptions are necessary for the pooled estimator to be a consistent estimator of β?

2. All of the suggested covariates in the model are time varying. Fit a random effects model and a fixed effects model (this can only be done by brute force – there is no conditional estimator for the probit model). Report your results.

3. We are interested in deciding which is preferred, fixed or random effects. I propose to use a variable addition test. Add the group means to the model, then carry out a likelihood ratio test of the hypothesis that the coefficients on the group means are all zero. What do you find? What do you conclude is the preferred model?

Hint: These commands can be used for parts 1. – 3.

```
Setpanel ; Group=id ; Pds=ti $ $
Namelist ; X = age,educ,hhninc,handper,working,newhsat $
Probit ; Lhs = public
       ; Rhs = one,X ; Panel ; Random ; HPT = 8$
Probit ; Lhs = public
       ; Rhs = one,X ; Panel ; FEM $
Probit ; Lhs = public
       ; Rhs = one,X,Gmn(X)  ; Panel; Random ; HPT = 8$
```

4. Suppose it were hypothesized that the previous year's choice of whether or not to choose public insurance were on the right hand side of the equation. That is,

$$\text{Prob}(y_{it} = 1 \mid x_{it}) = \Phi(c_i + \beta'x_{it} + \gamma y_{i,t-1})$$

What would this imply for how one (you) should go about estimating the parameters of the model. What issues should you be concerned with for a dynamic model?