

Discounted Cash Flow Valuation

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Discounted Cashflow Valuation: Basis for Approach

$$\text{Value} = \sum_{t=1}^{t=n} \frac{CF_t}{(1+r)^t}$$

where CF_t is the cash flow in period t , r is the discount rate appropriate given the riskiness of the cash flow and t is the life of the asset.

Proposition 1: For an asset to have value, the expected cash flows have to be positive some time over the life of the asset.

Proposition 2: Assets that generate cash flows early in their life will be worth more than assets that generate cash flows later; the latter may however have greater growth and higher cash flows to compensate.

Equity Valuation versus Firm Valuation

- Value just the equity stake in the business
- Value the entire business, which includes, besides equity, the other claimholders in the firm

I. Equity Valuation

- The value of equity is obtained by discounting expected cashflows to equity, i.e., the residual cashflows after meeting all expenses, tax obligations and interest and principal payments, at the cost of equity, i.e., the rate of return required by equity investors in the firm.

$$\text{Value of Equity} = \sum_{t=1}^{t=n} \frac{\text{CF to Equity}_t}{(1 + k_e)^t}$$

where,

CF to Equity_t = Expected Cashflow to Equity in period t

k_e = Cost of Equity

- The dividend discount model is a specialized case of equity valuation, and the value of a stock is the present value of expected future dividends.

II. Firm Valuation

- The value of the firm is obtained by discounting expected cashflows to the firm, i.e., the residual cashflows after meeting all operating expenses and taxes, but prior to debt payments, at the weighted average cost of capital, which is the cost of the different components of financing used by the firm, weighted by their market value proportions.

$$\text{Value of Firm} = \sum_{t=1}^{t=n} \frac{\text{CF to Firm}_t}{(1 + \text{WACC})^t}$$

where,

CF to Firm_t = Expected Cashflow to Firm in period t

WACC = Weighted Average Cost of Capital

Firm Value and Equity Value

- To get from firm value to equity value, which of the following would you need to do?
 - ❑ Subtract out the value of long term debt
 - ❑ Subtract out the value of all debt
 - ❑ Subtract the value of all non-equity claims in the firm, that are included in the cost of capital calculation
 - ❑ Subtract out the value of all non-equity claims in the firm
- Doing so, will give you a value for the equity which is
 - ❑ greater than the value you would have got in an equity valuation
 - ❑ lesser than the value you would have got in an equity valuation
 - ❑ equal to the value you would have got in an equity valuation

Cash Flows and Discount Rates

- Assume that you are analyzing a company with the following cashflows for the next five years.

Year	CF to Equity	Int Exp (1-t)	CF to Firm
1	\$ 50	\$ 40	\$ 90
2	\$ 60	\$ 40	\$ 100
3	\$ 68	\$ 40	\$ 108
4	\$ 76.2	\$ 40	\$ 116.2
5	\$ 83.49	\$ 40	\$ 123.49
Terminal Value	\$ 1603.0		\$ 2363.008

- Assume also that the cost of equity is 13.625% and the firm can borrow long term at 10%. (The tax rate for the firm is 50%.)
- The current market value of equity is \$1,073 and the value of debt outstanding is \$800.

Equity versus Firm Valuation

Method 1: Discount CF to Equity at Cost of Equity to get value of equity

- Cost of Equity = 13.625%
- PV of Equity = $50/1.13625 + 60/1.13625^2 + 68/1.13625^3 + 76.2/1.13625^4 + (83.49+1603)/1.13625^5 = \1073

Method 2: Discount CF to Firm at Cost of Capital to get value of firm

Cost of Debt = Pre-tax rate (1- tax rate) = 10% (1-.5) = 5%

WACC = $13.625\% (1073/1873) + 5\% (800/1873) = 9.94\%$

PV of Firm = $90/1.0994 + 100/1.0994^2 + 108/1.0994^3 + 116.2/1.0994^4 + (123.49+2363)/1.0994^5 = \1873

- PV of Equity = PV of Firm - Market Value of Debt
= \$ 1873 - \$ 800 = \$1073

First Principle of Valuation

- Never mix and match cash flows and discount rates.
- The key error to avoid is mismatching cashflows and discount rates, since discounting cashflows to equity at the weighted average cost of capital will lead to an upwardly biased estimate of the value of equity, while discounting cashflows to the firm at the cost of equity will yield a downward biased estimate of the value of the firm.

The Effects of Mismatching Cash Flows and Discount Rates

Error 1: Discount CF to Equity at Cost of Capital to get equity value

$$\text{PV of Equity} = 50/1.0994 + 60/1.0994^2 + 68/1.0994^3 + 76.2/1.0994^4 + (83.49+1603)/1.0994^5 = \$1248$$

Value of equity is overstated by \$175.

Error 2: Discount CF to Firm at Cost of Equity to get firm value

$$\text{PV of Firm} = 90/1.13625 + 100/1.13625^2 + 108/1.13625^3 + 116.2/1.13625^4 + (123.49+2363)/1.13625^5 = \$1613$$

$$\text{PV of Equity} = \$1612.86 - \$800 = \$813$$

Value of Equity is understated by \$ 260.

Error 3: Discount CF to Firm at Cost of Equity, forget to subtract out debt, and get too high a value for equity

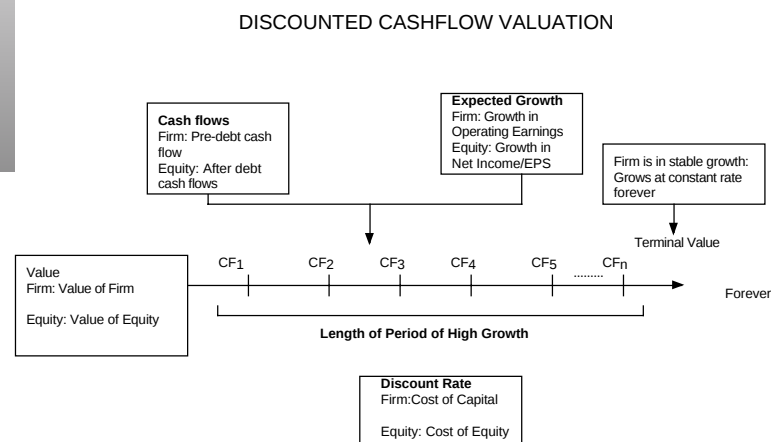
$$\text{Value of Equity} = \$1613$$

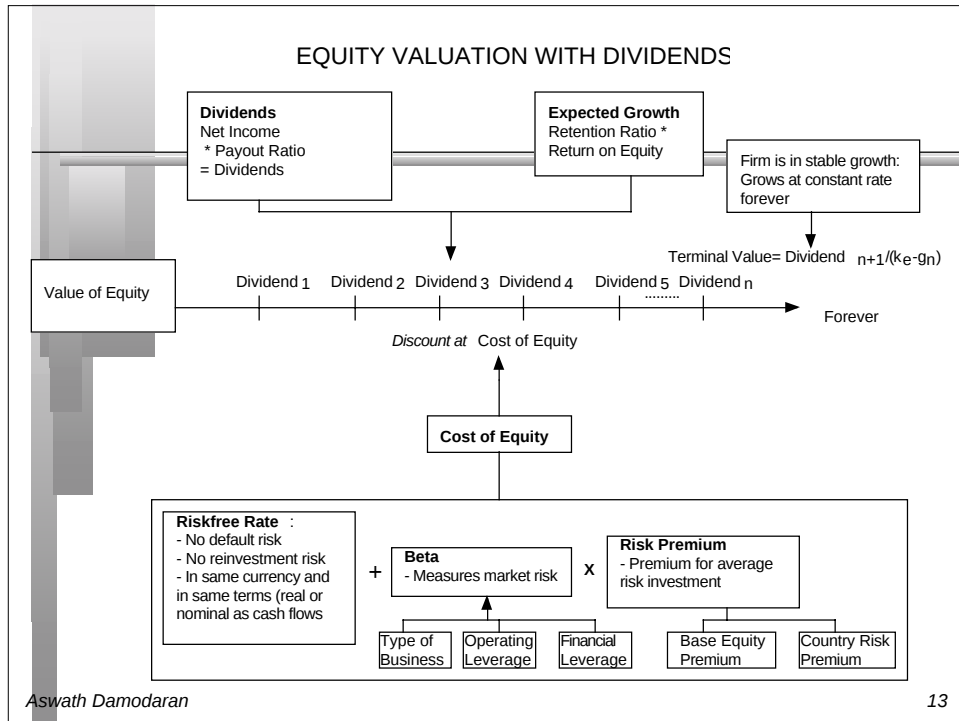
Value of Equity is overstated by \$ 540

Discounted Cash Flow Valuation: The Steps

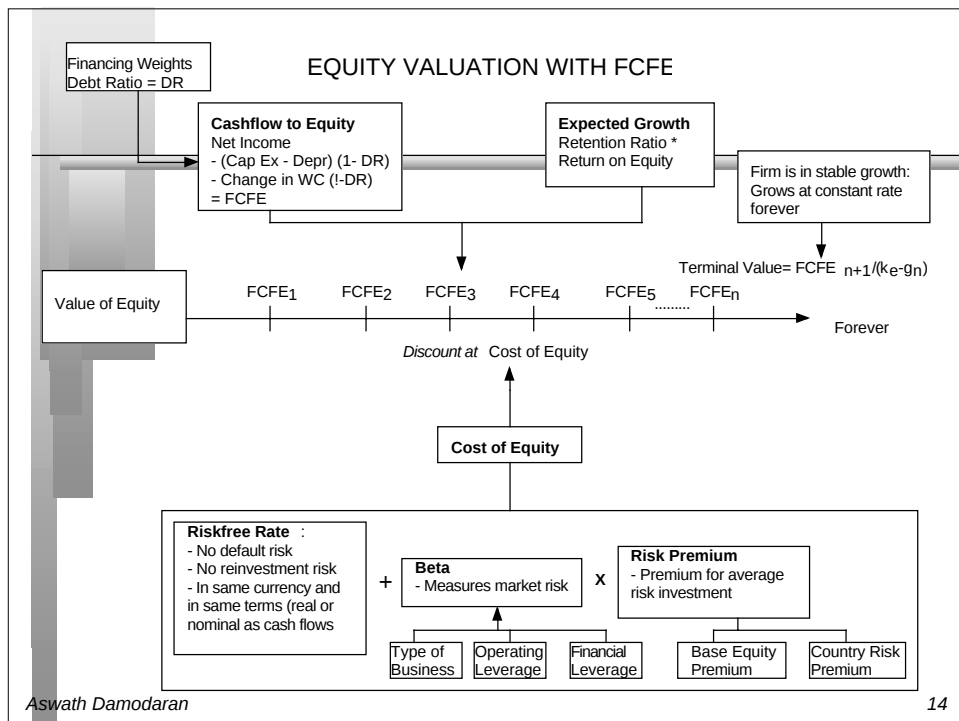
- Estimate the **discount rate** or rates to use in the valuation
 - Discount rate can be either a cost of equity (if doing equity valuation) or a cost of capital (if valuing the firm)
 - Discount rate can be in nominal terms or real terms, depending upon whether the cash flows are nominal or real
 - Discount rate can vary across time.
- Estimate the **current earnings** and **cash flows** on the asset, to either equity investors (CF to Equity) or to all claimholders (CF to Firm)
- Estimate the **future earnings and cash flows** on the firm being valued, generally by estimating an expected growth rate in earnings.
- Estimate **when** the firm will reach “**stable growth**” and what characteristics (risk & cash flow) it will have when it does.
- Choose the **right DCF model** for this asset and value it.

Generic DCF Valuation Model

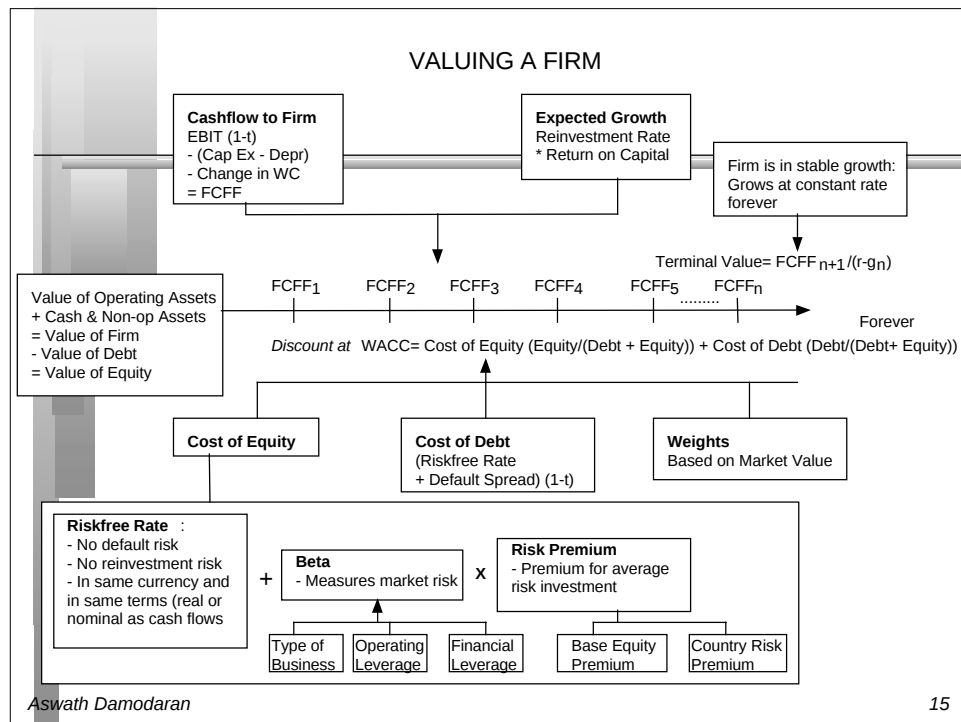




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Discounted Cash Flow Valuation: The Inputs

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I. Estimating Discount Rates

DCF Valuation

Estimating Inputs: Discount Rates

- **Critical ingredient** in discounted cashflow valuation. Errors in estimating the discount rate or mismatching cashflows and discount rates can lead to serious errors in valuation.
- At an intuitive level, the discount rate used should be consistent with both the **riskiness** and the **type of cashflow** being discounted.
 - Equity versus Firm: If the cash flows being discounted are cash flows to equity, the appropriate discount rate is a cost of equity. If the cash flows are cash flows to the firm, the appropriate discount rate is the cost of capital.
 - Currency: The currency in which the cash flows are estimated should also be the currency in which the discount rate is estimated.
 - Nominal versus Real: If the cash flows being discounted are nominal cash flows (i.e., reflect expected inflation), the discount rate should be nominal

Cost of Equity

- The cost of equity should be higher for riskier investments and lower for safer investments
- While risk is usually defined in terms of the variance of actual returns around an expected return, risk and return models in finance assume that the risk that should be rewarded (and thus built into the discount rate) in valuation should be the risk perceived by the marginal investor in the investment
- Most risk and return models in finance also assume that the marginal investor is well diversified, and that the only risk that he or she perceives in an investment is risk that cannot be diversified away (I.e, market or non-diversifiable risk)

The Cost of Equity: Competing Models

<i>Model</i>	<i>Expected Return</i>	<i>Inputs Needed</i>
CAPM	$E(R) = R_f + \beta (R_m - R_f)$	Riskfree Rate Beta relative to market portfolio Market Risk Premium
APM	$E(R) = R_f + \sum_{j=1} \beta_j (R_j - R_f)$	Riskfree Rate; # of Factors; Betas relative to each factor Factor risk premiums
Multi factor	$E(R) = R_f + \sum_{j=1..N} \beta_j (R_j - R_f)$	Riskfree Rate; Macro factors Betas relative to macro factors Macro economic risk premiums
Proxy	$E(R) = a + \sum_{j=1..N} b_j Y_j$	Proxies Regression coefficients

The CAPM: Cost of Equity

- Consider the standard approach to estimating cost of equity:
$$\text{Cost of Equity} = R_f + \text{Equity Beta} * (E(R_m) - R_f)$$
where,
 R_f = Riskfree rate
 $E(R_m)$ = Expected Return on the Market Index (Diversified Portfolio)
- In practice,
 - Short term government security rates are used as risk free rates
 - Historical risk premiums are used for the risk premium
 - Betas are estimated by regressing stock returns against market returns

Short term Governments are not riskfree

- On a riskfree asset, the actual return is equal to the expected return. Therefore, there is no variance around the expected return.
- For an investment to be riskfree, then, it has to have
 - No default risk
 - No reinvestment risk
- Thus, the riskfree rates in valuation will depend upon when the cash flow is expected to occur and will vary across time
- A simpler approach is to match the duration of the analysis (generally long term) to the duration of the riskfree rate (also long term)
- In emerging markets, there are two problems:
 - The government might not be viewed as riskfree (Brazil, Indonesia)
 - There might be no market-based long term government rate (China)

Estimating a Riskfree Rate

- Estimate a range for the riskfree rate in local terms:
 - Approach 1: Subtract default spread from local government bond rate:
Government bond rate in local currency terms - Default spread for Government in local currency
 - Approach 2: Use forward rates and the riskless rate in an index currency (say Euros or dollars) to estimate the riskless rate in the local currency.
- Do the analysis in real terms (rather than nominal terms) using a real riskfree rate, which can be obtained in one of two ways –
 - from an inflation-indexed government bond, if one exists
 - set equal, approximately, to the long term real growth rate of the economy in which the valuation is being done.
- Do the analysis in another more stable currency, say US dollars.

A Simple Test

- You are valuing Ambev, a Brazilian company, in U.S. dollars and are attempting to estimate a riskfree rate to use in the analysis. The riskfree rate that you should use is
 - ❑ The interest rate on a Brazilian Real denominated long term Government bond
 - ❑ The interest rate on a US \$ denominated Brazilian long term bond (C-Bond)
 - ❑ The interest rate on a US \$ denominated Brazilian Brady bond (which is partially backed by the US Government)
 - ❑ The interest rate on a US treasury bond

Everyone uses historical premiums, but..

- The historical premium is the premium that stocks have historically earned over riskless securities.
- Practitioners never seem to agree on the premium; it is sensitive to
 - How far back you go in history...
 - Whether you use T.bill rates or T.Bond rates
 - Whether you use geometric or arithmetic averages.
- For instance, looking at the US:

Historical Period	<i>Arithmetic average</i>		<i>Geometric Average</i>	
	Stocks -	Stocks -	Stocks -	Stocks -
	T.Bills	T.Bonds	T.Bills	T.Bonds
1928-2002	7.67%	6.25%	5.73%	4.53%
1962-2002	5.17%	3.66%	3.90%	2.76%
1992-2002	6.32%	2.15%	4.69%	0.95%

If you choose to use historical premiums....

- Go back as far as you can. A risk premium comes with a standard error. Given the annual standard deviation in stock prices is about 25%, the standard error in a historical premium estimated over 25 years is roughly:

$$\text{Standard Error in Premium} = 25\%/\sqrt{25} = 25\%/5 = 5\%$$

- Be consistent in your use of the riskfree rate. Since we argued for long term bond rates, the premium should be the one over T.Bonds
- Use the geometric risk premium. It is closer to how investors think about risk premiums over long periods.

Country Risk Premiums

- Historical risk premiums are almost impossible to estimate with any precision in markets with limited history - this is true not just of emerging markets but also of many Western European markets.
- For such markets, we can estimate a modified historical premium beginning with the U.S. premium as the base:
 - *Relative Equity Market approach*: The country risk premium is based upon the volatility of the market in question relative to U.S. market.

$$\text{Country risk premium} = \text{Risk Premium}_{\text{US}} * \sigma_{\text{Country Equity}} / \sigma_{\text{US Equity}}$$
 - *Country Bond approach*: In this approach, the country risk premium is based upon the default spread of the bond issued by the country.

$$\text{Country risk premium} = \text{Risk Premium}_{\text{US}} + \text{Country bond default spread}$$
 - *Combined approach*: In this approach, the country risk premium incorporates both the country bond spread and equity market volatility.

Step 1: Assessing Country Risk Using Country Ratings: Latin America - March 2001

Country	Rating	Typical Spread	Market Spread
Argentina	B1	450	563
Bolivia	B1	450	551
Brazil	B1	450	537
Colombia	Ba2	300	331
Ecuador	Caa2	750	787
Guatemala	Ba2	300	361
Honduras	B2	550	581
Mexico	Baa3	145	235
Paraguay	B2	550	601
Peru	Ba3	400	455
Uruguay	Baa3	145	193
Venezuela	B2	550	631

Step 2: From Bond Default Spreads to Equity Spreads

- Country ratings measure default risk. While default risk premiums and equity risk premiums are highly correlated, one would expect equity spreads to be higher than debt spreads.
 - One way to adjust the country spread upwards is to use information from the US market. In the US, the equity risk premium has been roughly twice the default spread on junk bonds.
 - Another is to multiply the bond spread by the relative volatility of stock and bond prices in that market. For example,
 - Standard Deviation in Bovespa (Equity) = 32.6%
 - Standard Deviation in Brazil C-Bond = 17.1%
 - Adjusted Equity Spread = $5.37\% (32.6/17.1\%) = 10.24\%$
- Ratings agencies make mistakes. They are often late in recognizing and building in risk.

Another Example: Assessing Country Risk Using Currency Ratings: Western Europe

Country	Rating	Typical Spread	Actual Spread
Austria	Aaa	0	
Belgium	Aaa	0	
Denmark	Aaa	0	
Finland	Aaa	0	
France	Aaa	0	
Germany	Aaa	0	
Greece	A3	95	50
Ireland	AA2	65	35
Italy	Aa3	70	30
Netherlands	Aaa	0	
Norway	Aaa	0	
Portugal	A3	95	55
Spain	Aa1	60	30
Sweden	Aa1	60	25
Switzerland	Aaa	0	

Greek Country Risk Premium

- Country ratings measure default risk. While default risk premiums and equity risk premiums are highly correlated, one would expect equity spreads to be higher than debt spreads.
 - One way to adjust the country spread upwards is to use information from the US market. In the US, the equity risk premium has been roughly twice the default spread on junk bonds.
 - Another is to multiply the bond spread by the relative volatility of stock and bond prices in that market. For example,
 - Standard Deviation in Greek ASE(Equity) = 40.5%
 - Standard Deviation in Greek GDr Bond = 26.1%
 - Adjusted Equity Spread = $0.95\% (40.5\%/26.1\%) = 1.59\%$
- Ratings agencies make mistakes. They are often late in recognizing and building in risk.

From Country Spreads to Corporate Risk premiums

- Approach 1: Assume that every company in the country is equally exposed to country risk. In this case,

$$E(\text{Return}) = \text{Riskfree Rate} + \text{Country Spread} + \text{Beta (US premium)}$$
 Implicitly, this is what you are assuming when you use the local Government's dollar borrowing rate as your riskfree rate.
- Approach 2: Assume that a company's exposure to country risk is similar to its exposure to other market risk.

$$E(\text{Return}) = \text{Riskfree Rate} + \text{Beta (US premium + Country Spread)}$$
- Approach 3: Treat country risk as a separate risk factor and allow firms to have different exposures to country risk (perhaps based upon the proportion of their revenues come from non-domestic sales)

$$E(\text{Return}) = \text{Riskfree Rate} + \beta (\text{US premium}) + \lambda (\text{Country Spread})$$

Estimating Company Exposure to Country Risk

- Different companies should be exposed to different degrees to country risk. For instance, a Brazilian firm that generates the bulk of its revenues in the United States should be less exposed to country risk in Brazil than one that generates all its business within Brazil.
- The factor “ λ ” measures the relative exposure of a firm to country risk. One simplistic solution would be to do the following:

$$\lambda = \% \text{ of revenues domestically}_{\text{firm}} / \% \text{ of revenues domestically}_{\text{avg firm}}$$

For instance, if a firm gets 35% of its revenues domestically while the average firm in that market gets 70% of its revenues domestically

$$\lambda = 35\% / 70\% = 0.5$$
- There are two implications
 - A company’s risk exposure is determined by where it does business and not by where it is located
 - Firms might be able to actively manage their country risk exposures

Estimating E(Return) for Embraer

- Assume that the beta for Embraer is 0.88, and that the riskfree rate used is 4.5%. (Real Riskfree Rate)
- Approach 1: Assume that every company in the country is equally exposed to country risk. In this case,

$$E(\text{Return}) = 4.5\% + 10.24\% + 0.88 (5.51\%) = 19.59\%$$
- Approach 2: Assume that a company’s exposure to country risk is similar to its exposure to other market risk.

$$E(\text{Return}) = 4.5\% + 0.88 (5.51\% + 10.24\%) = 18.36\%$$
- Approach 3: Treat country risk as a separate risk factor and allow firms to have different exposures to country risk (perhaps based upon the proportion of their revenues come from non-domestic sales)

$$E(\text{Return}) = 4.5\% + 0.88(5.51\%) + 0.50 (10.24\%) = 14.47\%$$

Embraer is less exposed to country risk than the typical Brazilian firm since much of its business is overseas.

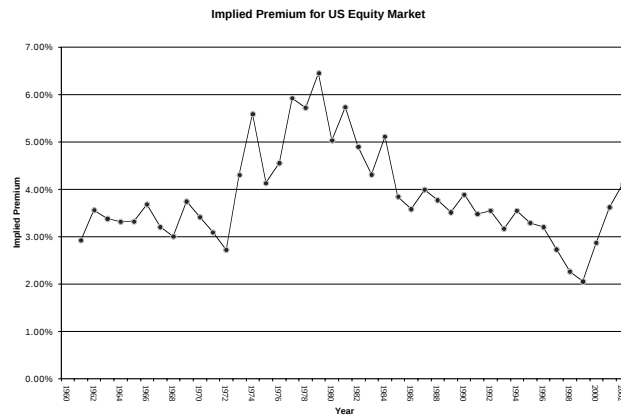
Implied Equity Premiums

- If we use a basic discounted cash flow model, we can estimate the implied risk premium from the current level of stock prices.
- For instance, if stock prices are determined by a variation of the simple Gordon Growth Model:
 - Value = Expected Dividends next year/ (Required Returns on Stocks - Expected Growth Rate)
 - Dividends can be extended to include expected stock buybacks and a high growth period.
 - Plugging in the current level of the index, the dividends on the index and expected growth rate will yield a “implied” expected return on stocks. Subtracting out the riskfree rate will yield the implied premium.
- This model can be extended to allow for two stages of growth - an initial period where the entire market will have earnings growth greater than that of the economy, and then a stable growth period.

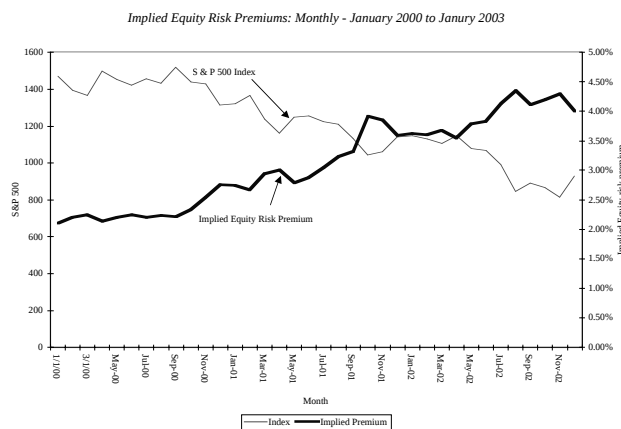
Estimating Implied Premium for U.S. Market: Jan 1, 2003

- Level of the index = 879.82
 - Treasury bond rate = 3.81%
 - Expected Growth rate in earnings (next 5 years) = 8% (Consensus estimate for S&P 500 earnings)
 - Expected growth rate after year 5 = 3.81%
 - Dividends + stock buybacks = 3.29% of index (in latest year)
- | | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 |
|---|---------------------------|---------|---------|---------|---------|
| Expected Dividends = | \$31.25 | \$33.75 | \$36.45 | \$39.37 | \$42.52 |
| + Stock Buybacks | | | | | |
| Expected dividends + buybacks in year 6 = | 42.52 (1.0381) = \$ 44.14 | | | | |
| $879.82 = 31.25/(1+r) + 33.75/(1+r)^2 + 36.45/(1+r)^3 + 39.37/(1+r)^4 + (42.52 + (44.14/(r - 0.0381)))/(1+r)^5$ | | | | | |
| Solving for r, r = 7.91%. (Only way to do this is trial and error) | | | | | |
| Implied risk premium = 7.91% - 3.81% = 4.10% | | | | | |

Implied Premiums: US- 1960 - 2002



Implied Premiums: US- 2000 - 2002



Implied Premium for Brazilian Market: March 1, 2001

- Level of the Index = 16417
- Dividends on the Index = 4.40% of (Used weighted yield)
- Other parameters
 - Riskfree Rate = 4.5% (real riskfree rate)
 - Expected Growth
 - Next 5 years = 13.5% (Used expected real growth rate in Earnings)
 - After year 5 = 4.5% (real growth rate in long term)
- Solving for the expected return:
 - Expected return on Equity = 11.16%
 - Implied Equity premium = $11.16\% - 4.5\% = 6.66\%$

The Effect of Using Implied Equity Premiums on Value

- Embraer's value per share (using historical premium + country risk adjustment) = 11.22 BR
- Embraer's value per share (using implied equity premium of 6.66%) = 20.02 BR
- Embraer's stock price (at the time of the valuation) = 15.25 BR

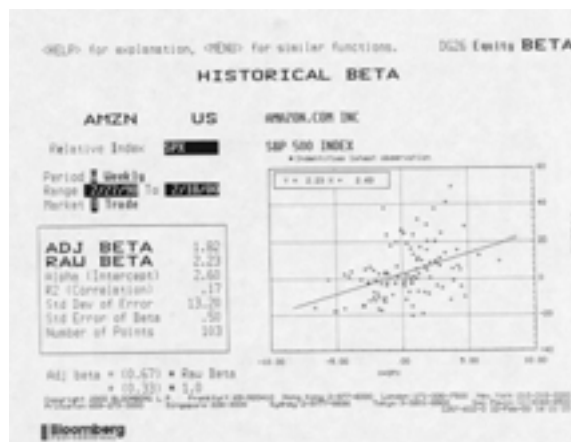
Estimating Beta

- The standard procedure for estimating betas is to regress stock returns (R_j) against market returns (R_m) -

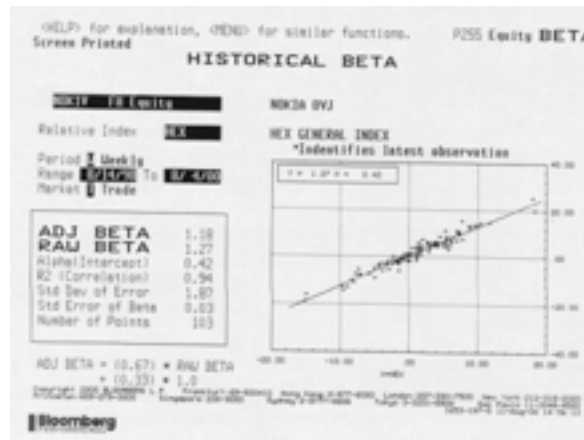
$$R_j = a + b R_m$$

- where a is the intercept and b is the slope of the regression.
- The slope of the regression corresponds to the beta of the stock, and measures the riskiness of the stock.
- This beta has three problems:
 - It has high standard error
 - It reflects the firm's business mix over the period of the regression, not the current mix
 - It reflects the firm's average financial leverage over the period rather than the current leverage.

Beta Estimation: The Noise Problem



Beta Estimation: The Index Effect



Determinants of Betas

- **Product or Service:** The beta value for a firm depends upon the sensitivity of the demand for its products and services and of its costs to macroeconomic factors that affect the overall market.
 - Cyclical companies have higher betas than non-cyclical firms
 - Firms which sell more discretionary products will have higher betas than firms that sell less discretionary products
- **Operating Leverage:** The greater the proportion of fixed costs in the cost structure of a business, the higher the beta will be of that business. This is because higher fixed costs increase your exposure to all risk, including market risk.
- **Financial Leverage:** The more debt a firm takes on, the higher the beta will be of the equity in that business. Debt creates a fixed cost, interest expenses, that increases exposure to market risk.

Equity Betas and Leverage

- The beta of equity alone can be written as a function of the unlevered beta and the debt-equity ratio

$$\beta_L = \beta_u (1 + ((1-t)D/E))$$

where

β_L = Levered or Equity Beta

β_u = Unlevered Beta (Asset Beta)

t = Corporate marginal tax rate

D = Market Value of Debt

E = Market Value of Equity

- While this beta is estimated on the assumption that debt carries no market risk (and has a beta of zero), you can have a modified version:

$$\beta_L = \beta_u (1 + ((1-t)D/E)) - \beta_{\text{debt}} (1-t) (D/E)$$

Solutions to the Regression Beta Problem

- Modify the regression beta by
 - changing the index used to estimate the beta
 - adjusting the regression beta estimate, by bringing in information about the fundamentals of the company
- Estimate the beta for the firm using
 - the standard deviation in stock prices instead of a regression against an index.
 - accounting earnings or revenues, which are less noisy than market prices.
- Estimate the beta for the firm from the bottom up without employing the regression technique. This will require
 - understanding the business mix of the firm
 - estimating the financial leverage of the firm
- Use an alternative measure of market risk that does not need a regression.

Bottom-up Betas

- The bottom up beta can be estimated by :

- Taking a weighted (by sales or operating income) average of the unlevered betas of the different businesses a firm is in.

$$\sum_{j=1}^{j=k} \beta_j \left[\frac{\text{Operating Income}_j}{\text{Operating Income}_{\text{Firm}}} \right]$$

(The unlevered beta of a business can be estimated by looking at other firms in the same business)

- Lever up using the firm's debt/equity ratio

$$\beta_{\text{levered}} = \beta_{\text{unlevered}} [1 + (1 - \text{tax rate}) (\text{Current Debt/Equity Ratio})]$$

- The bottom up beta will give you a better estimate of the true beta when

- It has lower standard error ($SE_{\text{average}} = SE_{\text{firm}} / \sqrt{n}$ (n = number of firms))
- It reflects the firm's current business mix and financial leverage
- It can be estimated for divisions and private firms.

Bottom-up Beta: Firm in Multiple Businesses Boeing in 1998

Segment	Estimated Value	Unlevered Beta	Segment Weight
Commercial Aircraft	30,160.48	0.91	70.39%
Defense	12,687.50	0.80	29.61%

Estimated Value = Revenues of division * Enterprise Value/Sales_{Business}

Unlevered Beta of firm = 0.91 (.7039) + 0.80 (.2961) = 0.88

Levered Beta Calculation

Market Value of Equity = \$ 33,401

Market Value of Debt = \$8,143

Market Debt/Equity Ratio = 24.38%

Tax Rate = 35%

Levered Beta for Boeing = 0.88 (1 + (1 - .35) (.2438)) = 1.02

Siderar's Bottom-up Beta

Siderar is an Argentine steel company.

Business	Unlevered D/E Ratio		Levered
	Beta		Beta
Steel	0.68	5.97%	0.71

Proportion of operating income from steel = 100%

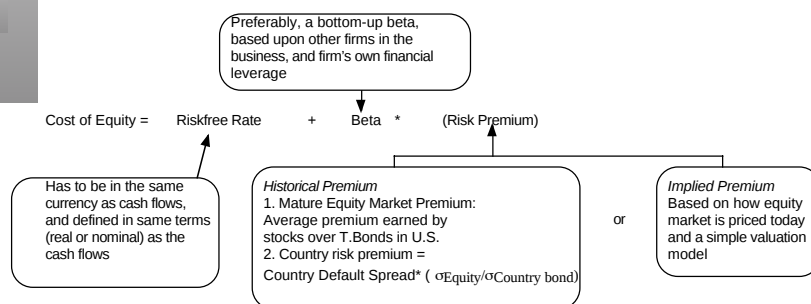
Levered Beta for Siderar = 0.71

Comparable Firms?

Can an unlevered beta estimated using U.S. steel companies be used to estimate the beta for an Argentine steel company?

- ☐ Yes
- ☐ No

The Cost of Equity: A Recap



Estimating the Cost of Debt

- The cost of debt is the rate at which you can borrow at currently, It will reflect not only your default risk but also the level of interest rates in the market.
- The two most widely used approaches to estimating cost of debt are:
 - Looking up the yield to maturity on a straight bond outstanding from the firm. The limitation of this approach is that very few firms have long term straight bonds that are liquid and widely traded
 - Looking up the rating for the firm and estimating a default spread based upon the rating. While this approach is more robust, different bonds from the same firm can have different ratings. You have to use a median rating for the firm
- When in trouble (either because you have no ratings or multiple ratings for a firm), estimate a synthetic rating for your firm and the cost of debt based upon that rating.

Estimating Synthetic Ratings

- The rating for a firm can be estimated using the financial characteristics of the firm. In its simplest form, the rating can be estimated from the interest coverage ratio

$$\text{Interest Coverage Ratio} = \text{EBIT} / \text{Interest Expenses}$$

- For Siderar, in 1999, for instance

$$\text{Interest Coverage Ratio} = 161/48 = 3.33$$

- Based upon the relationship between interest coverage ratios and ratings, we would estimate a rating of A- for Siderar. With a default spread of 1.25% (given the rating of A-)

- For Titan's interest coverage ratio, we used the interest expenses and EBIT from 2000.

$$\text{Interest Coverage Ratio} = 55,467 / 4028 = 13.77$$

Interest Coverage Ratios, Ratings and Default Spreads

If Coverage Ratio is	Estimated Bond Rating	Default Spread(1/99)	Default Spread(1/01)
> 8.50	AAA	0.20%	0.75%
6.50 - 8.50	AA	0.50%	1.00%
5.50 - 6.50	A+	0.80%	1.50%
4.25 - 5.50	A	1.00%	1.80%
3.00 - 4.25	A-	1.25%	2.00%
2.50 - 3.00	BBB	1.50%	2.25%
2.00 - 2.50	BB	2.00%	3.50%
1.75 - 2.00	B+	2.50%	4.75%
1.50 - 1.75	B	3.25%	6.50%
1.25 - 1.50	B -	4.25%	8.00%
0.80 - 1.25	CCC	5.00%	10.00%
0.65 - 0.80	CC	6.00%	11.50%
0.20 - 0.65	C	7.50%	12.70%
< 0.20	D	10.00%	15.00%

Cost of Debt computations

- Companies in countries with low bond ratings and high default risk might bear the burden of country default risk
 - For Siderar, the rating estimated of A- yields a cost of debt as follows:
Pre-tax Cost of Debt in 1999
= US T.Bond rate + Country default spread + Company Default Spread
= 6% + 5.25% + 1.25% = 12.50%
- The synthetic rating for Titan is AAA. The default spread in 2001 is 0.75%.
Pre-tax Cost of Debt
= Riskfree Rate + Company Default Spread+ Country Spread
= 5.10% + 0.75% + 0.95%= 6.80%

Synthetic Ratings: Some Caveats

- The relationship between interest coverage ratios and ratings, developed using US companies, tends to travel well, as long as we are analyzing large manufacturing firms in markets with interest rates close to the US interest rate
- They are more problematic when looking at smaller companies in markets with higher interest rates than the US.

Weights for the Cost of Capital Computation

- The weights used to compute the cost of capital should be the market value weights for debt and equity.
- There is an element of circularity that is introduced into every valuation by doing this, since the values that we attach to the firm and equity at the end of the analysis are different from the values we gave them at the beginning.
- As a general rule, the debt that you should subtract from firm value to arrive at the value of equity should be the same debt that you used to compute the cost of capital.

Estimating Cost of Capital: Titan Cements

- Equity
 - Cost of Equity = $5.10\% + 0.96 (4\% + 1.59\%) = 10.47\%$
 - Market Value of Equity = 739,217 million GDr (78.7%)
- Debt
 - Cost of debt = $5.10\% + 0.75\% + 0.95\% = 6.80\%$
 - Market Value of Debt = 199,766 million GDr (21.3 %)
- Cost of Capital

Cost of Capital = $10.47\% (.787) + 6.80\% (1 - .2449) (0.213) = 9.33\%$

Titan Cement: Book Value Weights

- Titan Cement has a book value of equity of 135,857 million GDR and a book value of debt of 200,000 million GDR. Estimate the cost of capital using book value weights instead of market value weights.

Estimating A U.S. Dollar Cost of Capital: Siderar - An Argentine Steel Company

- Equity
 - Cost of Equity = 6.00% + 0.71 (4% + 10.53%) = 16.32%
 - Market Value of Equity = 3.20 * 310.89 = 995 million (94.37%)
- Debt
 - Cost of debt = 6.00% + 5.25% (Country default) + 1.25% (Company default) = 12.5%
 - Market Value of Debt = 59 Mil (5.63%)
- Cost of Capital
 - Cost of Capital = 16.32 % (.9437) + 12.50% (1-.3345) (.0563))
= 16.32 % (.9437) + 8.32% (.0563)) = 15.87 %

Converting a Dollar Cost of Capital into a Peso cost of capital

- Approach 1: Use a peso riskfree rate in all of the calculations above. For instance, if the peso riskfree rate was 10%, the cost of capital would be computed as follows:
 - Cost of Equity = 10.00% + 0.71 (4% + 10.53%) = 20.32%
 - Cost of Debt = 10.00% + 5.25% (Country default) + 1.25% (Company default) = 16.5%

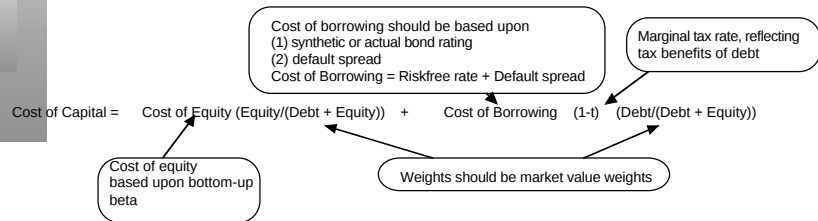
(This assumes the peso riskfree rate has no country risk premium embedded in it.)
- Approach 2: Use the differential inflation rate to estimate the cost of capital. For instance, if the inflation rate in pesos is 7% and the inflation rate in the U.S. is 3%

$$\begin{aligned} \text{Cost of capital} &= (1 + \text{Cost of Capital}_\$) \left[\frac{1 + \text{Inflation}_{\text{Peso}}}{1 + \text{Inflation}_\$} \right] \\ &= 1.1587 (1.07/1.03) = 1.2037 \rightarrow 20.37\% \end{aligned}$$

Dealing with Hybrids and Preferred Stock

- When dealing with hybrids (convertible bonds, for instance), break the security down into debt and equity and allocate the amounts accordingly. Thus, if a firm has \$ 125 million in convertible debt outstanding, break the \$125 million into straight debt and conversion option components. The conversion option is equity.
- When dealing with preferred stock, it is better to keep it as a separate component. The cost of preferred stock is the preferred dividend yield. (As a rule of thumb, if the preferred stock is less than 5% of the outstanding market value of the firm, lumping it in with debt will make no significant impact on your valuation).

Recapping the Cost of Capital



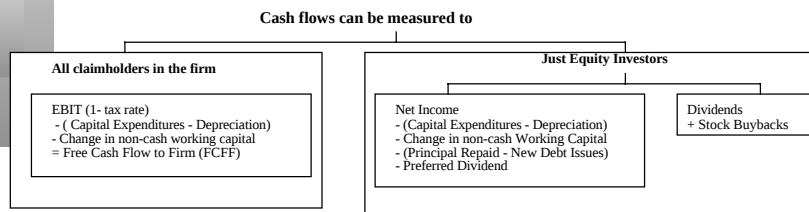
II. Estimating Cash Flows

DCF Valuation

Steps in Cash Flow Estimation

- Estimate the current earnings of the firm
 - If looking at cash flows to equity, look at earnings after interest expenses - i.e. net income
 - If looking at cash flows to the firm, look at operating earnings after taxes
- Consider how much the firm invested to create future growth
 - If the investment is not expensed, it will be categorized as capital expenditures. To the extent that depreciation provides a cash flow, it will cover some of these expenditures.
 - Increasing working capital needs are also investments for future growth
- If looking at cash flows to equity, consider the cash flows from net debt issues (debt issued - debt repaid)

Measuring Cash Flows



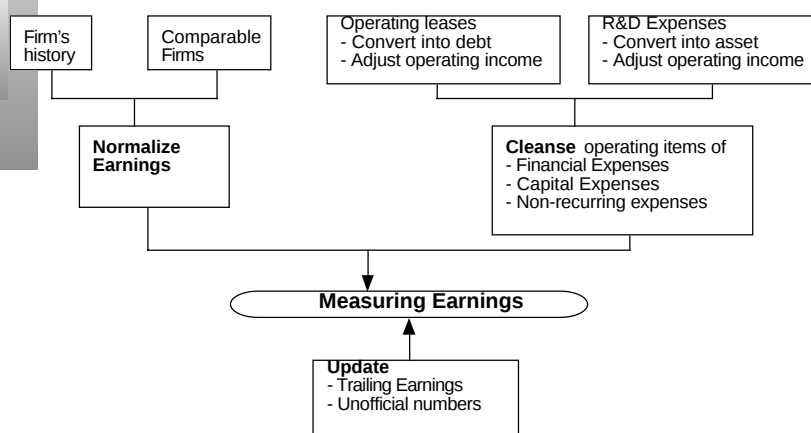
Measuring Cash Flow to the Firm

EBIT (1 - tax rate)

- (Capital Expenditures - Depreciation)
- Change in Working Capital
- = Cash flow to the firm

- Where are the tax savings from interest payments in this cash flow?

From Reported to Actual Earnings



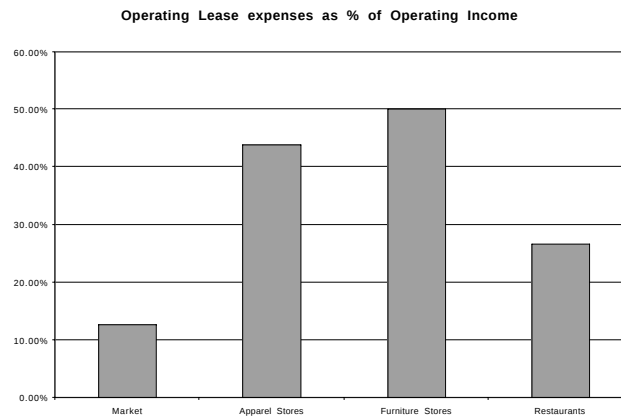
I. Update Earnings

- When valuing companies, we often depend upon financial statements for inputs on earnings and assets. Annual reports are often outdated and can be updated by using-
 - Trailing 12-month data, constructed from quarterly earnings reports.
 - Informal and unofficial news reports, if quarterly reports are unavailable.
- Updating makes the most difference for smaller and more volatile firms, as well as for firms that have undergone significant restructuring.

II. Correcting Accounting Earnings

- *The Operating Lease Adjustment:* While accounting convention treats operating leases as operating expenses, they are really financial expenses and need to be reclassified as such. This has no effect on equity earnings but does change the operating earnings
- *The R & D Adjustment:* Since R&D is a capital expenditure (rather than an operating expense), the operating income has to be adjusted to reflect its treatment.

The Magnitude of Operating Leases



Dealing with Operating Lease Expenses

- Operating Lease Expenses are treated as operating expenses in computing operating income. In reality, operating lease expenses should be treated as financing expenses, with the following adjustments to earnings and capital:
- Debt Value of Operating Leases = PV of Operating Lease Expenses at the pre-tax cost of debt
- Adjusted Operating Earnings
 - Adjusted Operating Earnings = Operating Earnings + Operating Lease Expenses - Depreciation on Leased Asset
 - As an approximation, this works:
Adjusted Operating Earnings = Operating Earnings + Pre-tax cost of Debt * PV of Operating Leases.

Operating Leases at The Home Depot in 1998

- The pre-tax cost of debt at the Home Depot is 6.25%

Yr	Operating Lease Expense	Present Value
1	\$ 294	\$ 277
2	\$ 291	\$ 258
3	\$ 264	\$ 220
4	\$ 245	\$ 192
5	\$ 236	\$ 174
6-15	\$ 270	\$ 1,450 (PV of 10-yr annuity)

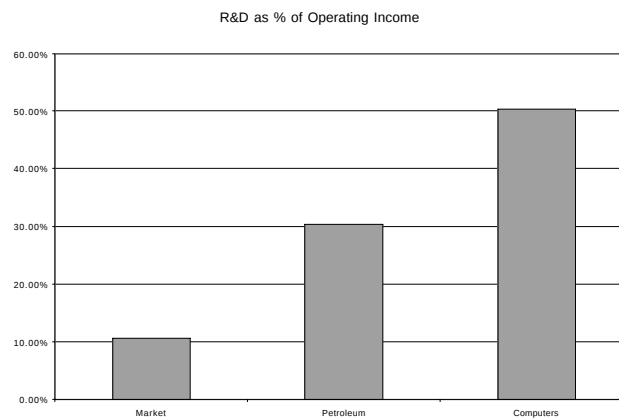
Present Value of Operating Leases = \$ 2,571

- Debt outstanding at the Home Depot = \$1,205 + \$2,571 = \$3,776 mil
(The Home Depot has other debt outstanding of \$1,205 million)
- Adjusted Operating Income = \$2,016 + 2,571 (.0625) = \$2,177 mil

The Effects of Capitalizing Operating Leases

- *Debt* : will increase, leading to an increase in debt ratios used in the cost of capital and levered beta calculation
- *Operating income*: will increase, since operating leases will now be before the imputed interest on the operating lease expense
- *Net income*: will be unaffected since it is after both operating and financial expenses anyway
- *Return on Capital* will generally decrease since the increase in operating income will be proportionately lower than the increase in book capital invested

The Magnitude of R&D Expenses



R&D Expenses: Operating or Capital Expenses

- Accounting standards require us to consider R&D as an operating expense even though it is designed to generate future growth. It is more logical to treat it as capital expenditures.
- To capitalize R&D,
 - Specify an amortizable life for R&D (2 - 10 years)
 - Collect past R&D expenses for as long as the amortizable life
 - Sum up the unamortized R&D over the period. (Thus, if the amortizable life is 5 years, the research asset can be obtained by adding up 1/5th of the R&D expense from five years ago, 2/5th of the R&D expense from four years ago....)

Capitalizing R&D Expenses: Cisco

- R & D was assumed to have a 5-year life.

Year	R&D Expense	Unamortized portion		Amortization this year
1999 (current)	1594.00	1.00	1594.00	
1998	1026.00	0.80	820.80	\$205.20
1997	698.00	0.60	418.80	\$139.60
1996	399.00	0.40	159.60	\$79.80
1995	211.00	0.20	42.20	\$42.20
1994	89.00	0.00	0.00	\$17.80
Total			\$ 3,035.40	\$ 484.60
Value of research asset =			\$ 3,035.4 million	
Amortization of research asset in 1998 =			\$ 484.6 million	
Adjustment to Operating Income = \$ 1,594 million - 484.6 million = 1,109.4 million				

The Effect of Capitalizing R&D

- Operating Income will generally increase, though it depends upon whether R&D is growing or not. If it is flat, there will be no effect since the amortization will offset the R&D added back. The faster R&D is growing the more operating income will be affected.
- Net income will increase proportionately, depending again upon how fast R&D is growing
- Book value of equity (and capital) will increase by the capitalized Research asset
- Capital expenditures will increase by the amount of R&D; Depreciation will increase by the amortization of the research asset; For all firms, the net cap ex will increase by the same amount as the after-tax operating income.

III. One-Time and Non-recurring Charges

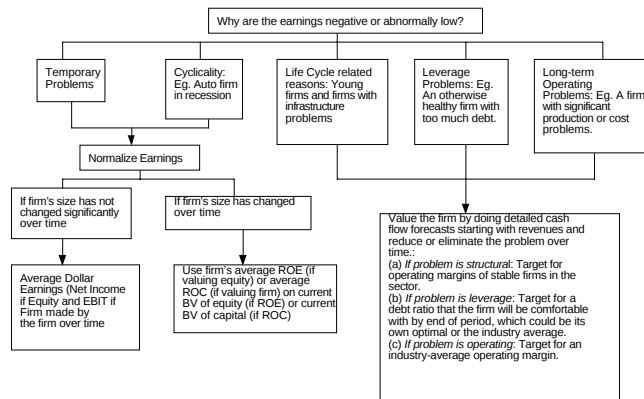
- Assume that you are valuing a firm that is reporting a loss of \$ 500 million, due to a one-time charge of \$ 1 billion. What is the earnings you would use in your valuation?
 - ❑ A loss of \$ 500 million
 - ❑ A profit of \$ 500 million
- Would your answer be any different if the firm had reported one-time losses like these once every five years?
 - ❑ Yes
 - ❑ No

IV. Accounting Malfeasance....

- Though all firms may be governed by the same accounting standards, the fidelity that they show to these standards can vary. More aggressive firms will show higher earnings than more conservative firms.
- While you will not be able to catch outright fraud, you should look for warning signals in financial statements and correct for them:
 - Income from unspecified sources - holdings in other businesses that are not revealed or from special purpose entities.
 - Income from asset sales or financial transactions (for a non-financial firm)
 - Sudden changes in standard expense items - a big drop in S,G &A or R&D expenses, for instance.

V. Dealing with Negative or Abnormally Low Earnings

A Framework for Analyzing Companies with Negative or Abnormally Low Earnings



What tax rate?

- The tax rate that you should use in computing the after-tax operating income should be
 - ❑ The effective tax rate in the financial statements (taxes paid/Taxable income)
 - ❑ The tax rate based upon taxes paid and EBIT (taxes paid/EBIT)
 - ❑ The marginal tax rate
 - ❑ None of the above
 - ❑ Any of the above, as long as you compute your after-tax cost of debt using the same tax rate

The Right Tax Rate to Use

- The choice really is between the effective and the marginal tax rate. In doing projections, it is far safer to use the marginal tax rate since the effective tax rate is really a reflection of the difference between the accounting and the tax books.
- By using the marginal tax rate, we tend to understate the after-tax operating income in the earlier years, but the after-tax tax operating income is more accurate in later years
- If you choose to use the effective tax rate, adjust the tax rate towards the marginal tax rate over time.

A Tax Rate for a Money Losing Firm

- Assume that you are trying to estimate the after-tax operating income for a firm with \$ 1 billion in net operating losses carried forward. This firm is expected to have operating income of \$ 500 million each year for the next 3 years, and the marginal tax rate on income for all firms that make money is 40%. Estimate the after-tax operating income each year for the next 3 years.

	Year 1	Year 2	Year 3
EBIT	500	500	500
Taxes			
EBIT (1-t)			
Tax rate			

Net Capital Expenditures

- Net capital expenditures represent the difference between capital expenditures and depreciation. Depreciation is a cash inflow that pays for some or a lot (or sometimes all of) the capital expenditures.
- In general, the net capital expenditures will be a function of how fast a firm is growing or expecting to grow. High growth firms will have much higher net capital expenditures than low growth firms.
- Assumptions about net capital expenditures can therefore never be made independently of assumptions about growth in the future.

Capital expenditures should include

- Research and development expenses, once they have been re-categorized as capital expenses. The adjusted net cap ex will be
$$\text{Adjusted Net Capital Expenditures} = \text{Net Capital Expenditures} + \text{Current year's R\&D expenses} - \text{Amortization of Research Asset}$$
- Acquisitions of other firms, since these are like capital expenditures. The adjusted net cap ex will be
$$\text{Adjusted Net Cap Ex} = \text{Net Capital Expenditures} + \text{Acquisitions of other firms} - \text{Amortization of such acquisitions}$$

Two caveats:

 1. Most firms do not do acquisitions every year. Hence, a normalized measure of acquisitions (looking at an average over time) should be used
 2. The best place to find acquisitions is in the statement of cash flows, usually categorized under other investment activities

Cisco's Acquisitions: 1999

Acquired	Method of Acquisition	Price Paid
GeoTel	Pooling	\$1,344
Fibex	Pooling	\$318
Sentient	Pooling	\$103
American Internert	Purchase	\$58
Summa Four	Purchase	\$129
Clarity Wireless	Purchase	\$153
Selsius Systems	Purchase	\$134
PipeLinks	Purchase	\$118
Amteva Tech	Purchase	\$159
		\$2,516

Cisco's Net Capital Expenditures in 1999

Cap Expenditures (from statement of CF)	= \$ 584 mil
- Depreciation (from statement of CF)	= \$ 486 mil
Net Cap Ex (from statement of CF)	= \$ 98 mil
+ R & D expense	= \$ 1,594 mil
- Amortization of R&D	= \$ 485 mil
+ Acquisitions	= \$ 2,516 mil
Adjusted Net Capital Expenditures	= \$3,723 mil

(Amortization was included in the depreciation number)

Working Capital Investments

- In accounting terms, the working capital is the difference between current assets (inventory, cash and accounts receivable) and current liabilities (accounts payables, short term debt and debt due within the next year)
- A cleaner definition of working capital from a cash flow perspective is the difference between non-cash current assets (inventory and accounts receivable) and non-debt current liabilities (accounts payable)
- Any investment in this measure of working capital ties up cash. Therefore, any increases (decreases) in working capital will reduce (increase) cash flows in that period.
- When forecasting future growth, it is important to forecast the effects of such growth on working capital needs, and building these effects into the cash flows.

Working Capital: General Propositions

- Changes in non-cash working capital from year to year tend to be volatile. A far better estimate of non-cash working capital needs, looking forward, can be estimated by looking at non-cash working capital as a proportion of revenues
- Some firms have negative non-cash working capital. Assuming that this will continue into the future will generate positive cash flows for the firm. While this is indeed feasible for a period of time, it is not forever. Thus, it is better that non-cash working capital needs be set to zero, when it is negative.

Volatile Working Capital?

	<i>Amazon</i>	<i>Cisco</i>	<i>Motorola</i>
Revenues	\$ 1,640	\$12,154	\$30,931
Non-cash WC	-419	-404	2547
% of Revenues	-25.53%	-3.32%	8.23%
Change from last year \$	(309)	(\$700)	(\$829)
Average: last 3 years	-15.16%	-3.16%	8.91%
Average: industry	8.71%	-2.71%	7.04%
<i>Assumption in Valuation</i>			
WC as % of Revenue	3.00%	0.00%	8.23%

Dividends and Cash Flows to Equity

- In the strictest sense, the only cash flow that an investor will receive from an equity investment in a publicly traded firm is the dividend that will be paid on the stock.
- Actual dividends, however, are set by the managers of the firm and may be much lower than the potential dividends (that could have been paid out)
 - managers are conservative and try to smooth out dividends
 - managers like to hold on to cash to meet unforeseen future contingencies and investment opportunities
- When actual dividends are less than potential dividends, using a model that focuses only on dividends will under state the true value of the equity in a firm.

Measuring Potential Dividends

- Some analysts assume that the earnings of a firm represent its potential dividends. This cannot be true for several reasons:
 - Earnings are not cash flows, since there are both non-cash revenues and expenses in the earnings calculation
 - Even if earnings were cash flows, a firm that paid its earnings out as dividends would not be investing in new assets and thus could not grow
 - Valuation models, where earnings are discounted back to the present, will over estimate the value of the equity in the firm
- The potential dividends of a firm are the cash flows left over after the firm has made any “investments” it needs to make to create future growth and net debt repayments (debt repayments - new debt issues)
 - The common categorization of capital expenditures into discretionary and non-discretionary loses its basis when there is future growth built into the valuation.

Estimating Cash Flows: FCFE

- Cash flows to Equity for a Levered Firm
$$\begin{aligned} &\text{Net Income} \\ &- (\text{Capital Expenditures} - \text{Depreciation}) \\ &- \text{Changes in non-cash Working Capital} \\ &- (\text{Principal Repayments} - \text{New Debt Issues}) \\ &= \text{Free Cash flow to Equity} \end{aligned}$$
 - I have ignored preferred dividends. If preferred stock exist, preferred dividends will also need to be netted out

Estimating FCFE when Leverage is Stable

Net Income

- $(1 - \delta)$ (Capital Expenditures - Depreciation)

- $(1 - \delta)$ Working Capital Needs

= Free Cash flow to Equity

δ = Debt/Capital Ratio

For this firm,

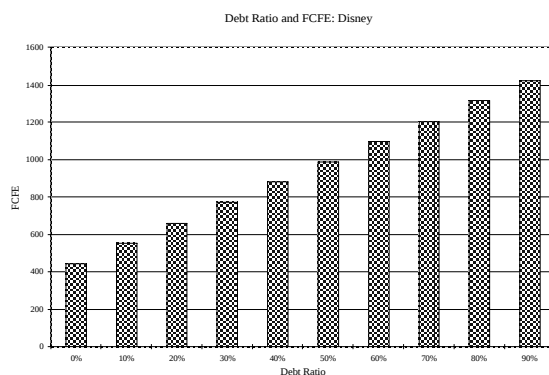
- Proceeds from new debt issues = Principal Repayments + δ (Capital Expenditures - Depreciation + Working Capital Needs)
- In computing FCFE, the book value debt to capital ratio should be used when looking back in time but can be replaced with the market value debt to capital ratio, looking forward.

Estimating FCFE: Disney

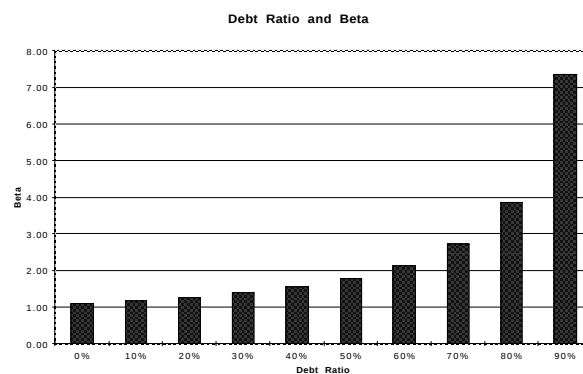
- Net Income = \$ 1533 Million
- Capital spending = \$ 1,746 Million
- Depreciation per Share = \$ 1,134 Million
- Increase in non-cash working capital = \$ 477 Million
- Debt to Capital Ratio = 23.83%
- Estimating FCFE (1997):

Net Income	\$1,533 Mil	
- (Cap. Exp - Depr)*(1-DR)	\$465.90	[(1746-1134)(1-.2383)]
Chg. Working Capital*(1-DR)	\$363.33	[477(1-.2383)]
= Free CF to Equity	\$ 704 Million	
- Dividends Paid** **\$ 345 Million**

FCFE and Leverage: Is this a free lunch?



FCFE and Leverage: The Other Shoe Drops



Leverage, FCFE and Value

- In a discounted cash flow model, increasing the debt/equity ratio will generally increase the expected free cash flows to equity investors over future time periods and also the cost of equity applied in discounting these cash flows. Which of the following statements relating leverage to value would you subscribe to?
 - ❑ Increasing leverage will increase value because the cash flow effects will dominate the discount rate effects
 - ❑ Increasing leverage will decrease value because the risk effect will be greater than the cash flow effects
 - ❑ Increasing leverage will not affect value because the risk effect will exactly offset the cash flow effect
 - ❑ Any of the above, depending upon what company you are looking at and where it is in terms of current leverage

Estimating FCFE: Brahma

- Net Income (1996) = 325 Million BR
- Capital spending (1996) = 396 Million
- Depreciation (1996) = 183 Million BR
- Increase in Non-cash Working capital Change (1996) = 12 Million BR
- Debt Ratio = 43.48%
- Estimating FCFE (1996):

Earnings per Share	325.00 Million BR
- (Cap Ex-Depr) (1-DR) = (396-183)(1-.4348) =	120.39 Million BR
- Change in Non-cash WC (1-DR) = 12 (1-.4348) =	6.78 Million BR
Free Cashflow to Equity	197.83 Million Br
 Dividends Paid	 232.00 Million BR

III. Estimating Growth

DCF Valuation

Ways of Estimating Growth in Earnings

- Look at the past
 - The historical growth in earnings per share is usually a good starting point for growth estimation
- Look at what others are estimating
 - Analysts estimate growth in earnings per share for many firms. It is useful to know what their estimates are.
- Look at fundamentals
 - Ultimately, all growth in earnings can be traced to two fundamentals - how much the firm is investing in new projects, and what returns these projects are making for the firm.

I. Historical Growth in EPS

- Historical growth rates can be estimated in a number of different ways
 - Arithmetic versus Geometric Averages
 - Simple versus Regression Models
- Historical growth rates can be sensitive to
 - the period used in the estimation
- In using historical growth rates, the following factors have to be considered
 - how to deal with negative earnings
 - the effect of changing size

Motorola: Arithmetic versus Geometric Growth Rates

	Revenues	% Change	EBITDA	% Change	EBIT	% Change
1994	\$ 22,245		\$ 4,151		\$ 2,604	
1995	\$ 27,037	21.54%	\$ 4,850	16.84%	\$ 2,931	12.56%
1996	\$ 27,973	3.46%	\$ 4,268	-12.00%	\$ 1,960	-33.13%
1997	\$ 29,794	6.51%	\$ 4,276	0.19%	\$ 1,947	-0.66%
1998	\$ 29,398	-1.33%	\$ 3,019	-29.40%	\$ 822	-57.78%
1999	\$ 30,931	5.21%	\$ 5,398	78.80%	\$ 3,216	291.24%
Arithmetic Average		7.08%		10.89%		42.45%
Geometric Average		6.82%		5.39%		4.31%
Standard deviation		8.61%		41.56%		141.78%

Cisco: Linear and Log-Linear Models for Growth

Year	EPS		ln(EPS)
1991	\$	0.01	-4.6052
1992	\$	0.02	-3.9120
1993	\$	0.04	-3.2189
1994	\$	0.07	-2.6593
1995	\$	0.08	-2.5257
1996	\$	0.16	-1.8326
1997	\$	0.18	-1.7148
1998	\$	0.25	-1.3863
1999	\$	0.32	-1.1394

- $EPS = -.066 + 0.0383 (t)$: EPS grows by \$0.0383 a year
Growth Rate = $\$0.0383/\$0.13 = 30.5\%$ (\$0.13: Average EPS from 91-99)
- $\ln(EPS) = -4.66 + 0.4212 (t)$: Growth rate approximately 42.12%

A Test

- You are trying to estimate the growth rate in earnings per share at Time Warner from 1996 to 1997. In 1996, the earnings per share was a deficit of \$0.05. In 1997, the expected earnings per share is \$ 0.25. What is the growth rate?
- ☐ -600%
- ☐ +600%
- ☐ +120%
- ☐ Cannot be estimated

Dealing with Negative Earnings

- When the earnings in the starting period are negative, the growth rate cannot be estimated. ($0.30/-0.05 = -600\%$)
- There are three solutions:
 - Use the higher of the two numbers as the denominator ($0.30/0.25 = 120\%$)
 - Use the absolute value of earnings in the starting period as the denominator ($0.30/0.05=600\%$)
 - Use a linear regression model and divide the coefficient by the average earnings.
- When earnings are negative, the growth rate is meaningless. Thus, while the growth rate can be estimated, it does not tell you much about the future.

The Effect of Size on Growth: Callaway Golf

<i>Year</i>	<i>Net Profit</i>	<i>Growth Rate</i>
1990	1.80	
1991	6.40	255.56%
1992	19.30	201.56%
1993	41.20	113.47%
1994	78.00	89.32%
1995	97.70	25.26%
1996	122.30	25.18%
Geometric Average Growth Rate = 102%		

Extrapolation and its Dangers

<i>Year</i>	<i>Net Profit</i>
1996	\$ 122.30
1997	\$ 247.05
1998	\$ 499.03
1999	\$ 1,008.05
2000	\$ 2,036.25
2001	\$ 4,113.23

- If net profit continues to grow at the same rate as it has in the past 6 years, the expected net income in 5 years will be \$ 4.113 billion.

II. Analyst Forecasts of Growth

- While the job of an analyst is to find under and over valued stocks in the sectors that they follow, a significant proportion of an analyst's time (outside of selling) is spent forecasting earnings per share.
 - Most of this time, in turn, is spent forecasting earnings per share in the next earnings report
 - While many analysts forecast expected growth in earnings per share over the next 5 years, the analysis and information (generally) that goes into this estimate is far more limited.
- Analyst forecasts of earnings per share and expected growth are widely disseminated by services such as Zacks and IBES, at least for U.S. companies.

How good are analysts at forecasting growth?

- Analysts forecasts of EPS tend to be closer to the actual EPS than simple time series models, but the differences tend to be small

Study	Time Period	Analyst Forecast Error	Time Series Model
Collins & Hopwood	Value Line Forecasts	31.7%	34.1%
Brown & Rozeff	Value Line Forecasts	28.4%	32.2%
Fried & Givoly	Earnings Forecaster	16.4%	19.8%

- The advantage that analysts have over time series models
 - tends to decrease with the forecast period (next quarter versus 5 years)
 - tends to be greater for larger firms than for smaller firms
 - tends to be greater at the industry level than at the company level
- Forecasts of growth (and revisions thereof) tend to be highly correlated across analysts.

Are some analysts more equal than others?

- A study of All-America Analysts (chosen by Institutional Investor) found that
 - There is no evidence that analysts who are chosen for the All-America Analyst team were chosen because they were better forecasters of earnings. (Their median forecast error in the quarter prior to being chosen was 30%; the median forecast error of other analysts was 28%)
 - However, in the calendar year following being chosen as All-America analysts, these analysts become slightly better forecasters than their less fortunate brethren. (The median forecast error for All-America analysts is 2% lower than the median forecast error for other analysts)
 - Earnings revisions made by All-America analysts tend to have a much greater impact on the stock price than revisions from other analysts
 - The recommendations made by the All America analysts have a greater impact on stock prices (3% on buys; 4.7% on sells). For these recommendations the price changes are sustained, and they continue to rise in the following period (2.4% for buys; 13.8% for the sells).

The Five Deadly Sins of an Analyst

- **Tunnel Vision:** Becoming so focused on the sector and valuations within the sector that you lose sight of the bigger picture.
- **Lemmingitis:** Strong urge felt to change recommendations & revise earnings estimates when other analysts do the same.
- **Stockholm Syndrome** (shortly to be renamed the Bre-X syndrome): Refers to analysts who start identifying with the managers of the firms that they are supposed to follow.
- **Factophobia** (generally is coupled with delusions of being a famous story teller): Tendency to base a recommendation on a “story” coupled with a refusal to face the facts.
- **Dr. Jekyll/Mr. Hyde:** Analyst who thinks his primary job is to bring in investment banking business to the firm.

Propositions about Analyst Growth Rates

- **Proposition 1:** There is far less private information and far more public information in most analyst forecasts than is generally claimed.
- **Proposition 2:** The biggest source of private information for analysts remains the company itself which might explain
 - why there are more buy recommendations than sell recommendations (information bias and the need to preserve sources)
 - why there is such a high correlation across analysts forecasts and revisions
 - why All-America analysts become better forecasters than other analysts after they are chosen to be part of the team.
- **Proposition 3:** There is value to knowing what analysts are forecasting as earnings growth for a firm. There is, however, danger when they agree too much (lemmingitis) and when they agree to little (in which case the information that they have is so noisy as to be useless).

III. Fundamental Growth Rates

$$\begin{array}{l}
 \boxed{\text{Investment in Existing Projects}} \quad \$1000 \quad \times \quad \boxed{\text{Current Return on Investment on Projects}} \quad 12\% \quad = \quad \text{Current Earnings} \quad \$120 \\
 \\
 \boxed{\text{Investment in Existing Projects}} \quad \$1000 \quad \times \quad \boxed{\text{Next Period's Return on Investment}} \quad 12\% \quad + \quad \boxed{\text{Investment in New Projects}} \quad \$100 \quad \times \quad \boxed{\text{Return on Investment on New Projects}} \quad 12\% \quad = \quad \text{Next Period's Earnings} \quad 132 \\
 \\
 \boxed{\text{Investment in Existing Projects}} \quad \$1000 \quad \times \quad \boxed{\text{Change in ROI from current to next period: 0\%}} \quad + \quad \boxed{\text{Investment in New Projects}} \quad \$100 \quad \times \quad \boxed{\text{Return on Investment on New Projects}} \quad 12\% \quad = \quad \text{Change in Earnings} \quad = \$12
 \end{array}$$

Growth Rate Derivations

In the special case where ROI on existing projects remains unchanged and is equal to the ROI on new projects

$$\begin{array}{l}
 \frac{\text{Investment in New Projects}}{\text{Current Earnings}} \quad \times \quad \text{Return on Investment} \quad = \quad \frac{\text{Change in Earnings}}{\text{Current Earnings}} \\
 \frac{100}{120} \quad \times \quad 12\% \quad = \quad \frac{\$12}{\$120} \\
 \text{Reinvestment Rate} \quad \times \quad \text{Return on Investment} \quad = \quad \text{Growth Rate in Earnings} \\
 83.33\% \quad \times \quad 12\% \quad = \quad 10\%
 \end{array}$$

in the more general case where ROI can change from period to period, this can be expanded as follows:

$$\frac{\text{Investment in Existing Projects} \times (\text{Change in ROI}) + \text{New Projects (ROI)}}{\text{Investment in Existing Projects} \times \text{Current ROI}} \quad = \quad \frac{\text{Change in Earnings}}{\text{Current Earnings}}$$

For instance, if the ROI increases from 12% to 13%, the expected growth rate can be written as follows:

$$\frac{\$1,000 \times (.13 - .12) + 100 (.13\%)}{\$1,000 \times .12} \quad = \quad \frac{\$23}{\$120} \quad = \quad 19.17\%$$

I. Expected Long Term Growth in EPS

- When looking at growth in earnings per share, these inputs can be cast as follows:
Reinvestment Rate = Retained Earnings/ Current Earnings = Retention Ratio
Return on Investment = ROE = Net Income/Book Value of Equity
- In the special case where the current ROE is expected to remain unchanged
 $g_{EPS} = \text{Retained Earnings}_{t-1} / \text{NI}_{t-1} * \text{ROE}$
 $= \text{Retention Ratio} * \text{ROE}$
 $= b * \text{ROE}$
- Proposition 1: The expected growth rate in earnings for a company cannot exceed its return on equity in the long term.

Estimating Expected Growth in EPS: ABN Amro

- Current Return on Equity = 15.79%
- Current Retention Ratio = $1 - \text{DPS}/\text{EPS} = 1 - 1.13/2.45 = 53.88\%$
- If ABN Amro can maintain its current ROE and retention ratio, its expected growth in EPS will be:
Expected Growth Rate = $0.5388 (15.79\%) = 8.51\%$

Expected ROE changes and Growth

- Assume now that ABN Amro's ROE next year is expected to increase to 17%, while its retention ratio remains at 53.88%. What is the new expected long term growth rate in earnings per share?
- Will the expected growth rate in earnings per share next year be greater than, less than or equal to this estimate?
 - ☐ greater than
 - ☐ less than
 - ☐ equal to

Changes in ROE and Expected Growth

- When the ROE is expected to change,
$$g_{EPS} = b * ROE_{t+1} + (ROE_{t+1} - ROE_t) / ROE_t$$
- Proposition 2: Small changes in ROE translate into large changes in the expected growth rate.
 - The lower the current ROE, the greater the effect on growth of changes in the ROE.
- Proposition 3: No firm can, in the long term, sustain growth in earnings per share from improvement in ROE.
 - Corollary: The higher the existing ROE of the company (relative to the business in which it operates) and the more competitive the business in which it operates, the smaller the scope for improvement in ROE.

Changes in ROE: ABN Amro

- Assume now that ABN's expansion into Asia will push up the ROE to 17%, while the retention ratio will remain 53.88%. The expected growth rate in that year will be:

$$\begin{aligned} g_{EPS} &= b * ROE_{t+1} + (ROE_{t+1} - ROE_t) / ROE_t \\ &= (.5388)(.17) + (.17 - .1579) / (.1579) \\ &= 16.83\% \end{aligned}$$

- Note that 1.21% improvement in ROE translates into almost a doubling of the growth rate from 8.51% to 16.83%.

ROE and Leverage

- $ROE = ROC + D/E (ROC - i (1-t))$

where,

$$ROC = EBIT_t (1 - \text{tax rate}) / \text{BV of Capital}_{t-1}$$

$$D/E = \text{BV of Debt} / \text{BV of Equity}$$

$$i = \text{Interest Expense on Debt} / \text{BV of Debt}$$

$$t = \text{Tax rate on ordinary income}$$

- Note that BV of capital = BV of Debt + BV of Equity.

- BV: Book Value

Decomposing ROE: Brahma

- Real Return on Capital = $687 (1-.32) / (1326+542+478) = 19.91\%$
 - This is assumed to be real because both the book value and income are inflation adjusted.
- Debt/Equity Ratio = $(542+478)/1326 = 0.77$
- After-tax Cost of Debt = $8.25\% (1-.32) = 5.61\%$ (Real BR)
- Return on Equity = $ROC + D/E (ROC - i(1-t))$
 $19.91\% + 0.77 (19.91\% - 5.61\%) = 30.92\%$

Decomposing ROE: Titan Watches (India)

- Return on Capital = $713 (1-.25)/(1925+2378+1303) = 9.54\%$
- Debt/Equity Ratio = $(2378 + 1303)/1925 = 1.91$
- After-tax Cost of Debt = $13.5\% (1-.25) = 10.125\%$
- Return on Equity = $ROC + D/E (ROC - i(1-t))$
 $9.54\% + 1.91 (9.54\% - 10.125\%) = 8.42\%$

II. Expected Growth in Net Income

- The limitation of the EPS fundamental growth equation is that it focuses on per share earnings and assumes that reinvested earnings are invested in projects earning the return on equity.
- A more general version of expected growth in earnings can be obtained by substituting in the equity reinvestment into real investments (net capital expenditures and working capital):
Equity Reinvestment Rate = (Net Capital Expenditures + Change in Working Capital) (1 - Debt Ratio) / Net Income
Expected Growth_{Net Income} = Equity Reinvestment Rate * ROE

III. Expected Growth in EBIT And Fundamentals: Stable ROC and Reinvestment Rate

- When looking at growth in operating income, the definitions are
Reinvestment Rate = (Net Capital Expenditures + Change in WC) / EBIT(1-t)
Return on Investment = ROC = EBIT(1-t) / (BV of Debt + BV of Equity)
- Reinvestment Rate and Return on Capital
$$g_{\text{EBIT}} = (\text{Net Capital Expenditures} + \text{Change in WC}) / \text{EBIT}(1-t) * \text{ROC}$$
$$= \text{Reinvestment Rate} * \text{ROC}$$
- **Proposition: The net capital expenditure needs of a firm, for a given growth rate, should be inversely proportional to the quality of its investments.**

No Net Capital Expenditures and Long Term Growth

- You are looking at a valuation, where the terminal value is based upon the assumption that operating income will grow 3% a year forever, but there are no net cap ex or working capital investments being made after the terminal year. When you confront the analyst, he contends that this is still feasible because the company is becoming more efficient with its existing assets and can be expected to increase its return on capital over time. Is this a reasonable explanation?
 - Yes
 - No
 - Explain.

Estimating Growth in EBIT: Cisco versus Motorola

Cisco's Fundamentals

- Reinvestment Rate = 106.81%
- Return on Capital = 34.07%
- Expected Growth in EBIT = $(1.0681)(.3407) = 36.39\%$

Motorola's Fundamentals

- Reinvestment Rate = 52.99%
- Return on Capital = 12.18%
- Expected Growth in EBIT = $(.5299)(.1218) = 6.45\%$

IV. Operating Income Growth when Return on Capital is Changing

- When the return on capital is changing, there will be a second component to growth, positive if the return on capital is increasing and negative if the return on capital is decreasing.
- If ROC_t is the return on capital in period t and ROC_{t+1} is the return on capital in period $t+1$, the expected growth rate in operating income will be:

$$\text{Expected Growth Rate} = ROC_{t+1} * \text{Reinvestment rate} + (ROC_{t+1} - ROC_t) / ROC_t$$

- If the change is over multiple periods, the second component should be spread out over each period.

Motorola's Growth Rate

- Motorola's current return on capital is 12.18% and its reinvestment rate is 52.99%.
- We expect Motorola's return on capital to rise to 17.22% over the next 5 years (which is half way towards the industry average)

Expected Growth Rate

$$\begin{aligned} &= ROC_{\text{New Investments}} * \text{Reinvestment Rate}_{\text{current}} + \{ [1 + (ROC_{\text{In 5 years}} - ROC_{\text{Current}}) / ROC_{\text{Current}}]^{1/5} - 1 \} \\ &= .1722 * .5299 + \{ [1 + (.1722 - .1218) / .1218]^{1/5} - 1 \} \\ &= .174 \text{ or } 17.40\% \end{aligned}$$

One way to think about this is to decompose Motorola's expected growth into

Growth from new investments: $.1722 * .5299 = 9.12\%$

Growth from more efficiently using existing investments: $17.40\% - 9.12\% = 8.28\%$

{Note that I am assuming that the new investments start making 17.22% immediately, while allowing for existing assets to improve returns gradually}

V. Estimating Growth when Operating Income is Negative or Margins are changing

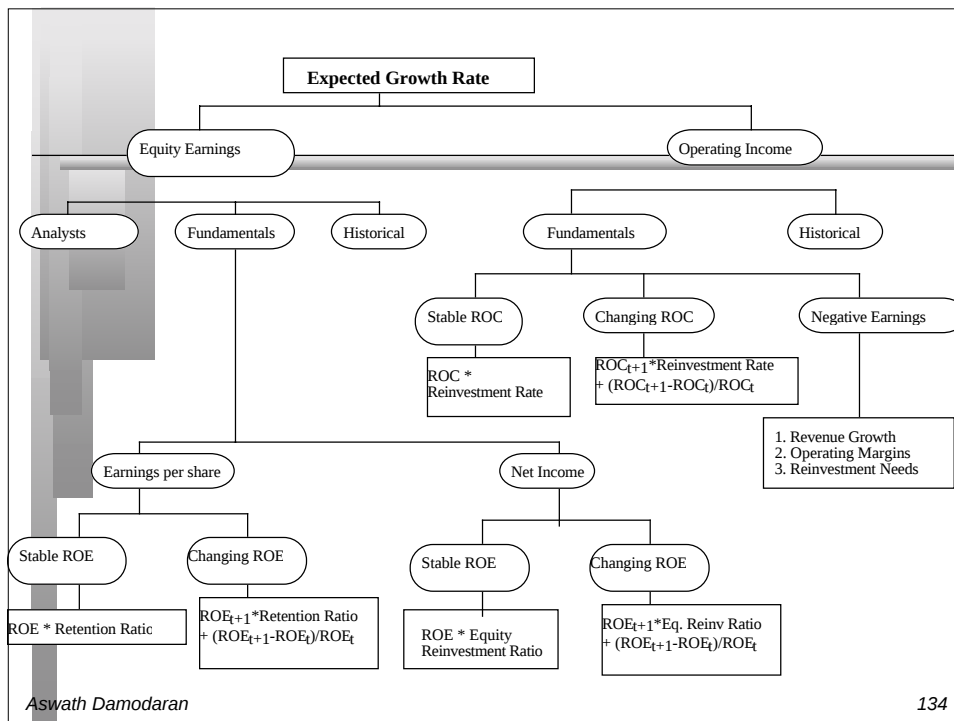
- When operating income is negative or margins are expected to change over time, we use a three step process to estimate growth:
 - Estimate growth rates in revenues over time
 - Use historical revenue growth to get estimates of revenue growth in the near future
 - Decrease the growth rate as the firm becomes larger
 - Keep track of absolute revenues to make sure that the growth is feasible
 - Estimate expected operating margins each year
 - Set a target margin that the firm will move towards
 - Adjust the current margin towards the target margin
 - Estimate the capital that needs to be invested to generate revenue growth and expected margins
 - Estimate a sales to capital ratio that you will use to generate reinvestment needs each year.

Commerce One: Revenues and Revenue Growth

Year	Growth Rate	Revenues	Operating Margin	Operating Income
Current		\$537	-79.62%	-\$428
1	50.00%	\$806	-48.17%	-\$388
2	100.00%	\$1,611	-27.21%	-\$438
3	80.00%	\$2,900	-13.23%	-\$384
4	60.00%	\$4,640	-3.91%	-\$182
5	40.00%	\$6,496	2.30%	\$149
6	35.00%	\$8,770	6.44%	\$565
7	30.00%	\$11,401	9.20%	\$1,049
8	20.00%	\$13,681	11.04%	\$1,510
9	10.00%	\$15,049	12.27%	\$1,846
10	5.00%	\$15,802	13.08%	\$2,068

Commerce One: Reinvestment Needs

Year	Revenues	ΔRevenues	Sales/Capital	Reinvestment	Capital	ROC
Current	\$537				\$2,744	
1	\$806	\$269	2.20	\$122	\$2,866	-14.14%
2	\$1,611	\$806	2.20	\$366	\$3,232	-15.30%
3	\$2,900	\$1,289	2.20	\$586	\$3,818	-11.87%
4	\$4,640	\$1,740	2.20	\$791	\$4,609	-4.76%
5	\$6,496	\$1,856	2.20	\$844	\$5,452	3.24%
6	\$8,770	\$2,274	2.20	\$1,033	\$6,486	10.36%
7	\$11,401	\$2,631	2.20	\$1,196	\$7,682	16.17%
8	\$13,681	\$2,280	2.20	\$1,036	\$8,718	14.17%
9	\$15,049	\$1,368	2.20	\$622	\$9,340	13.76%
10	\$15,802	\$752	2.20	\$342	\$9,682	14.39%
Industry average =						15%



IV. Closure in Valuation

Discounted Cashflow Valuation

Getting Closure in Valuation

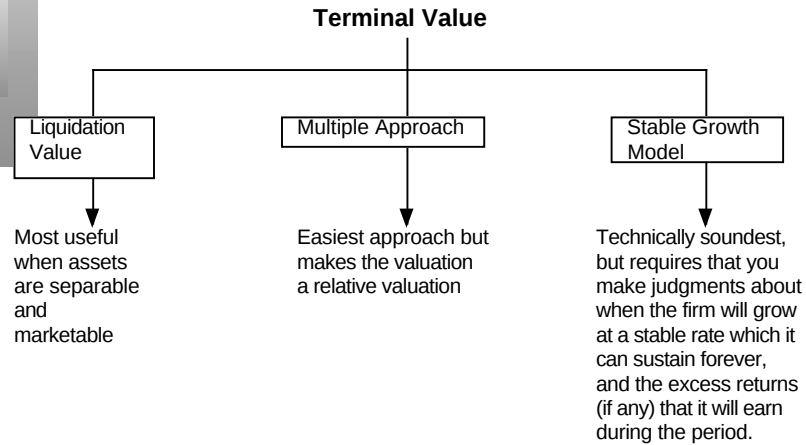
- A publicly traded firm potentially has an infinite life. The value is therefore the present value of cash flows forever.

$$\text{Value} = \sum_{t=1}^{t=\infty} \frac{CF_t}{(1+r)^t}$$

- Since we cannot estimate cash flows forever, we estimate cash flows for a “growth period” and then estimate a terminal value, to capture the value at the end of the period:

$$\text{Value} = \sum_{t=1}^{t=N} \frac{CF_t}{(1+r)^t} + \frac{\text{Terminal Value}}{(1+r)^N}$$

Ways of Estimating Terminal Value



Stable Growth and Terminal Value

- When a firm's cash flows grow at a "constant" rate forever, the present value of those cash flows can be written as:
$$\text{Value} = \text{Expected Cash Flow Next Period} / (r - g)$$

where,
 r = Discount rate (Cost of Equity or Cost of Capital)
 g = Expected growth rate
- This "constant" growth rate is called a stable growth rate and cannot be higher than the growth rate of the economy in which the firm operates.
- While companies can maintain high growth rates for extended periods, they will all approach "stable growth" at some point in time.
- When they do approach stable growth, the valuation formula above can be used to estimate the "terminal value" of all cash flows beyond.

Limits on Stable Growth

- The stable growth rate cannot exceed the growth rate of the economy but it can be set lower.
 - If you assume that the economy is composed of high growth and stable growth firms, the growth rate of the latter will probably be lower than the growth rate of the economy.
 - The stable growth rate can be negative. The terminal value will be lower and you are assuming that your firm will disappear over time.

Growth Patterns

- A key assumption in all discounted cash flow models is the period of high growth, and the pattern of growth during that period. In general, we can make one of three assumptions:
 - there is no high growth, in which case the firm is already in stable growth
 - there will be high growth for a period, at the end of which the growth rate will drop to the stable growth rate (2-stage)
 - there will be high growth for a period, at the end of which the growth rate will decline gradually to a stable growth rate (3-stage)
 - Each year will have different margins and different growth rates (n stage)

Determinants of Growth Patterns

- Size of the firm
 - Success usually makes a firm larger. As firms become larger, it becomes much more difficult for them to maintain high growth rates
- Current growth rate
 - While past growth is not always a reliable indicator of future growth, there is a correlation between current growth and future growth. Thus, a firm growing at 30% currently probably has higher growth and a longer expected growth period than one growing 10% a year now.
- Barriers to entry and differential advantages
 - Ultimately, high growth comes from high project returns, which, in turn, comes from barriers to entry and differential advantages.
 - The question of how long growth will last and how high it will be can therefore be framed as a question about what the barriers to entry are, how long they will stay up and how strong they will remain.

Stable Growth and Fundamentals

- The growth rate of a firm is driven by its fundamentals - how much it reinvests and how high project returns are. As growth rates approach “stability”, the firm should be given the characteristics of a stable growth firm.

<i>Model</i>	<i>High Growth Firms usually</i>	<i>Stable growth firms usually</i>
DDM	1. Pay no or low dividends 2. Have high risk 3. Earn high ROC	1. Pay high dividends 2. Have average risk 3. Earn ROC closer to WACC
FCFE/ FCFF	1. Have high net cap ex 2. Have high risk 3. Earn high ROC 4. Have low leverage	1. Have lower net cap ex 2. Have average risk 3. Earn ROC closer to WACC 4. Have leverage closer to industry average

The Dividend Discount Model: Estimating Stable Growth Inputs

- Consider the example of ABN Amro. Based upon its current return on equity of 15.79% and its retention ratio of 53.88%, we estimated a growth in earnings per share of 8.51%.
- Let us assume that ABN Amro will be in stable growth in 5 years. At that point, let us assume that its return on equity will be closer to the average for European banks of 15%, and that it will grow at a nominal rate of 5% (Real Growth + Inflation Rate in NV)
- The expected payout ratio in stable growth can then be estimated as follows:
$$\text{Stable Growth Payout Ratio} = 1 - g / \text{ROE} = 1 - .05 / .15 = 66.67\%$$
$$g = b (\text{ROE})$$
$$b = g / \text{ROE}$$
$$\text{Payout} = 1 - b$$

The FCFE/FCFF Models: Estimating Stable Growth Inputs

- The soundest way of estimating reinvestment rates in stable growth is to relate them to expected growth and returns on capital:
$$\text{Reinvestment Rate} = \text{Growth in Operating Income} / \text{ROC}$$
- For instance, Cisco is expected to be in stable growth 13 years from now, growing at 5% a year and earning a return on capital of 16.52% (which is the industry average). The reinvestment rate in year 13 can be estimated as follows:
$$\text{Reinvestment Rate} = 5\% / 16.52\% = 30.27\%$$
- If you are consistent about estimating reinvestment rates, you will find that it is not the stable growth rate that drives your value but your excess returns. If your return on capital is equal to your cost of capital, your terminal value will be unaffected by your stable growth assumption.

V. Beyond Inputs: Choosing and Using the Right Model

Discounted Cashflow Valuation

Summarizing the Inputs

- In summary, at this stage in the process, we should have an estimate of the
 - the current cash flows on the investment, either to equity investors (dividends or free cash flows to equity) or to the firm (cash flow to the firm)
 - the current cost of equity and/or capital on the investment
 - the expected growth rate in earnings, based upon historical growth, analysts forecasts and/or fundamentals
- The next step in the process is deciding
 - which cash flow to discount, which should indicate
 - which discount rate needs to be estimated and
 - what pattern we will assume growth to follow

Which cash flow should I discount?

- Use Equity Valuation
 - (a) for firms which have stable leverage, whether high or not, and
 - (b) if equity (stock) is being valued
- Use Firm Valuation
 - (a) for firms which have leverage which is too high or too low, and expect to change the leverage over time, because debt payments and issues do not have to be factored in the cash flows and the discount rate (cost of capital) does not change dramatically over time.
 - (b) for firms for which you have partial information on leverage (eg: interest expenses are missing..)
 - (c) in all other cases, where you are more interested in valuing the firm than the equity. (Value Consulting?)

Given cash flows to equity, should I discount dividends or FCFE?

- Use the Dividend Discount Model
 - (a) For firms which pay dividends (and repurchase stock) which are close to the Free Cash Flow to Equity (over a extended period)
 - (b) For firms where FCFE are difficult to estimate (Example: Banks and Financial Service companies)
- Use the FCFE Model
 - (a) For firms which pay dividends which are significantly higher or lower than the Free Cash Flow to Equity. (What is significant? ... As a rule of thumb, if dividends are less than 80% of FCFE or dividends are greater than 110% of FCFE over a 5-year period, use the FCFE model)
 - (b) For firms where dividends are not available (Example: Private Companies, IPOs)

What discount rate should I use?

- Cost of Equity versus Cost of Capital
 - If discounting cash flows to equity -> Cost of Equity
 - If discounting cash flows to the firm -> Cost of Capital
- What currency should the discount rate (risk free rate) be in?
 - Match the currency in which you estimate the risk free rate to the currency of your cash flows
- Should I use real or nominal cash flows?
 - If discounting real cash flows -> real cost of capital
 - If nominal cash flows -> nominal cost of capital
 - If inflation is low (<10%), stick with nominal cash flows since taxes are based upon nominal income
 - If inflation is high (>10%) switch to real cash flows

Which Growth Pattern Should I use?

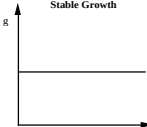
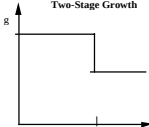
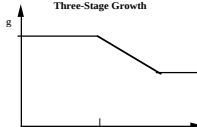
- If your firm is
 - large and growing at a rate close to or less than growth rate of the economy, *or*
 - constrained by regulation from growing at rate faster than the economy
 - has the characteristics of a stable firm (average risk & reinvestment rates)

Use a Stable Growth Model
- If your firm
 - is large & growing at a moderate rate ($\leq$ Overall growth rate + 10%) *or*
 - has a single product & barriers to entry with a finite life (e.g. patents)

Use a 2-Stage Growth Model
- If your firm
 - is small and growing at a very high rate ($>$ Overall growth rate + 10%) *or*
 - has significant barriers to entry into the business
 - has firm characteristics that are very different from the norm

Use a 3-Stage or n-stage Model

The Building Blocks of Valuation

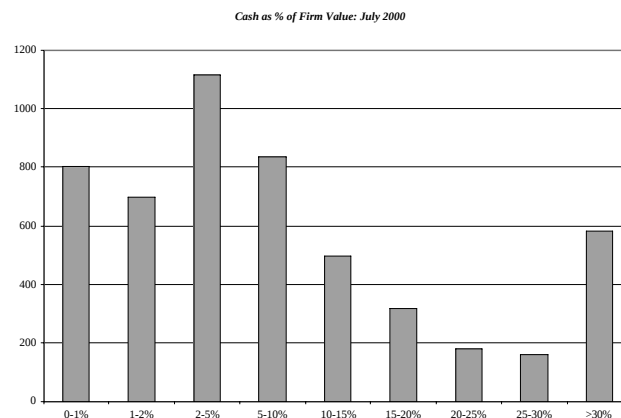
Choose a	Dividends	Cashflows to Equity	Cashflows to Firm
Cash Flow	Expected Dividends to Stockholders	Net Income - (1- δ) (Capital Exp. - Deprec'n) - (1- δ) Change in Work. Capital = Free Cash flow to Equity (FCFE) [δ = Debt Ratio]	EBIT (1- tax rate) - (Capital Exp. - Deprec'n) - Change in Work. Capital = Free Cash flow to Firm (FCFF)
& A Discount Rate	<i>Cost of Equity</i> - Basis: The riskier the investment, the greater is the cost of equity. - Models: CAPM: Riskfree Rate + Beta (Risk Premium) APM: Riskfree Rate + Σ Beta_i (Risk Premium): n factors		<i>Cost of Capital</i> $WACC = k_E (E / (D+E)) + k_D (D / (D+E))$ k_D = Current Borrowing Rate (1-t) E, D: Mkt Val of Equity and Debt
& a growth pattern	 Stable Growth	 Two-Stage Growth High Growth Stable	 Three-Stage Growth High Growth Transition Stable

6. Tying up Loose Ends

Dealing with Cash and Marketable Securities

- The simplest and most direct way of dealing with cash and marketable securities is to keep it out of the valuation - the cash flows should be before interest income from cash and securities, and the discount rate should not be contaminated by the inclusion of cash. (Use betas of the operating assets alone to estimate the cost of equity).
- Once the firm has been valued, add back the value of cash and marketable securities.
 - If you have a particularly incompetent management, with a history of overpaying on acquisitions, markets may discount the value of this cash.

How much cash is too much cash?



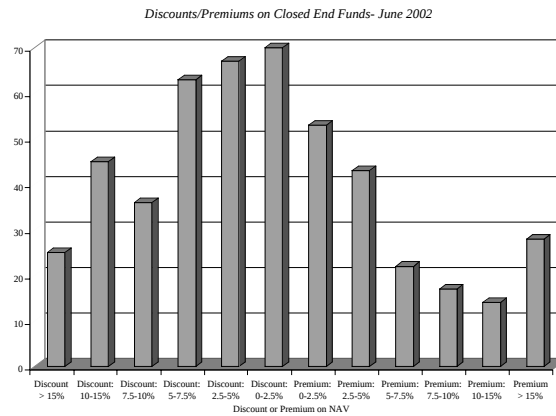
The Value of Cash

- Implicitly, we are assuming here that the market will value cash at face value. Assume now that you are buying a firm whose only asset is marketable securities worth \$ 100 million. Can you ever consider a scenario where you would not be willing to pay \$ 100 million for this firm?
 - Yes
 - No
- What is or are the scenario(s)?

The Case of Closed End Funds

- Closed end funds are mutual funds, with a fixed number of shares. Unlike regular mutual funds, where the shares have to trade at net asset value (which is the value of the securities in the fund), closed end funds shares can and often do trade at prices which are different from the net asset value.
- The average closed end fund has always traded at a discount on net asset value (of between 10 and 20%) in the United States.

Closed End Funds: Price and NAV



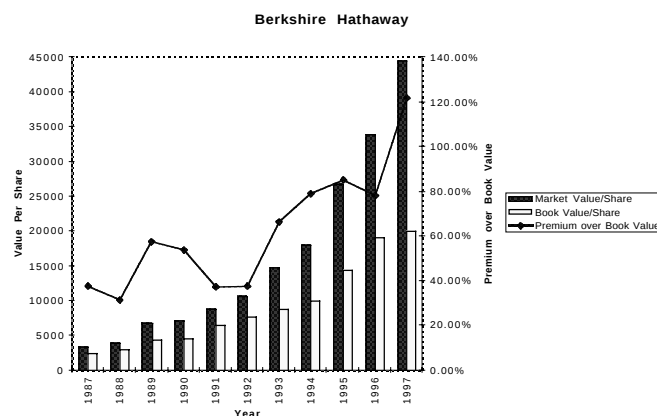
A Simple Explanation for the Closed End Discount

- Assume that you have a closed-end fund that invests in ‘average risk’ stocks. Assume also that you expect the market (average risk investments) to make 11.5% annually over the long term. If the closed end fund underperforms the market by 0.50%, estimate the discount on the fund.

A Premium for Marketable Securities

- Some closed end funds trade at a premium on net asset value. For instance, the Thai closed end funds were trading at a premium of roughly 40% on net asset value and the Indonesian fund at a premium of 80%+ on NAV on December 31, 1997. Why might an investor be willing to pay a premium over the value of the marketable securities in the fund?

Berkshire Hathaway



Dealing with Holdings in Other firms

- Holdings in other firms can be categorized into
 - Minority passive holdings, in which case only the dividend from the holdings is shown in the balance sheet
 - Minority active holdings, in which case the share of equity income is shown in the income statements
 - Majority active holdings, in which case the financial statements are consolidated.

How to value holdings in other firms

<i>Fin Statement</i>	<i>Valuing</i>	<i>What to do...</i>
Not consolidated	Equity	Value equity in subsidiary and take share of holding.
Not consolidated	Firm	Value subsidiary as a firm and add portion of firm value. Add portion of debt in subsidiary to the debt in estimating equity value.
Consolidated	Firm	Strip operating income of subsidiary and value subsidiary separately. Add portion of this value to value of parent firm.

How some deal with subsidiaries...

- When financial statements are consolidated, some analysts value the firm with the consolidated operating income and then subtract minority interests from the firm value to arrive at the value of the equity in the firm. What is wrong with this approach?

Equity Value and Per Share Value: A Test

- Assume that you have done an equity valuation of Microsoft. The total value for equity is estimated to be \$ 400 billion and there are 5 billion shares outstanding. What is the value per share?

An added fact

- In 1999, Microsoft had 500 million options outstanding, granted to employees over time. These options had an average exercise price of \$ 20 (the current stock price is \$ 80). Estimate the value per share.

Equity Value and Per Share Value

- The conventional way of getting from equity value to per share value is to divide the equity value by the number of shares outstanding. This approach assumes, however, that common stock is the only equity claim on the firm.
- In many firms, there are other equity claims as well including:
 - warrants, that are publicly traded
 - management and employee options, that have been granted, but do not trade
 - conversion options in convertible bonds
 - contingent value rights, that are also publicly traded.
- The value of these non-stock equity claims has to be subtracted from the value of equity before dividing by the number of shares outstanding.

Warrants

- A warrant is a security issued by a company that provides the holder with the right to buy a share of stock in the company at a fixed price during the life of the warrant.
- A warrant is therefore a long term call option on the equity of the firm and can be valued using option pricing models.
- Warrants and other equity options issued by the firm are claims on the equity of the firm and have to be treated as equity, which has relevance for:
 - estimating debt and equity for the leverage calculation
 - estimating per share value from total equity value

Convertible Bonds

- A convertible bond is a bond that can be converted into a pre-determined number of shares, at the option of the bond holder.
- While it generally does not pay to convert at the time of the bond issue, conversion becomes a more attractive option as stock prices increase.
- A convertible bond can be considered to be made up of two securities - a straight bond and a conversion option.
- Firms generally add conversions options to bonds to lower the interest rate paid on the bonds.

The Straight Bond Component

- Embedded in every convertible bond is a straight bond component.
- The easiest way to value the straight bond component is to act as if the conversion option does not exist and value the bond. This can be accomplished as follows:
 - Step 1: Obtain the coupon rate on the convertible bond (which will generally be low because of the conversion option)
 - Step 2: Estimate the interest rate that the company would have had to pay if it had issued a straight bond. This can be obtained either from other bonds that the company has outstanding or from its bond rating.
 - Step 3: Using the maturity of the convertible bond, the coupon rate and the market interest rate, estimate the value of the bond as:
$$\text{Value of Bond} = \text{PV of coupons at market interest rate} + \text{PV of face value of bond at market interest rate}$$
- The straight bond component is clearly debt.

Factors in Using Option Pricing Models to Value Convertibles and Warrants

- Option pricing models can be used to value the conversion option with three caveats –
 - conversion options are long term, making the assumptions about constant variance and constant dividend yields much shakier,
 - conversion options result in stock dilution, and
 - conversion options are often exercised before expiration, making it dangerous to use European option pricing models.
- These problems can be partially alleviated by using a binomial option pricing model, allowing for shifts in variance and early exercise, and factoring in the dilution effect

Steps in Getting to Value Per Share

- Step 1: Value the firm, using discounted cash flow or other valuation models.
- Step 2: Subtract out the value of the outstanding debt to arrive at the value of equity. Alternatively, skip step 1 and estimate the value of equity directly.
- Step 3: Subtract out the market value (or estimated market value) of other equity claims:
 - Value of Warrants = Market Price per Warrant * Number of Warrants : Alternatively estimate the value using option pricing model
 - Value of Conversion Option = Market Value of Convertible Bonds - Value of Straight Debt Portion of Convertible Bonds
- Step 4: Divide the remaining value of equity by the number of shares outstanding to get value per share.

An Example: Valuing Sterling Software

- The equity in Sterling Software was valued at \$2,036 million, based upon projected cash flows
- The firm has two equity options outstanding:
 - The firm has 115,000 bonds outstanding, each of which can be converted into 20 shares of stock. The market price of each convertible bond is \$1,522 and the face value is \$ 1000; coupon rate of 5.75%; expires in 8 years; Bond Rating is A-; Interest rate on comparable debt = 7.50%;
 - The firm has 1.8 million warrants outstanding, with a strike price of \$ 55 per share; these are trading at \$ 30 per share

Estimating the Value of Options

- Convertible Debt has market value of \$ 175 million; face value of \$ 115 million; coupon rate of 5.75%; expires in 8 years;
 - Bond Rating is A-; Interest rate on comparable debt = 7.50%;
 - Coupon on Convertible Debt = $.0575 * 115 \text{ million} = \$ 6.6125 \text{ million}$
 - Value of Straight Debt Portion of Convertible Debt = \$ 6.6125 (PV of Annuity, 7.5%, 8 years) + $\$ 115 \text{ million} / 1.075^8 = \$ 103.21 \text{ million}$
 - Value of Conversion Option in Debt = Market Value of Convertible Debt - Straight Debt Portion = $\$ 175 - \$ 103 = \$ 72 \text{ million}$:Equity
- Value of Warrants = Number of warrants * Warrant Price = $1.8 \text{ million warrants} * \$ 30 = \$ 54 \text{ million}$

Value Per Share: Sterling Software

Value of Equity	= \$2,036 million
- Value of Equity in Convertible Debt	= \$ 72 million
- Value of Equity in Warrants	= \$ 54 million
Value of Equity in Common Stock	= \$ 1,910 million
/ Number of Shares outstanding	= 25.50 million
Value per Share	= \$ 74.90

A Comparison to Other Approaches

- The Conservative Approach: Estimate the total number of shares outstanding, including those in the options.

$$\begin{aligned}\text{Value of Equity per share} &= \text{Value of Equity} / \text{Fully diluted \# of shares} \\ &= \$ 2,036 / (25.50 + 2.3 + 1.8) = \$ 68.78\end{aligned}$$

- The Treasury Stock Approach: Add the expected proceeds from exercise to the numerator before dividing by the number of shares outstanding:

$$\begin{aligned}\text{Value of Equity per share} &= (\text{Value of Equity} + \text{Proceeds from Exercise}) / \\ &\quad \text{Fully diluted number of shares} \\ &= (2036 + 115 + 1.8 \cdot 55) / (25.5 + 2.3 + 1.8) = \$ 76.01\end{aligned}$$

Valuations

Aswath Damodaran

Companies Valued

<i>Company</i>	<i>Model Used</i>	<i>Remarks</i>
Con Ed	Stable DDM	Dividends=FCFE, Stable D/E, Low g
ABN Amro	2-Stage DDM	FCFE=?, Regulated D/E, $g > \text{Stable}$
S&P 500	2-Stage DDM	Collectively, market is an investment
Sony	Stable FCFE	Understated Earnings?
Nestle	2-Stage FCFE	Dividends $\neq$ FCFE, Stable D/E, High g
Tsingtao	3-Stage FCFE	Dividends $\neq$ FCFE, Stable D/E, High g
DaimlerChrysler	Stable FCFF	Normalized Earnings; Stable Sector
The Home Depot	2-stage FCFF	Capitalizing Operating Leases
Global Crossing	2-stage FCFF	Dealing with Distress
Amazon.com	n-stage FCFF	Varying margins over time

General Information

- The risk premium that I will be using in the latest valuations for mature equity markets is 4%. This is the average implied equity risk premium from 1960 to 2002.
- For the valuations from 1998 and earlier, I use a risk premium of 5.5%.

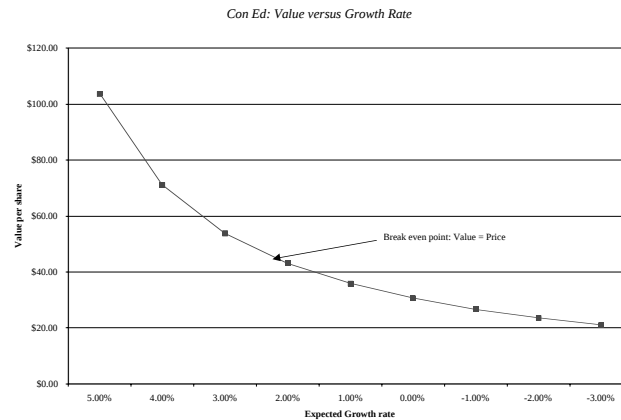
Con Ed: Rationale for Model

- The firm is in stable growth; based upon size and the area that it serves. Its rates are also regulated; It is unlikely that the regulators will allow profits to grow at extraordinary rates.
- Firm Characteristics are consistent with stable, DDM model firm
 - The beta is 0.80 and has been stable over time.
 - The firm is in stable leverage.
 - The firm pays out dividends that are roughly equal to FCFE.
 - Average Annual FCFE between 1997 and 2002 = \$585 million
 - Average Annual Dividends between 1997 and 2002 = \$ 577 million
 - Dividends as % of FCFE = 99%

Con Ed: A Stable Growth DDM: December 31, 2002

- Earnings per share for 2002 = \$ 3.10 (Fourth quarter estimate used)
- Dividend Payout Ratio over 2002 = 71.61%
- Dividends per share for 2002 = \$2.22
- Expected Growth Rate in Earnings and Dividends = 3%
- Con Ed Beta = 0.80 (Bottom-up beta estimate)
- Cost of Equity = $4.05\% + 0.80 \times 4\% = 7.20\%$
Value of Equity per Share = $\$2.22 \times 1.03 / (.072 - .03) = \$ 53.80$
The stock was trading at \$ 42.90 on December 31, 2002

Con Ed: Break Even Growth Rates



Estimating Implied Growth Rate

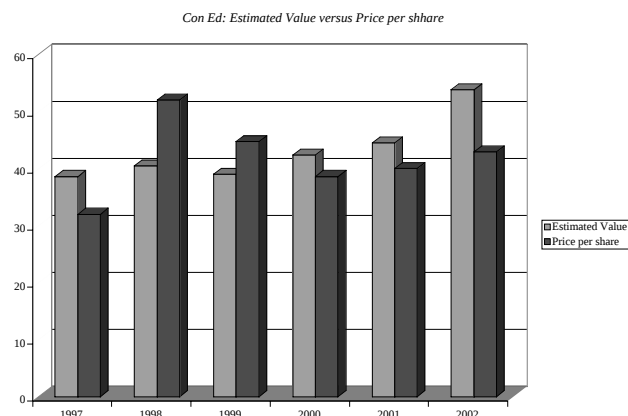
- To estimate the implied growth rate in Con Ed's current stock price, we set the market price equal to the value, and solve for the growth rate:
 - Price per share = \$ 42.90 = $\$2.22 \times (1+g) / (.072 - g)$
 - Implied growth rate = 1.96%
- Given its retention ratio of 28.39% and its return on equity in 2002 of 11%, the fundamental growth rate for Con Ed is:

$$\text{Fundamental growth rate} = (.2839 \times .11) = 3.12\%$$
- You could also frame the question in terms of a break-even return on equity.
 - Break even Return on equity = $g / \text{Retention ratio} = .0196 / .2839 = 6.91\%$

Implied Growth Rates and Valuation Judgments

- When you do any valuation, there are three possibilities. The first is that you are right and the market is wrong. The second is that the market is right and that you are wrong. The third is that you are both wrong. In an efficient market, which is the most likely scenario?
- Assume that you invest in a misvalued firm, and that you are right and the market is wrong. Will you definitely profit from your investment?
 - ☐ Yes
 - ☐ No

Con Ed: A Look Back



ABN Amro: Rationale for 2-Stage DDM

- As a financial service institution, estimating FCFE or FCFF is very difficult.
- The expected growth rate based upon the current return on equity of 16% and a retention ratio of 42% is 6.7%. This is higher than what would be a stable growth rate (roughly 4% in Euros)

ABN Amro: Summarizing the Inputs

- Market Inputs
 - Long Term Riskfree Rate (in Euros) = 4.95%
 - Risk Premium = 4% (U.S. premium : Netherlands is AAA rated)
 - Current Earnings Per Share = 1.54 Eur; Current DPS = 0.90 Eur;
- | Variable | High Growth Phase | Stable Growth Phase |
|------------------|--------------------------|------------------------------|
| Length | 5 years | Forever after yr 5 |
| Return on Equity | 16.00% | 8.95% (Set = Cost of equity) |
| Payout Ratio | 58.44% | 55.31% (1 - 4/8.95) |
| Retention Ratio | 41.56% | 44.69% (b=g/ROE=4/8.95) |
| Expected growth | .16*.4156=.0665 | 4% (Assumed) |
| Beta | 0.95 | 1.00 |
| Cost of Equity | 4.95%+0.95(4%)
=8.75% | 4.95%+1.00(4%)
= 8.95% |

ABN Amro: Valuation

Year	EPS	DPS	PV at 8.75%
1	1.64	0.96	0.88
2	1.75	1.02	0.87
3	1.87	1.09	0.85
4	1.99	1.16	0.83
5	2.12	1.24	0.82

Expected EPS in year 6 = $2.12(1.04) = 2.21$ Eur

Expected DPS in year 6 = $2.21 * 0.5531 = 1.22$ Eur

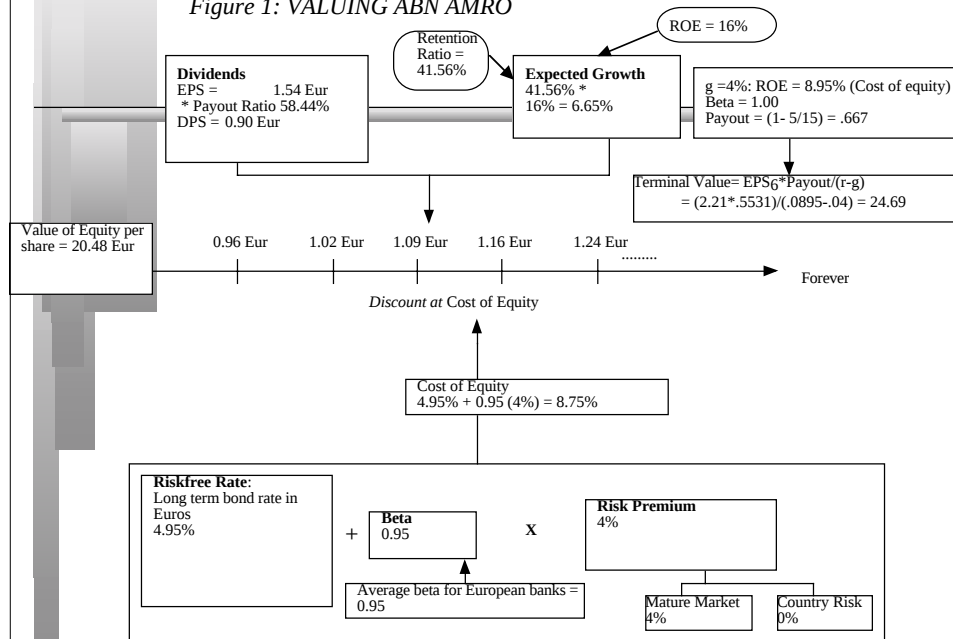
Terminal Price (in year 5) = $1.22 / (.0895 - .04) = 24.69$ Eur

PV of Terminal Price = $24.69 / (1.0875)^5 = 16.23$ Eur

Value Per Share = $0.88 + 0.87 + 0.85 + 0.83 + 0.82 + 16.23 = 20.48$ Eur

The stock was trading at 15.99 Euros on December 31, 2002

Figure 1: VALUING ABN AMRO



The Value of Growth

- In any valuation model, it is possible to extract the portion of the value that can be attributed to growth, and to break this down further into that portion attributable to “high growth” and the portion attributable to “stable growth”. In the case of the 2-stage DDM, this can be accomplished as follows:

$$P_0 = \underbrace{\left\{ \sum_{t=1}^n \frac{DPS_t}{(1+r)^t} + \frac{P_n}{(1+r)^n} \right\}}_{\text{Value of High Growth}} - \underbrace{\frac{DPS_0 * (1+g_n)}{(r-g_n)}}_{\text{Value of Stable Growth}} + \underbrace{\frac{DPS_0}{r}}_{\text{Assets in Place}}$$

DPS_t = Expected dividends per share in year t

r = Cost of Equity

P_n = Price at the end of year n

g_n = Growth rate forever after year n

ABN Amro: Decomposing Value

- Value of Assets in Place = Current DPS/Cost of Equity
= 0.90 Euros/.0895
= 10.06 Euros
- Value of Stable Growth = 0.90 (1.04)/(.0895-.04) - 10.06 Euros
= 8.85 Euros

(A more precise estimate would have required us to use the stable growth payout ratio to re-estimate dividends)

- Value of High Growth = Total Value - (10.06+8.85)
= 20.48 - (10.06+8.85) = 1.57 Euros

S & P 500: Rationale for Use of Model

- While markets overall generally do not grow faster than the economies in which they operate, there is reason to believe that the earnings at U.S. companies (which have outpaced nominal GNP growth over the last 5 years) will continue to do so in the next 5 years. The consensus estimate of growth in earnings (from Zacks) is roughly 8% (with top-down estimates)
- Though it is possible to estimate FCFE for many of the firms in the S&P 500, it is not feasible for several (financial service firms). The dividends during the year should provide a reasonable (albeit conservative) estimate of the cash flows to equity investors from buying the index.

S & P 500: Inputs to the Model (12/31/02)

- General Inputs
 - Long Term Government Bond Rate = 4.05%
 - Risk Premium for U.S. Equities = 4%
 - Current level of the Index = 879.82
- Inputs for the Valuation

	High Growth Phase	Stable Growth Phase
Length	5 years	Forever after year 5
Dividend Yield	1.81%	1.81%
Expected Growth	8%	4.05% (Nominal g)
Beta	1.00	1.00

S & P 500: 2-Stage DDM Valuation

	1	2	3	4	5
Expected Dividends =	\$17.24	\$18.62	\$20.11	\$21.71	\$23.45
Expected Terminal Value =					\$610.01
Present Value =	\$15.95	\$15.95	\$15.94	\$15.93	\$430.12
Intrinsic Value of Index =	\$493.89				

Cost of Equity = $4.05\% + 1(4\%) = 8.05\%$

Terminal Value = $23.45 * 1.0405 / (.0805 - .0405) = 610.01$

Explaining the Difference

- The index is at 880, while the model valuation comes in at 494. This indicates that one or more of the following has to be true.
 - The dividend discount model understates the value because dividends are less than FCFE.
 - The expected growth in earnings over the next 5 years will be much higher than 8%.
 - The risk premium used in the valuation (4%) is too high
 - The market is overvalued.

A More Realistic Valuation of the Index

- We estimated the free cashflows to equity for each firm in the index and averaged the free cashflow to equity as a percent of market cap. The average FCFE yield for the index was about 3.15% in 2001.
- The average implied risk premium between 1960 and 2002 of 4% is used in the valuation.
- With these inputs in the model:

	1	2	3	4	5
Expected Dividends =	\$29.93	\$32.33	\$34.91	\$37.71	\$40.72
Expected Terminal Value =					\$1,059.27
Present Value =	\$27.70	\$27.69	\$27.68	\$27.66	\$746.90
Intrinsic Value of Index =	\$857.63				

At a level of 880, the market is overvalued by about 3%.

Sony: Background on Japanese firms

- Japanese firms have proved to be among the most difficult to value for several reasons:
 - The earnings in 2001 for most Japanese firms was depressed relative to earnings earlier in the decade and in the 1980s, reflecting the Japanese economy.
 - Japanese accounting standards tend to understate earnings and overstate book value of equity, as firms are allowed to set aside provisions for unspecified expenses
 - The earnings of many export oriented Japanese firms tends to be heavily influenced by exchange rate movements
 - The cross holdings that Japanese firms have in other firms, and the lack of transparency in these holdings, makes it difficult to value these holdings.

Valuing Sony: August 2000

- Sony had net income of 31 billion JPY in 1999, down from 76 billion JPY in 1997 and 38 billion in 1998. The return on equity at Sony dropped from 5.25% in 1997 to 2.13% in 1999. The firm paid out dividends of 21 billion JPY in 1999.
- Capital expenditures in 1999 amounted to 103 billion JPY, whereas depreciation is 76 billion JPY.
- Non-cash working capital at Sony in 1999 was 220 billion JPY on revenues of 2593 billion yet, yielding a non-cash working capital to revenue ratio of 8.48%.
- The long term government bond rate in Japan was 2% at the time of this valuation.

Sony: Rationale for Model

- We will normalize earnings to reflect the fact that current earnings are depressed. To normalize earnings, we will use the return on equity of 5.25%, which is the return on equity that Sony had last year and is close to return on equity it used to earn in the early 1990s.
- We will assume that the firm's dominant market share will keep it from posting high growth. Over the last 5 years, the growth rate in revenues has been 3.5%. We will assume a long term stable growth rate of 3% (higher than the Japanese economy due to global exposure)
- We will assume that the net capital expenditures will grow at the same rate and that non-cash working capital will stay at 8.48% of revenues
- Sony's current book debt to capital ratio is 25.8%; we will assume that they will finance reinvestment with this ratio (rather than the market value)
- We will use a beta of 1.10, to reflect the unlevered beta of electronic firms (globally) and Sony's market value debt to equity ratio (16%)

Estimating the Inputs

- Normalized Earnings:
 - Book Value of Equity (3/1999) = 1795 billion JPY
 - Estimated Return on Equity = 5.25%
 - Normalized Net Income next year = 1795 billion * .0525 = 94.24 billion
- Reinvestment Needs
 - Current Net Capital Expenditures = (103 - 76) = 27 billion JPY
 - Expected Net Capital Expenditures = 27 billion (1.03) = 27.81 billion
 - Current Revenues = 2593 billion
 - Expected Revenues next year = 2593(1.03) = 2671 billion
 - Expected Change in non-cash Working Capital = (2671 - 2593)*.0848 = 6.60 billion JPY
- Book Value Debt Ratio = 25.8%
- Cost of Equity = 2% + 1.10 (4%) = 6.40%

The Valuation

- Expected FCFE next year

Expected Net Income	= 94.24 billion
- (Net Cap Ex) (1- Debt Ratio)= 27.81 (1-.258)	= 20.64
- (Δ Non-cash WC) (1-Debt ratio) = 6.6 (1-.258)	= 4.89
FCFE	=68.71 billion JPY
 - Valuation
 - Cost of Equity = 6.4%; Stable growth rate = 3%;
 - Value of Equity = 68.71 billion / (.064 - .03) = 2021 billion JPY
- Sony was trading at a market value of equity of 7146 billion JPY

The Effect of Cross-holdings

- When firms have minority passive holdings in other companies, they report only the dividends they receive from these holdings as part of net income.
- Consequently, we tend to understate the value of these crossholdings in valuations.
- To value them right, we have to estimate the value of the companies in which these holdings are, and then take the percentage of the value of these firms owned by the firm you are valuing.

Nestle: Rationale for Using Model - January 2001

- Earnings per share at the firm has grown about 5% a year for the last 5 years, but the fundamentals at the firm suggest growth in EPS of about 11%. (Analysts are also forecasting a growth rate of 12% a year for the next 5 years)
- Nestle has a debt to capital ratio of about 37.6% and is unlikely to change that leverage materially. (How do I know? I do not. I am just making an assumption.)
- Like many large European firms, Nestle has paid less in dividends than it has available in FCFE.

Nestle: Summarizing the Inputs

■ General Inputs

- Long Term Government Bond Rate (Sfr) = 4%
- Current EPS = 108.88 Sfr; Current Revenue/share =1,820 Sfr
- Capital Expenditures/Share=114.2 Sfr; Depreciation/Share=73.8 Sfr

	<i>High Growth</i>	<i>Stable Growth</i>
Length	5 years	Forever after yr 5
Beta	0.85	0.85
Return on Equity	23.63%	16%
Retention Ratio	65.10% (Current)	NA
Expected Growth	15.38%	5.00%
WC/Revenues	9.30% (Existing)	9.30% (Grow with earnings)
Debt Ratio	37.60%	37.60%
Cap Ex/Deprecn	Current Ratio	150%

Estimating the Risk Premium for Nestle

	<i>Revenues</i>	<i>Weight</i>	<i>Risk Premium</i>
North America	17.5	24.82%	4.00%
South America	4.3	6.10%	12.00%
Switzerland	1.1	1.56%	4.00%
Germany/France/UK	18.4	26.10%	4.00%
Italy/Spain	6.4	9.08%	5.50%
Asia	5.8	8.23%	9.00%
Rest of W. Europe	13	18.44%	4.00%
Eastern Europe	4	5.67%	8.00%
Total	70.5	100.00%	5.26%

- The risk premium that we will use in the valuation is 5.26%
- Cost of Equity = 4% + 0.85 (5.26%) = 8.47%

Nestle: Valuation

	1	2	3	4	5
Earnings	\$125.63	\$144.95	\$167.25	\$192.98	\$222.66
- (Net CapEX)*(1-DR)	\$29.07	\$33.54	\$38.70	\$44.65	\$51.52
-Δ WC*(1-DR)	\$16.25	\$18.75	\$21.63	\$24.96	\$28.79
Free Cashflow to Equity	\$80.31	\$92.67	\$106.92	\$123.37	\$142.35
Present Value	\$74.04	\$78.76	\$83.78	\$89.12	\$94.7

Earnings per Share in year 6 = $222.66(1.05) = 231.57$

Net Capital Ex₆ = Deprec'n₆ * 0.50 = $73.8(1.1538)^5(1.05)(.5) = 78.5$ Sfr

Chg in WC₆ = $(Rev_6 - Rev_5)(.093) = 1820(1.1538)^5(.05)(.093) = 13.85$ Sfr

FCFE₆ = $231.57 - 78.5(1-.376) - 13.85(1-.376) = 173.93$ Sfr

Terminal Value per Share = $173.93/(.0847-.05) = 3890.16$ Sfr

Value = $\$74.04 + \$78.76 + \$83.78 + \$89.12 + \$94.7 + 3890/(1.0847)^5 = 3011$ Sfr

The stock was trading 2906 Sfr on December 31, 1999

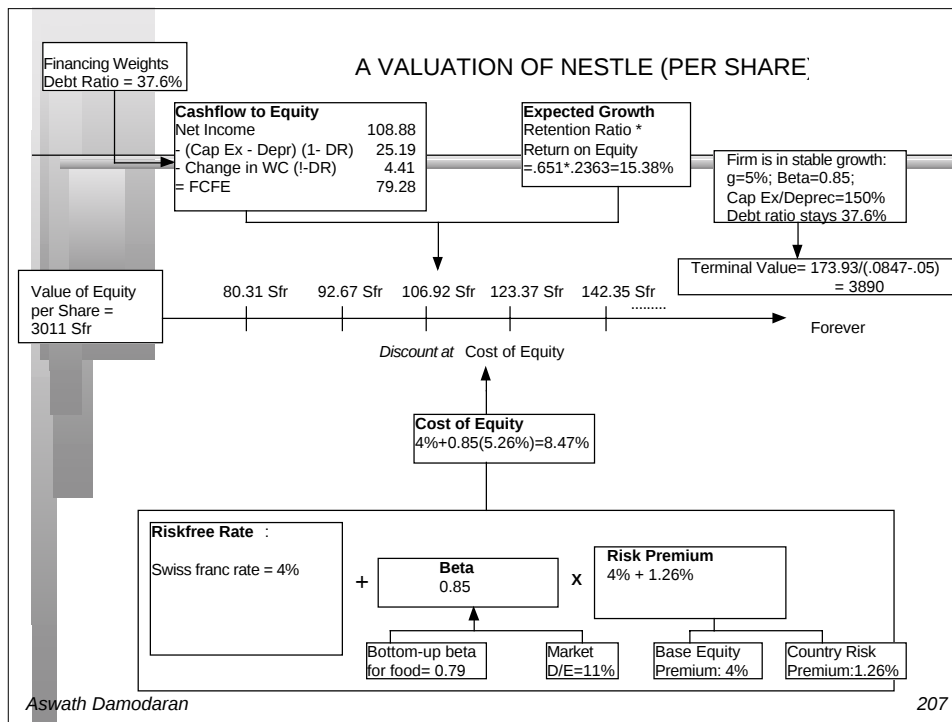
Nestle: The Net Cap Ex Assumption

- In our valuation of Nestle, we assumed that cap ex would be 150% of depreciation in steady state. If, instead, we had assumed that net cap ex was zero, as many analysts do, the terminal value would have been:

FCFE₆ = $231.57 - 13.85(1-.376) = 222.93$ Sfr

Terminal Value per Share = $222.93/(.0847-.05) = 4986$ Sfr

Value = $\$74.04 + \$78.76 + \$83.78 + \$89.12 + \$94.7 + 4986/(1.0847)^5 = 3740.91$ Sfr



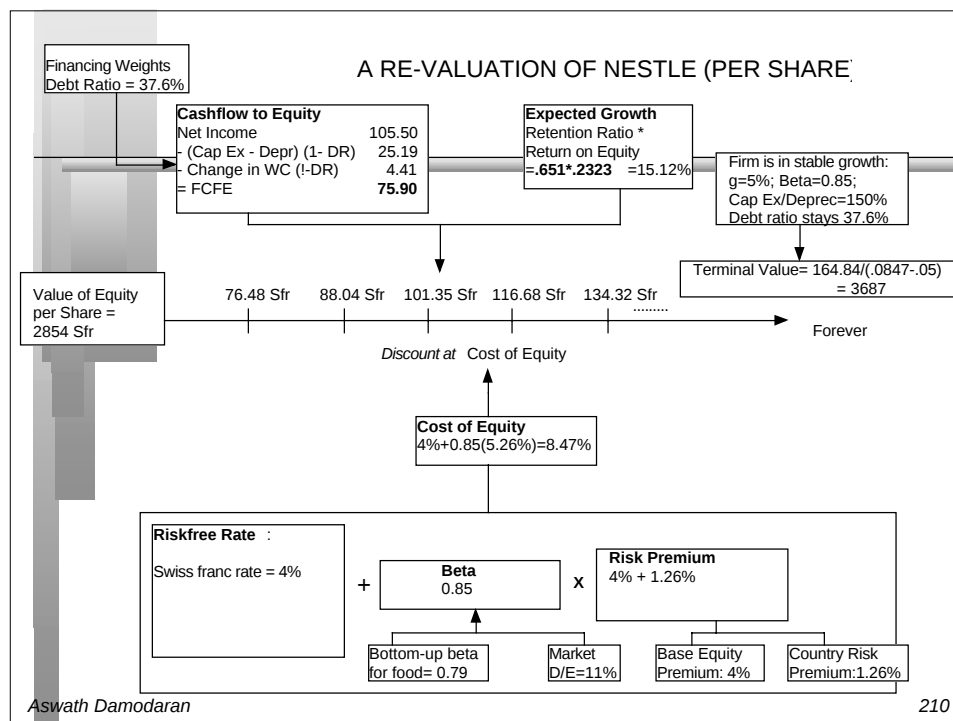
The Effects of New Information on Value

- No valuation is timeless. Each of the inputs to the model are susceptible to change as new information comes out about the firm, its competitors and the overall economy.
 - Market Wide Information
 - Interest Rates
 - Risk Premiums
 - Economic Growth
 - Industry Wide Information
 - Changes in laws and regulations
 - Changes in technology
 - Firm Specific Information
 - New Earnings Reports
 - Changes in the Fundamentals (Risk and Return characteristics)

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Nestle: Effects of an Earnings Announcement

- Assume that Nestle makes an earnings announcement which includes two pieces of news:
 - The earnings per share come in lower than expected. The base year earnings per share will be 105.5 Sfr instead of 108.8 Sfr.
 - Increased competition in its markets is putting downward pressure on the net profit margin. The after-tax margin, which was 5.98% in the previous analysis, is expected to shrink to 5.79%.
- There are two effects on value:
 - The drop in earnings will make the projected earnings and cash flows lower, even if the growth rate remains the same
 - The drop in net margin will make the return on equity lower (assuming turnover ratios remain unchanged). This will reduce expected growth.



Tsingtao Breweries: Rationale for Using Model: June 2001

- *Why three stage?* Tsingtao is a small firm serving a huge and growing market – China, in particular, and the rest of Asia, in general. The firm's current return on equity is low, and we anticipate that it will improve over the next 5 years. As it increases, earnings growth will be pushed up.
- *Why FCFE?* Corporate governance in China tends to be weak and dividends are unlikely to reflect free cash flow to equity. In addition, the firm consistently funds a portion of its reinvestment needs with new debt issues.

Background Information

- In 2000, Tsingtao Breweries earned 72.36 million CY (Chinese Yuan) in net income on a book value of equity of 2,588 million CY, giving it a return on equity of 2.80%.
- The firm had capital expenditures of 335 million CY and depreciation of 204 million CY during the year.
- The working capital changes over the last 4 years have been volatile, and we normalize the change using non-cash working capital as a percent of revenues in 2000:

$$\text{Normalized change in non-cash working capital} = \frac{(\text{Non-cash working capital}_{2000} / \text{Revenues}_{2000}) - (\text{Non-cash working capital}_{1999} / \text{Revenues}_{1999})}{\text{Revenues}_{2000} - \text{Revenues}_{1999}} = (180/2253) - (1598/2253) = 52.3 \text{ million CY}$$

$$\text{Normalized Reinvestment} = \text{Capital expenditures} - \text{Depreciation} + \text{Normalized Change in non-cash working capital}$$

$$= 335 - 204 + 52.3 = 183.3 \text{ million CY}$$
- As with working capital, debt issues have been volatile. We estimate the firm's book debt to capital ratio of 40.94% at the end of 1999 and use it to estimate the normalized equity reinvestment in 2000.

Inputs for the 3 Stages

	High Growth	Transition Phase	Stable Growth
Length	5 years	5 years	Forever after yr 10
Beta	0.75	Moves to 0.80	0.80
Risk Premium	4%+2.28%	-->	4+0.95%
ROE	2.8%->12%	12%->20%	20%
Equity Reinv.	149.97%	Moves to 50%	50%
Expected Growth	44.91%	Moves to 10%	10%

■ We will assume that

Equity Reinvestment Ratio = Reinvestment (1- Debt Ratio) / Net Income
 = 183.3 (1-.4094) / 72.36 = 149.97%

Expected growth rate- next 5 years

= Equity reinvestment rate * $ROE_{New} + [1 + (ROE_5 - ROE_{today}) / ROE_{today}]^{1/5} - 1$
 = 1.4997 * .12 + [(1 + (.12 - .028) / .028)^{1/5} - 1] = 44.91%

Tsingtao: Projected Cash Flows

Year	Expected Growth	Net Income	Equity Reinvestment Rate	FCFE	Cost of Equity	Present Value
Current		CY72.36	149.97%			
1	44.91%	CY104.85	149.97%	(CY52.40)	14.71%	(CY45.68)
2	44.91%	CY151.93	149.97%	(CY75.92)	14.71%	(CY57.70)
3	44.91%	CY220.16	149.97%	(CY110.02)	14.71%	(CY72.89)
4	44.91%	CY319.03	149.97%	(CY159.43)	14.71%	(CY92.08)
5	44.91%	CY462.29	149.97%	(CY231.02)	14.71%	(CY116.32)
6	37.93%	CY637.61	129.98%	(CY191.14)	14.56%	(CY84.01)
7	30.94%	CY834.92	109.98%	(CY83.35)	14.41%	(CY32.02)
8	23.96%	CY1,034.98	89.99%	CY103.61	14.26%	CY34.83
9	16.98%	CY1,210.74	69.99%	CY363.29	14.11%	CY107.04
10	10.00%	CY1,331.81	50.00%	CY665.91	13.96%	CY172.16
Sum of the present values of FCFE during high growth =						(\$186.65)

Tsingtao: Terminal Value

- Expected stable growth rate = 10%
- Equity reinvestment rate in stable growth = 50%
- Cost of equity in stable growth = 13.96%
- Expected FCFE in year 11
 - = $\text{Net Income}_{11} \times (1 - \text{Stable period equity reinvestment rate})$
 - = $\text{CY } 1331.81 (1.10)(1-.5) = \text{CY } 732.50 \text{ million}$
- Terminal Value of equity in Tsingtao Breweries
 - = $\text{FCFE}_{11} / (\text{Stable period cost of equity} - \text{Stable growth rate})$
 - = $732.5 / (.1396 - .10) = \text{CY } 18,497 \text{ million}$

Tsingtao: Valuation

- Value of Equity
 - = PV of FCFE during the high growth period + PV of terminal value
 - = $\text{CY } 186.65 + \text{CY } 18,497 / (1.1471^5 \times 1.1456 \times 1.1441 \times 1.1426 \times 1.1411 \times 1.1396)$
 - = $\text{CY } 4,596 \text{ million}$
- Value of Equity per share = Value of Equity / Number of Shares
 - = $\text{CY } 4,596 / 653.15 = \text{CY } 7.04 \text{ per share}$
- The stock was trading at 10.10 Yuan per share, which would make it overvalued, based upon this valuation.

DaimlerChrysler: Rationale for Model

June 2000

- DaimlerChrysler is a mature firm in a mature industry. We will therefore assume that the firm is in stable growth.
- Since this is a relatively new organization, with two different cultures on the use of debt (Daimler has traditionally been more conservative and bank-oriented in its use of debt than Chrysler), the debt ratio will probably change over time. Hence, we will use the FCFF model.

Daimler Chrysler: Inputs to the Model

- In 1999, Daimler Chrysler had earnings before interest and taxes of 9,324 million DM and had an effective tax rate of 46.94%.
- Based upon this operating income and the book values of debt and equity as of 1998, DaimlerChrysler had an after-tax return on capital of 7.15%.
- The market value of equity is 62.3 billion DM, while the estimated market value of debt is 64.5 billion
- The bottom-up unlevered beta for automobile firms is 0.61, and Daimler is AAA rated.
- The long term German bond rate is 4.87% (in DM) and the mature market premium of 4% is used.
- We will assume that the firm will maintain a long term growth rate of 3%.

Daimler/Chrysler: Analyzing the Inputs

- Expected Reinvestment Rate = $g / \text{ROC} = 3\% / 7.15\% = 41.98\%$
- Cost of Capital
 - Bottom-up Levered Beta = $0.61 (1 + (1 - .4694)(64.5/62.3)) = 0.945$
 - Cost of Equity = $4.87\% + 0.945 (4\%) = 8.65\%$
 - After-tax Cost of Debt = $(4.87\% + 0.20\%) (1 - .4694) = 2.69\%$
 - Cost of Capital = $8.65\%(62.3/(62.3+64.5)) + 2.69\% (64.5/(62.3+64.5)) = 5.62\%$

Daimler Chrysler Valuation

- Estimating FCFF
 - Expected EBIT $(1-t) = 9324 (1.03) (1 - .4694) = 5,096$ mil DM
 - Expected Reinvestment needs = $5,096(.42) = 2,139$ mil DM
 - Expected FCFF next year = $2,957$ mil DM
- Valuation of Firm
 - Value of operating assets = $2957 / (.056 - .03) = 112,847$ mil DM
 - + Cash + Marketable Securities = $18,068$ mil DM
 - Value of Firm = $130,915$ mil DM
 - Debt Outstanding = $64,488$ mil DM
 - Value of Equity = $66,427$ mil DM

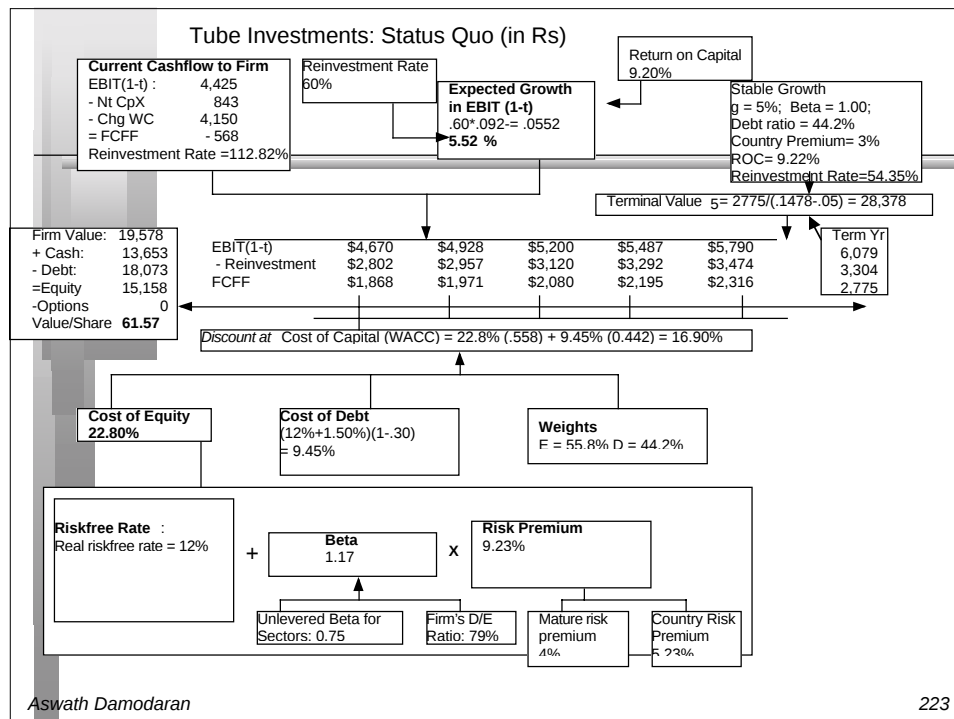
Value per Share = 72.7 DM per share
Stock was trading at 62.2 DM per share on June 1, 2000

Circular Reasoning in FCFF Valuation

- In discounting FCFF, we use the cost of capital, which is calculated using the market values of equity and debt. We then use the present value of the FCFF as our value for the firm and derive an estimated value for equity. Is there circular reasoning here?
 - Yes
 - No
- If there is, can you think of a way around this problem?

Tube Investment: Rationale for Using 2-Stage FCFF Model - June 2000

- Tube Investments is a diversified manufacturing firm in India. While its growth rate has been anemic, there is potential for high growth over the next 5 years.
- The firm's financing policy is also in a state of flux as the family running the firm reassesses its policy of funding the firm.



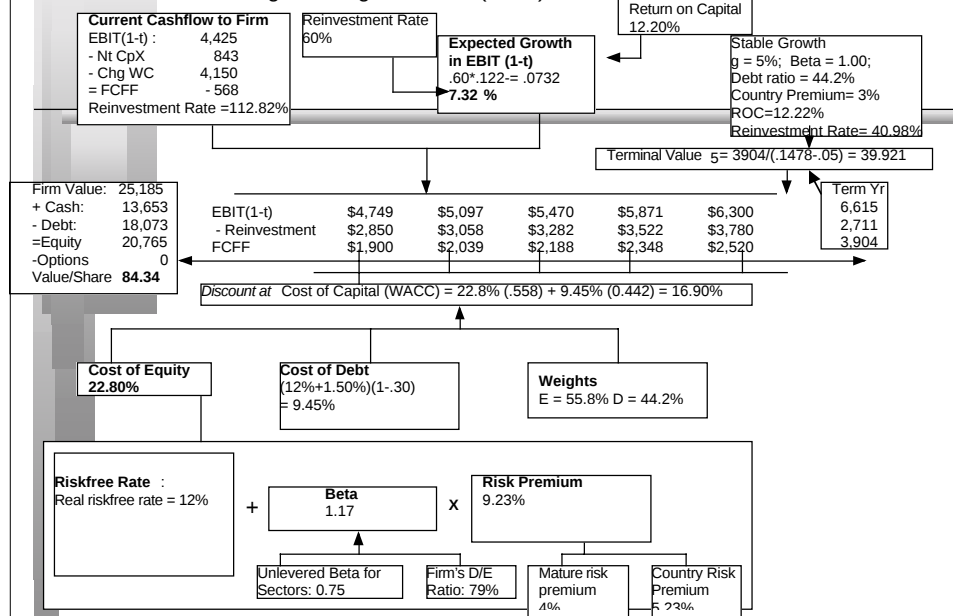
Stable Growth Rate and Value

- In estimating terminal value for Tube Investments, I used a stable growth rate of 5%. If I used a 7% stable growth rate instead, what would my terminal value be? (Assume that the cost of capital and return on capital remain unchanged.)

The Effects of Return Improvements on Value

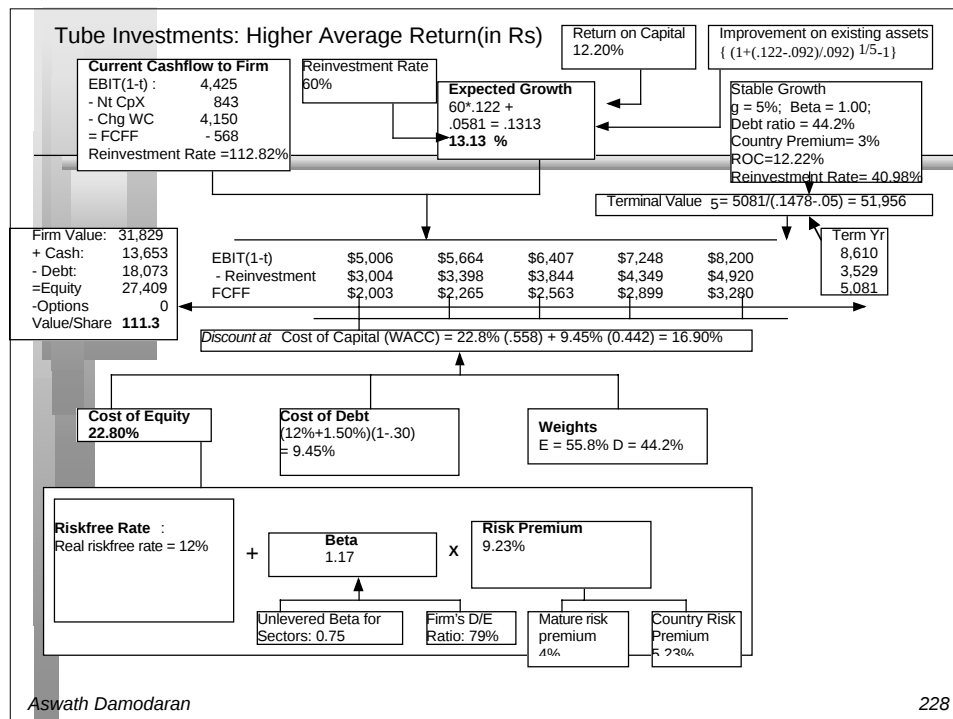
- The firm is considering changes in the way in which it invests, which management believes will increase the return on capital to 12.20% on just new investments (and not on existing investments) over the next 5 years.
- The value of the firm will be higher, because of higher expected growth.

Tube Investments: Higher Marginal Return(in Rs)



Return Improvements on Existing Assets

- If Tube Investments is also able to increase the return on capital on existing assets to 12.20% from 9.20%, its value will increase even more.
- The expected growth rate over the next 5 years will then have a second component arising from improving returns on existing assets:
- Expected Growth Rate = $.122 * .60 + \{ (1 + (.122 - .092) / .092)^{1/5} - 1 \}$
= .1313 or 13.13%



Tube Investments and Tsingtao: Should there be a corporate governance discount?

- Stockholders in Asian, Latin American and many European companies have little or no power over the managers of the firm. In many cases, insiders own voting shares and control the firm and the potential for conflict of interests is huge. Would you discount the value that you estimated to allow for this absence of stockholder power?
 - ☐ Yes
 - ☐ No.

Dealing with Operating Leases: A Valuation of the Home Depot

- The Home Depot does not carry much in terms of traditional debt on its balance sheet. However, it does have significant operating leases.
- When doing firm valuation, these operating leases have to be treated as debt. This, in turn, will mean that operating income has to get restated.

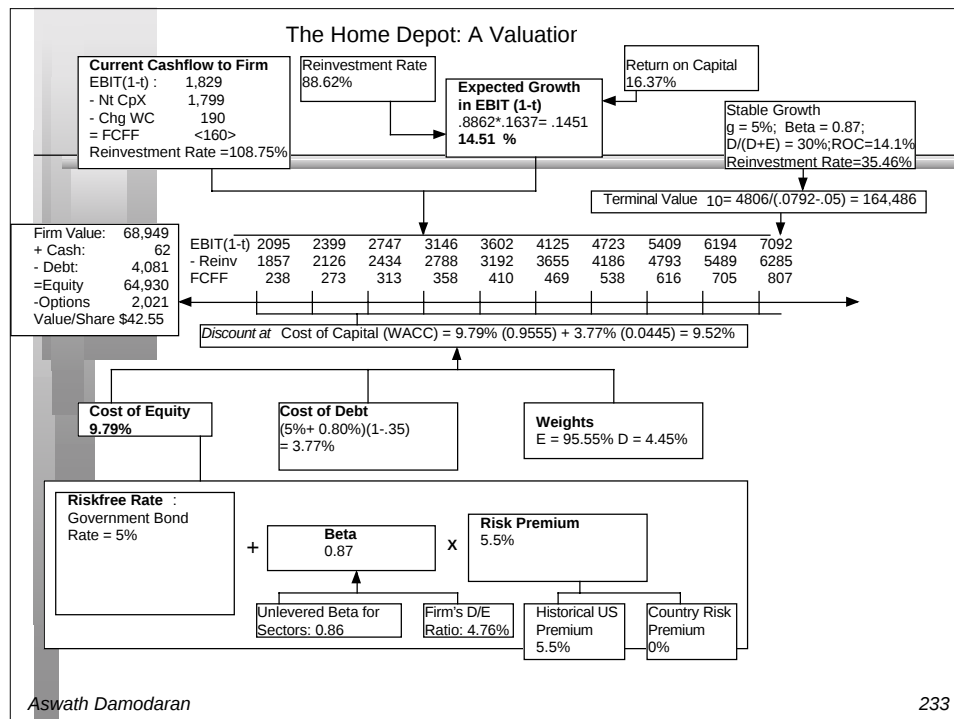
Operating Leases at The Home Depot in 1998

- The pre-tax cost of debt at the Home Depot is 5.80%

Year	Commitment	Present Value
1	\$ 294.00	\$277.88
2	\$ 291.00	\$259.97
3	\$ 264.00	\$222.92
4	\$ 245.00	\$195.53
5	\$ 236.00	\$178.03
6 and beyond	\$ 270.00	\$1,513.37
■ Debt Value of leases =		\$ 2,647.70

Other Adjustments from Operating Leases

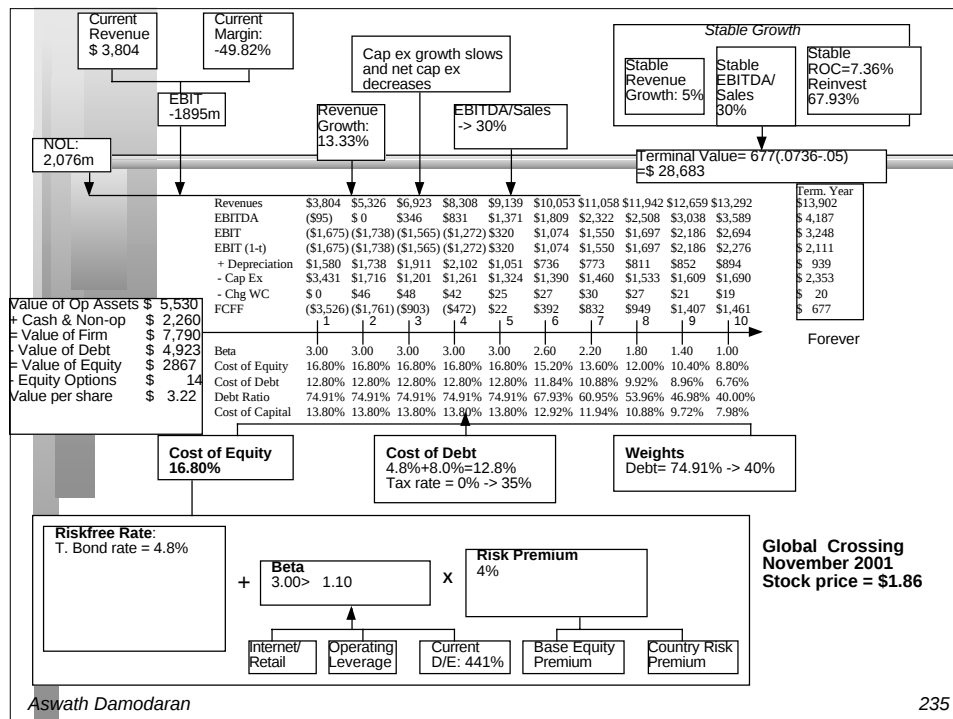
	Operating Lease Expensed	Operating Lease converted to Debt
EBIT	\$ 2,661mil	\$ 2,815 mil
EBIT (1-t)	\$1,730 mil	\$1,829 mil
Debt	\$1,433 mil	\$ 4,081 mil



Dealing with Distress

- A DCF valuation values a firm as a going concern. If there is a significant likelihood of the firm failing before it reaches stable growth and if the assets will then be sold for a value less than the present value of the expected cashflows (a distress sale value), DCF valuations will understate the value of the firm.
- Value of Equity = DCF value of equity (1 - Probability of distress) + Distress sale value of equity (Probability of distress)
- There are three ways in which we can estimate the probability of distress:
 - Use the bond rating to estimate the cumulative probability of distress over 10 years
 - Estimate the probability of distress with a probit
 - Estimate the probability of distress by looking at market value of bonds..
- The distress sale value of equity is usually best estimated as a percent of book value (and this value will be lower if the economy is doing badly and there are other firms in the same business also in distress).

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Valuing Global Crossing with Distress

■ Probability of distress

- Price of 8 year, 12% bond issued by Global Crossing = \$ 653

$$653 = \sum_{t=1}^{t=8} \frac{120(1 - \pi_{\text{Distress}})^t}{(1.05)^t} + \frac{1000(1 - \pi_{\text{Distress}})^8}{(1.05)^8}$$

- Probability of distress = 13.53% a year
- Cumulative probability of survival over 10 years = $(1 - .1353)^{10} = 23.37\%$

■ Distress sale value of equity

- Book value of capital = \$14,531 million
- Distress sale value = 15% of book value = $.15 * 14531 = \$2,180$ million
- Book value of debt = \$7,647 million
- Distress sale value of equity = \$ 0

■ Distress adjusted value of equity

- Value of Global Crossing = $\$3.22 (.2337) + \$0.00 (.7663) = \$0.75$

Dealing with R&D: Bristol Myers

- Bristol Myers, like most pharmaceutical firms, has a significant amount of research and development expenses. These expenses, though treated as operating expenditures, by accountants, are really capital expenditures.
- When R&D expenses are reclassified as capital expenditures, there is a ripple effect on the following:
 - Operating income
 - Capital Expenditures
 - Depreciation and Amortization
 - Reinvestment Rates
 - Return on Capital

Converting R&D Expenses to Capital Expenses

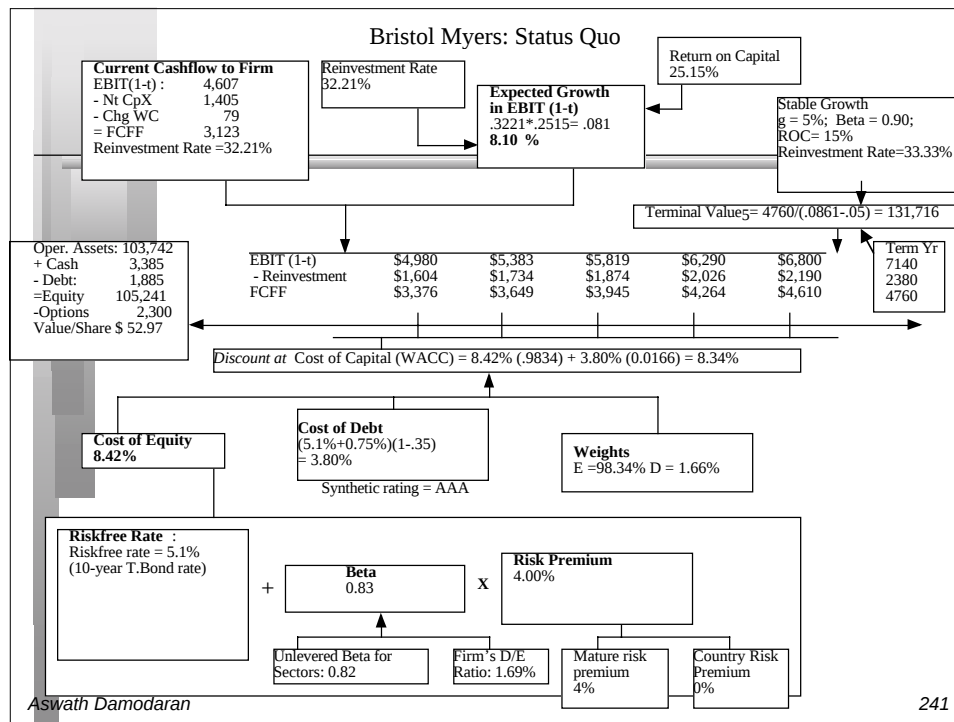
Year	R&D Expense	Unamortized portion	Amortization this year	
Current	1939.00	1.00	1939.00	
-1	1759.00	0.90	1583.10	\$175.90
-2	1577.00	0.80	1261.60	\$157.70
-3	1385.00	0.70	969.50	\$138.50
-4	1276.00	0.60	765.60	\$127.60
-5	1199.00	0.50	599.50	\$119.90
-6	1108.00	0.40	443.20	\$110.80
-7	1128.00	0.30	338.40	\$112.80
-8	1083.00	0.20	216.60	\$108.30
-9	983.00	0.10	98.30	\$98.30
-10	881.00	0.00	0.00	\$88.10
Value of Research Asset =			\$8,214.80	
Amortization this year =				\$1,237.90

The Consequences of a Research Asset

- Amortization of asset for current year = \$ 1,238 million
- Adjustment to Operating Income :
 - Add back the R& D Expenses \$1,939 million
 - Subtract out the amortization \$1,238 million
 - Increase in Operating Income \$ 701 million (Increase)
- Tax Effect of R&D Expensing
 - The entire R&D expense of \$1,939 million is tax-deductible, rather than just the amortization of \$1,238 million
 - This creates a tax benefit that can be computed as follows:
 Additional tax benefit of expensing = $(1939-1238) (.35) = \$ 245$ million

Capitalizing R& D: The Effects

	R&D expensed	R&D capitalized	Effect
EBIT =	\$ 6,009 mil	\$6,710 mil	Increase \$ 701
EBIT (1-t)	\$ 3,906 mil	\$4,607 mil	Increase \$ 701
Capital spending =	\$1,505 mil	\$ 3,444 mil	Increase \$1939
Depreciation =	\$ 801 mil	\$ 2,039 mil	Increase \$ 1238
Net Cap Ex	\$ 704mil	\$1,405 mil	Increase \$ 701
Non-cash WC Chg =	\$ 79 mil	\$ 79 mil	Unchanged
Reinvestment Rate	20.04%	32.21%	Increase
BV of Equity	\$ 10,105 mil	\$ 18,320 mil	Increase
ROC	38.65%	25.21%	Decrease



The Dark Side of Valuation

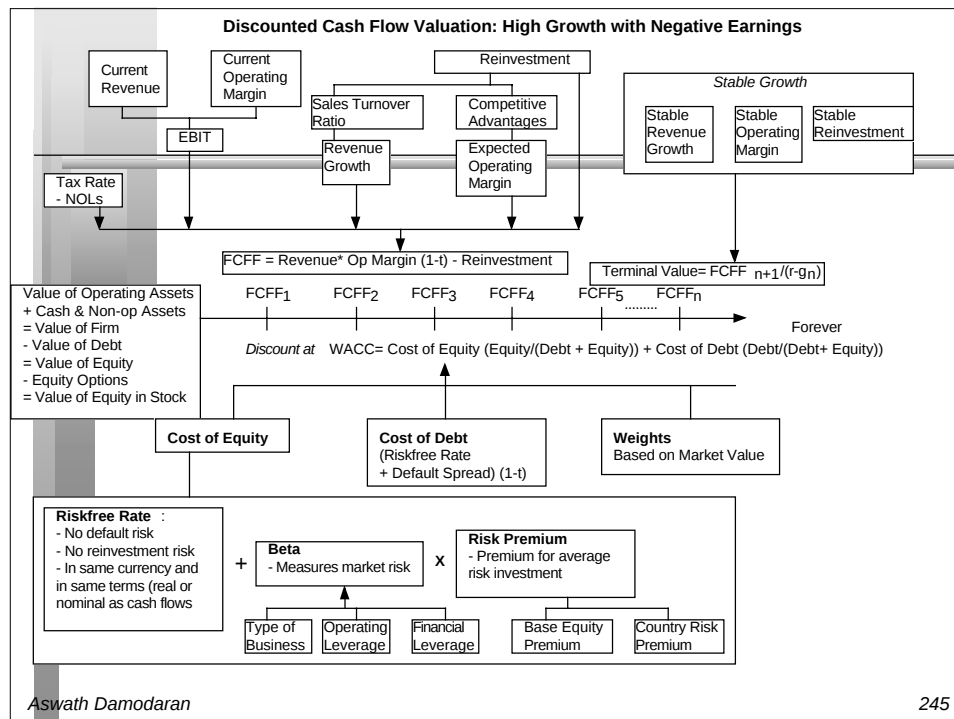
Aswath Damodaran
<http://www.stern.nyu.edu/~adamodar>

To make our estimates, we draw our information from..

- The firm's current financial statement
 - How much did the firm sell?
 - How much did it earn?
- The firm's financial history, usually summarized in its financial statements.
 - How fast have the firm's revenues and earnings grown over time? What can we learn about cost structure and profitability from these trends?
 - Susceptibility to macro-economic factors (recessions and cyclical firms)
- The industry and comparable firm data
 - What happens to firms as they mature? (Margins.. Revenue growth... Reinvestment needs... Risk)
- We often substitute one type of information for another; for instance, in valuing Ford, we have 70 years+ of historical data, but not too many comparable firms; in valuing a software firm, we might not have too much historical data but we have lots of comparable firms.

The Dark Side...

- Valuation is most difficult when a company
 - Has negative earnings and low revenues in its current financial statements
 - No history
 - No comparables (or even if they exist, they are all at the same stage of the life cycle as the firm being valued)



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Amazon's Bottom-up Beta

Unlevered beta for firms in internet retailing = 1.60

Unlevered beta for firms in specialty retailing = 1.00

- Amazon is a specialty retailer, but its risk currently seems to be determined by the fact that it is an online retailer. Hence we will use the beta of internet companies to begin the valuation but move the beta, after the first five years, towards the beta of the retailing business.

Estimating Synthetic Ratings and cost of debt

- The rating for a firm can be estimated using the financial characteristics of the firm. In its simplest form, the rating can be estimated from the interest coverage ratio

$$\text{Interest Coverage Ratio} = \text{EBIT} / \text{Interest Expenses}$$

- Amazon.com has negative operating income; this yields a negative interest coverage ratio, which should suggest a low rating. We computed an average interest coverage ratio of 2.82 over the next 5 years. This yields an average rating of BBB for Amazon.com for the first 5 years. (In effect, the rating will be lower in the earlier years and higher in the later years than BBB)

Estimating the cost of debt

- The synthetic rating for Amazon.com is BBB. The default spread for BBB rated bonds is 1.50%
- Pre-tax cost of debt = Riskfree Rate + Default spread
= 6.50% + 1.50% = 8.00%
- After-tax cost of debt right now = 8.00% (1 - 0) = 8.00%: The firm is paying no taxes currently. As the firm's tax rate changes and its cost of debt changes, the after tax cost of debt will change as well.

	1	2	3	4	5	6	7	8	9	10
Pre-tax	8.00%	8.00%	8.00%	8.00%	8.00%	7.80%	7.75%	7.67%	7.50%	7.00%
Tax rate	0%	0%	0%	16.1%	35%	35%	35%	35%	35%	35%
After-tax	8.00%	8.00%	8.00%	6.71%	5.20%	5.07%	5.04%	4.98%	4.88%	4.55%

Estimating Cost of Capital: Amazon.com

- Equity
 - Cost of Equity = $6.50\% + 1.60 (4.00\%) = 12.90\%$
 - Market Value of Equity = $\$84/\text{share} \times 340.79 \text{ mil shs} = \$28,626 \text{ mil (98.8\%)}$
- Debt
 - Cost of debt = $6.50\% + 1.50\% (\text{default spread}) = 8.00\%$
 - Market Value of Debt = $\$349 \text{ mil (1.2\%)}$
- Cost of Capital

Cost of Capital = $12.9\% (.988) + 8.00\% (1 - 0.988) = 12.84\%$
- Amazon.com has a book value of equity of \$138 million and a book value of debt of \$349 million. Shows you how irrelevant book value is in this process.

Calendar Years, Financial Years and Updated Information

- The operating income and revenue that we use in valuation should be updated numbers. One of the problems with using financial statements is that they are dated.
- As a general rule, it is better to use 12-month trailing estimates for earnings and revenues than numbers for the most recent financial year. This rule becomes even more critical when valuing companies that are evolving and growing rapidly.

	<i>Last 10-K</i>	<i>Trailing 12-month</i>
Revenues	\$ 610 million	\$1,117 million
EBIT	- \$125 million	- \$ 410 million

Are S, G & A expenses capital expenditures?

- Many internet companies are arguing that selling and G&A expenses are the equivalent of R&D expenses for a high-technology firms and should be treated as capital expenditures.
- If we adopt this rationale, we should be computing earnings before these expenses, which will make many of these firms profitable. It will also mean that they are reinvesting far more than we think they are. It will, however, make not their cash flows less negative.
- Should Amazon.com's selling expenses be treated as cap ex?

Amazon.com's Tax Rate

Year	1	2	3	4	5
EBIT	-\$373	-\$94	\$407	\$1,038	\$1,628
Taxes	\$0	\$0	\$0	\$167	\$570
EBIT(1-t)	-\$373	-\$94	\$407	\$871	\$1,058
Tax rate	0%	0%	0%	16.13%	35%
NOL	\$500	\$873	\$967	\$560	\$0

After year 5, the tax rate becomes 35%.

Estimating FCFF: Amazon.com

- EBIT (Trailing 1999) = -\$ 410 million
- Tax rate used = 0% (Assumed Effective = Marginal)
- Capital spending (Trailing 1999) = \$ 243 million (includes acquisitions)
- Depreciation (Trailing 1999) = \$ 31 million
- Non-cash Working capital Change (1999) = - 80 million
- Estimating FCFF (1999)

Current EBIT * (1 - tax rate) = - 410 (1-0)	= - \$410 million
- (Capital Spending - Depreciation)	= \$212 million
- Change in Working Capital	= -\$ 80 million
Current FCFF	= - \$542 million

Growth in Revenues, Earnings and Reinvestment: Amazon

Year	Revenue Growth	Chg in Revenue	New Investment	Sales/Capital	ROC
1	150.00%	\$1,676	\$559	3.00	-76.62%
2	100.00%	\$2,793	\$931	3.00	-8.96%
3	75.00%	\$4,189	\$1,396	3.00	20.59%
4	50.00%	\$4,887	\$1,629	3.00	25.82%
5	30.00%	\$4,398	\$1,466	3.00	21.16%
6	25.20%	\$4,803	\$1,601	3.00	22.23%
7	20.40%	\$4,868	\$1,623	3.00	22.30%
8	15.60%	\$4,482	\$1,494	3.00	21.87%
9	10.80%	\$3,587	\$1,196	3.00	21.19%
10	6.00%	\$2,208	\$736	3.00	20.39%

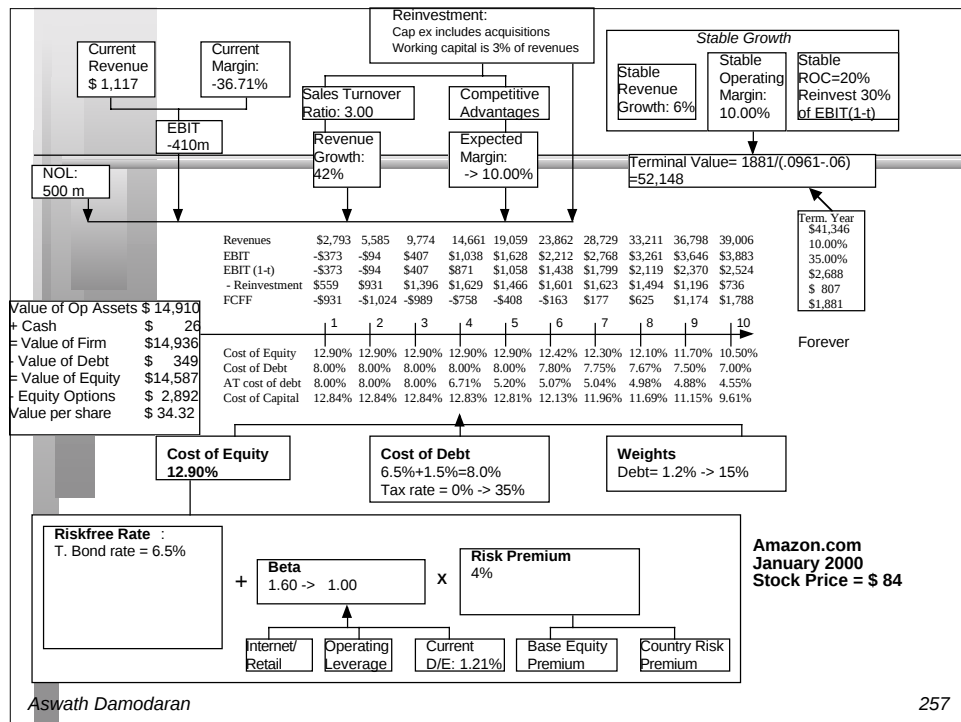
The sales/capital ratio of 3.00 was based on what Amazon accomplished last year and the averages for the industry.

Amazon.com: Stable Growth Inputs

	High Growth	Stable Growth
• Beta	1.60	1.00
• Debt Ratio	1.20%	15%
• Return on Capital	Negative	20%
• Expected Growth Rate	NMF	6%
• Reinvestment Rate	>100%	6%/20% = 30%

Estimating the Value of Equity Options

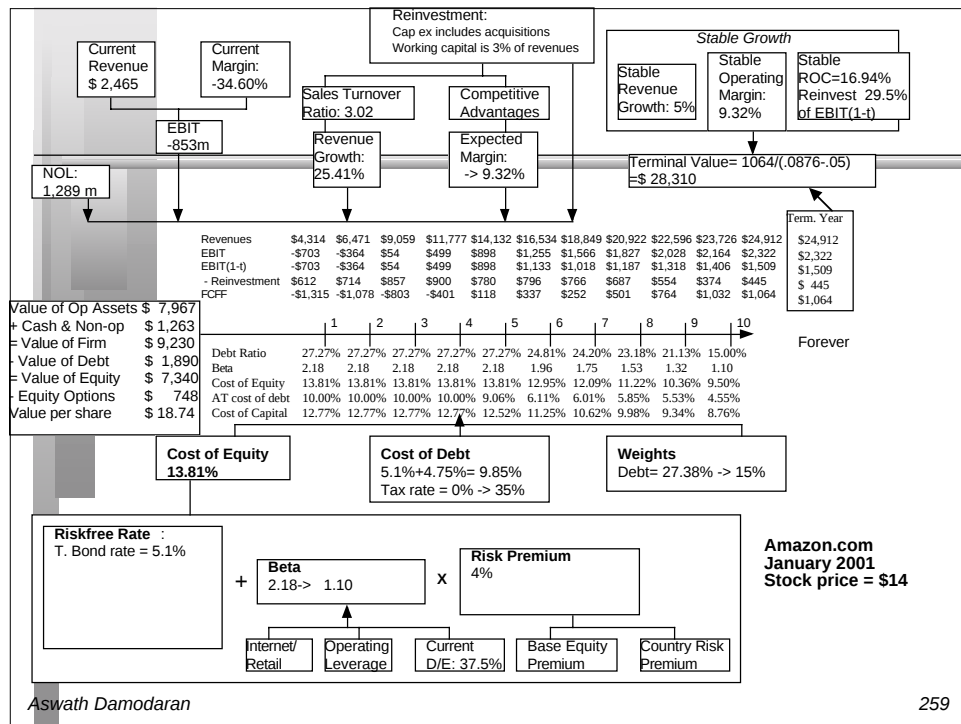
- Details of options outstanding
 - Average strike price of options outstanding = \$ 13.375
 - Average maturity of options outstanding = 8.4 years
 - Standard deviation in ln(stock price) = 50.00%
 - Annualized dividend yield on stock = 0.00%
 - Treasury bond rate = 6.50%
 - Number of options outstanding = 38 million
 - Number of shares outstanding = 340.79 million
- Value of options outstanding (using dilution-adjusted Black-Scholes model)
 - Value of equity options = \$ 2,892 million



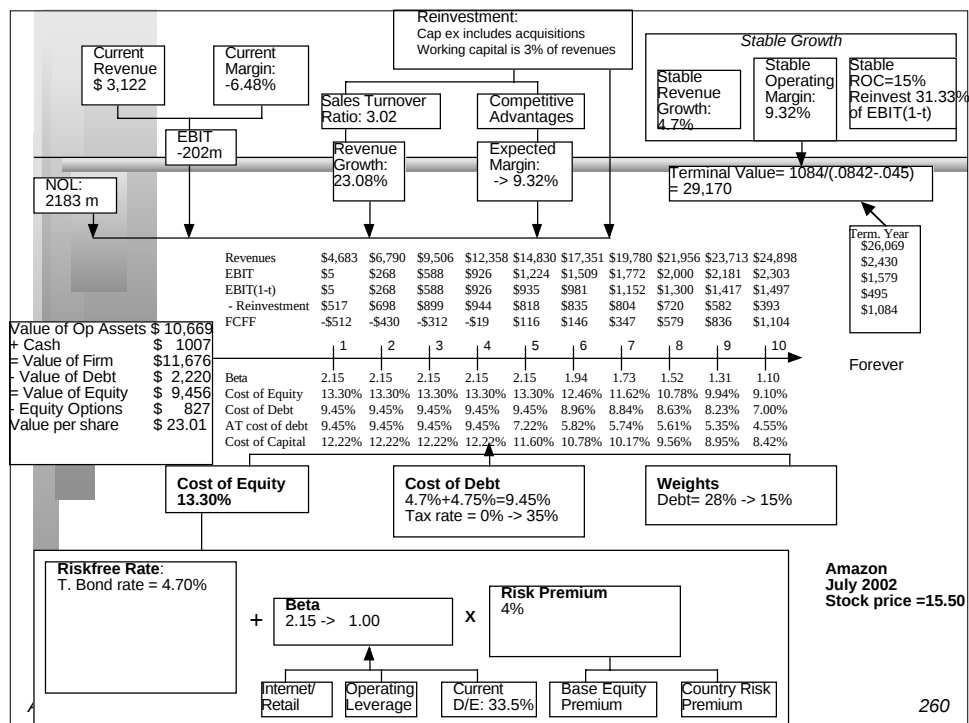
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What do you need to break-even at \$ 84?

	6 %	8 %	10 %	12 %	14 %
30 %	\$ (1.94)	\$ 2.95	\$ 7.84	\$ 12.71	\$ 17.57
35 %	\$ 1.41	\$ 8.37	\$ 15.33	\$ 22.27	\$ 29.21
40 %	\$ 6.10	\$ 15.93	\$ 25.74	\$ 35.54	\$ 45.34
45 %	\$ 12.59	\$ 26.34	\$ 40.05	\$ 53.77	\$ 67.48
50 %	\$ 21.47	\$ 40.50	\$ 59.52	\$ 78.53	\$ 97.54
55 %	\$ 33.47	\$ 59.60	\$ 85.72	\$ 111.84	\$ 137.95
60 %	\$ 49.53	\$ 85.10	\$ 120.66	\$ 156.22	\$ 191.77



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