Analyzing Pricing Strategies for Online-Services with Network Externalities

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Abstract

In this study, we model firms that sell a product and a complementary online service, where only the latter involves network externalities. That is, the value each consumer derives from the service increases with the total number of consumers that subscribe to the service. We consider two pricing strategies: 1) bundle pricing, in which the firm charges a single price for the product and the service; and 2) separate pricing, in which the firm sets the prices of the product and the service separately and consumers self-select whether to buy both or only the product. We show that, in contrast to the common result in the bundling literature, often the monopolist chooses not to offer the bundle (he either sells the service separately or not at all) while bundling would increase consumer surplus and social welfare. Thus, under-provision of the service can be the market outcome. Considering a competitive market with two firms, we find that in some cases the firms are caught in a Prisoner's Dilemma, as in equilibrium both offer the bundle while they would be better off if the service was not offered or sold separately. We show that the firms' profit decreases as the extent of network externalities increases, and that even in the duopoly case, under-provision of the service may occur in equilibrium.

Keywords: Bundling, Network Externalities, Price Discrimination, Online Services, Online Game Industry

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All Appendices are available at: http://www-personal.umich.edu/~noticeme/wise2009/

1. Introduction

Advances in information and communication technologies, such as Web 2.0 and 'Social Technologies' (Li and Bernoff 2008), enable firms to offer a variety of online services, such as blogs, peer-to-peer file sharing, social networking, and online games, to their customers. Most of these online services promote relationship building and interactivity among the users, and thus these services often exhibit positive network externalities. That is, the value of the service to a user increases with the number of customers that subscribe to the service. In this paper, we consider firms that sell a product and can offer a complementary online service. Offering the service can increase the firm's profit in two ways; it can generate direct profit from service subscribers and, in addition, it can increase the demand for the firm's products. We analyze the firms' optimal provision and pricing of such online services and derive strategic and welfare implications.

One industry to which our study applies is the online game industry. Until the early 1990s, most PC and video games had none or little network connectivity, and thus customers had to play scenarios or missions predefined in the game package and battle hypothetical competitors with artificial intelligence. Due to significant progress in network and Internet technologies and the increase in broadband presence since the late 1990s, PC game vendors started to incorporate network and Internet connectivity in their game products and to operate online game services where subscribers can play the games with remote human players. Thus, customers can now buy a game package, which supports playing against hypothetical competitors with predefined strategies, but in addition they can subscribe to the online service and play against remote human players. The set of pre-programmed game strategies included in the game package is narrower than the set of strategies used by human players. Therefore, the utility from playing games while being connected to the online service is considerably higher.

Online communities are another online service that involves network externalities. Many firms

facilitate the creation of online communities for customers who buy their products. Dell is an exemplar company that successfully utilizes online communities in its business (Li and Bernoff 2008). In Dell's online community [1], Dell's customers can share technical information and knowledge that they learn while using Dell's products. Its online community not only saves Dell millions of dollars in customer support, but it generates additional value for its customers from communicating with peers. Oracle [2] and IBM [3] also operate online communities for users and developers of their products. Microsoft operates an online community, Zune Social, where it's Zune MP3 player users can share music they purchase.

Access to most online communities is still offered free of charge, but some web sites specializing in specific topics require a subscription fee to access their online communities. For example, ValueForum.com, a Web site for individual stock investors, sells stock market information and investment tools as well as a 3 months subscription to its stock investor community for $85. The value from joining the community clearly increases with the number of other subscribers.

In this paper, we examine an industry in which firms sell a product and in addition can offer a complementary online service. We assume that the product has zero marginal cost, as is the case for most information goods, while the cost of operating the service increases with the number of subscribers. The firm may either bundle the service with the product and sell both for a single price or charge a separate service subscription fee, allowing some customers to buy only the product. The firm may also choose not to operate the service at all. We provide answers to the following research questions: 1) Should the firm offer a complementary online service that exhibits network externalities, and if so, should it sell the product and service as a bundle or sell the service separately? 2) Can the bundling of service and product increase consumer surplus? 3) Are online services being

[1] http://www.dell.com/community
[2] For example, Oracle E-Business Suite Application Community - http://www.oracle.com/applications/community/e-business-community.html
[3] IBM developerWorks http://www.ibm.com/developerworks/

over- or under-supplied in the market? 4) Do network externalities affect the answers to the previous questions and if so, how? And 5) how does the proliferation of online services affect the competitive landscape?

Both pricing strategies (bundling and selling the service separately) can be observed in the online game industry. Blizzard Entertainment, for instance, sells the games *Starcraft* and *Diablo II* each for $19.98. A customer can play these games either in the single-play mode, where he enjoys predefined scenarios, or in the multi-play mode, where he connects via the Internet to *Battle.net*, Blizzard's online game service, and plays in real-time with other players from all over the world. The single payment includes unlimited access to *Battle.net*. On the other hand, Blizzard sells the base package of *World of Warcraft*, which supports only the single play mode, for $36.99, and requires players to pay a monthly subscription fee of $15 to play in the multi-play mode. In this paper, we examine when the firm should choose one strategy over the other and whether the strategy that maximizes the firm's profit also maximizes social welfare and consumer surplus.

Our investigation of the monopoly case reveals that the service may be supplied less than is socially optimal. Especially, we find that under certain conditions, bundling is preferred by consumers and maximizes social welfare, but the monopoly chooses to sell the service separately or not at all. This result differs from the common findings reported in the bundling literature, according to which the monopoly uses bundling to extract surplus from consumers, and consumers cannot benefit from bundling (Adams and Yellon 1976, Bakos and Brynjolfsson 1999). We show that network externalities might be a reason why the service is often under-provided compared to the social optimal level. In contrast, we find no instances in which the service is over-supplied by the monopoly when the market is not covered.

We also examine the case in which the monopoly can exerts third degree price discrimination, offering the bundle at different prices to different consumer groups. This might be the case when a

firm sells its product in different countries and can identify the origin of the customers. We show that surprisingly, third degree price discrimination can increase both the monopoly's profit and consumer surplus. This happens if the monopoly chooses to sell the bundle (at different prices for the different consumer groups) when he can price-discriminate, but chooses to sell the service separately otherwise.

Finally, we consider a market with two competing firms selling differentiated products. Each firm can sell a complementary service to its customers, and may choose one of the three strategies presented in the monopoly model (bundling of service with product, selling the service separately, or not offering it at all). In addition, the firms may differ in their marginal cost of offering the service. We find that in some cases the two firms are caught in a Prisoner's Dilemma. Specifically, in equilibrium both firms choose to sell a bundle, but they would be better off when neither offers the service. In addition, we show that an increase in the extent of network externalities, which would increase firms' profits if holding prices fixed, leads to lower equilibrium profits due to the intensification of price competition. Lastly, we show that under-provision of the service may occur even under competition.

2. Literature Review

Our study is related to the literature on network externalities and on bundling. Network externalities arise when the utility that a user derives from a product increases with the number of other consumers that use the same or compatible product (Katz and Shapiro 1985). Therefore, a customer's utility from a product with network externalities is usually modeled as a function of the product's inherent value and of the number of customers using the product (Ellison and Fudenberg 2000). In addition, most models consider the network effect to be linear in the size of the user-base (Katz and Shapiro 1986, Fudenberg and Tirole 2000, Lee and Mendelson 2007).

In this paper we show that a service with network externalities is often under-supplied compared

to the socially optimal level. This finding is in accordance with arguments in the network externalities literature that network goods may be adopted less than is socially optimum (Katz and Shapiro 1986, Farrell and Klemperer 2007). For example, Katz and Shapiro (1994) state that at the presence of network externalities, social marginal benefits from an increase of one unit in network size exceed private (i.e. the firm's) marginal benefits, and thus the equilibrium network size is smaller than the socially optimal network size. Farrell and Saloner (1985) suggest that excess inertia or excess momentum may be the explanation for inefficient adoptions of goods with network externalities.

Our work is also closely related to the bundling literature. The seminal paper by Adams and Yellon (1976) provides a two-good bundling model and examines three pricing strategies: pure components, pure bundling, and mixed bundling in which each individual component as well as the bundle are sold to consumers. While Adams and Yellen (1979) consider negative correlation between the valuations for the two goods, McAfee et al. (1989) allow for positive correlation or independence of valuations. Schmalensee (1984) considers a case in which one of two products is provided competitively while the firm is the sole vendor of the other product. Schmalensee (1984) and McAfee et al. (1989) show that mixed bundling always dominates offering only a bundle, and examine under which conditions the mixed bundling strategy outperforms the pure component strategy. Both show that, although in some cases the mixed bundling strategy is optimal for the monopolist, the pure component strategy always yields higher consumer surplus.

Our model differs from those in the above papers in several ways. First, in our model one of the components, i.e. the service, has no stand-alone value, and thus consumers never subscribe to the service without buying the product; they either buy both or only the product. Therefore, in our setup, the seller needs to consider only two strategies: pure bundling, forcing all customers who buy the product to also buy the service, or pure components, selling the two separately and thus allowing

some consumers to buy only the product. There is no practical difference between *pure component* and *mixed bundling*, as there is never a group that buys only the service[4]. Second, we assume that the valuations for the two goods (product and service) are independent of each other. Lastly, unlike in Adams and Yellen (1976), in our model, the seller cannot execute first-degree price discrimination.

Bakos and Brynjolfsson (1999) model a monopolist selling a large number of information goods and show that bundling enables the firm to capture most of the consumer surplus, as it reduces the variance in consumers' valuations and makes the demand more elastic. In their following study (Bakos and Brynjolfsson 2000), they introduce a positive marginal production cost and a distribution cost for delivering the bundle to each individual customer. They show that these two components of costs play a key role in the monopolist's decision whether or not to bundle the goods. In our model, the marginal cost of providing the service has a similar effect to the production cost from Bakos and Brynjolfsson (2000), i.e., the firm is more likely to offer a bundle of product and service if the marginal cost of the service decreases.

The literature generally agrees that bundling enables a seller to capture more value from consumers and thus it reduces the consumer surplus. However, several studies find that both the consumers and the seller may benefit from bundling (Salinger 1995, Dansby and Conrad 1984). In addition, Dewan and Freimer (2003) show that in some cases consumers prefer a bundle of base software and add-in, but the monopoly chooses to sell the products separately. This result hangs on the assumptions that the add-in has no stand alone value and that some users incur a penalty when the add-in is bundled with the base software.

Our model differs from those in previous studies that show consumers can benefit from bundling. First, we do not consider the presence of cost-saving in bundling (Salinger 1995) nor the superadditive value of a bundle (Dansby and Conrad 1984). Second, we assume that one of the two

[4] In our model, a mixed bundling strategy with price f for the service, p for the product and p_b for a bundle of the two, where $p_b<p+f$, is identical to a pure component strategy with prices p for the product and $(p_b - p)$ for the service.

components, i.e. the complementary service, exhibits positive network externalities. Third, while in Dewan and Freimer (2003) both the base software and the add-in have zero marginal cost (both are information goods), in our model providing the service is costly for the firm. When more users register to the service, the firm has to invest in increasing communication capabilities, network capacity, and so on. Finally, in our model consumers never incur a penalty due to the fact that the service is sold bundled with the product.

Though our setting and focus differ from those in Dewan and Freimer (2003), we reach similar results. Consumers often prefer the service to be bundled with the product, while the monopoly does not find this pricing strategy optimal. This might happen even when two firms compete in the market. In our setup, one reason for this under-provision of the service is, surprisingly, the presence of network externalities.

3. The Monopoly

We consider a monopoly that sells a product and can provide a complementary online service to customers who buy the product. We model the case in which the product has zero marginal production cost (a common assumption for most information goods), while the cost of providing the service increases with the number of subscribers[5]. For instance, an online-game provider needs to operate a larger service system (e.g., servers, network facilities, and so forth) to serve an increasing number of subscribers[6]. In addition, we assume the service has positive network externalities, and thus consumers' valuation for the service is a function of its intrinsic value and the number of service subscribers.

3.1. The Model

There are *N* customers who are heterogeneous in terms of their valuations for the product and for the

[5] If the product has a positive marginal cost, then the model presented here still applies if consumers' valuations of the product are taken net of the marginal cost.

[6] Blizzard reports that it invested more than $200 million in its service infrastructure for *World of Warcraft* online users.

service. We assume that the valuation for the product, θ, is uniformly distributed on $[0, \overline{\theta}]$. In addition, a consumer's valuation for the service is given by $\alpha(s+n_s)$, where α represents the consumer type, n_s is the total number of customers who subscribe to the service, and s is the intrinsic value of the service independent on the number of service users (for example, s can be the value of the product's functionalities which are activated only when a user subscribes to the service). This functional form is based on the assumption that a customer who has a higher intrinsic value for the service also assigns more importance to the size of the network of service subscribes (Ellison and Fudenberg 2000).

The heterogeneity in the valuation for the service is modeled as follows. To simplify the analysis, we divide customers into two groups. βN customers (Group 1) have $\alpha=\alpha_1$, while the remaining $(1-\beta)N$ customers (Group 2) have $\alpha=\alpha_2$. We assume that $0 < \beta \leq 1$ and $\alpha_1 > \alpha_2 \geq 0$, so that Group 1's customers have a higher valuation for the service and for the size of the network. In this section, we assume that the vendor cannot identify a customer's valuation for the product or for the service. This assumption is relaxed in Section 3.4, where we assume that the vendor can identify the valuation for the service (α_i). We also assume that θ and α are independent. Lastly, the monopolist has to incur a marginal cost of c to serve each service subscriber. Table 1 summarizes the notations used in the monopoly model.

The monopoly has three options as follows. He can sell only the product; this is likely to be his choice when consumers do not value the service highly enough to justify its provision cost. Alternatively, he can choose to offer the product and the service as a bundle, in which case all customers that buy the product also gain access to the service. Finally, he may sell the service separately, in which case there can be a group of customers who buy the product but do not subscribe to the service. Table 2 lists the consumers' utility and the seller's profit for each of these three

strategies.

N	The number of customers
θ	Consumers' valuation of the product, drawn from the uniform distribution on $[0,\bar{\theta}]$
α_i	The coefficient for service valuation for Group i's customers, where $\alpha_1 > \alpha_2 \geq 0$
β	The proportion of Group 1 customers
s	The intrinsic value of the complementary service
c	The marginal cost for providing the service
p_N	The product price when the service is not offered
p_B	The bundle price
p_S	The product price when the service is sold separately
f	The service subscription fee when the service is sold separately
n_p	Total demand for the product
n_S	Total demand for the service
π_B	Profit from selling a bundle
π_N	Profit from selling only the product (no service)
π_S	Profit from selling the service separately only to Group 1 customers

Table 1. Notation in the Monopoly model

Firm's Strategy	**Consumers' Utility**	**Firm's Profit**
Sell only product	$u_1=u_2=\theta-p_N$	$\pi_N=n_p\, p_N$
Sell a bundle	$u_1=\theta+\alpha_1(s+n_S)-p_B \quad u_2=\theta+\alpha_2(s+n_S)-p_B$	$\pi_B=n_S\,(p_B-c)$
Sell service separately	$u_1=\theta+\alpha_1(s+n_S)-p_S-f \quad u_2=\theta-p_S$	$\pi_S=n_p\, p_S+n_S\,(f-c)$

Table 2. The consumer's utility and the monopoly's profit for the different pricing strategies. u_i is the utility of a Group i's customer and is a function of his product valuation θ.

If users from both groups subscribe to the service when the monopoly sells the service separately, then there is no difference between the two strategies of selling the service separately and selling a bundle. Thus, when the monopoly sells the service separately, we maximize the profit gained when only customers from Group 1, who have higher valuation for the service, subscribe to the service. In this case, the two following incentive compatibility conditions should be met.

$$\alpha_1(s+n_s) \geq f \tag{1}$$

$$\alpha_2(s+n_s+1) \leq f \tag{2}$$

Eq. 1 specifies the condition required so that no single Group 1's customer has incentive to

unsubscribe from the service. Eq. 2 specifies the condition required so that a Group 2 customer has no incentive to deviate and subscribe to the service.

Before embarking the analysis, we make parameter assumptions as follows.

Assumption 1. *i)* $\bar{\theta} > \alpha_0 N$

$$ii)\ \ c > -s\alpha_0 - \bar{\theta} + \frac{2(N+s)\alpha_1\bar{\theta}}{N(\alpha_1-\alpha_2)(1-\beta)+\bar{\theta}}$$

$$iii)\ \ c > \alpha_1(s+2\beta N) - \bar{\theta}$$

where $\alpha_0 \triangleq \beta\alpha_1 + (1-\beta)\alpha_2$

Condition (*i*) in Assumption 1 guarantees the profit function to be concave for the two strategies in which the monopoly sells the service. Conditions (*ii*) and (*iii*) guarantee that not all Group 1 customers buy the product when the monopoly sells the service. Thus, in what follows we focus on cases in which, under the monopoly's optimal strategy, the market is not covered. Table 3 presents the optimal prices and the resulting profit for the three strategies described above. The derivations are presented in Appendix 1[7]

Firm's Strategy	**Prices**	**Firm's Profit**
Sell only product	$p_N^* = \frac{\bar{\theta}}{2}$	$\pi_N^* = \frac{\bar{\theta}N}{4}$
Sell a bundle	$p_B^* = \frac{\bar{\theta}+\alpha_0 s + c}{2}$	$\pi_B^* = \frac{N(\bar{\theta}+\alpha_0 s - c)^2}{4(\bar{\theta}-\alpha_0 N)}$
Sell service separately	$p_S^* = \frac{\bar{\theta}}{2}$, $f^* = \frac{\alpha_1 s + c}{2}$	$\pi_S^* = \frac{N}{4}\left(\frac{\beta(\bar{\theta}+\alpha_1 s - c)^2}{\bar{\theta}-\alpha_1\beta N} + (1-\beta)\bar{\theta}\right)$

Table 3. Optimal prices and resulting profit for the monopoly model for the considered strategies

From the results presented in Table 3, we derive the following proposition regarding the monopoly's optimal strategy. In Proposition 1, $c_{i,j}$ represents the value of c at which $\pi_i^* > \pi_j^*$ if and only if $c < c_{i,j}$ (where the profit expressions, π_i^* and π_j^*, are given in Table 3).

[7] all appendices are available at http://www-personal.umich.edu/~noticeme/wise2009/

Proposition 1. *The monopoly chooses its strategy as follows.*

- *When* $s \leq s_0$ *, the monopoly chooses the bundle pricing if and only if* $c<c_{B,N}$ *and does not offer the service otherwise.*
- *When* $s > s_0$ *, the monopoly chooses the bundle pricing if and only if* $c<c_{B,S}$*, the separate pricing if and only if* $c_{B,S} < c < c_{S,N}$*, and does not operate the service otherwise,*

where

$$s_0 = \frac{\sqrt{\bar{\theta}}\left(\sqrt{\bar{\theta}-\alpha_1\beta N}-\sqrt{\bar{\theta}-\alpha_0 N}\right)}{(\alpha_1-\alpha_2)(1-\beta)} \; ; \; c_{B,N} = \bar{\theta}\left(1-\frac{\sqrt{\bar{\theta}-\alpha_0 N}}{\sqrt{\bar{\theta}}}\right)+\alpha_0 s \; ; \; c_{S,N} = \bar{\theta}\left(1-\frac{\sqrt{\bar{\theta}-\alpha_1\beta N}}{\sqrt{\bar{\theta}}}\right)+\alpha_1 s; \text{ and}$$

$$c_{B,S} = \bar{\theta} - \frac{\alpha_2\bar{\theta}s-\sqrt{(\bar{\theta}-\alpha_1\beta N)(\bar{\theta}-\alpha_0 N)(\beta s^2(\alpha_1-\alpha_2)^2+\bar{\theta}\left(\bar{\theta}-\beta N(\alpha_1-\alpha_2)\right)}}{\bar{\theta}-\beta N(\alpha_1-\alpha_2)}$$

Proofs of all the propositions are given in Appendix 2.

Figure 1 describes the vendor's pricing strategy choice according to Proposition 1. If $s \leq s_0$, then the monopoly never chooses to sell the service separately, and he offers the bundle only when the marginal cost of service, *c*, is lower than a threshold value given by $c_{B,N}$. Notice that $c_{B,N}$ is always positive and is increasing in *s*. Thus as *s* increases (but is still below s_0), the range in which the seller offers the bundle increases. When $s > s_0$, if the marginal cost of offering the service is lower than a threshold value given by $c_{B,S}$, it is optimal for the vendor to sell a bundle of product and service If the marginal cost is between $c_{B,S}$ and $c_{S,N}$, the monopolist chooses the separate pricing strategy, and if the marginal cost is higher than $c_{S,N}$, the service is not offered.

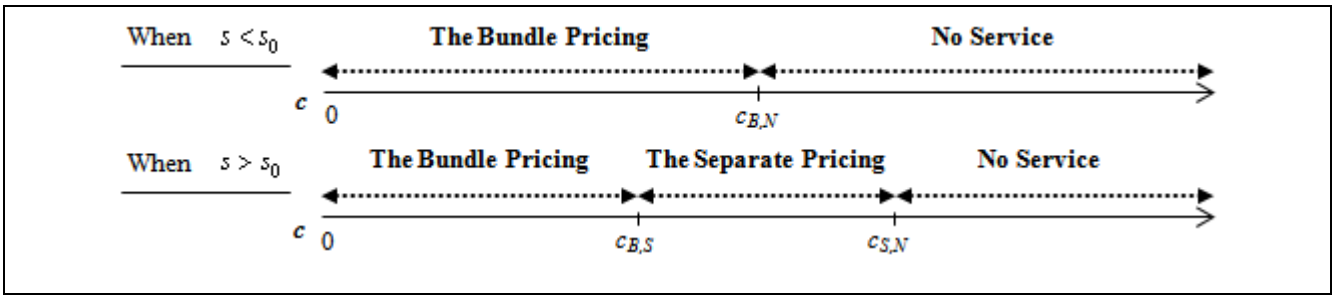


Figure 1. The monopoly's optimal pricing strategy

According to Proposition 1, the intrinsic value of the service, *s*, has to reach a threshold value for separate pricing to be chosen by the monopoly. This finding is consistent with the literature on price discrimination (Mussa and Rosen 1978, Deneckere and McAfee 1996). Specifically, only when the two consumer groups are differentiated enough in their service valuations (the difference between the valuations, $s(\alpha_1 - \alpha_2)$, is increasing in *s*), the monopoly chooses to use second-degree price discrimination where consumers self-select what to buy, both product and service or only the product.

In addition, according to Proposition 1, as the marginal cost of service, *c*, increases, the separate pricing strategy might outperform the bundling strategy. This finding is consistent with a real-life example. According to a 2004 survey, users of *Starcraft*, *Warcraft III*, and *Diablo II*, spend 10.5 minutes a day on average in *Battle.net*, Blizzard's online gaming system; for all three games, online access is bundled with the game package[8]. On the other hand, the average online playing-time per day of a user of *World of Warcraft*, is 3.24 hours[9]. This shows that Blizzard has to incur a higher cost in online systems to serve a *World of Warcraft* player than to serve a player of *Starcraft*, *Warcraft III*, or *Diablio II*. As predicted by our model, access to the online platform of *World of Warcraft* is sold separately from the game package.

3.2. Social Welfare and Consumer Surplus

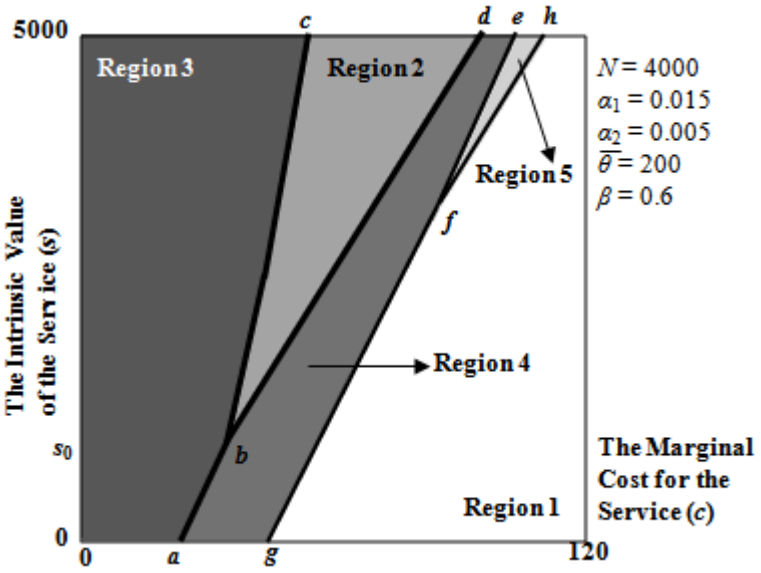
In this section we examine how the monopolist's choice affects social welfare and consumer surplus. We start with a typical numerical example, presented in Figure 2, which demonstrates that the complementary service may be supplied less than is socially optimal.

Figure 2 displays the strategy that maximizes the monopoly's profit and the strategy that maximizes social welfare for different values of the marginal cost (*c*) and the intrinsic value of the

[8] http://www.eff.org/files/filenode/Blizzard_v_bnetd/20040930BNETDOrder.pdf. According to this article (page 4), as of September 2004, the number of active users is approximately 12 million, and they spend more than 2.1 million hours a day. By dividing 2.1 million hours by 12 million, we get 10.5 minutes.

[9] http://www.nickyee.com/daedalus/archives/001365.php. This page reports that on average, a player spends 22.7 hours per week, equivalent to 3.24 hours per day.

service (s), when the proportion of Group 1 customers, β, is 0.6. Notice that s_0 from Proposition 1 corresponds to the value of s at point b in Figure 2. In addition, the lines a-b, b-c, and b-d in Figure 2 correspond to the thresholds $c_{B,N}$, $c_{B,S}$, and $c_{S,N}$ from Proposition 1, respectively.



Region	Monopoly's Choice	Social Optimum	Inefficiency
1	No Service	No Service	N/A
2	The Separate Pricing	The Bundle Pricing	Under-provision
3	The Bundle Pricing	The Bundle Pricing	N/A
4	No Service	The Bundle Pricing	Under-provision
5	No Service	The Separate Pricing	Under-provision

Figure 2. The monopoly's profit maximizing strategy vs. the strategy that maximizes social welfare

Most of the bundling literature shows that bundling enables the monopoly to extract higher surplus from the consumers (Bakos and Brynjolfsson 1999, 2000). Our finding of Region 2 in Figure 2 in which a bundle is socially optimal, but the monopolist sells the service separately (which implies that consumer surplus is maximized with a bundle), is similar to the main finding from Dewan and Freimer (2003). The main result in Dewan and Freimer (2003) is driven by the existence of a group of customers that incur a penalty when they are forced to buy the add-in bundled with the base product. We show that their result is more general, because a bundle may maximize consumer surplus but the monopoly chooses to sell the add-in/service separately even in the absence of such a

penalty. This happens when the add-in/service has positive network externalists.

The cases presented in Regions 4 and 5 in Figure 2 are not reported in Dewan and Freimer (2003). In their paper, the add-in has no marginal cost and thus it is always being offered, either in a bundle or separately. In our model, on the other hand, there is a cost for providing the service which increases with the number of subscribers. If the cost is high enough, the vendor may choose not to provide the service. Thus, in some cases the monopoly does not offer the service while it is socially optimal to offer the service either in a bundle or separately. While clearly under-provision of the service is feasible (Regions 2, 4, and 5), we could not find any numerical exmples in which the service is over-provided compared to the socially optimal level[10]. Numerical investigation was necessary because the inequalites that need to be satisfied for over-provision of service to be the market outcome are too complex to analtyically verify whether they can all be met simultenously. The fact that we could not find instances for which over-provision of service is the market outcome might be because we are considering only cases in which, under the monopoly's optimal strategy, the market is not covered .

3.3. Network Externalities and the Under-Provision of the Service

Under-provision of the service can be a result of the monopoly's incentive to price-discriminate. In such cases, the monopoly chooses to sell the service separately and price it so that only consumers who value it highly, i.e. only Group 1's customers, buy it; however, consumer surplus and social welfare would be higher if the service was sold to consumers from both groups, bundled with the product. Under-provision of service can also happen when the seller chooses not to offer the service.

In this section we examine the relationship between the existence of network externalities and the fact that under-provision of service prevails. To do so, we focus on the case in which consumers

[10] By comparing social welfare from the startegy that maximizes the seller's profit to the social welfare from other strategies, we searched for cases in which sevice is over-supplied. We searched over the following range of parameter valeus: β in [0, 1] in increments of 0.1; N in [0, 10000] in increments of 500; α_1 in [0, 1], with increments of 0.05; α_2 in [0 , α_1] in increments of 0.05; $\bar{\theta}$ in [$\alpha_0 N$, 10000] with increments of 100; s in [0, 20000] in increments of 500.

are homogeneous in terms of their service valuation, so that price discrimination cannot be the cause of under-provision of the service. We first compare the model with network externalities to a model without network externalities, showing that the service can be under-provided in the former, but this does not happen in the latter. We then show that in the presence of network externalities, the value created for consumers from a decrease in the marginal cost of the service or from an increase in the value of the service cannot be fully captured by the monopoly, which explains the under-provision of the service.

When consumers have homogeneous service valuation, the seller either sells the service to everyone that buys the product (sells a bundle) or does not offer the service. In Proposition 2, we specify the conditions under which under-provision of the service prevails when the service has network externalities and all consumers have the same service valuation, which is given by $\alpha(s+n_s)$.

Proposition 2. *If the service exihibts netwrok externalits, and service valuation is given by* $\alpha(s+n_s)$*, the monopoly does not sell the service although the service would increase social welfare when* $d_{B,N}<c<d_{S0}$*, where*

$$d_{B,N} = \overline{\theta}\left(1 - \frac{\sqrt{\overline{\theta}-\alpha N}}{\sqrt{\overline{\theta}}}\right) + \alpha s \qquad , \; d_{S0} = \overline{\theta}\left(1 - \frac{(\overline{\theta}-\alpha N)\sqrt{\overline{\theta}+2}}{\sqrt{\overline{\theta}}\sqrt{\overline{\theta}(\overline{\theta}+2)-\alpha N}}\right) + \alpha s$$

According to Proposition 2, for $c < d_{B,N}$, the monopoly offers a bundle and this strategy also maximizes social welfare. For $d_{B,N} <c< d_{S0}$, the monopoly does not offer the service, although social welfare and consumer surplus would be maximized with a bundle, and for $c>d_{S0}$, the monopoly does not offer the service, a choice which also maximizes social welfare.

We now analyze an alternative model in which the complementary service does not involve network externalities. In this model, the utility from the service is αs and it does not depend on the number of service subscribers. The monopoly offers the bundle at a price of $(\overline{\theta}+ c + \alpha s)/2$ when $c < \alpha s$ and does not offer the service otherwise, a decision which is socially optimal. Thus, when the

service does not create network effects, there is no under-provision of the service.

We note that although there is no under-provision of service in the setting with no network externalities, the range of c values where the service is provided is smaller in this case (i.e., $d_{B,N} > \alpha s$). The reason is that if we hold s fixed, the total value of the service to consumers is lower in the case with no network externalities.

The following two propositions give additional insights as to why the service may be supplied less than is socially optimal at the presence of network externalities. Notice that Proposition 3 applies also to the more general case, in which there are two groups of consumers with different valuations for the service. The result for the special case of homogeneous service valuations can be obtained by substituting $\alpha_1 = \alpha_2 = \alpha$ and $\beta=1$

Proposition 3. *When $\alpha_0 > 1/(2N)$ and $c < \overline{\theta} + \alpha_0 s$, consumer surplus and social welfare under the bundle strategy decrease in the marginal cost of the service to a greater extent than the monopolist's profit does. Specifically,* $\frac{\partial}{\partial c} CS_B < \frac{\partial}{\partial c} \pi_B$.

Proposition 4. *When consumers have homogeneous service valuation, $\alpha(s+n_s)$, such that $\alpha > 1/(2N)$ and $c < \overline{\theta} + \alpha s$, consumer surplus and social welfare under the bundle strategy increase in α to a greater extent than the monopolist's profit does. Specifically,* $\frac{\partial}{\partial \alpha} CS_B > \frac{\partial}{\partial \alpha} \pi_B$.

Propositions 3 and 4 explain why in some cases the monopoly chooses not to offer the service while consumer surplus is maximized with a bundle. Technological developments can cause a reduction in the operating cost of the service (i.e., a decrease in c) or enable the vendor to offer a service that provides higher utility and involves a greater degree of network externalities (i.e., an increase in α). According to Proposition 3 and 4, as c decreases or as α increases, consumer surplus from a bundle my exceeds their surplus from only the product *before* the vendor's profit from offering the bundle exceeds his profit from selling only the product.

Notice that $\alpha>1/(2N)$ is a sufficient condition which holds when α or N are large enough. Interestingly, an increase in either of these two values represents an increase in the degree of network externalities. In addition, Proprositons 3 and 4 examine only the range of c values where bundling can be profitable for the monpoly (if $\overline{\theta} + \alpha_0 s < c$, then bundling can not be profitable because $p_B^* < c$).

3.4. Price Discrimination with Identifiable Groups

In the preceding model, we assume that the monopoly cannot identify each customer's valuation for the service (α_i). In this section, we relax this assumption and instead consider the case in which the vendor can identify to which group a customer belongs. This enables the vendor to charge different bundle prices from the two groups, executing third-degree price discrimination. For example, an online game service provider which offers its service globally can identify a user's nationality by identifying her IP address. Knowing that users' valuations for the service vary across geographic locations, the vendor may set different bundle prices in different countries[11].

The utility functions of Group 1 and Group 2 customers when the monopoly offers a bundle at different prices (p_{B1} to Group 1 and p_{B2} to Group 2) are as follows.

$$u_1 = \theta + \alpha_1(s + n_s) - p_{B1} \quad (3)$$

$$u_2 = \theta + \alpha_2(s + n_s) - p_{B2} \quad (4)$$

The monopolist's profit in this case is given by:

$$\pi_B = n_{S1}(p_{B1} - c) + n_{S2}(p_{B2} - c), \quad (5)$$

where n_{S1} and n_{S2} are the number of customers from Group 1 and 2, respectively, that buy the bundle. The utility and profit functions when the service is not offered and when the seller chooses the separate pricing strategy, so that only Group 1 customers subscribe to the service, are the same as in

[11] The service subscription fee for World of Warcraft differs across the world. As of 2008, the fee in the U.S. is $14.99 for one month, while it is approximately $3.76 per 60 hours play in China.

Section 3.1.

In Appendix 3, Lemma 2 describes the monopoly's optimal pricing strategy choice in this case. Similarly to the case without price discrimination, we find a threshold value s_0^{PD} so that if *s* is below the threshold, the monopoly chooses to offer the bundle at different prices to the two groups when *c* is lower than a threshold value, $c_{B,N}^{PD}$, and chooses not to offer the service otherwise. If *s* is larger than the threshold value s_0^{PD}, as *c* increases the monopoly's optimal strategy changes from offering the bundle at different prices to the two groups ($c < c_{B,S}^{PD}$), to selling the service separately ($c_{B,S}^{PD} < c < c_{S,N}^{PD}$), to not selling the service. Thus, the structure of the vendor's strategy choice with identifiable groups is similar to that with unidentifiable groups, while only the threshold values (for *s* and for *c*) change. Specifically, $c_{B,N}^{PD} > c_{B,N}$ and $c_{B,S}^{PD} > c_{B,S}$ while $c_{S,N}^{PD} = c_{S,N}$, so that bundling is offered for a larger range of *c* values when the seller can price-discriminate.

How does the ability to price-discriminate affect consumer surplus and social welfare? Clearly, when selling the bundle the monopoly is able to enjoy higher profit with price discrimination, while consumer surplus is reduced. However, our next numerical example shows that in some cases the seller's ability to price-discriminate can benefit consumers.

Figure 4 shows a numerical example in which, when the monopoly can identify customers' valuation for the service, he chooses to sell the bundle when $c < 61.49$ and sell the service separately for $61.49 < c < 93.89$. However, when the monopoly cannot identify consumers' valuation for the service, he sells the bundle only when $c < 54.31$ and sells the service separately for $54.31 < c < 93.89$. That is, when the monopoly can identify groups, the bundle pricing is the optimal strategy for a larger range of *c* values. Figure 5 demonstrates that even though consumers prefer bundling without price discrimination to bundling with price discrimination, they prefer the latter to having the service sold separately. Thus, when $54.31 < c < 61.49$, consumers benefit from the seller's ability to price-discriminate. In this range, the seller chooses the bundle pricing when he can price-

discriminate but the separate pricing otherwise.

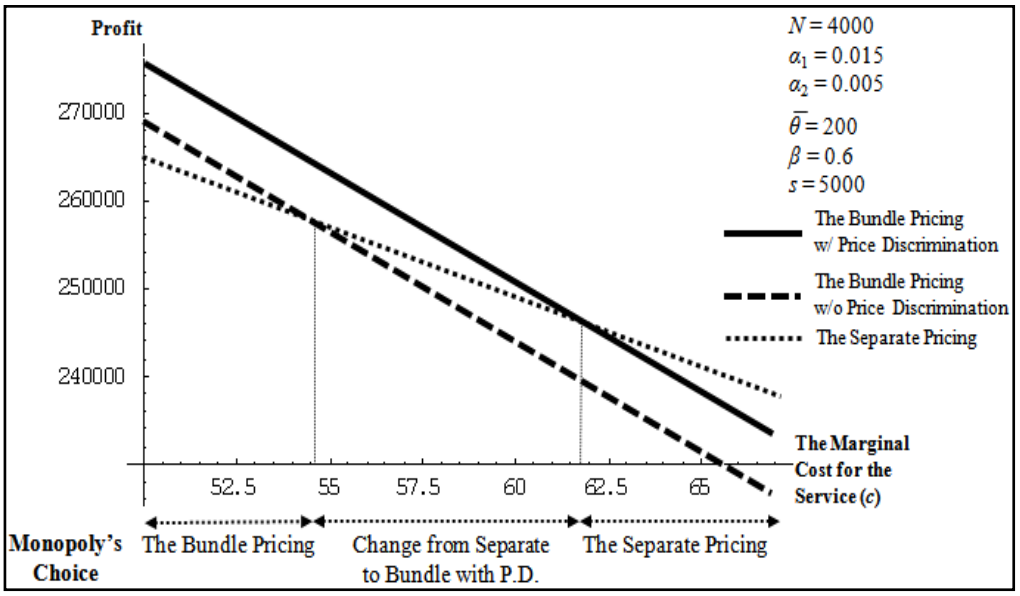


Figure 4. The monopoly's strategy choice with and without price discrimination.

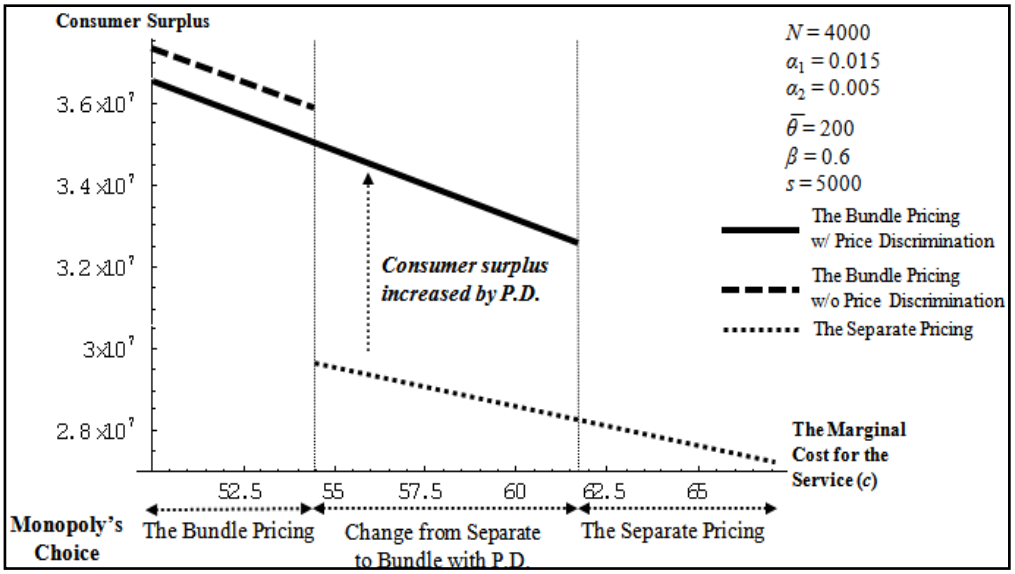


Figure 5. Consumer surplus with and without price discrimination

The ability to price-discriminate can only increase the vendor's profit. His profit from bundling with price discrimination is higher than his profit from bundling without price discrimination, and the profit is unchanged if he sells the service only to Group 1 or not at all. In addition, when third-degree price discrimination is feasible, the range of the marginal cost in which the vendor chooses to provide the bundle extends. Thus, if without price discrimination the seller chooses separate pricing,

but with the ability to price-discriminate he chooses to offer the bundle, consumers surplus may increase due to the seller's ability to price-discriminate. We conclude that in some cases, price discrimination may be Pareto improvement.

4. The Duopoly Model

In this section we explore the characteristics of market equilibrium when there are two firms in the market and each firm sells a product and can provide a complementary service to customers who purchase its product. As before, the service exhibits network externalities and there are two groups of consumers, Group 1 with marginal service valuation α_1 and Group 2 with marginal valuation α_2.

The two firms are differentiated in terms of the product they offer. We use the classical model of horizontal differentiation (Hotelling 1929) where Firm A's product is located at point 0, Firm B's product is located at point $\bar{\theta}$, and consumers' ideal product locations are uniformly distributed on the line between the two firms, i.e., on the $[0, \bar{\theta}]$ interval. The utility a consumer gets when being able to obtain his ideal product is $\bar{\theta}$.[12] When the consumer buys a product that differs from his ideal product, he incurs a misfit cost which is increasing in the distance between his ideal product and the product he buys, and the misfit cost per unit distance is t. Thus, a consumer with ideal product location θ has the following valuations for the products sold by Firm A and by Firm B respectively:

$$V^{A}(\theta)= \bar{\theta} - t\theta \,, \tag{6}$$

$$V^{B}(\theta)=\bar{\theta} - t(\bar{\theta}-\theta) \tag{7}$$

Each consumer is interested in purchasing only one of the two products, and perhaps a complementary service. We assume consumers are risk neutral and thus choose the option that maximizes their expected utility (detailed utility functions under the different sellers' strategies are provided in Appendix 4). The two firms incur c_A and c_B, respectively, in serving each service user.

[12] All the results presented in this section (existence of Prisoner dilemma, under- and over- provision of service etc.) still hold if the reservation value is r and is independent of the length of the interval.

Therefore, in the duopoly model, the two firms are symmetric except for the values of c_A and c_B.

We assume each of the two firms can adopt one of the three strategies presented in Section 3: selling only the product, selling the service to both groups (the bundle strategy), or selling the service only to Group 1 customers for a subscription fee (the separate pricing strategy). Here, we also introduce the following two assumptions. First, we assume that a consumer is able to benefit from the service only if he has the product sold by the same firm offering the service (i.e., the service offered by Firm *B* is worthless to a consumer that has the product from Firm *A*). Second, we assume that the value a consumer derives from the service depends only on the number of users of that service, and not on the number of users of the service offered by the other firm.

In what follows we consider only cases in which, regardless of what strategies the firms choose, the two firms compete for the marginal consumer from each group. Thus, in the cases examined, every consumer buys at least a product from one of the two firms. Notice that if the firms do not compete for the marginal customer, then the market is not covered and in fact each firm behaves as a local monopoly. In Appendix 4, we show how we derive the firms' profits for each combination of strategists.

4.1. The Market Equilibrium

In the duopoly market, equilibrium consists of a pair of strategies, one for each of the two firms, such that neither firm has incentive to deviate to another strategy. Table 4 presents the market equilibriums for different combinations of the cost parameters (c_A, c_B), when β is 0.3. For each pair of cost parameters (i.e., for each cell in Table 4), we used the profit expressions from Appendix 4 to create the payoff matrix for the game between the two firms, and determined which pair of strategies consist an equilibrium (see examples of payoff matrixes in Tables 5-8). The parameter values for Table 4 were chosen so that, regardless of which strategies the firms choose, the market is covered and the firms compete for the marginal customer.

c_B \ c_A	0	10	20	30	40	50	60	70	80
0	**AB**	**AB**	**AB**	aB	aB	aB	**B**	**B**	**B**
10	**AB**	**AB**	**AB**	aB	aB	aB	**B**	**B**	**B**
20	**AB**	**AB**	**AB**	aB	aB	aB	**B**	**B**	**B**
30	Ab	Ab	Ab	Ab | aB	aB	aB	**B**	**B**	**B**
40	Ab	Ab	Ab	Ab	*ab*	*ab*	*ab*	*b*	*b*
50	Ab	Ab	Ab	Ab	*ab*	*ab*	*ab*	*b*	*b*
60	**A**	**A**	**A**	**A**	*ab*	*ab*	*ab*	*b*	*b*
70	**A**	**A**	**A**	**A**	*a*	*a*	*a*	0	0
80	**A**	**A**	**A**	**A**	*a*	*a*	*a*	0	0

Table 4. The market equilibrium when N=1000, $\overline{\theta} = 200$, t = 1/2, s=1000, α_1=0.07, α_2= 0.05, β=0.3.

- Letter of the firm's name in upper case: the firm sells a bundle.
- Letter of the firm's name in lower case: the firm sells the service separately.
- Letter of firm's name does not appear in the cell: the firm sells only the product.
- Several equilibriums for the same cell are separated by | .
- 0 – No firm offers the service in equilibrium

From Table 4, we learn that Firm i offers a bundle for low c_i values, sells the service separately for mid-range c_i values and does not sell the service for large c_i values. The boundary values (for c_i) may increase as the competitor's marginal cost, c_j, increases. For example, in Table 4, when c_B=10, Firm A offers the bundle for $c_A \leq 20$, sells the service separately for $c_A \in [30, 50]$ and does not sell the service for $c_A \geq 60$. However, when c_B=70, Firm A offers the bundle for $c_A \leq 30$, and sells the service separately for $c_A \in [40, 60]$. Thus, as Firm B's cost decreases, Firm A might stop offering the service or it might switch from selling a bundle to selling the service separately. Interestingly, if Firm A were a monopoly, and the parameter values were the same as in Table 4, Firm A would choose to sell a bundle for $c_A < 112.8$; That is, it would sell the bundle for all c_A values listed in Table 4.[13] Thus, due to the competition, Firm A is less likely to sell the service than if it were a monopoly.

[13] Notice, that in the duopoly model with t=0.5, as in Tables 4, consumers valuations (net of misfit cost) for the product sold by Firm A are uniformly distributed in $[\overline{\theta}/2, \overline{\theta}]$. Thus, for comparison purposes, we derived Firm A's optimal strategy if it were a monopoly when consumers valuations for its product are uniformly distributed on $[\overline{\theta}/2, \overline{\theta}]$. This is a generalization of the monopoly model presented in Section 3, in which valuations were assumed to be uniformly distributed on $[0, \overline{\theta}]$.

In general, an equilibrium in which one firm sells a bundle while the other does not sell the service prevails when the former firm has a significant cost advantage. An equilibrium in which one firm sells a bundle while the other sells the service separately (cells with 'Ab' or 'aB') prevails when the former firm has a low marginal cost while the latter has a mid-range cost. An equilibrium in which one firm sells the service separately while the other does not sell the service (cells with '*a*' or '*b*') prevails when the former firm has a mid-range cost and the latter has a high cost. Finally, when both firms incur low cost of offering the service, there is equilibrium in which both sell the bundle and when both firms have a high marginal cost, there is an equilibrium in which neither sells the service. Next, we examine the entire payoff matrix for a few of the cells from Table 4.

In the case presented in Table 5, the two firms have the same low cost of offering the service, $c_A=c_B$ =10, and in the unique Nash equilibrium both firms sell a bundle of product and service, which also maximizes social welfare. Interestingly, this example illustrates a situation of Prisoner's dilemma. The two firms' profit in equilibrium (22k) is less than the profit when neither firm offers the service (50k) or when both sell the service only to Group 1 customers (~46k). Due to the low marginal cost, offering a bundle is a dominant strategy for each of the firms. That is, bundling maximizes the firm's profit regardless of the strategy chosen by the other firm. However, when both firms offer a bundle they are both worse off. Notice that when both firms sell a bundle, they lose a market share of Group 2 consumers with low product valuation (so that their combined valuation for the product and the service is low).

Firm A \ **Firm B**	**No Service**	**Bundle**	**Separate**	$\bar{\theta}$ = 200 N = 1000 s = 1000 α_1 = 0.07 α_2 = 0.05 β = 0.3 t = 1/2
No Service	50000, 50000	15559, 64892	41197, 56924	
Bundle	64892, 15559	**22000, 22000**	53724, 18190	
Separate	56924, 41197	18190, 53724	45777, 45777	

Table 5. The payoff matrix for the duopoly when $c_A=c_B=10$; an example of a Prisoner Dilemma

In Table 6, c_A=c_B=30 and in equilibrium one firm chooses to provide the service bundled with the product, while the other sells the service only to Group 1 customers. Interestingly, the two firms choose different pricing strategies even though their marginal cost is same. Notice that if one firm offers a bundle and the other sells the service separately or not at all, than the first firm targets mainly Group 1 customers (bundle price is lower than the sum of the two prices set separately), while the latter firm targets Group 2 customers who might want to buy only the product. Thus, when only one firm bundles the service with the product, the firms are better differentiated in their offerings than when both choose the same strategy.

Firm A \ Firm B	**No Service**	**Bundle**	**Separate**	$\bar{\theta}$ = 100 N = 1000 s = 1000 α_1 = 0.07 α_2 = 0.05 β = 0.3 t = 1
No Service	25000, 25000	20250, 56250	23510, 32574	
Bundle	56250, 20250	22000, 22000	**48252, 22111**	
Separate	32574, 23510	**22111, 48252**	29132, 29123	

Table 6. The payoff matrix for the duopoly when c_A=c_B=30; an example with two symmetric Nash equilibriums: one firm offers a bundle and the other sells service separately

In the example presented in Table 6, if the two firms choose the bundle pricing, then their bundle price is 74. On the other hand, if Firm A and Firm B choose the bundle pricing and the separate pricing, respectively, then the optimal bundle price of Firm A is 111, and Firm B's product and service prices are 47.42 and 55.63. The reason both firms can increase their prices is that now the two firms are better differentiated. In the case presented in Tables 6, consumer surplus and social welfare are maximized when both firms sell a bundle. Thus the service is under-supplied. This example shows that even in a duopoly, when the market is covered (everyone buys at least a product), the service may be under-supplied.

4.2. Social Welfare

In this section, we examine analytically the case in which consumers have homogeneous service valuations, and thus each firm either sells a bundle of product and service or does not sell the service.

We limit our analysis to cases in which, regardless of which strategy each firm chooses, the market is covered (i.e., everyone buys at least a product) and each firm has a positive demand. The required parameter conditions are derived in the proof of Proposition 6.

Table 7 lists the payoff (profit) matrix for the game between the two firms when consumers have homogeneous service valuations. The profit equations in Table 7 were derived by substituting $\alpha_1 = \alpha$ and β=1 in the profit equations from Appendix 4 (for the four relevant cases). The existence conditions for the different Nash equilibriums are specified in Proposition 6.

Firm A \ **Firm B**	**No Service**	**Bundle**
No Service	$\pi_A^0 = \frac{Nt\bar{\theta}}{2}$ $\pi_B^0 = \frac{Nt\bar{\theta}}{2}$	$\pi_A^B = \frac{N(3t\bar{\theta} - \alpha(2N+s) + c_B)^2}{9(2t\bar{\theta} - \alpha N)}$ $\pi_B^B \frac{N(3t\bar{\theta} - \alpha(N-s) - c_B)^2}{9(2t\bar{\theta} - \alpha N)}$
Bundle	$\pi_A^A = \frac{N(3t\bar{\theta} - \alpha(N-s) - c_A)^2}{9(2\bar{\theta} - \alpha N)}$ $\pi_B^A = \frac{N(3t\bar{\theta} - \alpha(N+2s) + c_A)^2}{9(2t\bar{\theta} - \alpha N)}$	$\pi_A^1 = \frac{N(3t\bar{\theta} - 3\alpha N - c_A + c_B)^2}{18(t - \alpha N)}$ $\pi_B^1 = \frac{N(3\bar{\theta} - 3\alpha N + c_A - c_B)^2}{18(\bar{\theta} - \alpha N)}$

Table 7. The profits of the firms for each pair of strategies, when consumers have homogeneous service valuations. A superscript 0 indicates that neither firm sells the service. A superscript *i* indicates that only Firm *i* sells the service. A superscript 1 indicates that both sell the service.

Proposition 6*. With two competing firms and homogenous service valuations, the market structure is as follows*

i) *Both firms offer a bundle is equilibrium iff* $c_A < c_B X + Y$ *and* $c_B < c_A X + Y$

ii) *Both firms offer only product is equilibrium iff* $c_A > \bar{c}$ *and* $c_B > \bar{c}$,

iii) *Only firm A offers a bundle is equilibrium iff* $c_A < \bar{c}$ *and* $c_B > c_A X + Y$

iv) *Only firm B offers a bundle is equilibrium iff* $c_B < \bar{c}$ *and* $c_A > c_B X + Y$

where: $X = 1 - \frac{\sqrt{2t\bar{\theta} - 2\alpha N}}{\sqrt{2t\bar{\theta} - \alpha N}}$, $Y = 3(t\bar{\theta} - \alpha N) + \frac{((2N+s)\alpha - 3t\bar{\theta})\sqrt{2t\bar{\theta} - 2\alpha N}}{\sqrt{2t\bar{\theta} - \alpha N}}$, *and*

$$\bar{c} = 3t\bar{\theta} - \alpha(N-s) - \frac{3}{\sqrt{2}}\sqrt{t\bar{\theta}(2t\bar{\theta} - \alpha N)}$$

Figure 6 exhibits the resulting market equilibrium over the range of the two firms' marginal costs when $\bar{c}X+Y < \bar{c}$ (which also implies $Y< \bar{c}$). We see that in the range *c*-e-*f*-*d*, two equilibriums are feasible: an equilibrium in which only Firm A offers the bundle and an equilibrium in which only Firm B offers the Bundle. In this range, each of the firms does not have incentive to sell the service when the competitor sells the service; however, each firm finds it optimal to sell the service when it is the only one doing so.

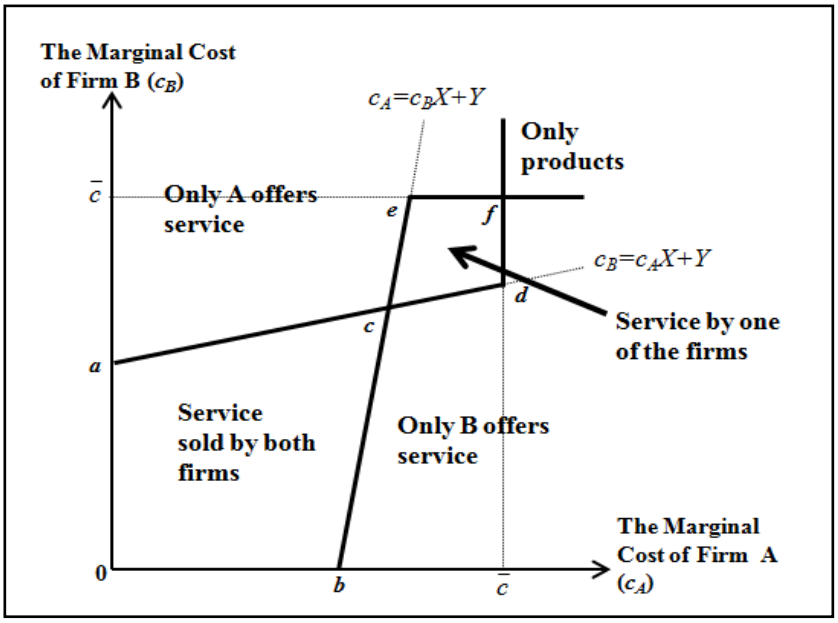


Figure 6. The market equilibrium when consumers have homogeneous service valuations. Two equilibriums exist in *c-e-f-d*: either Firm A or Firm B sells the service.

Perhaps more interesting is the case in which the parameter values are such that $\bar{c}X+Y > \bar{c}$, which is presented in Figure 7. We see that in the range *f-d-c-e*, two equilibriums are feasible: either both firms offer a bundle, or both firms sell only the product. Here, if the competitor sells a bundle, the firm is better off selling a bundle as well because it will find it too difficult to compete when selling only the product (this can happen, for example, when network effects are strong). However, if the competitor does not sell a bundle, the firm would choose to do the same. In such cases, it can be important for social planners to induce the market equilibrium that yields higher social welfare.

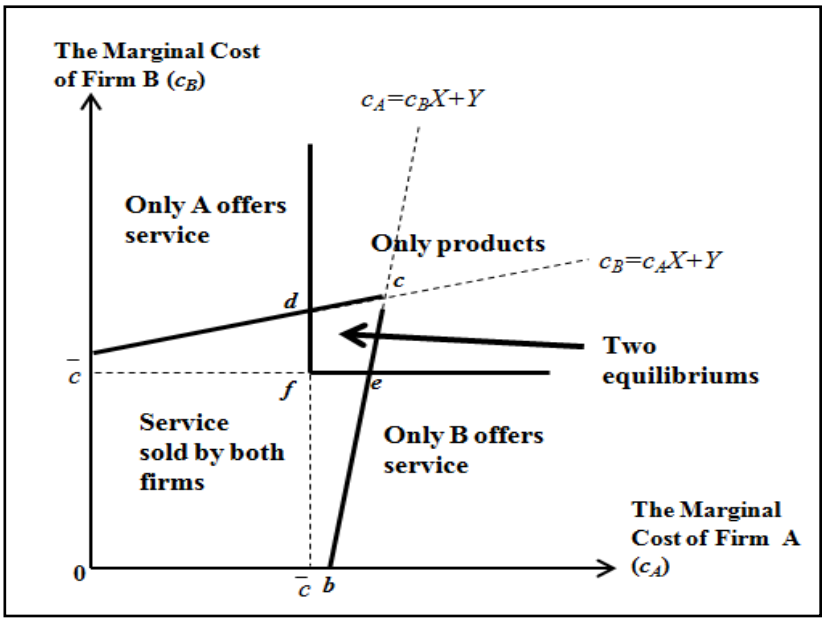


Figure 7. The market equilibrium when consumers have homogeneous service valuations. Two feasible equilibriums in *c-e-f-d*: either both firms sell a bundle or neither firm sells the service.

Numerical examples in the preceding section show that in some cases, the firms offer the service less than is socially optimal. Proposition 7 lists sufficient conditions under which the service is under- or over-provided in the market when $c_A < \bar{c}$ and $c_B < \bar{c}$ (the two conditions guarantee that in equilibrium at least one firm sells the service).

Proposition 7. *When $c_A < \bar{c}$ and $c_B < \bar{c}$:*

i) In equilibrium, the service is under-provided compared to social optimum if $F^A(c_A, c_B) > 0$, $F^B(c_A, c_B) > 0$, and at least one of the following conditions hold: $c_A > c_BX + Y$ or $c_B > c_AX + Y$.

ii) In equilibrium, the service is over-provided compared to the social optimum if $c_A < c_BX + Y$, $c_B < c_AX + Y$, so that both firms sell the bundle, and $F^A(c_A, c_B) < 0$ or $F^B(c_A, c_B) < 0$ (or both)

where X and Y are as given in Proposition 6 and

$$F^i(c_i, c_j) = \frac{\alpha N^2\left(4\alpha^2N^2+4t^2\bar{\theta}^2(\bar{\theta}+2)-3\alpha Nt\bar{\theta}(\bar{\theta}+4)\right)}{36\left(2t\bar{\theta}-\alpha N\right)^2\left(t\bar{\theta}-\alpha N\right)^2}c_i^2 +$$

$$\frac{c_i}{18}\left(\frac{N\left(4\alpha N-t\bar{\theta}(\bar{\theta}+4)\right)}{18\left(t\bar{\theta}-\alpha N\right)^2}c_j + \frac{\alpha N\left(4st\bar{\theta}(\bar{\theta}+4)-\alpha N^2(4+3\bar{\theta})+8N\left(t\bar{\theta}(\bar{\theta}+1)-\alpha s\right)\right)}{18\left(\bar{\theta}-\alpha N\right)^2}\right) + \frac{N(t\bar{\theta}(\bar{\theta}+4)-4\alpha N)}{36\left(t\bar{\theta}-\alpha N\right)^2}c_j^2 - \frac{c_jN\bar{\theta}}{2} +$$

$$\frac{\alpha N\left(2\alpha^2 N^3(15\bar{\theta}-8)-4st\bar{\theta}(s\alpha(4+\bar{\theta})-18t\bar{\theta}^2)+4N\left(2s^2\alpha^2+9t^2\bar{\theta}^2(3\bar{\theta}-2)-2\alpha st\bar{\theta}(2+11\bar{\theta})\right)+\alpha N\left(t\bar{\theta}(68-115\bar{\theta})+8s(\alpha+3\alpha\bar{\theta})\right)\right)}{36(2\bar{\theta}-\alpha N)^2}$$

Part (i) of Proposition 7 specifies the conditions required so that social welfare when both firms sell the bundle is higher than the social welfare when only one firm sells a bundle ($F^i(c_A, c_B) > 0$ is the same as $\pi_A^1 + \pi_B^1 + CS^1 > \pi_A^i + \pi_B^i + CS^i$) , but in equilibrium at least one firm does not offer the bundle. Part (ii) of Proposition 7 specifies the conditions required so that in equilibrium both firms offer the bundle, but social welfare is higher if one of the firms does not offer the bundle.

Figure 8 exhibits the results from Proposition 7 for a specific set of parameter values. The service is offered by the two firms in the region of *0-a-b-d*; only one firm operates the service in the other regions. However, it is socially optimal for both firms to offer the service only within the two curves of *f-g* and *e-h*. Therefore, there is under-provision of the service in the horizontally striped area. On the other hand, the service is excessively operated in the two vertically striped areas. In these areas, a sufficiently low marginal cost enables two firms to compete selling the service, even though it is socially optimal that the firm with the higher marginal cost does not operate the service.

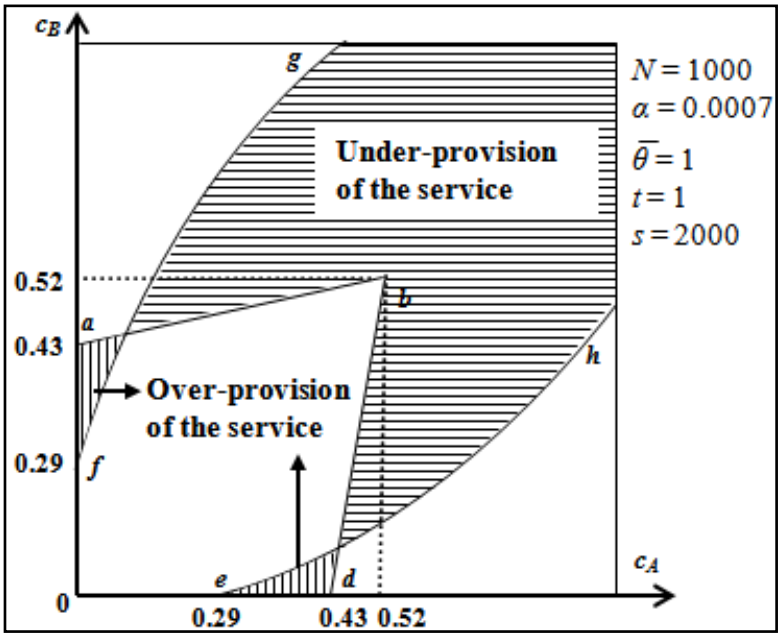


Figure 8. Social inefficiencies in the duopoly model when consumers have homogenous service valuations

The following proposition shows that an increase in service valuations, i.e. an increase in α, decreases the prices charged by the two firms and reduces the firms' profits when both offer a bundle. Thus, although consumers benefit from stronger network effects, the firms not only cannot capture the additional surplus created, but engage in stronger price competition leading to lower profits. This partly explains cases of under-provision of the service at the presence of network externalities.

Proposition 8. *When both firms offer the bundle in equilibrium,*

$\frac{\partial p_A}{\partial \alpha} = \frac{\partial p_B}{\partial \alpha} = -N < 0$ *and* $\frac{\partial \pi_A}{\partial \alpha} = \frac{\partial \pi_B}{\partial \alpha} < 0$ *, implying that an increase in the degree of network externalities decrease the bundle price charged by the firms as well as their profits.*

5. Conclusions

In this paper we study firms who can offer a complementary online service to consumers who buy their product. We consider services with positive network externalities, a characteristic that is common to many online services. We examine under which conditions the firm should sell a bundle of service and product rather than sell the service separately, or not at all. In addition, we examine social welfare and consumer surplus and determine whether online services might be under- or over-supplied in the market.

We show that often consumer surplus is maximized by a bundle of product and service, but the monopoly chooses to sell the service separately or not at all. This finding is in contrast to common contentions in the bundling literature, according to which bundling allows the monopoly to extract more consumers surplus, but it is consistent with findings from recent papers such as Dewan and Freimer (2003), which show that a bundle is provided less than is socially optimal.

Surprisingly, we find that the presence of network externalities may explain the under-provision of the service. We show that when consumers have homogenous service valuations, the service might be under-supplied only if it exhibits network effects. In addition, we show that as technology

progresses, lowering the cost of the service and increasing the value of the service and its network, the adoption of online services by firms may lag behind the socially optimal level. Our findings imply that social planners or policy makers may have to provide the monopolist with incentives to offer online services.

In most industries, price discrimination hurts consumers and increases the firm's profits. We show that this is not necessarily true when considering the provision of online services with network externalities. Specifically, if the monopoly can exert third-degree price discrimination (selling the bundle at different prices to different market segments based on their service valuations), in some cases both consumer surplus and the firm's profit increase. This can happen when with third-degree price discrimination the monopoly chooses to sell a bundle, while without the ability to price-discriminate, he sells the service separately.

While under-provision of service can occur when there is only one firm in the market, it seems less plausible to be the market outcome when two firms compete with each other. We thus investigate market equilibrium and strategic interactions in a horizontally differentiated duopoly and show that even with two firms, services can be under-supplied. Specifically, we show examples in which consumer surplus is maximized if both firms sell a bundle but in equilibrium only one of the firms sells a bundle while the other offers the service separately or not at all.

Our numerical investigation of the duopoly model also demonstrates that in some cases the firms engage in a Prisoner's Dilemma, where both firms sell a bundle, but they would be better off if both sell the service separately or not at all. In addition, we show that in some cases, by choosing different strategies for providing the service (i.e., one firm sells a bundle while the other sells the service separately), the firms can differentiate their offerings and target different market segments, which leads to higher prices and profits compared to when both choose the same strategy. Thus, coordination can often increase the entire industry's profit. Finally, we show that an increase in

network externalities can intensify price competition, reducing prices and profits.

In summary, based on our analytical and numerical investigation, firms who consider offering online services should follow the following guidelines: 1) Bundling the service with the product is not always optimal. As the cost of providing the service or the value of the service to customers increases, selling the service separately at a subscription fee might yield greater profit. 2) In a competitive environment, collaboration and alliances between firms can often increases the industry's profit. 3) Third-degree price discrimination can increase the firm's profit, but can also increase consumer surplus, a win-win situation. 4) In a competitive environment, investments in technology that are expected to increase the degree of network externalities should be considered carefully if the technology cannot be patented. While generating more value to consumers, an increase in the degree of network externalities can lead to lower prices and profits.

An interesting extension to our work would be to examine the duopoly case when consumers can be interested in buying both products. or when the service offered by one firm can be valuable to consumers who buy the competitor's product. It is not clear whether we would still find under-provision of the service and whether an increase in the extent of network externalities would still have a negative effect on profits. We predict the answer to these questions to be yes, because if the service sold by one firm is valuable to consumers of both products, competition will only intensify and network effects have the potential of being stronger (combing two networks). As online services proliferate and become strategically important for firms, it is important for researchers to investigate how firms should strategically profit from offering such services.

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