

Designing Clearinghouse Default Funds*

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Abstract

A prominent post financial crisis reform to reduce counterparty risk in over-the-counter markets is the adoption of clearinghouses. Current standards require clearing members to contribute to a loss-mutualizing default fund so as to cover the liquidation costs imposed by the default of two members, the “Cover II” rule. We show that such an arrangement is intrinsically vulnerable: although the default funds allow members to share risk ex-post, an inherent externality induces members to take excessive risk ex-ante. We design a default fund level that trades off ex-post risk-sharing with ex-ante risk-shifting, thus providing regulators an optimal cover rule for default fund collection.

Keywords: Central counterparties (CCPs), default funds, loss mutualization, externality, risk-taking

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1 Introduction

Heightened counterparty risk during the great recession has led policymakers to mandate central counterparties (CCPs) in the over-the-counter (OTC) derivatives markets. Through a process called novation, after a trade is established between two parties, the contractual obligations are replaced by equivalent positions between the two original parties and the clearinghouse. In the event of one party’s default, the original counterparty is insulated from losses as his contractual position is now with the clearinghouse. In this respect, the clearinghouse is referred to as a central counterparty (CCP); see also Pirrong (2011) for a comprehensive review of clearinghouse functioning mechanisms. Both the Dodd-Frank Wall Street Reform Act in the United States and the European Market Infrastructure Regulation in Europe mandate the central clearing of all standardized OTC derivatives contracts. The class of products being centrally cleared is rising steadily—around 80% of all interest rate and credit derivatives in the United States are now centrally cleared (Financial Stability Board Report, 2015).

There is, however, still considerable debate over the optimal design of clearinghouse arrangements (see, e.g., Dudley, 2014, and *Economist*, 2014). One aspect of the arrangements is that members are required to contribute to a loss-mutualizing default fund. Currently, the required contribution by each member is such that the default fund is sufficient to cover the liquidation costs of two defaulting members, the “Cover II” rule.¹ The default loss of an institution that exceeds its initial margins and its default fund contribution are absorbed by the CCP equity capital and the default fund contributions of the surviving members. Typically, the CCP’s equity capital is the first to absorb losses. Residual losses are then allocated to surviving members on a pro-rata basis.²

¹CPSS-IOSCO regulatory guidelines require CCPs to maintain a default fund sufficient to cover the liquidation costs caused by the default of two members, in extreme yet plausible market scenarios. The recent European Market Infrastructure Regulation requires each CCP to cover the default of the clearing member to which it has the largest exposure, or of the second and third largest clearing members if the sum of their exposures is larger.

²There is not yet a universally agreed upon loss allocation rule. Ice Clear Credit, the leading US CDS clearinghouse, distributes losses to non-defaulting clearing members on a pro-rata basis for cleared futures and options contracts (see ICE, 2016). Similarly, for cleared credit default swaps, it allocates losses among members on a pro-rata basis corresponding to the uncollateralized stress losses of each individual member.

In this paper, we study the optimal design of the default fund contributions. First, we show that the loss-mutualization arrangement by the CCPs is intrinsically vulnerable: While the default funds allow members to share risk *ex post*, an inherent externality induces members to take excessive risk *ex-ante*. Second, we show that the excessive risk-taking behavior can be mitigated by regulating the amount in the default fund. Thirdly, we design a default fund level that trades off *ex post* risk-sharing with *ex-ante* risk-shifting, thus providing regulators an optimal cover rule for default fund collection. In particular, we show that as the number of clearing members grows large, the optimal default fund should be designed to cover the default costs of a fixed fraction of the members rather than a fixed number of clearing members, as is done, for instance, in the currently implemented “Cover II” rule.

The economic forces under the loss-mutualization arrangement work as follows. Under the pro-rata rule, CCPs redistribute counterparty risk through mutualization among all members. This achieves *ex post* risk-sharing. However, this risk-sharing benefit comes with a flip side. Sharing the common pool of default funds creates a dependency among members. When members can choose to take excessive risk, a typical negative externality arises. Notably, the size of the default fund contribution directly determines the extent of the externality: While a lower default fund reduces the opportunity costs of the members, it leads to larger negative externalities among members, reducing total welfare through excessive risk-taking. As such, a regulator faces a tradeoff in collecting default fund contributions between (1) reducing the counterparty risk of clearing members and (2) generating excessive risk taking. We study this tradeoff and identify the right balance in designing the clearinghouse default funds. To the best of our knowledge, our study is the first to address this mechanism.

We develop a game-theoretical model to study the optimal design of default funds. In the model, a regulator decides what the size of the default fund contribution should be in order to maximize social welfare, defined as the aggregate value of clearing members and CCP. Given the required default fund amount, members of the clearinghouse decide on the riskiness of the undertaken projects. More specifically, each clearing member maximizes its expected total utility, taking into account the costs the member incurs to absorb losses generated by other

members’ defaults. Thus the strategic interaction between the regulator and its members is modeled through a Stackelberg game with a coordination problem, in which the regulator is the leader and the members of the CCP are the followers.

Using the model, we study the positive and normative implications of default fund requirements. Our positive analysis focuses on the social welfare implications of the “Cover II” rule.³ We show that a collective default fund under the “Cover II” rule might lead to moral hazard problems by reducing the incentives of a clearing member to avoid default. A member may decide to engage in excessively risky activities because the cost of its own default would be jointly borne by the other participants through their default fund contributions. This generates an inefficiency, especially in the setting of central clearing where mitigating systemic risk is critical. From a normative perspective, our results shed light on the optimal cover number for the default fund rule—i.e., the one that is most socially desirable. We illustrate that, under some circumstances, the inefficiency could be mitigated if the regulator were allowed to decide the size of the contribution by the clearing members to the default fund. By regulating the design of default funds, the regulator can give members an incentive to take less risk, hence improving social welfare.

A major novelty of our study is to analytically derive an optimal default fund level. Our model predicts that, as the number of clearing members increases, the optimal default fund level converges to a fixed fraction of the default cost, provided that the opportunity cost of default fund is not too high. In this case, it is socially optimal to mandate a default fund sufficiently high to cover a proportion of the members in the network. The Pareto dominating equilibrium associated with this default fund level prescribes that all members behave safely; i.e., they run away from the externality imposed by the risk-taking activities of the others. To the extent that the opportunity cost of default fund is associated with the cost of funding the collateral position and thus with the prevailing interest rates, our model predicts that default fund levels should be higher than those prescribed by the Cover II rule in the current

³The CPSS-IOSCO’s Principles for Financial Market Infrastructures (PFMI) published in 2012 state that CCPs should maintain financial resources to cover the default of two participants that would potentially cause the largest aggregate credit exposure for the CCP, in extreme but plausible market conditions.

low interest rate environment. Our results are in line with [ISDA \(2013\)](#), which shows that default fund levels are quite conservative for their sample, and that on average less than 20% of the default funds are used to cover defaults that occur under stressed scenarios.

Our results support the Cover II rule when the costs of funding collateral are high. In this case, the socially optimal choice is to require members to contribute a lower amount to the default fund. Such an action induces members to engage in risky activities, resulting in a higher expected number of defaults than what could be covered by the members’ default fund levels. Hence, our analysis suggests that the Cover II rule should be viewed in a different perspective: it is optimal when funding illiquidity (associated with marginal opportunity costs of default fund) is so high that it becomes socially preferable to induce a higher number of defaults in the system, than to subject members to the very high costs of raising collateral.

The paper proceeds as follows. [Section 2](#) reviews the literature. [Section 3](#) introduces the baseline model with binary risk and demonstrates members’ incentives for excess risk-taking. [Section 4](#) analyzes the game between the regulator and the clearing members; we demonstrate that although members’ risk-taking is unobservable, it can be supervised when the regulator strategically chooses the default fund contribution. [Section 5](#) generalizes the environment to the case of continuous risk choice and compares the social benefit and cost of increasing the size of the default fund. [Section 6](#) concludes. Proofs of technical results are in the Appendix.

2 Literature Review

Our main contribution to the literature on clearinghouses is to develop a tractable model that delivers explicit “Cover type” rules, accounting for the main economic forces at play—i.e., default costs, opportunity costs of posting collateral, and the risk-return trade-offs of the investments. These rules can be readily employed by clearinghouse supervising authorities to perform macro-stress testing, and compared with the currently employed Cover II rule. To the best of our knowledge, our study is the first to theoretically investigate the design of collateral resources further down the waterfall, namely the default levels.

Following the Dodd-Frank mandatory clearing for standardized OTC derivatives, several studies have analyzed the extent to which central clearing reduces counterparty credit risk. The seminal paper by [Duffie and Zhu \(2011\)](#) shows that netting benefits exist only if a clearinghouse nets across different asset classes, while counterparty credit risk may arise if the clearing process is fragmented across multiple clearinghouses. These predictions are empirically confirmed by [Duffie, Scheicher, and Vuillemeys \(2015\)](#) using bilateral exposures data from the credit default swap market.⁴ [Biais, Heider, and Hoerova \(2016\)](#) study how hedging with derivatives can introduce moral hazard originating from the fact that the trading counterparties neglect risk management. They show that margin calls can be optimally designed to mitigate the moral hazard problem thereby enhancing risk-sharing. While these studies consider initial margins—i.e. collateral resources designed to absorb the losses of an individual member—our paper focuses on the optimal degree of risk-sharing for the determination of default fund requirements.

Other studies have analyzed the risk-management implications of transparency introduced by a central clearinghouse. [Acharya and Bisin \(2014\)](#) illustrate one type of counterparty externality arising from the lack of portfolio transparency in OTC markets and show that it can be corrected if trades are centrally cleared. [Zawadowski \(2013\)](#) shows that the establishment of a clearinghouse can improve efficiency as it effectively forces banks to contribute ex-ante to bail-out defaulting counterparties, thus reducing the hedging losses of a bank. In his model, defaults are caused by informational effects that induce runs on banks if they have experienced hedging losses. [Antinolfi, Carapella, and Carli \(2016\)](#) show that central clearing can be socially inefficient because loss-mutualization may weaken the incentives to acquire and reveal information about counterparty risk, whereas bilateral trading typically encourages assessment of counterparty risk. Different from these works, we consider a symmetric information model and highlight a different form of inefficiency due to loss mutualization. Like [Zawadowski \(2013\)](#), we show that default fund requirement is a tool that can be used to correct for inefficiency; unlike him, however, we find that the domino effects of defaults

⁴[Loon and Zhong \(2014\)](#) and [Bernstein, Hughson, and Weidenmier \(2014\)](#) both find evidence consistent with central clearing reducing counterparty risk, using CDS spreads data and historical data, respectively.

plays the prominent role, and the inefficiency is corrected by balancing the benefits of ex-post risk-sharing with the costs of ex-ante risk-shifting.

Our paper contributes to a nascent literature focusing on default fund requirements. [Menkveld \(2017\)](#) analyzes systemic liquidation within a crowded trades setting and sets the default fund as the minimum level of funds needed to cover default losses in extreme yet plausible conditions. [Ghamami and Glasserman \(2017\)](#) provide a calibration framework and show that lower default fund requirements reduce the cost of clearing but make CCPs less resilient. While these studies follow risk-measure based rules to determine default funds, the latter are endogenously determined in our model. Other studies focus on the default fund design explicitly. [Amini, Filipović, and Minca \(2015\)](#) analyze systemic risk under central clearing in an Eisenberg-Noe type clearing network, and propose an alternative structure for a default fund to reduce liquidation costs. In contrast, we solve for the optimal default fund in a symmetric equilibrium setting, taking members' incentives into account. [Capponi, Cheng, and Sethuraman \(2017\)](#) analyze the incentives behind the determination of default fund and clearinghouse equity in the default waterfall structure. In their model, the mass of safe and risky members is exogenously specified, and their objective is to study the optimal balance of equity and default fund requirements from the clearinghouse point of view. In our model, members instead endogenously choose the risk profile, and we focus on the socially optimal cover rule.

In emphasizing an intrinsic vulnerability from loss mutualization, our paper joins the literature that highlights various aspects of inefficiency and “unintended consequences” associated with central clearing. The model proposed by [Koepl and Monnet \(2010\)](#) predicts that, though CCP clearing can induce traders to take the socially optimal level of counterparty risk, it affects liquidity across the OTC markets in a way that not all traders universally benefit. Building on this model, [Koepl \(2012\)](#) shows that higher collateral requirements lower default risk, but also reduce market liquidity, which in turn amplifies collateral costs. [Pirrong \(2014\)](#) argues that central clearing reforms may redistribute risk rather than reduce it, and that expanding clearing makes the financial system more connected and transforms credit

risk into liquidity risk. [Arnold \(2017\)](#) shows that, under current market regulations, central clearing may have unintended consequences such as a higher number of issued loans, but a lower credit quality of these loans. A closely related paper by [Biais, Heider, and Hoerova \(2012\)](#) shows that the main advantage of centralized clearing is loss mutualization, which fully insures members against idiosyncratic risk, but not against aggregate risk. They argue that CPP should be designed to incentivize members to search for solid counterparties under aggregate risk. In contrast, we show that the inefficiency arises even when the source of risk is idiosyncratic, and we demonstrate how this inefficiency can be mitigated by the optimal design of default fund contributions.

3 Baseline Model

In this section we introduce our baseline model in which clearing members have a binary choice of risk. By comparing the (first-best) risk level that maximizes social welfare with the one that maximizes individual members' profit, we demonstrate that members have an incentive for excessive risk-taking due to an inherent externality associated with loss mutualization.

3.1 The Environment

We consider a two-period model, $t = 0, 1$. The economy consists of N homogeneous clearing members and the regulator. The clearing members are risk-neutral. At $t = 0$, the regulator chooses the required level of the default fund and the members make investment decisions. At $t = 1$, payoffs are realized.

Clearing Members. Clearing members may differ in their choice of investments such as whether to invest in a high-risk or low-risk project. Project types can model engagement in risky investments, choice of weak trading counterparties before novation, lower effort in risk management, or reduced hedging of counterparty exposure. Such choices are unobservable and capture the risk-return tradeoff faced by member institutions.

The payoffs from investing in a high-risk project and a low-risk project are denoted, respectively, by R_h and R_l . The expected payoff from the investment depends on the risk level. Let $\mu = \mathbb{E}[R]$ and assume $\mu_h > \mu_l$. We denote by q_h and q_l the default probability of a member who chooses, respectively, the high-risk and low-risk project. Defaults are costly, and we use c to denote the constant default cost. We analyze the economic role of default funds, and do not model initial margins which usually serve as the first line of defense against default losses. Hence, we can view the default costs as describing the losses that exceed initial margin requirements. The following assumption formalizes the risk-return tradeoff.

Assumption 1 *A member who chooses the high-risk project has a higher expected return but a higher default probability than a member who chooses the low-risk project – i.e.*

$$0 < \mu_l < \mu_h, \quad 0 < q_l < q_h. \quad (1)$$

Default Fund. Clearing members derive risk-sharing benefits from entering into a *loss mutualization* arrangement with other member's through the CCP. A *positive default fund*, $F > 0$, sets an upper bound on the loss given the default of a member. We consider a default waterfall structure in which the clearinghouse capital has seniority over the default funds of surviving members in absorbing losses.^{c0} After the funds of a defaulted clearing member are exhausted, the loss mutualization mechanism ensures that the remaining losses are allocated proportionally to the available default funds of the surviving members. This is the risk-sharing mechanism.^{c0} On the other hand, the default fund increases a surviving members chances of incurring a loss when others default, creating externality among members.

Specifically, the CCP charges a default fund amount F to each member upfront. Because the default fund is segregated, each member will incur an opportunity cost of $r_f F$, where

^{c0}A well known example is the Korean CCP KRX. The default of a clearing member in December 2013 generated losses that exceeded the defaulter's collateral. According to the KRX's rules, the remaining losses were allocated first to the default fund contributions of surviving participants.

^{c0}In the contractual agreement between the clearinghouse and the clearing members, the initial margins of a defaulted member are first used to cover losses arising at liquidation. In our analysis, we do not model the initial margins in the clearinghouse default waterfall to focus entirely on the risk-shifting incentives triggered by the default fund resources. This comes without loss of generality, however, as we can view the default costs as the losses that exceed initial margin requirements.

$r_f \in (0, 1)$ denotes the risk-free rate.

A member is willing to participate in such a loss mutualization fund only if the sum of his default fund contribution and the resulting opportunity cost does not exceed his default cost. To summarize, it is rational for individual members to participate in loss mutualization only if

$$0 < F + r_f F < c. \quad (2)$$

Investment Choice of a Clearing Member. For a given default fund requirement F , and for given investment choice $a^{-i} = (a_1, a_2, \dots, a_{i-1}, a_{i+1}, \dots, a_N)$ made by all clearing members except i , member i decides on the riskiness of his investment to maximize the following objective function:

$$V_i(a^{-i}) = \max_{a_i \in \{h, l\}} \mathbb{E} \left[R_{a_i} - \mathbf{1}_{i \text{ defaults}} F - \mathbf{1}_{i \text{ survives}} \min \left(F, \frac{\sum_{j \neq i} \mathbf{1}_{j \text{ defaults}}}{1 + \sum_{j \neq i} \mathbf{1}_{j \text{ survives}}} (c - F) \right) - r_f F \right]. \quad (3)$$

The objective function (3) can be understood by analyzing two possible scenarios:

1. If i defaults, i does not contribute to the loss mutualization, but i 's default fund F will be used to cover its loss. i 's total cash flow will thus include the value of its investment minus the default fund and the associated opportunity cost:

$$R_{a_i} - F - r_f F \quad (4)$$

2. If i survives, the default fund of i will be used to absorb the losses generated by defaulted members, if any; hence, the cash flow is given by

$$R_{a_i} - F \min \left(1, \frac{\sum_{j \neq i} \mathbf{1}_{j \text{ defaults}} (c - F)}{F + \sum_{j \neq i} \mathbf{1}_{j \text{ survives}} F} \right) - r_f F. \quad (5)$$

The cost to a surviving member is the highest if all other clearing members default because that member will have to bear all costs totaling $\min[F, \sum_{j \neq i} (c - F)]$. If no clearing member defaults, the cost incurred by i will be zero. More generally, if k members default, $k < N$, all N members will collectively contribute an amount equal to $\min(NF, kc)$ to absorb the default

losses; if $k \geq \frac{F}{c}N$, then the default funds of all members, totaling NF , will be used to cover the default costs.

Assumption 2 *The aggregate default fund contribution satisfies a Cover II constraint:*

$$2c \leq NF. \quad (6)$$

The Cover II rule provides that the default fund contribution collected from all members, NF , is always sufficient to cover the default costs of two members.

Assumption 2 and the participation constraint in Eq. (2) together set an upper and lower bound for F : $\frac{2c}{N} < F < \frac{c}{1+r_f}$. The *Cover II* constraint is the currently imposed requirement that CCPs should maintain financial resources sufficient to cover a wide range of potential stress scenarios, including the default of two members.

If more than two members default, the total default fund contribution, NF , may not be enough to cover the default costs. In this case, the rest of the default costs are covered by CCP's capital.

Assumption 3 *The CCP contributes with his own capital and incurs an equity loss of $(\mathcal{N}_d \cdot c - N \cdot F)^+$, where \mathcal{N}_d is the number of defaults.*

Assumption 3 guarantees that there are always enough resources in the system to pay for the default costs of members in every possible state.

3.2 First-best Benchmark: optimal investment

For a given default fund contribution F , the choice of risky investments $\{a_i^*\}_{i=1,\dots,N}$ that maximizes the aggregate values of all agents in the economy (clearing members and the CCP) is given by

$$\{a^*(F)\} = \arg \max \mathbb{E} \left[\sum_i (R_{a_i} - c \mathbf{1}_{i \text{ defaults}} - r_f F) \right]. \quad (7)$$

To solve for the socially optimal risk profile, it is sufficient to consider the risk and return tradeoff of a representative clearing member by comparing the return differential $\mu_h - \mu_l$ with the expected default cost differential $c(q_h - q_l)$.

Proposition 1 *The socially optimal risk profile is given by*

$$a_i^*(F) = \begin{cases} l, & \frac{\mu_h - \mu_l}{q_h - q_l} \leq c \\ h, & \frac{\mu_h - \mu_l}{q_h - q_l} > c \end{cases} \quad (8)$$

The focus of this paper is on the members' *ex ante* incentive to take excessive risk. Therefore, for the rest of Section 3, we consider the case that the socially optimal risk profile is the low-risk project and impose the following:

Assumption 4 *The parameters $\{\mu_h, \mu_l, q_h, q_l, c\}$ satisfy the condition*

$$\frac{\mu_h - \mu_l}{q_h - q_l} \leq c. \quad (9)$$

3.3 Equilibrium Investment

In this section, we solve for the best response of members to a given choice of default fund F . Our objective is to compare the investment choice of the profit-maximizing members in equilibrium with the socially optimal choice. We call *risk-shifting* the situation in which the first-best outcome prescribes that all members choose the low-risk project but the equilibrium response of members is to instead undertake the high-risk project. This is socially inefficient in the sense that by taking excessive risk members obtain a lower total value.

Definition 1 *Under Assumption 2, and if F satisfies (2), the risk profile $(a_1, a_2, \dots, a_N) \in \{h, l\}^N$ is a Nash equilibrium if for all i , it holds that*

$$V_i(a^{-i}) = \mathbb{E} \left[R_i^{a_i} - \mathbf{1}_{i \text{ defaults}} F - \mathbf{1}_{i \text{ survives}} \min \left(F, \frac{\sum_{j \neq i} \mathbf{1}_{j \text{ defaults}}}{1 + \sum_{j \neq i} \mathbf{1}_{j \text{ survives}}} (c - F) \right) - r_f F \right].$$

A Nash equilibrium (a_1, a_2, \dots, a_N) is Pareto dominating, if, for any other Nash equilib-

rium (b_1, b_2, \dots, b_N) , it holds that

$$V_i(a^{-i}) \geq V_i(b^{-i}), \quad i = 1, 2, \dots, N,$$

and the above inequality holds strictly for at least one i .

Suppose member i survives, and g of the remaining $N - 1$ members choose the low-risk project. Then, for any given admissible choice of default fund, $F \in [\frac{2c}{N}, \frac{c}{1+r_f}]$, the expected contribution of member i to other members' defaults is given by

$$\mathbb{E} \left[\min \left(F, \frac{N - \mathcal{N}_s}{\mathcal{N}_s} (c - F) \right) \middle| \text{member } i \text{ survives} \right] := F - \psi(g; F), \quad (10)$$

where \mathcal{N}_s is the number of surviving members. The left-hand side of (10) is the expected contribution of member i to other members' defaults, given that i survives: If $N - \mathcal{N}_s$ is the number of defaulted members, the total cost of their defaults is $c(N - \mathcal{N}_s)$. Each defaulted member will first absorb the losses using his own default fund, and the remaining cost will be shared equally by the surviving members. Hence, each member i will be charged, on average, a cost equal to $(c - F) \frac{N - \mathcal{N}_s}{\mathcal{N}_s}$ capped at maximum amount F that i can contribute. The quantity $F - \psi(g; F)$ on the right-hand side of Eq. (10) is the cost due to loss mutualization imposed by the members of the network on i . A lower value of $\psi(g; F)$ means that the negative externalities on i caused by the risky project choices of the other members are high. The next proposition summarizes our main results on the investment choice of clearing members.

Proposition 2 *For a given default fund requirement F satisfying Assumption 2 and condition (2), if Assumption 4 holds, then all possible equilibrium risk profiles are given by*

$$a^e = \begin{cases} h, \forall i & F < \hat{F} \\ h, \forall i, \text{ or } l, \forall i & \hat{F} \leq F \leq \bar{F} \\ l, \forall i & \bar{F} < F \end{cases} \quad (11)$$

Moreover, $\psi(\cdot; \cdot)$ is a strictly increasing function in both the first and second argument. The

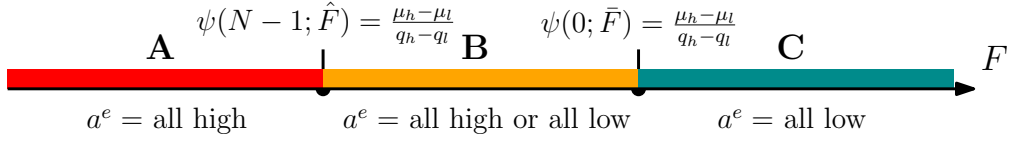


Figure 1. Equilibrium Investment Choices. This figure illustrates how the equilibrium investment choices change as we vary the size F of the default fund. a^e is the equilibrium risk profile from individual members' strategic behavior. In region A, the unique equilibrium differs from the first-best benchmark and features risk-shifting; in region B, we have multiple equilibria; in region C, the unique equilibrium coincides with the first-best.

parameters \hat{F} and \bar{F} are implicitly defined as

$$\psi(N-1; \hat{F}) = \frac{\mu_h - \mu_l}{q_h - q_l}, \quad \psi(0; \bar{F}) = \frac{\mu_h - \mu_l}{q_h - q_l}. \quad (12)$$

If $\hat{F} \leq F \leq \bar{F}$, the “all low”-risk equilibrium Pareto dominates the “all high”-risk equilibrium.

The result in Proposition 2 states that the members' incentives in shifting risk decrease as the required default level of F rises. If the profitability of risk-shifting measured by excess return per unit of default probability, $\frac{\mu_h - \mu_l}{q_h - q_l}$, is exceeded by the negative externalities generated from the risk-shifting behavior, $\psi(0, F)$, or equivalently $F > \bar{F}$, then each member decides to run away from the externalities and chooses the low-risk project. Figure 1 illustrates the equilibrium risk profile as a function of F .

3.4 Inefficiency in Investment Choice: risk-shifting

In this section, we show that there exist cases in which the loss mutualization mechanism induces members to take excessive risk *ex ante* due to an inherent externality among members.

Corollary 2 *If $F < \hat{F}$, the first-best benchmark is not an equilibrium, and in fact, “all high”-risk is the unique inefficient equilibrium. More generally, if $F \leq \bar{F}$, “all high”-risk is an inefficient equilibrium.*

Under $F < \hat{F}$, if all other members choose low-risk, member i strategically deviates to choose high-risk. Under loss mutualization, the expected liquidation cost $\psi(N-1; F)$ is lower

than c . This wedge shifts a member's incentive from choosing low-risk to high-risk, thereby creating a *risk-shifting* problem.

In the presence of risk-shifting incentives, it is still possible to achieve the socially optimal risk-taking in equilibrium. This happens when the default fund contribution F is close enough to the default cost such that the externality is restrained.

Corollary 3 *If $\bar{F} < F$, the “all low”-risk equilibrium is the unique equilibrium and is first-best. Moreover, the “all low”-risk equilibrium is the unique Pareto dominant equilibrium if $\hat{F} < F$.*

4 Equilibrium between Clearing Members and the Regulator

We analyze how to design a default fund that mitigates the inefficiency arising from excessive risk-taking. Corollary 3 indicates that a default fund can potentially correct members' risk-shifting incentives. In this section, we develop a game theoretic analysis to show that the regulator can correct the inherent externality by optimally choosing a default fund, balancing the *ex post* risk-sharing benefit and *ex ante* risk-shifting cost.

Definition 4 *A Nash equilibrium between clearing members and the regulator is a set of members' risk profiles $a^e := (a_1^e, \dots, a_N^e)$ and a default fund contribution F^e set by the regulator, such that:*

1. *Taking the default fund F^e and other members' risk profile a_{-i}^e as given, a_i^e solves the optimization problem of clearing member i given in (3).*
2. *Taking as given the risk profile of clearing members a^e , the regulator chooses a feasible default fund level F^e , satisfying assumptions (2)–(3) and condition (2) to maximize the aggregate value of the members and the equity of CCP:*

$$F^e = \arg \max_F \sum_i V_i - \mathbb{E} [(\mathcal{N}_d \cdot c - N \cdot F)^+]. \quad (13)$$

In the above equation, V_i is given by Eq. (3), and $\mathcal{N}_d = \sum_i \mathbf{1}_{i \text{ defaults}}$ is the number of defaulted members.

4.1 Default Fund: a tool to mitigate risk-shifting

We solve for the optimal default fund level that satisfies the criterion in Equation (13), taking the functional dependence of a^e on F into account. Given the assumption of independent defaults, the objective function of the regulator in (13) may be rewritten as follows:

$$W(F) = \mathbb{E} \left[\sum_i V_i - (\mathcal{N}_d \cdot c - N \cdot F)^+ \right] = \mathbb{E} \left[\sum_i R_{a_i} - \mathcal{N}_d \cdot c - N r_f F \right]. \quad (14)$$

Thanks to Proposition 2, it suffices to consider either “all low” or “all high”-risk profiles. We base our analysis on the equilibrium refinement concept of Pareto dominance: From Proposition 2 and Corollary 3, members all choose low-risk when $\hat{F} \leq F$ because it is either the unique equilibrium (region C in Figure 1) or the Pareto dominating equilibrium (in region B). Hence the equilibrium risk profile *switches* from high to low at the boundary between Regions A and B. To obtain the threshold value of \hat{F} at this boundary, define the linear function of F : $I(F) = \psi(N-1; F) - \frac{\mu_h - \mu_l}{q_h - q_l}$. By Lemma 8 in Appendix A, $I(F)$ is a strictly increasing function. Moreover, $\psi(N-1; c) = c$. Thus if the following condition holds,

$$\psi \left(N-1; \frac{2c}{N} \right) < \frac{\mu_h - \mu_l}{q_h - q_l} c^0, \quad (15)$$

there is a unique threshold value \hat{F} such that $I(\hat{F}) = 0$, i.e. $\psi(N-1; \hat{F}) = \frac{\mu_h - \mu_l}{q_h - q_l}$.

The function $W(F)$ may be defined piecewisely, with just one discontinuity at \hat{F} , as follows:

$$W(F) = N \left(W^l(F) \mathbf{1}_{F \geq \hat{F}} + W^h(F) \mathbf{1}_{F < \hat{F}} \right), \quad W^a(F) = \mu_a - c q_a - r_f F, \quad a = l, h. \quad (16)$$

Because the ‘low-risk’ equilibrium is socially optimal, $W(F)$ exhibits a positive jump as we increase F : the equilibrium switches from “all high” to “all low”-risk at \hat{F} . Denote the size

^{c0}When the inequality fails to hold, the aforementioned switch from “all low”-risk equilibrium to “all high”-risk equilibrium does not occur. The reason is that the lower bound set by the Cover II rule may already be large enough that choosing high-risk is not attractive for an individual member. In the sequel, we only focus on the (interesting) case that the inequality (15) holds.

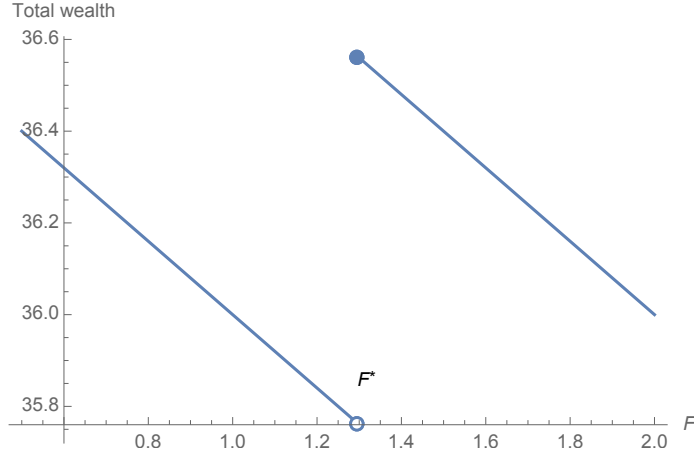


Figure 2. Regulator's Objective Function $W(F)$. This figure plots the total wealth $W(F)$ as a function of F , where F ranges from the lower bound $\frac{2c}{N}$ that satisfies the Cover II requirement to the upper bound that equals the default cost c . The graph shows that a default fund given by the Cover II requirement may not be socially optimal. Rather, a higher value of F that corrects the risk-shifting yields the highest total value. Model parameters: $\mu_h = 5, \mu_l = 4.8, N = 8, c = 2, q_h = 0.2, q_l = 0.05$, and $r_f = 0.1$. The optimal default fund $F^e = \hat{F} = 1.30$, and $W(F^e) - W(\frac{2}{N}c) = 0.16$.

of the upward jump at \hat{F} by Δ . Then

$$\Delta \equiv W^l(\hat{F}) - W^h(\hat{F}) = \mu_l - \mu_h - c(q_l - q_h) > 0. \quad (17)$$

Moreover, $W^h(F)$ and $W^l(F)$ are both strictly decreasing in F . Thus the maximum of $W(F)$ over the set of feasible values for F is attained either at the lower bound $\frac{2}{N}c$, or at the switch point \hat{F} . We summarize this result in the following proposition (see also Figure 2 for an illustration).

Proposition 3 *Suppose the inequality (15) holds. Among all feasible values of F – i.e., those satisfying Assumption 2 and condition (2) – the default fund level that maximizes the regulator's objective function (13) is given by*

$$F^e = \begin{cases} \hat{F}, & \text{if } \Delta > r_f \left(\hat{F} - \frac{2}{N}c \right), \\ \frac{2}{N}c, & \text{else.} \end{cases} \quad (18)$$

In conclusion, the objective of the regulator is to maximize the total value of the agents

in the system, taking members' incentives for risk-shifting into consideration. The choice of default fund yields the following tradeoff. On the one hand, the benefit of having a higher value of F is to correct for risk-shifting: as F increases to \hat{F} , we move from region A to region B in Figure 1, where members' risk choices switch from high to low. Hence, by mandating a high enough default fund F , the regulator can give the member an incentive to choose risk with a total social value equal to $N\Delta$. On the other hand, increasing F raises the opportunity cost of each member from $r_f \frac{2}{N}c$ to $r_f \hat{F}$. If the benefit exceeds the cost, it is optimal to set the default fund at \hat{F} , a higher value than required by the Cover II rule; otherwise, the regulator will find it optimal to use the Cover II rule.

4.2 Cover X: the optimal covering number

The previous sections have shown that the Cover II rule is not necessarily socially optimal, when we consider the role of mitigating risk-shifting. As the number of clearing members increases, how many members' defaults should the default funds be ready to cover? We design a Cover X rule so that the implied default fund achieves the efficiency of \hat{F} in Equation (18).

The *generalized Cover X rule* for any given number N of participating clearing members is

$$x(N) := \frac{N \cdot F^e(N)}{c}, \quad (19)$$

where $F^e(N)$ is the default fund level that maximizes the regulator's objective given in Proposition 3, when N is the number of members in the CCP. When $x(N) > 2$, our model provides a rationale to charge a default fund more than the current regulatory requirement prescribed by the Cover II rule; see also Figure 3 for an illustration.

Interestingly, the exact Cover X rule depends on the number of members. If $N = 3$, then $x(N)$ is approximately equal to two, so Cover II is able to prevent risk-shifting. However, if, for example, $N = 8$, the optimal requirement becomes Cover 5.18. Under the Cover II requirement, clearing members would engage in excessive risk-taking and deviate from the social optimum, resulting in a higher total cost of default. In contrast, the proposed Cover 5.18 requirement induces members to choose low-risk projects, thereby effectively mitigating

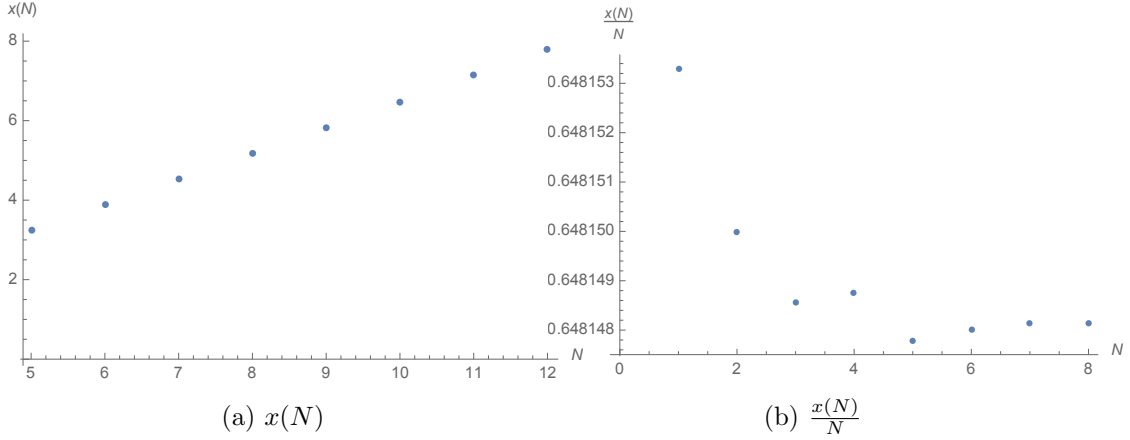


Figure 3. Optimal Covering Number. This figure shows the optimal covering number $x(N)$ given in equation (19) as a function of the number of clearing members N . We use the same parameter settings for project return and default probability as in Figure 2. Left panel: the optimal covering numbers $x(N)$; Right panel: the ratio $\frac{x(N)}{N}$.

risk-shifting.

While the optimal covering number clearly depends on the number of members N , the ratio $x(N)/N$ shows little variation with respect to N , if N is sufficiently large. The next proposition characterizes the asymptotic behavior of the optimal coverage ratio, as the number of clearing members grows large. Specifically, it shows that the ratio between the optimal default fund level $F^e(N)$ and the default cost c , or equivalently, the optimal proportion of covered members, $x(N)/N$, converges to a constant as the size of the CCP network tends to infinity. If the marginal opportunity cost r_f is sufficiently low, then this limit is a positive number in $(0, 1)$, meaning that the optimal covering number should be proportional to the size of network N (at least for large N); otherwise, this limit is 0, implying that it is optimal to cover only a small portion of the CCP network.

Hence, as the number of clearing members grows large, the cover rule that the regulator should adopt is simple—rather than covering a fixed number of clearing members as prescribed, for instance, by the Cover II rule, the regulator should cover a fixed fraction of the members. Considering that the major U.S. derivative clearinghouses consist of more than 30 members (this is the case, for instance, for the major CDS derivative clearinghouse ICE Clear Credit, and interest rates swaps clearinghouse LCH), our result implies that the default fund

rule is robust with respect to entry and exit of the members in the clearing business.^{c0}

Proposition 4 *In the large CCP network limit – i.e., as $N \rightarrow \infty$ – we have*

$$\frac{F^e(N)}{c} = \frac{x(N)}{N} \rightarrow \begin{cases} q_l + (1 - q_l) \frac{1}{c} \frac{\mu_h - \mu_l}{q_h - q_l}, & \text{if } \mu_l - \mu_h - c(q_l - q_h) > r_f(q_l c + (1 - q_l) \frac{\mu_h - \mu_l}{q_h - q_l}), \\ 0, & \text{else.} \end{cases}$$

It is immediate from Proposition 4 that if the differential ratio $\frac{\mu_h - \mu_l}{q_h - q_l}$ is much smaller than the default cost c (i.e., the risky project is not sufficiently profitable to compensate for the costs incurred at default), then the members are more inclined to switch from risky to safe investments as the default fund level increases. Thus the regulator can use a lower default fund level to prevent the risk-shifting.

5 Continuous Choice of Risk-Taking

Having illustrated the simple logic of how a default fund can alleviate risk-taking for two possible choices of risk, in the rest of the paper we extend the analysis to the more general case of a continuous choice of taken risks. Such a setup allows us to track the marginal impact of setting a higher default fund contribution on the risk-taking behavior of a member.

5.1 The Environment

Member i has a continuous choice of risk-levels. The member invests a fraction $a_i \in [0, 1]$ of its resources in the risky project and allocates the remaining fraction $1 - a_i$ to a risk-free project. The risk-free project has a guaranteed payoff of $(1 + r_f)$ times the invested amount and can be thought as an investment in risk-free assets (e.g., U.S. Treasury bonds). The risky project could include a portfolio of loans to firms in the corporate sector or a portfolio of mortgages, exposing the member to volatile returns. The payoff, denoted by \tilde{R}_i , is realized

^{c0}For instance, in May 2014, the Royal Bank of Scotland announced the wind down of its clearing business due to increasing operational costs. This was followed by State Street, BNY Mellon, and more recently Nomura, each of whom shut down part or all of their clearing business.

at $t = 1$ and assumed to be a random variable:

$$\tilde{R}_i = \begin{cases} R, & \text{with probability } 1 - a_i \\ r, & \text{with probability } a_i, \end{cases} \quad (20)$$

R can be viewed as the notional value of the loan/mortgage. Let $R > 1 + r_f > r$: in the good state, the realized payoff is higher than that of the risk-free project, whereas in the bad state the payoff is lower than the return from the same investment in the risk-free project. Similar to the setup in [Holmstrom and Tirole \(2001\)](#) and [Acharya, Shin, and Yorulmazer \(2010\)](#), the risky technology has diminishing returns to scale with risk-taking; i.e., the probability of a good state decreases with a_i .^{c0} In particular, the probability of observing the good state is $1 - a_i$.

Member i defaults if the value of the realized risky project is r :

$$\mathbf{1}_{i \text{ defaults}} \Leftrightarrow a_i \tilde{R}_i + (1 - a_i)(1 + r_f) < 1 + r_f \Leftrightarrow \tilde{R}_i = r. \quad (21)$$

The default probability of member i is equal to a_i : the fraction invested by i in the risky project. Default can always be avoided if member i invests entirely in the risk-free project. Defaults are costly, and we use $c > 0$ to denote the cost of a default. Let $R > 1 + r_f + c$, indicating that members prefer to invest a non-zero fraction in the risky project. Notice that at $a = 0$, the marginal profit of risk-taking is R , whereas the marginal cost is $1 + r_f$ (forgone return from the risk-free project) plus c (marginal cost of default). We assume that the realizations \tilde{R}_i , $i = 1, \dots, N$, are independent across members, which implies that defaults are independent.

Strategic Investment Choice of a Member. Consider a CCP, and assume a default fund level F satisfying the Cover II requirement: given investment strategies a_{-i} chosen by

^{c0}The assumption of diminishing returns to scale is not essential for our results, but helps to obtain a neat expression for the default probability.

other members except i , the expected payoff of member i is given by

$$V_i(a_{-i}) = \sup_{a_i \in [0,1]} E \left[a_i \tilde{R}_i + (1-a_i) - \mathbf{1}_{i \text{ defaults}} F - \mathbf{1}_{i \text{ survives}} \min \left(F, \frac{\sum_{j \neq i} \mathbf{1}_{j \text{ defaults}}}{1 + \sum_{j \neq i} \mathbf{1}_{j \text{ survives}}} (c - F) \right) - r_f F \right]. \quad (22)$$

Each clearing member chooses his own investment strategically to maximize his expected payoff. If all other members choose the same investment strategy $a_{-i} \in [0, 1]$, we have

$$V_i(a_{-i}) = \sup_{a \in [0,1]} [-(R-r)a^2 + (R-1)a + 1 - F + (1-a)\phi(a_{-i}; F) - r_f F], \quad (23)$$

where $\phi(a_{-i}; F) \equiv \psi(0; F)$, and $\psi(0; F)$ is obtained by evaluating the function $\psi(g; F)$ defined in Eq. (A1) choosing $q_h \equiv a_{-i}$ therein. Because of our symmetric configuration of clearing members, we restrict our attention to symmetric equilibria. A *symmetric equilibrium* among members is $a^e \in [0, 1]$ such that, member i 's best response when all other members choose a^e is also a^e ; i.e., there is no unilateral deviation. Taking the first-order condition of the objective function in (23), and then setting a_{-i} with a_i , we obtain the following result.

Proposition 5 *For a large enough differential return $R - r > 0$, given a default fund level $F \in [\frac{2c}{N}, c)$, there exists a unique symmetric response of members; i.e., $a_i = a^e \forall i$, which satisfies*

$$\frac{R - 1 - \phi(a^e, F)}{2(R - r)} - a^e = 0. \quad (24)$$

Moreover, a^e is a strictly decreasing function of F ; for $F \neq \frac{lc}{N}$, $l = 2, 3, \dots, N-1$, a^e is an infinitely differentiable function of F and $\frac{da^e}{dF} < 0$; for $F = \frac{lc}{N}$, $l = 3, 4, \dots, N-1$, $\frac{da^e}{dF}(F+) - \frac{da^e}{dF}(F-) < 0$, where $\frac{da^e}{dF}(F+)$ and $\frac{da^e}{dF}(F-)$ are respectively the right and left derivatives of $a^e(F)$ at F .

Risk-shifting. We demonstrate that the risk-shifting pattern shown in the binary case also manifests in the continuous risk-taking setup. As a first-best benchmark, we solve for the socially optimal risky asset investment a^* that maximizes the aggregate value of all members

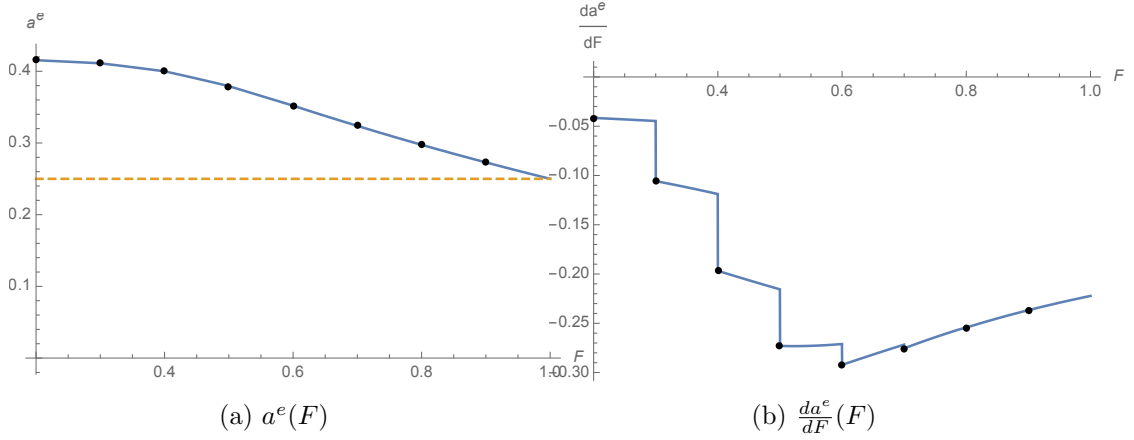


Figure 4. Members' Strategic Response to the Default Fund Level. Panel 4a plots the symmetric strategic response a^e as a function of F (blue solid line). a^e is strictly decreasing in F . The socially optimal invested fraction in the risky project is shown in the amber dashed line and is equal to $a^* = 0.25$. Panel 4b plots the derivative of a^e with respect to F . There are downward jumps at the kinks $\frac{l}{N}$ for $l = 3, 4, \dots, N-1$. Kinks are indicated by black dots. The parameters used are: $R = 3.5$, $r = 0.5$, $N = 10$, and $c = 1$.

net of the CCP's equity loss. Equivalently, a^* maximizes the expected payoff of a representative member:

$$a^* = \arg \max_{a_i} \mathbb{E}[a_i \tilde{R}_i + (1 - a_i) - c \mathbf{1}_{i \text{ defaults}} - r_f F]. \quad (25)$$

While a^* balances the socially desirable tradeoff between risk and return, members have incentives for risk-shifting under the loss mutualization scheme, as shown by the following proposition:

Proposition 6 *Assume $R > 1 + c$. Then the socially optimal investment a^* in the risky project satisfies*

$$0 < a^* = \frac{R - 1 - c}{2(R - r)} < \frac{1}{2}. \quad (26)$$

The privately optimal investment choice a^e of the clearing members is given by the solution to Eq. (24) and satisfies

$$a^* < a^e. \quad (27)$$

A direct comparison of the privately optimal investment choice a^e (solution to Eq. (24)) and the socially optimal outcome a^* (solution to Eq. (26)) immediately reveals the economic

mechanism. In the risk-return tradeoff faced by the social planner, a member faces marginal cost-due-to-default c ; in the decentralized risk-return tradeoff under loss mutualization, the marginal cost-due-to-default becomes $\phi(a; F)$. As $\phi(a; F) < c$ (which directly results from risk-sharing via the CCP), members strategically chooses higher risk than what is socially optimal. Figure 4 plots $a^e(F)$, a^* , and $\frac{da^e}{dF}$: the decreasing pattern of $a^e(F)$ is clearly seen from the figure.

5.2 Equilibrium between Clearing Members and the Regulator

In this section, we show that a default fund higher than the Cover II requirement can be used to regulate members' risk-taking. As in Section 4, the regulator selects the default fund level F , which maximizes the social value of the system (including all members and the CCP), anticipating the risk-taking activities chosen by the members in response to his choice. The members' response has been characterized in Proposition 5. Next, we describe the game theoretical setting:

Definition 5 *For a given number N of clearing members, a symmetric Nash equilibrium between all clearing members and the CCP is the set of members' risk profiles $\{a_i^e\}_{i=1}^n$, and the default fund contribution F^e set by the CCP such that,*

1. *Taking the default fund F^e and other members' risk profile a_{-i}^e as given, a_i^e solves the optimization problem of clearing member i given in (22).*
2. *Taking as given the risk profile of clearing members a^e , the regulator chooses a feasible default fund level F^{c0} to maximize the total value of the system $W(F)$:*

$$W(F) = \sum_i V_i - \mathbb{E}[(\mathcal{N}_d \cdot c - N \cdot F)^+] = N \cdot (B(F) - r_f F), \quad (28)$$

$$F^e = \arg \max_F W(F) = \arg \max_F (B(F) - r_f F), \quad (29)$$

^{c0}We recall that feasible means that the assumptions (2)-(3) and condition (2) are satisfied.

where V_i is the payoff of member i given by (22), \mathcal{N}_d is the number of defaults, and $B(F) = -(R - r)(a^e(F))^2 + (R - 1 - c)a^e(F) + 1$ is the part of the representative member's value which does not account for the opportunity cost of default fund F .

To solve for the tradeoff in choosing the default fund, we analyze the differential properties of the various components of the objective function. First, recall from Proposition 5 that $a^e(F)$ is continuously differentiable in F except over the set of kinks $\{\frac{lc}{N}; l = 2, \dots, N - 1\}$. Thus the same property holds for $B(F)$. To see how $B(F)$ changes with F , consider the quadratic function $U(a) = -(R - r)a^2 + (R - 1 - c)a + 1$, which achieves its maximum at $a = a^*$. An application of the chain rule shows that the marginal benefit of increasing F is given by

$$B'(F) = \frac{\partial U}{\partial a} \frac{\partial a^e}{\partial F} = -2(R - r)(a^e(F) - a^*) \frac{da^e}{dF}, \quad \forall F \neq \frac{lc}{N}, l = 2, \dots, N - 1. \quad (30)$$

Recall from propositions 5 and 6 that members exhibit risk-shifting ($a^e(F) > a^*$), and that a higher default fund can mitigate risk-shifting ($\frac{da^e}{dF} < 0$, $\forall F \neq \frac{lc}{N}$, $l = 2, \dots, N - 1$). Therefore, $B(F)$ increases with F ; that is, a higher default fund increases value $B(F)$ by steering members' risk-taking closer to the socially optimal level.^{c0}

Proposition 7 *The equilibrium default fund F^e set by the regulator is either the Cover II level $\frac{2c}{N}$ or a solution to the first-order condition of Equation (29). Formally, $F^e \in \{\bar{F}^e, \frac{2c}{N}\}$ where \bar{F}^e satisfies*

$$B'(\bar{F}^e) = r_f, \quad B''(\bar{F}^e) \leq 0, \quad \bar{F}^e \neq \frac{lc}{N}, l = 2, \dots, N. \quad (31)$$

Proposition 7 offers a simple algorithm for the regulator to pin down the equilibrium (constrained optimal) default fund. To maximize the social objective value function (29), the regulator balances the marginal benefit $B'(\bar{F}^e)$ and marginal cost of capital r_f . While a higher default fund increases the social value by mitigating risk-shifting, it is also costly because the fund needs to be segregated. If the marginal benefit and marginal cost cross paths

^{c0}Formally, Equation (30) implies that the marginal benefit of increasing F is positive; i.e., $B'(F) > 0$, $\forall F \in [\frac{2c}{N}, c) \setminus \{\frac{lc}{N}; l = 2, \dots, N - 1\}$. Moreover, as F crosses a kink $\frac{lc}{N}$ from below, Proposition 6 indicates that $B'(\frac{lc}{N}+) - B'(\frac{lc}{N}-) = -2(R - r)(a^e(\frac{lc}{N}) - a^*)[\frac{da^e}{dF}(\frac{lc}{N}+) - \frac{da^e}{dF}(\frac{lc}{N}-)] > 0$. Hence, at each kink value of F , the marginal benefit of increasing F increases with a positive jump. Taken together, $B(F)$ increases with F .

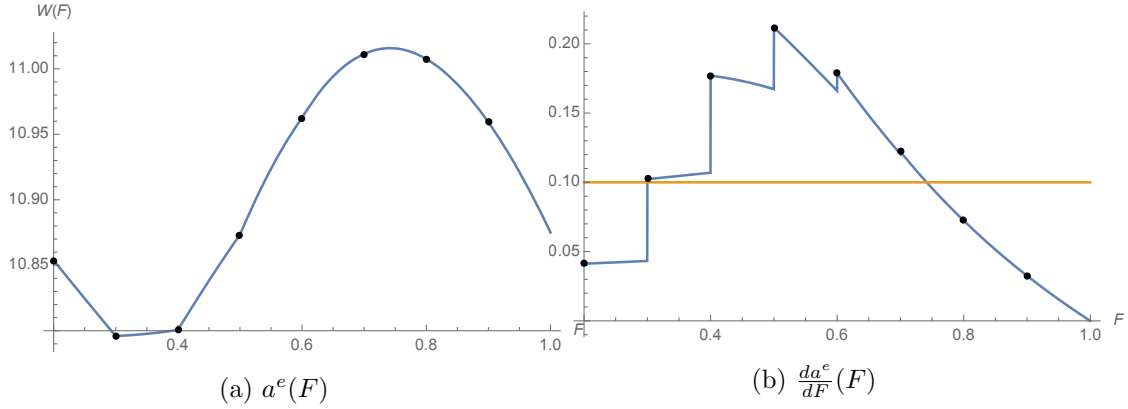


Figure 5. The Social Value Function and the Marginal Value. Panel (a) plots the total wealth $W(F)$. Panel (b) plots the marginal values: the marginal benefit $B'(F)$ (in blue), and the marginal cost r_f (in amber). The parameters used are: $R = 3.5, r = 0.5, N = 10, c = 1$ and $r_f = 0.1$. The optimal default fund is $F^e \approx 0.74$.

at a fund value higher than the Cover II lower bound, and the condition $B''(\bar{F}^e) \leq 0$ holds, then \bar{F}^e is the constrained optimal level. In other words, if the marginal value of increasing F is higher than the opportunity cost at the lower bound $\frac{2c}{N}$ ($B'(\frac{2c}{N}) > 0$), then $\frac{2c}{N} < F^e < c$. That is, the optimal default fund level exceeds the critical level for the Cover II requirement. Figure 5 considers a scenario in which the Cover II rule is not socially optimal. As the default fund level increases, the social value function increases and peaks at $F^e = 0.74$. This value corresponds to a Cover 7.4 rule instead for a 10-member clearing arrangement.

An Example with Three Members We consider a simplified setting consisting of three clearing members, in which closed-form expressions for $a^e(F)$ and F^e can be obtained. For a given choice of F that satisfies the Cover II requirement, the equilibrium investment in the risky asset is given by $a^e(F) = \frac{R-r}{c-F} - \frac{1}{2} - \sqrt{\left(\frac{R-r}{c-F} - \frac{1}{2}\right)^2 - \frac{R-1-F}{c-F}}$, and obtained by solving Equation (24). It is immediate to see that $a^e(F)$ is strictly decreasing in F . We can explicitly compute the unique equilibrium default fund F^e . In particular, let $h(a) = a - a^* - r_f \frac{1+a^*+2a^*a-a^2}{(1+a+a^2)^2}$, which is a strictly increasing function over $[0, 1]$, and satisfies $h(a^*) < 0$. Then

$$F^e = \begin{cases} \frac{2c}{3}, & \text{if } h(a^e(\frac{2c}{3})) \leq 0, \\ c - 2(R-r) \frac{a_0 - a^*}{1 + a_0 + a_0^2}, & \text{if } h(a^e(\frac{2c}{3})) > 0. \end{cases} \quad (32)$$

where a_0 is the unique root to $h(a_0) = 0$ over the interval $(a^*, a^e(\frac{2c}{3}))$. This example serves to illustrate that even in a simple central clearing setup consisting of three members, neither Cover II nor Cover III may be the optimal default fund allocation. In fact, $2c/3 < F^e < c$, and thus an intermediary level for the default fund level might be optimal.

Analogous to the binary case, we can show that the ratio between the optimal default fund level $F^e(N)$ and the default cost c , or equivalently, the optimal proportion of covered members, $x(N)/N$, also converges to a constant as the number of members N grows large. This limit is a positive number in $(0, 1)$ when the marginal opportunity cost r_f is sufficiently low. In particular, paralleling the result in Proposition 4, we have the following asymptotic result.

Proposition 8 *In the large CCP network limit – i.e., as $N \rightarrow \infty$ – the unique symmetric response of members, $a_i = a^e$ is given by*

$$a^e(F) \rightarrow \begin{cases} \frac{R-1}{2(R-r)}, & \text{if } \frac{F}{c} \leq \frac{R-1}{2(R-r)}, \\ \frac{(1+a^*) - \sqrt{(1+a^*)^2 - 2\frac{R-1-F}{R-r}}}{2}, & \text{if } \frac{F}{c} > \frac{R-1}{2(R-r)}. \end{cases} \quad (33)$$

Letting $F(a) = R-1 + \frac{R-r}{2}[(1+a^* - 2a)^2 - (1+a^*)^2]$ be the inverse of line two in (33), $a_\infty := \frac{(1+r_f)a^* + r_f}{1+2r_f}$, and $\hat{F}_\infty := F(a_\infty)$. Then we have

$$\frac{F^e(N)}{c} = \frac{x(N)}{N} \rightarrow \begin{cases} \frac{\hat{F}_\infty}{c}, & \text{if } B(\hat{F}_\infty) - r_f \hat{F}_\infty > B(0) \text{ and } a_\infty < \frac{R-1}{2(R-r)}, \\ 0, & \text{else,} \end{cases} \quad (34)$$

where $B(\cdot)$ is defined in Definition 5.

From Proposition 8, we see that for a CCP consisting of many members, the highest risk choice a^e converges $\frac{R-1}{2(R-r)}$. When members take this level of risk, then the optimal response of the regulator is to set the default fund level to zero because the members' risk choice is independent of F . If the marginal opportunity cost of default fund is low, the regulator may attain a higher per member social welfare by charging a larger default fund requirement \hat{F}_∞ to members, that in turn incentivize them to take a lower level risk level a_∞ .

6 Conclusion and Policy Implications

The problem of the optimal determination of default fund levels contributed by members of a clearinghouse has been the subject of extensive regulatory debate. Current regulatory requirements prescribe that default fund contributions should guarantee that the clearinghouse is able to continue its services in the event that its two largest clearing members default. There is, however, no economic analysis of the conditions under which this rule is socially optimal, or of alternative designs that are welfare improving. Our paper fills this important gap and introduces a parsimonious model to study the main economic incentives behind the determination of the default fund requirements. While default funds allow members to effectively share counterparty risk *ex post*, we highlight a novel mechanism related to loss mutualization that induces members to take excessive risk *ex ante* owing to an inherent externality among them. Our analysis shows that the CCPs can mitigate the inefficiency generated by members' excessive risk-taking activities through an optimal choice of the default fund level. Such a choice balances the *ex post* risk-sharing and the *ex ante* risk-taking of members.

Our analysis shows that if the clearinghouse consists of a sufficiently high number of clearing members, then the optimal cover rule should be to cover the default costs of a constant fraction of members. This finding contrasts with the currently imposed Cover II requirements, whose optimality is supported by our analysis only if the marginal opportunity costs of collateral posting are high.

Our results have important policy implications. They point to the need to design a simple rule that guarantees the coverage of the costs generated by the default of a proportion of clearing members. This simple covering requirement is robust to the size of the participating member base. The optimal proportion depends, in an explicit way, on the relation between the premium earned by the member who undertakes high-risk projects and the costs incurred at default. These parameters can be accessed by clearinghouses and their supervisory authorities who typically have detailed information on the risk profile of their members. Owing to its simplicity, the proposed rule can also serve as a benchmark against more complex rules based

on simulated scenario stress testing.

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A Proof of Proposition 2

In this Appendix, we prove Proposition 2. We first present some technical lemmas to fix preliminary results and notations.

Lemma 6 *Suppose member i is alive, and g of the remaining $N - 1$ members choose the low risk project. Then, for any given $F \in [\frac{2c}{N}, \frac{c}{1+r_f}]$, we have that*

$$\psi(g; F) := \sum_{k=N-1-\lfloor \frac{NF}{c} \rfloor}^{N-1} f_g(k) \left(c - \frac{N(c-F)}{k+1} \right). \quad (\text{A1})$$

Here $\lfloor \cdot \rfloor$ denotes the floor function (giving the greatest integer less than or equal to the argument), and

$$f_g(k) := \sum_{m=0}^k \binom{g}{m} (1-q_l)^m q_l^{g-m} \times \binom{N-1-g}{k-m} (1-q_h)^{k-m} q_h^{N-1-g-(k-m)}$$

are positive constant.

Proof of Lemma 6. Suppose that the default fund F is such that

$$\frac{lc}{N} \leq F < \frac{(l+1)c}{N} \text{ or equivalently } 1 - \frac{1+l}{N} < 1 - \frac{F}{c} \leq 1 - \frac{l}{N}, \quad (\text{A2})$$

for some integer $l = 2, 3, \dots, N-1$. Then member i 's contribution to other members' default when himself does not default is given by

$$\begin{aligned} \min \left(F, \frac{N - \mathcal{N}_s}{\mathcal{N}_s} (c - F) \right) &= F \min \left(1, \left(\frac{N}{\mathcal{N}_s} - 1 \right) \left(\frac{c}{F} - 1 \right) \right) \\ &= (c - F) \left(\frac{N}{\mathcal{N}_s} - 1 \right) 1_{\mathcal{N}_s \geq (1 - \frac{F}{c})N} + F 1_{\mathcal{N}_s < (1 - \frac{F}{c})N} \end{aligned}$$

For F in the range (A2), we have

$$N - (l+1) < \left(1 - \frac{F}{c} \right) N \leq N - l$$

Thus $\mathcal{N}_s \geq (1 - \frac{F}{c})N$ if and only if $\mathcal{N}_s \geq n - l$. In other words, among the remaining $N - 1$ members, if there are less than or equal to l defaults, member i will pay less than F . But if there are l or more defaults and $F = \frac{lc}{N}$ then member i 's default fund will be exhausted completely.

Suppose all members except member i choose the low risk project, then if member i

survives, his expected contribution is

$$\begin{aligned}
& (c - F) \mathbb{E} \left[\left(\frac{N}{\mathcal{N}_s} - 1 \right) 1_{\mathcal{N}_s \geq N-l} \middle| \text{member } i \text{ survives} \right] + F \cdot \Pr(\mathcal{N}_s < N-l | \text{member } i \text{ survives}) \\
&= (c - F) \sum_{k=N-(l+1)}^{N-1} \binom{N-1}{k} \frac{N-1-k}{k+1} (1-q_l)^k q_l^{N-1-k} + F \sum_{k=0}^{N-(l+2)} \binom{N-1}{k} (1-q_l)^k q_l^{N-1-k} \\
&= (c - F) \sum_{k=N-(l+1)}^{N-2} \binom{N-1}{k+1} (1-q_l)^k q_l^{N-1-k} + F \sum_{k=0}^{N-(l+2)} \binom{N-1}{k} (1-q_l)^k q_l^{N-1-k}. \quad (\text{A3})
\end{aligned}$$

Likewise, if all members except member i choose the high risk project, then if member i survives, his expected contribution is

$$\begin{aligned}
& (c - F) \mathbb{E} \left[\left(\frac{N}{\mathcal{N}_s} - 1 \right) 1_{\mathcal{N}_s \geq N-l} \middle| \text{member } i \text{ survives} \right] + F \cdot \Pr(\mathcal{N}_s < N-l | \text{member } i \text{ survives}) \\
&= (c - F) \sum_{k=N-(l+1)}^{N-2} \binom{N-1}{k+1} (1-q_h)^k q_h^{N-1-k} + F \sum_{k=0}^{N-(l+2)} \binom{N-1}{k} (1-q_h)^k q_h^{N-1-k}. \quad (\text{A4})
\end{aligned}$$

In general, if there are g members among the remaining $N-1$ choosing the low risk project, for $g = 0, 1, \dots, N-1$, then the number of surviving ones among these $N-1$ members, $\mathcal{N}_s - 1$, is the sum of the number of the survived ones choosing the low risk project and that of the survived choosing the high risk project. Specifically, the probability that there are k survived ones is given by

$$f_g(k) := \sum_{m=0}^k \binom{g}{m} (1-q_l)^m q_l^{g-m} \times \binom{N-1-g}{k-m} (1-q_h)^{k-m} q_h^{N-1-g-(k-m)}.$$

It follows that, if member i survives, his expected contribution is

$$\begin{aligned}
& (c - F) \mathbb{E} \left[\left(\frac{N}{\mathcal{N}_s} - 1 \right) 1_{\mathcal{N}_s \geq N-l} \middle| \text{member } i \text{ survives} \right] + F \cdot \Pr(\mathcal{N}_s < N-l | \text{member } i \text{ survives}) \\
&= (c - F) \sum_{k=N-(l+1)}^{N-1} f_g(k) \frac{N-1-k}{1+k} + F \sum_{k=0}^{N-(l+2)} f_g(k) \\
&= \sum_{k=N-(l+1)}^{N-1} f_g(k) \left(\frac{N(c-F)}{k+1} - c \right) + F, \quad (\text{A5})
\end{aligned}$$

where the last line comes from the total probability $\sum_{k=0}^{N-1} f_g(k) = 1$. ■

Lemma 7 For any given $F \in [\frac{2c}{N}, c)$, the function $\psi(g; F)$ is strictly decreasing in g , i.e.

$$0 < \psi(0; F) < \psi(1; F) < \dots < \psi(N-1; F) < F < c.$$

Proof of Lemma 7. Suppose member i survives, and $\mathcal{N}_s - 1$ is the number of survivals

except member i . Given how we define a default event, we have

$$\mathcal{N}_s - 1 = \sum_{j \neq i} 1_{\text{member } j \text{ defaults}} \quad (\text{A6})$$

By Lemma 6, we only need to show that

$$g \mapsto \mathbb{E} \left[\min \left(F, \frac{N - \mathcal{N}_s}{\mathcal{N}_s} (c - F) \right) \right] \equiv F - \psi(g; F)$$

is strictly decreasing in g for $g = 0, 1, 2, \dots, n - 1$, where g is the number of members other than member i , who chooses the low risk project. Because the expression $\psi(g; F)$ only depends on the default probabilities p_h, p_l , not how defaults occur, we can choose a probability model for the defaults that is convenient to our analysis. More precisely, suppose for each member i , there is an independent random variable ϵ_i , with a uniform distribution on $(0, 1)$, such that, if this member has chosen the low risk project, he will default at time 1 if and only if $\epsilon_i < p_l$; on the other hand, if this member has chosen the high risk project, then he will default at time 1 if and only if $\epsilon_i < p_h$. Because the event $\{\epsilon_i < p_l\}$ implies $\{\epsilon_i < p_h\}$, we see that, in this probability model of defaults, increasing g , the number of remaining members (other than member i) choosing the low risk project, always makes \mathcal{N}_s non-increasing, and can makes \mathcal{N}_s strictly increasing with a positive probability (equal to $p_h - p_l$ in this particular specification of default).

Similarly, there is a positive chance that \mathcal{N}_s may decrease from 1 to N as g increases from 0 to $N - 1$. As \mathcal{N}_s varies between 1 and N , $\frac{N - \mathcal{N}_s}{\mathcal{N}_s} (c - F)$ varies between $(N - 1)(c - F)$ and 0, hence the random variable $\min \left(F, \frac{N - \mathcal{N}_s}{\mathcal{N}_s} (c - F) \right)$ is non-decreasing with g , and there is a positive chance that it is strictly decreasing with g . As a result, we know that the mapping

$$g \mapsto \mathbb{E} \left[\min \left(F, \frac{N - \mathcal{N}_s}{\mathcal{N}_s} (c - F) \right) \right], \quad g = 0, 1, \dots, N - 1$$

is strictly decreasing.

Lastly, because the expected contribution $F - \psi(N - 1; F)$ is positive (member i has to contribute as long as there is at least one other member default), we have $\psi(N - 1; F) < F$. Moreover, the expectation contribution $F - \psi(0; F)$ is strictly less than F (member i contributes less than F when there is no default). ■

Lemma 8 *For any fixed $g = 0, 1, \dots, N - 1$, the function $\psi(g; F)$ is piecewise linear and strictly increasing in F , in the interval $[\frac{2}{N}c, \frac{c}{1+r_f}]$. In particular, $\psi(g; c) = c$.*

Proof of Lemma 8. From (A1), $\psi(g; F)$ is linear and strictly increasing for $F \in (\frac{l}{N}, \frac{l+1}{N})$ with $l = 2, 3, \dots, N - 1$. Moreover, the nonnegative random variable $\min(F, \frac{N - \mathcal{N}_s}{\mathcal{N}_s} (c - F))$ is almost surely continuous in F , and is bounded by c . By the dominated convergence theorem, we know that $\psi(g; F)$ is continuous. Therefore, the function $\psi(g; F)$ is strictly increasing for all $F \in [\frac{2}{N}c, \frac{c}{1+r_f}]$. The value of $\psi(g; c)$ follows directly from (10). ■

To prove Proposition 2, we next analyze different scenarios for a member's investment choice, taken as given the default fund and other members' investment choices. In particular,

we characterize conditions such that member i chooses high risk given all possible combinations of risk profile of other members. When we check those conditions for all members, we will see that “all high risk” and “all low risk” strategy are the only possible equilibria.

Suppose that the other members do not choose the same risk level: There are g members who choose low risk projects and the rest $N - 1 - g$ choose high risk projects, where $g \in \{0, \dots, N - 1\}$.

If member i chooses low risk, his expected utility is (by Lemma 6)

$$\begin{aligned} & \mathbb{E} \left[R_i^l - \mathbf{1}_{i \text{ defaults}} F - \mathbf{1}_{i \text{ survives}} \min \left(F, \frac{\sum_{j \neq i} \mathbf{1}_{j \text{ defaults}} (c - F)}{1 + \sum_{j \neq i} \mathbf{1}_{j \text{ survives}}} \right) - r_f F \right] \\ &= \mu_l - q_l \psi(g; F) - F + \psi(g; F) - r_f F. \end{aligned}$$

If member i chooses instead high risk, his expected utility is

$$\begin{aligned} & \mathbb{E} \left[R_i^h - \mathbf{1}_{i \text{ defaults}} F - \mathbf{1}_{i \text{ survives}} \min \left(F, \frac{\sum_{j \neq i} \mathbf{1}_{j \text{ defaults}} (c - F)}{1 + \sum_{j \neq i} \mathbf{1}_{j \text{ survives}}} \right) - r_f F \right] \\ &= \mu_h - q_h \psi(0; F) - F + \psi(0; F) - r_f F. \end{aligned}$$

Hence, member i chooses high (low, resp.) risky project when g members choose low and $N - 1 - g$ choose high if and only if

$$\frac{\mu_h - \mu_l}{q_h - q_l} > (<, \text{resp.}) \psi(g; F). \quad (\text{A7})$$

When (A7) takes an equality, member i is indifferent to choosing high or low risky project. As a consequence,

1. If $\frac{\mu_h - \mu_l}{q_h - q_l} > \psi(N - 1; F)$, every member will choose the high risky project, regardless of other members' choice. Hence, the “all high risk” strategy is the unique equilibrium among members.
2. If $\frac{\mu_h - \mu_l}{q_h - q_l} < \psi(0; F)$, every member will choose the low risky project, regardless of other members' choice. Hence, the “all low risk” strategy is the unique equilibrium among members.
3. If $\psi(0; F) \leq \frac{\mu_h - \mu_l}{q_h - q_l} \leq \psi(N - 1; F)$, it is straightforward to verify that both the “all high risk” strategy and the “all low risk” strategy are equilibria among members. To prove there cannot be any other forms of equilibrium, suppose there is an equilibrium which is consisted of g low and $(N - g)$ high, for some $g = 1, 2, \dots, N - 1$. Then for any member choosing high, he faces g choosing low and $(N - g - 1)$ choosing high, so in order for him to stay high as well, it must hold that

$$\frac{\mu_h - \mu_l}{q_h - q_l} \geq \psi(g; F). \quad (\text{A8})$$

Yet, for any member choosing low, he faces $g - 1$ low and $N - g$ high, so for this member to stay low, it must hold that

$$\frac{\mu_h - \mu_l}{q_h - q_l} \leq \psi(g - 1; F). \quad (\text{A9})$$

However, (A8) and (A9) cannot hold simultaneously because $\psi(g-1; F) < \psi(g; F)$ (see Lemma 7).

Finally, we prove that the all low risk profile is Pareto dominating the all high risk profile when $\psi(0; F) \leq \frac{\mu_h - \mu_l}{q_h - q_l} \leq \psi(N-1; F)$ holds. Recall that member i 's expected utility under the all low risk profile is

$$\mu_l - q_l \psi(N-1; F) - F + \psi(N-1) - r_f F.$$

Likewise, his expected utility under the all high risk profile is

$$\mu_h - q_h \psi(0; F) - F + \psi(0; F) - r_f F.$$

The difference is then given by

$$\begin{aligned} & [\mu_l - q_l \psi(N-1; F) - F + \psi(N-1) - r_f F] - [\mu_h - q_h \psi(0; F) - F + \psi(0; F) - r_f F] \\ &= \mu_l - \mu_h + (1 - q_l) \psi(N-1; F) - (1 - q_h) \psi(0; F). \end{aligned}$$

When $\mu_h - \mu_l \leq (q_h - q_l) \psi(N-1; F)$, the above expression is bounded from below by $(1 - q_h) (\psi(N-1; F) - \psi(0; F)) > 0$, due to Lemma 7. Therefore, among the two possible equilibriums, the all low risk profile is Pareto dominating.

B Proof of Proposition 5

Recall that a symmetric equilibrium under cover-II requirement is given by $a_i^e = a^e$ for all $i = 1, 2, \dots, N$, with a^e being the root to the follow equation:

$$f(a^e; F) = 0, \text{ where } f(a; F) = \frac{R-1-\phi(a; F)}{2(R-r)} - a.$$

We first show the existence of such a root. To that end, recall that (10) implies

$$\mathbb{E} \left[\min \left(F, \frac{N - \mathcal{N}_s}{\mathcal{N}_s} (c - F) \right) \middle| \text{member } i \text{ survives} \right] = F - \phi(a; F),$$

where \mathcal{N}_s is the number of surviving members, each of who has a default probability of a . It follows that when $a = 0$, we have $\mathcal{N}_s = N$ almost surely, so $0 = F - \phi(0; F)$, which implies that

$$\phi(0; F) = F.$$

Therefore we have

$$f(0; F) = \frac{R-1-F}{2(R-r)} > \frac{R-1-c}{2(R-r)} = a^* > 0. \quad (\text{A10})$$

On the other hand, because $\min(F, \frac{N - \mathcal{N}_s}{\mathcal{N}_s} (c - F)) \leq F$ holds almost surely, we know that $\phi(a; F) \geq 0$ for all $a \in [0, 1]$. Therefore, we have

$$f(1; F) = \frac{R-1-\phi(1; F)}{2(R-r)} - 1 \leq \frac{R-1}{2(R-r)} - 1 < \frac{1}{2} - 1 = -\frac{1}{2} < 0. \quad (\text{A11})$$

Hence, from (A10) and (A11) we can conclude that, for any fixed $F \in [\frac{2c}{N}, \frac{c}{1+r_f})$, there exists at least one a^e such that $f(a^e; F) = 0$.

To prove the uniqueness of a^e , we demonstrate that, when $R - r > 0$ is sufficiently large, $f(a; F)$ is strictly decreasing in q over $(0, 1)$ for every F . Indeed, Lemma 7 indicates that $\phi(q_h; F) = \psi(0; F) < \psi(N - 1; F) = \phi(q_l; F)$ for $0 < q_l < q_h < 1$, so $\phi(a; F)$ is strictly decreasing in a . Therefore, the monotonicity of $f(a; F)$ in a is completely determined by the strictly increasing function $-\frac{1}{2(R-r)}\phi(a; F)$ and the strictly decreasing function $-a$. Intuitively, as $(R - r)$ becomes large, we have $-\frac{1}{2(R-r)}\frac{\partial}{\partial a}\phi(a; F) - 1 < 0$, meaning $f(a; F)$ is strictly decreasing in a . In Lemma 9 below we formally bound $|\frac{\partial}{\partial a}\phi(a; F)|$ from above, which effectively gives a sufficient lower bound for $(R - r)$ such that $f(a; F)$ is strictly decreasing, which implies the uniqueness of a^e .

To prove the monotonicity of $a^e(F)$ in F , we use lemma 8 to know that, for each fixed $a \in (0, 1)$, $\phi(a; F)$ is strictly increasing for all $F \in [\frac{2c}{N}, \frac{c}{1+r_f})$, so $f(a; F)$ is strictly decreasing in F over the same domain. Let $\frac{2c}{N} \leq F_1 < F_2 < c$, we have

$$0 = f(a^e(F_2); F_2) = f(a^e(F_1); F_1) > f(a^e(F_1); F_2).$$

Because $f(q; F_2)$ is strictly decreasing in q when $R - r > 0$ is sufficiently large, we must have $a^e(F_2) < a^e(F_1)$. This proves the monotonicity of $a^e(F)$ on F .

The infinite differentiability of $a^e(F)$ in F follows from implicit differentiation and Lemma 8, from which we know that $f(a; F)$ is infinitely differentiable in F if $F \neq \frac{lc}{N}$ for $l = 2, 3, \dots, N - 1$. This, in conjunction with the monotonicity of $a^e(F)$ in F , implies that $\frac{da^e}{dF}(F) < 0$ for any $F \in (\frac{2c}{N}, c)$ but $F \neq \frac{lc}{N}$, $l = 3, 4, \dots, N - 1$.

Lastly, to prove that $a^e(F)$ is not differentiable at $F = \frac{lc}{N}$, and with a greater left derivative, we use implicit differentiation to obtain that, for any $F \in (\frac{lc}{N}, \frac{(l+1)c}{N})$ with $l = 3, \dots, N - 2$,

$$\frac{da^e}{dF}(F) = -\frac{\frac{\partial f}{\partial F}}{\frac{\partial f}{\partial a}} \Big|_{(a,F)=(a^e(F),F)} = -\frac{u_l^{(N)}(a^e(F))}{2(R-r) + c \cdot v_l^{(N)'}(a^e(F)) - (c-F)u_l^{(N)'}(a^e(F))},$$

where $v_l^{(N)}$ and $u_l^{(N)}$ are functions defined in the proof of Lemma 9 below. Thus, the right derivative at $F = \frac{lc}{N}$ is given by

$$\frac{da^e}{dF}(\frac{lc}{N}+) = -\frac{u_l^{(N)}(a^e(\frac{lc}{N}))}{2(R-r) + c \cdot v_l^{(N)'}(a^e(\frac{lc}{N})) - (c - \frac{lc}{N})u_l^{(N)'}(a^e(\frac{lc}{N}))}. \quad (\text{A12})$$

Similarly, for the left derivative at $F = \frac{lc}{N}$ we obtain that

$$\frac{da^e}{dF}(\frac{lc}{N}-) = -\frac{u_{l-1}^{(N)}(a^e(\frac{lc}{N}))}{2(R-r) + c \cdot v_{l-1}^{(N)'}(a^e(\frac{lc}{N})) - (c - \frac{lc}{N})u_{l-1}^{(N)'}(a^e(\frac{lc}{N}))}. \quad (\text{A13})$$

To compare (A12) with (A13), we first show that the denominators for $\frac{da^e}{dF}(\frac{lc}{N}+)$ and $\frac{da^e}{dF}(\frac{lc}{N}-)$

are the same. To that end, we have

$$\begin{aligned}
& \left(c \cdot v_l^{(N)'}(a^e(\frac{lc}{N})) - c \frac{N-l}{N} u_l^{(N)'}(a^e(\frac{lc}{N})) \right) - \left(c \cdot v_{l-1}^{(N)'}(a^e(\frac{lc}{N})) - c \frac{N-l}{N} u_{l-1}^{(N)'}(a^e(\frac{lc}{N})) \right) \\
&= c \left(\binom{N-1}{N-1-l} [(1-a)^{N-1-l} a^l]' - \frac{N-l}{N} \binom{N}{N-l} [(1-a)^{N-l-1} a^l]' \right) \Big|_{a=a^e(\frac{lc}{N})} \\
&= c [(1-a)^{N-1-l} a^l]'|_{a=a^e(F)} \cdot \left(\binom{N-1}{l} - \frac{N-l}{N} \binom{N}{l} \right) = 0.
\end{aligned}$$

On the other hand, recall that when $R-r$ is large enough, both $\frac{da^e}{dF}(\frac{lc}{N}+)$ and $\frac{da^e}{dF}(\frac{lc}{N}-)$ are negative. Given that $u_{l-1}^{(N)}(a^e(\frac{lc}{N})), u_l^{(N)}(a^e(\frac{lc}{N})) > 0$, we know from (A12) and (A13) that the common denominator for $\frac{da^e}{dF}(\frac{lc}{N}+)$ and $\frac{da^e}{dF}(\frac{lc}{N}-)$ must be positive. Thus,

$$\begin{aligned}
\frac{da^e}{dF}(\frac{lc}{N}+) - \frac{da^e}{dF}(\frac{lc}{N}-) &= \frac{-[u_l^{(N)}(a^e(\frac{lc}{N})) - u_{l-1}^{(N)}(a^e(\frac{lc}{N}))]}{2(R-r) + c \cdot v_l^{(N)'}(a^e(\frac{lc}{N})) - c \frac{N-1}{N} u_l^{(N)'}(a^e(\frac{lc}{N}))} \\
&= \frac{-(\binom{N}{N-1})(1-a)^{N-l-1} a^l|_{a=a^e(\frac{lc}{N})}}{2(R-r) + c \cdot v_l^{(N)'}(a^e(\frac{lc}{N})) - c \frac{N-1}{N} u_l^{(N)'}(a^e(\frac{lc}{N}))} < 0.
\end{aligned}$$

Lemma 9 For any $F \in [\frac{2c}{N}, \frac{c}{1+r_f})$, we have that

$$\sup_{a \in [0,1]} \left| \frac{\partial}{\partial a} \phi(a; F) \right| < \infty.$$

Proof.

Without loss of generality, let us suppose $F \in [\frac{lc}{N}, \frac{(l+1)c}{N})$ for some $l = 2, 3, \dots, N-1$, so that $\lfloor \frac{NF}{c} \rfloor = l$. Using (A1), we have

$$\begin{aligned}
\phi(a; F) &= \sum_{k=N-1-l}^{N-1} \binom{N-1}{k} (1-a)^k a^{N-1-k} \left(c - \frac{N(c-F)}{k+1} \right) \\
&= c \cdot v_l^{(N)}(a) - (c-F) u_l^{(N)}(a),
\end{aligned} \tag{A14}$$

where

$$v_l^{(N)}(a) = \sum_{k=N-1-l}^{N-1} \binom{N-1}{k} (1-a)^k a^{N-1-k}, \quad u_l^{(N)}(a) = \sum_{k=N-l}^N \binom{N}{k} (1-a)^{k-1} a^{N-k}.$$

Functions $v_l^{(N)}(a)$ and $u_l^{(N)}(a)$ do not depend on F and c , and apparently both of them have continuous first order derivative in a over $[0, 1]$. This completes the proof. ■

C Proof of Proposition 6

We first study the first best. To that end, recall from (25) that

$$a^* = \arg \max [-(R-r)a^2 + (R-1-c)a + 1 - r_f F].$$

Using the first order condition, we obtain that $a^* = \frac{R-1-c}{2(R-r)}$.

To prove (27), the upper bound follows immediately from (24) and the fact that $\phi(a; F) \geq 0$:

$$a^e = \frac{R-1-\phi(a^e; F)}{2(R-r)} \leq \frac{R-1}{2(R-r)}.$$

To prove the lower bound, let us suppose for the moment that F satisfies a cover- $(N-1)$ rule, i.e. $\frac{(N-1)c}{N} \leq F < c$. In this case, we have $\phi(a; F) = F - (c-F)\frac{a(1-a^{N-1})}{1-a} < c$ for all $0 < a < 1$ and $\frac{(N-1)c}{N} \leq F < c$. On the other hand, using Lemma 8 we know that, for each fixed $a \in (0, 1)$, $\phi(a; F)$ is strictly increasing for all $F \in [\frac{2c}{N}, \frac{c}{1+r_f})$, so that $\phi(a; F) < c$ for all $\frac{2c}{N} \leq F < c$. Comparing a^* and a^e in expressions (26) and (24), we have that

$$a^e = \frac{R-1-\phi(a^e; F)}{2(R-r)} > \frac{R-1-c}{2(R-r)} = a^*.$$

This completes the proof.

D Proof of Proposition 7

The objective function $B(F) - r_f F$ is differentiable in F off the set of kinks $\{\frac{lc}{N}; l = 2, 3, \dots, N-1\}$. Thus, if the equilibrium F^e is not at one of the kinks, it must solve the regulator's tradeoff between the marginal benefit and marginal cost, i.e. the marginal benefit of increase F in mitigating risk-shifting should equal to the marginal cost of opportunity of default fund segregation. However, as F increases over the kink $\frac{lc}{N}$ for some $l = 3, 4, \dots, N-1$, the marginal cost r_f increases continuously, but the marginal benefit $B'(F)$ increases abruptly. Thus, it is never optimal to choose a default fund level F at one of such kinks.

Next we show that if the marginal value of increasing F is higher than the opportunity cost at the lower bound $\frac{2c}{N}$ ($B'(\frac{2c}{N}) > r_f$), then $\frac{2c}{N} < F^e < c$. We claim that

$$B'(\frac{2c}{N}) > r_f \leftrightarrow \frac{(a^e(\frac{2c}{N}) - a^*) \cdot u_2^{(N)}(a^e(\frac{2c}{N}))}{1 + \frac{c}{2(R-r)}[v_2^{(N),'}(a^e(\frac{2c}{N})) - \frac{N-2}{N}u_2^{(N),'}(a^e(\frac{2c}{N}))]} > r_f. \quad (\text{A15})$$

where $v_2^{(N)}(a) = \sum_{k=N-3}^{N-1} \binom{N-1}{k} (1-a)^k a^{N-1-k}$, $u_2^{(N)}(a) = \sum_{k=N-2}^N \binom{N}{k} (1-a)^k a^{N-k}$. To see (A15), notice that for any $F \neq \frac{lc}{N}$,

$$B'(F) = [-2(R-r)a^e(F) + R-1-c] \frac{da^e}{dF}(F) = -2(R-r)(a^e(F) - a^*) \frac{da^e}{dF}(F). \quad (\text{A16})$$

Moreover, using implicit differentiation we have

$$-\frac{da^e}{dF}(F) = \frac{\frac{\partial \phi}{\partial F}}{2(R-r) + \frac{\partial \phi}{\partial a}} = \frac{u_2^{(N)}(a^e(F))}{2(R-r) + c[v_2^{(N)}(a^e(F)) - \frac{N-2}{N}u_2^{(N)}(a^e(F))]}.$$
 (A17)

From (A16) and (A17) we have

$$B'(\frac{2c}{N}+) = \frac{u_2^{(N)}(a^e(\frac{2c}{N}))}{1 + \frac{c}{2(R-r)}[v_2^{(N)}(a^e(\frac{2c}{N})) - \frac{N-2}{N}u_2^{(N)}(a^e(\frac{2c}{N}))]}.$$
 (A18)

Therefore, the condition in (A15) is equivalent to $B'(\frac{2c}{N}+) - r_f > 0$, so the objective function $B(F) - r_f F$ is locally increasing in a small right neighborhood of $\frac{2c}{N}$. Similarly, because $a^e(c-) = a^*$ (see Proposition 6), we know that $B'(c-) - r_f = -r_f < 0$. That is, the objective function $B(F) - r_f F$ is locally decreasing in a small left neighborhood of c . It follows that the maximum $F^e < c$.

E Proof of Equation (32)

When $N = 3$, the first order equation is a quadratic equation

$$f(a; F) = \frac{R - 1 - F + (c - F)(a + a^2)}{2(R - r)} - a = 0.$$
 (A19)

Its solution is given by

$$a_{\pm} = \frac{R - r}{c - F} - \frac{1}{2} \pm \sqrt{\left(\frac{R - r}{c - F} - \frac{1}{2}\right)^2 - \frac{R - 1 - F}{c - F}}.$$

Since $R - r > \frac{c}{2}$, we know that $a_- > 0$ and $a_+ < 0$. Moreover, one can easily verify that $a_- < 1$ by plugging $a = 1$ into (A19) and using the condition that $2c - 3F \leq 0$ (the cover-II requirement). Hence, a_- gives the formula for $a^e(F)$.

To demonstrate the monotonicity of $a^e(F)$, we notice that, for any $\frac{2c}{3} \leq F < c$ and $0 \leq a \leq 1$,

$$\begin{aligned} \frac{\partial f}{\partial a} &= \frac{(c - F)(1 + 2a)}{2(R - r)} - 1 \leq \frac{\frac{c}{3}(1 + 2)}{2(R - r)} - 1 = \frac{c}{2(R - r)} - 1 < 0, \\ \frac{\partial f}{\partial F} &= -\frac{1}{2(R - r)}(1 + a + a^2) < 0. \end{aligned}$$

By the implicit differentiation theorem, we know that $a^e(F)$ is strictly decreasing and differentiable in F .

Now that we have established that the mapping $F \mapsto a^e(F)$ is one-to-one and decreasing, to obtain the equilibrium F^e , we consider a change of variable: $a^e \mapsto F(a^e)$, namely, the inverse of $a^e(F)$:

$$F(a) = c - 2(R - r) \frac{a - a^*}{1 + a + a^2}.$$

Then we know that $F^e = F(\hat{a})$, where

$$\begin{aligned}\hat{a} &= \arg \max_{a \in [a^*, a^e(\frac{2c}{3})]} [-(R-r)a^2 + (R-1-c)a + 1 - r_f F(a)] \\ &= \arg \max_{a \in [a^*, a^e(\frac{2c}{3})]} [-(R-r)a^2 + (R-1-c)a + 1 - r_f F(a)].\end{aligned}\quad (\text{A20})$$

To fix \hat{a} , we calculate the derivative of the objective function of (A20) with respect to a :

$$-2(R-r)a + R-1-c - 2r_f(R-r) \frac{1-a^2+a^*+2a^*a}{(1+a+a^2)^2} = -2(R-r)h(a). \quad (\text{A21})$$

Moreover, notice that

$$h'(a) = 1 + r_f \frac{1-a^3+3a^*a^2+2(1+a^*)a}{(1+a+a^2)^3} > 1, \forall a \in [0, 1]. \quad (\text{A22})$$

Therefore, the first order condition equation $h(a) = 0$ can have at most one root. On the other hand, one clearly has $h(a^*) < 0$, so if $h(a^e(\frac{2c}{3})) \leq 0$, then the objective function of (A20) is strictly increasing in a over $[a^*, a^e(\frac{2c}{3})]$. It follows that $\hat{a} = a^e(\frac{2c}{3})$ and $F^e = F(a^e(\frac{2c}{3})) = \frac{2c}{3}$. On the other hand, if $h(a^e(\frac{2c}{3})) > 0$, then there is a unique maximizer $\hat{a} = a_0 \in (a^*, a^e(\frac{2c}{3}))$, which solves the first order condition equation $h(a_0) = 0$. Hence, the equilibrium default fund $F^e = F(a_0)$.

F The large CCP network limit: proofs of Proposition 4 and Proposition 8

In this section, we derive the limit of $\psi(N-1; F)$ and $\phi(a; F)$ as $N \rightarrow \infty$, which will imply the limit of \hat{F} and $a^e(F)$ as the number of members in the CCP network grows without bound. To that end, we recall (A14) that,

$$\phi(a; F) = c v_l^{(N)}(a) - (c - F) u_l^{(N)}(a),$$

where $l = \lfloor \frac{NF}{c} \rfloor$, and functions $v_l^{(N)}$ and $u_l^{(N)}$ have the following representation: suppose X follows a Binomial distribution with parameter $(N-1, a)$ and Y follows a Binomial distribution with parameter (N, a) , then

$$v_l^{(N)}(a) = \mathbb{P}(X \leq l) = \mathbb{P}(\sqrt{N-1}(\frac{X}{N-1} - a) \leq \sqrt{N-1}(\frac{l}{N-1} - a)), \quad (\text{A23})$$

$$u_l^{(N)}(a) = \frac{\mathbb{P}(Y \leq l)}{1-a} = \mathbb{P}(\sqrt{N}(\frac{Y}{N} - a) \leq \sqrt{N}(\frac{l}{N} - a)). \quad (\text{A24})$$

By the central limit theorem, both $\sqrt{N-1}(\frac{X}{N-1} - a)$ and $\sqrt{N}(\frac{Y}{N} - a)$ converges in distribution to a normal distribution with mean 0 and variance $a(1-a)$. On the other hand, from

$\frac{NF-c}{c} < \lfloor \frac{NF}{c} \rfloor \leq \frac{NF}{c}$ we know that,

$$\lim_{N \rightarrow \infty} \sqrt{N-1} \left(\frac{l}{N-1} - a \right) = \lim_{N \rightarrow \infty} \sqrt{N} \left(\frac{l}{N} - a \right) = \begin{cases} \infty, & \text{if } \frac{F}{c} > a, \\ 0, & \text{if } \frac{F}{c} = a, \\ -\infty, & \text{if } \frac{F}{c} < a \end{cases}.$$

It follows that

$$\lim_{N \rightarrow \infty} v_l^{(N)}(a) = 1_{\{\frac{F}{c} \geq a\}} + \frac{1}{2} 1_{\{\frac{F}{c} = a\}}, \quad \lim_{N \rightarrow \infty} u_l^{(N)}(a) = \frac{1_{\{\frac{F}{c} \geq a\}} + \frac{1}{2} 1_{\{\frac{F}{c} = a\}}}{1-a},$$

and

$$\lim_{N \rightarrow \infty} \phi(a; F) = c \cdot 1_{\{\frac{F}{c} > a\}} \left(1 - \frac{1 - \frac{F}{c}}{1-a} \right).$$

Returning to the definition of $\phi(a; F)$, we know that

$$\lim_{N \rightarrow \infty} \psi(N-1; F) = c \cdot 1_{\{\frac{F}{c} > q_l\}} \left(1 - \frac{1 - \frac{F}{c}}{1-q_l} \right).$$

Therefore, in the binary case, the switching point \hat{F} , which is defined as the unique root to $\psi(N-1; \hat{F}) = \frac{\mu_h - \mu_l}{q_h - q_l}$, converges to the solution to the equation:

$$c \cdot 1_{\{\frac{F}{c} > q_l\}} \left(1 - \frac{1 - \frac{F}{c}}{1-q_l} \right) = \frac{\mu_h - \mu_l}{q_h - q_l}.$$

We notice that, as the number of members $N \rightarrow \infty$, the cover-II requirement for F , now becomes $F > 0$. Hence, if $\frac{\mu_h - \mu_l}{q_h - q_l} \leq c$, then we obtain that

$$\hat{F} = q_l c + (1 - q_l) \frac{\mu_h - \mu_l}{q_h - q_l}.$$

Hence, by (18) we know that, as $N \rightarrow \infty$,

$$\frac{x^e(N)}{N} = \frac{F^e(N)}{c} \rightarrow \begin{cases} \frac{\hat{F}}{c}, & \text{if } \mu_l - \mu_h - c(q_l - q_h) > r_f(q_l c + (1 - q_l) \frac{\mu_h - \mu_l}{q_h - q_l}), \\ 0, & \text{else.} \end{cases} \quad (\text{A25})$$

This proves Proposition 4.

In the continuous case, recall that the optimal risk preference $a^e(F)$ and F satisfies the first order condition equation:

$$a = \frac{R - 1 - \phi(a; F)}{2(R - r)}.$$

As $N \rightarrow \infty$, we notice that the above equation converges to

$$a = \begin{cases} \frac{R-1}{2(R-r)}, & \text{if } \frac{F}{c} \leq a, \\ a^* + \frac{c-F}{2(R-r)(1-a)}, & \text{if } \frac{F}{c} > a. \end{cases}$$

In other words,

$$a^e(F) = \begin{cases} \frac{R-1}{2(R-r)}, & \text{if } \frac{F}{c} \leq \frac{R-1}{2(R-r)}, \\ \frac{(1+a^*) - \sqrt{(1+a^*)^2 - 2\frac{R-1-F}{R-r}}}{2}, & \text{if } \frac{F}{c} > \frac{R-1}{2(R-r)}, \end{cases} \quad (\text{A26})$$

which is a continuous function that is strictly decreasing over $(\frac{R-1}{2(R-r)}c, c)$, with limits $a^e(c-) = a^*$. From (29) we know that, as $N \rightarrow \infty$, we have

$$\frac{x^e(N)}{N} = \frac{F^e(N)}{c} \rightarrow \frac{1}{c} \arg \max_F \left(-(R-r)(a^e(F))^2 + (R-1-c)a^e(F) + 1 - r_f F \right). \quad (\text{A27})$$

To find the maximizer for the right hand side of (A27), we first notice that

$$\arg \max_{0 < F \leq \frac{R-1}{2(R-r)}c} \left(-(R-r)(a^e(F))^2 + (R-1-c)a^e(F) + 1 - r_f F \right) = 0,$$

with maximum

$$-(R-r) \left(\frac{R-1}{2(R-r)}c \right)^2 + (R-1-c) \frac{R-1}{2(R-r)}c + 1 = -\frac{(R-1)^2 c^2}{4(R-r)} + a^*(R-1)c + 1. \quad (\text{A28})$$

On the other hand, the range of the objective function in (A27) for $F \in (\frac{R-1}{2(R-r)}c, c)$ is the same as that of

$$G(a) := -(R-r)a^2 + (R-1-c)a + 1 - r_f F(a), \quad a \in (a^*, \frac{R-1}{2(R-r)}),$$

where $F(a)$ is the inverse of $a^e(F)$ in (A26) for $F \in (\frac{R-1}{2(R-r)}c, c)$, given by

$$F(a) = R-1 + \frac{[(1+a^*-2a)^2 - (1+a^*)^2](R-r)}{2}. \quad (\text{A29})$$

From

$$G'(a) = -2(R-r)[(1+2r_f)a - ((1+r_f)a^* + r_f)], \quad (\text{A30})$$

we know that $G'(a^*) > 0$. So

$$\sup_{a \in (a^*, \frac{R-1}{2(R-r)})} G(a) = \begin{cases} G(\frac{(1+r_f)a^* + r_f}{1+2r_f}), & \text{if } \frac{(1+r_f)a^* + r_f}{1+2r_f} < \frac{R-1}{2(R-r)}, \\ G(\frac{R-1}{2(R-r)}), & \text{else.} \end{cases}$$

Thus, we know that

$$\begin{aligned} & \arg \max_F (-(R-r)(a^e(F))^2 + (R-1-c)a^e(F) + 1 - r_f F) \\ &= \begin{cases} F(\frac{(1+r_f)a^* + r_f}{1+2r_f}), & \text{if } G(\frac{(1+r_f)a^* + r_f}{1+2r_f}) > -\frac{(R-1)^2 c^2}{4(R-r)} + a^*(R-1)c + 1 \text{ and} \\ & \frac{(1+r_f)a^* + r_f}{1+2r_f} < \frac{R-1}{2(R-r)}, \\ 0, & \text{else.} \end{cases} \end{aligned} \tag{A31}$$

This proves Proposition 8. Finally, since $a^* < \frac{R-1}{2(R-r)}$ and a^* maximizes $G(a)$ when $r_f = 0$, we know that we are in the first case of (A31) if $r_f > 0$ is sufficiently small.