Trading Volume, Illiquidity and Commonalities in FX Markets*

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Abstract

We provide a unified model for foreign exchange (FX), trading volume, and volatility in a multi-currency environment. Tied by no-arbitrage conditions, FX rate movements are determined by common information and differences in traders' reservation prices, or disagreement, that induce trading. Using unique (intraday) data representative for the global FX spot market, the empirical analysis validates our theoretical predictions. We find that (i) volume and volatility are driven by disagreement, (ii) our volatility-volume ratio as in Amihud (2002) is an effective measure of FX illiquidity, and (iii) the commonalities of FX global volume, volatility, and illiquidity that vary across currencies and time can be explained by no-arbitrage.

Keywords: FX Trading Volume, Volatility, Illiquidity, Commonalities, Co-Jumps, Arbitrage

J.E.L. classification: C15, F31, G12, G15

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1 Introduction

Since the demise of the post-war Bretton Woods system in the 1970s, the international financial system has witnessed a growing capital mobility and wider movements of foreign exchange (FX) rates. In such a regime of floating FX rates and open economies, anyone dealing with a currency other than that of the base currency is concerned with the (adverse) evolution of FX rates, their volatility, and market dynamics such as trading volume and illiquidity. It is thus a natural question how FX rates, volatility, and trading volume interrelate.

In this paper, we provide a simple theoretical framework to jointly explain FX rates, trading volume, and volatility in a multi-currency environment. Tied together by triangular no-arbitrage conditions, FX rate movements are determined by common information and differences in traders' reservation prices, or disagreement, that induce trading. In such a unified setting, our model outlines two main drivers within and across currencies: First, investors' disagreement is the common determinant of trading volume and volatility of each FX rate. Second, the no-arbitrage condition is the "glue" across currencies creating commonality in trading volume, volatility, and illiquidity. Our model also provides an intuitive closed-form solution for measuring illiquidity in terms of price impact (Amihud, 2002). Using new and unique intraday data representative for the global FX spot market, the empirical analysis validates our main theoretical predictions, that is, (i) more disagreement increases FX trading volume, volatility, and illiquidity, (ii) stronger commonalities pertain to more efficient (arbitrage-free) currencies, and (iii) our illiquidity proxy is effective in measuring FX illiquidity.

The joint analysis of FX volume and volatility is important for at least three reasons. First, the FX market is the world largest financial market with USD 5.1 trillion of daily traded volume (Bank of International Settlements, 2016). Despite its importance and apparent enormous liquidity, an in-depth understanding of FX volume is still missing. This can be explained by at least two reasons. On the one hand, FX rates are commonly traded over-the-counter, which is notoriously opaque and fragmented.[1] On the other hand, there has been a paucity of comprehensive volume data at a global scale. Second, FX rates are key for pricing many assets including international

[1] The microstructure of the FX market is explained in detail in e.g. Lyons (2001) and King et al. (2012). The recent developments of the FX markets are discussed in Rime and Schrimpf (2013) and Moore et al. (2016).

stocks, bonds, and derivatives, and for assessing their risk. They are also relevant for policy making such as conducting (unconventional) monetary policy and FX interventions. A better understanding of whether and how FX volume, volatility and illiquidity determine FX rates can improve all these tasks. Third, distressed markets such as currency crises are characterized by sudden FX rates movements, drops in liquidity, and raises in volatility. It could thus be supportive of financial stability to highlight the sources of volatility and illiquidity, how they reinforce each other, and across currencies.

Our analysis proceeds in two steps: theory and empirics. Our theory builds upon an equilibrium model in which the evolution of the FX rate is driven by the arrival of new information and by the trading activity. The trading volume is induced by the deviation of individual agent's reservation prices from the observed market price. The continuous-time feature of the model allows us to obtain consistent measurements of the underlying unobservable quantities, such as volatility and illiquidity, and to relate them to the trading volume. Furthermore, agents trade in a multi-currency environment in which direct FX rates are tied to cross rates by triangular no-arbitrage conditions. This implies that direct and arbitrage-related (or synthetic) rates must equate in equilibrium, while the trading volume reflects the dependence on the aggregated information flows across FX rates. Thus, trading volume is the driving force processing information and reservation prices in currency values and attracting FX rates to arbitrage-free prices.

Three basic propositions arise from our theoretical framework: First, trading volume and volatility are driven by traders' disagreement. Second, the combination of volatility and volume provides a closed-form intuitive expression for measuring illiquidity in terms of price impact such as the widespread proxy proposed in Amihud (2002). Third, trading volume, volatility and liquidity across FX rates are linked by no-arbitrage conditions, which lead to the commonalities across FX rates. Since arbitrage passes through the trading activity (volume), more liquid currencies should reveal stronger commonalities and price efficiency (in terms of smaller deviations from triangular arbitrage condition).

Set against this background, we test the main empirical predictions derived from our theory. To do this, we utilize two data sets. First, trading volume data come from CLS Bank International (CLS), which operates the largest payment-versus-payment (PVP) settlement service in the world. Hasbrouck and Levich (2017) provide a very comprehensive description of the CLS institutional setting and Gargano et al. (2019) show that CLS data cover around 50% of the FX global turnover compared to the BIS triennial surveys. Trading volume is measured at the hourly level, across 29 currency pairs over a 5-year period from November 2011 to November 2016.[2] For the same FX panel, we obtain intraday spot rates from Olsen data. For each FX rate and each minute of our sample, we observe the following quotes: ask, bid, low, high, close, and midquote. By merging these two data sets, we can analyze the hourly time series of trading volume, realized volatility, and FX rate and bid-ask spread evolutions.

To test the empirical predictions, we carry out the following analysis. First, we perform a descriptive analysis that uncovers some (new) stylized facts. For instance, we find that FX trading volume and illiquidity follow intraday patterns and seasonalities indicating market fragmentation across geographical areas and FX rates consistent with the OTC nature of the FX global market. Then, we perform various regressions to test the three above-mentioned theoretical propositions. Three main results emerge: First, trading volume and volatility are linked by a very strong positive relationship both within and across FX rates. To provide more direct evidence that both are governed by disagreement between the agents, we show that volume and volatility increase with heterogeneous beliefs as measured in Beber et al. (2010). In contrast, large and directional FX moves associated with little disagreement identified by co-jumps (Caporin et al., 2017) do not generate abnormal trading volume, while being associated with above-average volatility. Consistent with our theory, this finding suggests that new common information such as macroeconomic announcements (see e.g. Bollerslev et al., 2016) on which everyone agrees might give rise to above-average volatility but not abnormal trading volume. Second, we provide evidence that our illiquidity measure is effective in capturing FX illiquidity episodes and correlate with well-accepted measures of FX illiquidity. Finally, using three methods, namely the Principal Component Analysis, regression analysis, and the connectedness index of Diebold and Yilmaz (2014), we perform a comprehensive analysis of commonalities in FX volume, volatility, and illiquidity. After documenting and measuring them, we provide evidence that more liquid currencies have stronger commonalities and obey more to (triangular) arbitrage conditions.

Our paper contributes to two strands of the literature: First, we contribute to prior research

[2]The entire set includes 33 currency pairs but the Hungarian forint (HUF) joined the CLS system later. Therefore, EURHUF and USDHUF are available only since 07 November 2015.

on trading and liquidity in financial markets. While most of the previous studies on volume has mainly focused on stocks,[3] there is a growing literature on trading and liquidity in FX markets (e.g. Mancini et al., 2013 and Karnaukh et al., 2015). Most previous studies focus on specific aspects of FX liquidity such as transaction costs[4] or order flow, which is as the net of buyer-initiated and seller-initiated orders. Following the seminal paper by Evans and Lyons (2002), order flow has drawn much attention as the main determinant of FX rate formation.[5] In contrast, the literature on trading volume is scant due to the paucity of comprehensive data on the FX global volume. Prior research has focused on the interdealer segment in which Electronic Broking Services (EBS) and Reuters are the two predominant platforms. For instance, Evans (2002) uses Reuters D2000-1 data, Payne (2003) analyze data from D2000-2 while Mancini et al. (2013) and Chaboud et al. (2007) utilize data from EBS.[6] Only with the recent access to CLS data, research on FX global volume at relatively high frequencies (e.g. daily) became possible.[7] Fischer and Ranaldo (2011) look at global FX trading around central bank decisions. Hasbrouck and Levich (2017) measure FX illiquidity using volume and volatility data. Gargano et al. (2019) analyze the profitability of FX trading strategies exploiting the predictive ability of FX volume. We add to the extant literature theoretically and empirically. On the one hand, we build a continuous-time model in a multiple-currency setting, which serves the purpose of defining a theoretical foundation for FX price determination in connection to FX volume, volatility, and illiquidity. Although abstracting from some market "imperfections" such as liquidity frictions, our model provides a closed-form and intuitive solution for illiquidity in terms of price impact proxies such as in Amihud (2002). Furthermore, we are the first providing a joint empirical analysis of intraday FX global volume, (realized) volatilities, and illiquidity that support two empirical predictions from our theory: First, disagreement drives trading volume and volatility; Second, our FX measure in

[3]For a recent literature survey, see Vayanos and Wang (2013).

[4]Transaction costs are typically measured in terms of bid-ask spreads that tend to increase with volatility. FX transaction costs in spot and future markets are studied in Bessembinder (1994), Bollerslev and Melvin (1994), Christiansen et al. (2011), Ding (1999), Hartmann (1999), Huang and Masulis (1999), Hsieh and Kleidon (1996), Mancini et al. (2013).

[5]Among others, order flow is studied in Bjønnes and Rime (2005), Berger et al. (2008), Frömmel et al. (2008), Breedon and Ranaldo (2013), Evans and Lyons (2002), Evans (2002), Mancini et al. (2013), Payne (2003), and Rime et al. (2010).

[6]Other sources of trading volume data are proprietary data sets from some specific banks (see e.g. Bjønnes and Rime (2005) and Menkhoff et al. (2016)), central banks, or FX futures or forward contracts (see e.g. Bjønnes et al. (2003), Galati et al. (2007), Grammatikos and Saunders (1986), Levich (2012), and Bech (2012)).

[7]Except from CLS, the only source of global FX trading volume is the triennial survey of central banks conducted by the BIS. It provides a snapshot of FX market volume on a given day once every three-years.

the spirit of Amihud proxy is effective in measuring FX illiquidity.

Second, we contribute to the literature on commonalities in liquidity, which has extensively studied liquidity co-movements of stocks (e.g. Chordia et al., 2000, Hasbrouck and Seppi, 2001, and Karolyi et al., 2012). In FX markets, this issue is empirically analyzed in Mancini et al. (2013) and Karnaukh et al. (2015). We contribute to this strand of literature by studying commonality in trading *volume* and the proposed FX illiquidity measure, as well as some pricing implications stemming from commonality in FX liquidity. Prior research has also provided some theoretical explanations for liquidity commonality. For instance, when dealers are active in two markets (or assets), they tend to reduce their liquidity supply in case of trading losses (Kyle and Xiong, 2001) or under funding constraints (Cespa and Foucault, 2014). From an asset pricing perspective, investors require higher expected returns and invest less in assets exposed to liquidity risk (e.g. Acharya and Pedersen, 2005); additionally, illiquidity and low asset prices might endogenously result from erosion of arbitrageurs' wealth (Kondor and Vayanos, 2018). Even if our theory abstracts from these frictions, commonality in trading volume naturally arises from agents' disagreement and arbitrage trading. Empirically, we find consistent results with the adage that "liquidity begets liquidity" (e.g. Foucault et al., 2013) and that liquidity begets price efficiency in the sense that more liquid currencies have stronger commonality and are less subject to arbitrage deviations.

This paper is organized as follows. Section 2 presents the simple theoretical setting for an unified analysis of volatility, volume and illiquidity on the FX rates, and their commonalities. Section 3 introduces the dataset and discusses summary statistics. Section 4 presents the empirical analysis. Section 5 concludes the paper.

2 A unified model for FX rates, volatility and volume

We depart from the Mixture-of-Distribution Hypothesis (MDH) of Clark (1973) and Tauchen and Pitts (1983), which provides a stylized representation of the supply/demand mechanism on the market at the intradaily level.[8] Let's first consider a world with two currencies, x (base) and y (quote). We assume that the market consists of a finite number $J \geq 2$ of active traders, who take

[8] See also the empirical analysis in Andersen (1996) and the survey in Karpoff (1987). According to Bauwens et al. (2006) only one out of the 19 studies of MDH is on exchange rates.

long or short positions on the FX rate $x|y$. Within a given trading period of unit length (e.g. an hour, a day, a week), the market for the currency pair $x|y$ passes through a sequence of $i = 1, \ldots I$ equilibria. The evolution of the equilibrium price is motivated by the arrival of new information to the market. At intra-period i, the desired position of the j-th trader ($j = 1, \ldots, J$) on the FX rate $x|y$ is given by

$$q_{i,j}^{x|y}(t) = \xi^{x|y}(p_{i,j}^{x|y,*} - p_i^{x|y}), \quad \xi^{x|y} > 0 \tag{1}$$

where $p_{i,j}^{x|y,*}$ is the reservation price of the j-th trader and $p_i^{x|y}$ is the current market price (both measured in logs). The reservation price of each trader might reflect individual preferences, liquidity issues, asymmetries in information sets and/or different expectations about the fundamental values of the FX rate. In general, the reservation price can deviate from the market price because of idiosyncratic reasons inducing the j-th trader to trade. The term $\xi^{x|y}$ is a positive constant capturing the market depth: The larger $\xi^{x|y}$, the larger quantities of x can be exchanged for y (and viceversa) for a given difference $p_{i,j}^{x|y,*} - p_i^{x|y}$. In other words, $\xi^{x|y}$ measures the capacity of the market to allow large quantities to be exchanged at the intersection between the demand and supply side, thus recalling the concept of resilience. Figure 1 illustrates the demand/supply mechanism of the j-th trader for the $x|y$ FX rates. If $(p_{i,j}^{x|y,*} - p_i^{x|y}) > 0$, this means that the j-th trader believes that the equilibrium trading price of $x|y$ is too low, i.e. currency x should be more expansive relatively to y, so he will buy x and sell y. On the contrary, if $(p_{i,j}^{x|y,*} - p_i^{x|y}) < 0$, the j-th trader will buy y and sell x. The amount associated with a unit change of $p_{i,j}^{x|y,*} - p_i^{x|y}$ is given by the slope $\xi^{x|y}$. The baseline assumptions of the MDH (linearity of the trading function and

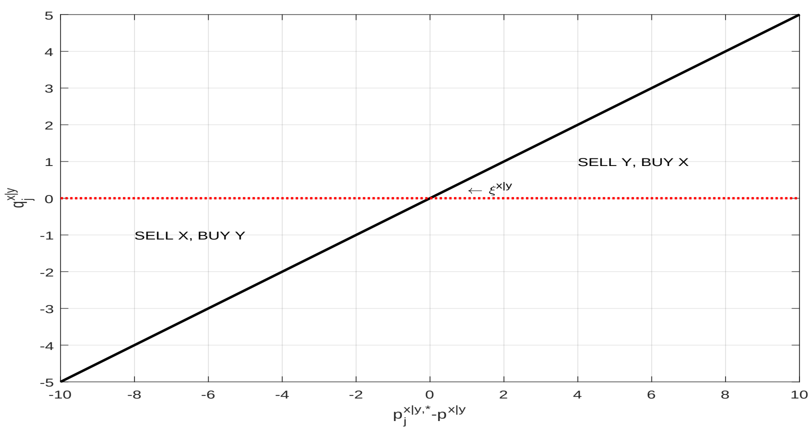


Figure 1: Trading function for the j-th trader for $x|y$ with $\xi^{x|y} = 0.5$.

constant number of active traders) are inevitably very stylized. As for the form of the equilibrium function in (1), note that the trades take place on short intradaily intervals of length $\Delta = 1/I$ and they are generally associated with small price variations. Therefore, it is not restrictive to assume the equilibrium function to be linear on small price changes. Furthermore, the assumption of J active traders observe one market price is more consistent with a centralized market or a fragmented one but with a reference price accessible to trading community, whereas FX rates can be dispersed and heterogeneous outside the interdealer segment, as emphasized by Evans and Rime (2016).

As new information arrives, the traders adjust their reservation prices, resulting in a change in the market price given by the average of the increments of the reservation prices. This means that the equilibrium condition is $\sum_j q_{i,j}^{x|y} = 0$. Hence, the average of the reservation prices clears the market, that is $p_i^{x|y} = \frac{1}{J}\sum_{j=1}^{J} p_{i,j}^{x|y,*}$, and the generated trading volume is

$$v_i^{x|y} = \frac{\xi^{x|y}}{2}\sum_{j=1}^{J} |\Delta p_{i,j}^{x|y,*} - \Delta p_i^{x|y}|,$$

where $\Delta p_{i,j}^{x|y,*} = p_{i,j}^{x|y,*} - p_{i-1,j}^{x|y,*}$ and $\Delta p_{i,j}^{x|y} = p_{i,j}^{x|y} - p_{i-1,j}^{x|y}$. The increments of the reservation log-prices are given by

$$\Delta p_{i,j}^{x|y,*} = \phi_i^{x|y} + \psi_{i,j}^{x|y}, \quad \text{with} \quad j = 1, \ldots, J,$$

where $\phi_i^{x|y}$ is the common information component about the FX rate $x|y$, stemming from public information events, such as those associated with central banks' announcements. The common term $\phi_i^{x|y}$could also be related to events that trigger common directional expectations among the practitioners about a specific currency. The term $\psi_{i,j}^{x|y}$ represents the investor's specific component about the FX rate between x and y. We assume the following continuous time version of the model to form the basis for volatility measurement, where the dynamics of the the investor-specific component about the FX rate is given by

$$d\psi_j^{x|y}(t) = \mu_j^{x|y}(t)dt + \sigma_j^{x|y}(t)dW_j^{x|y}(t), \quad j = 1, \ldots, J \tag{2}$$

where $W_j(t)$ is a Wiener process that is independent between each trader, i.e. $W_l(t) \perp\!\!\!\perp W_m(t)\ \forall l \neq m$ and the term $\sigma_j^{x|y}(t) \geq 0$ is the stochastic volatility process of the j-th trader which is assumed to have locally square integrable sample paths. The term $\mu_j(t)$ is a predictable and finite variation drift process, which might represent the long-run expectation of the j-th trader about the FX rate and it could be function of fundamental quantities like interest rates differentials and long-term macroeconomic views. By also allowing $\sigma_j^{x|y}$ to be different across traders, we are implicitly introducing heterogeneity among them. This also reconciles with many realistic features including the evidence of long-memory in volatility that is obtained by the superposition of traders operating at different frequencies, see for instance the heterogeneous autoregressive model of Müller et al. (1997) and Corsi (2009). This setup is coherent with a representation of a frictionless market where each trader participates through its reservation price to the price discovery process by carrying new information. On the i-th discrete sub-interval of length $\Delta = \frac{1}{I}$,[9]

$$\psi_{i,j}^{x|y} = \int_{\Delta(i-1)}^{\Delta i} \mu_j(s)ds + \int_{\Delta(i-1)}^{\Delta i} \sigma_j^{x|y}(s)dW_j^{x|y}(s). \tag{3}$$

Proposition 1. *Over an interval of unit length (e.g. a day or a month), the trading volume,* $v = \sum_{i=1}^{I} v_i^{x|y}$*, and the aggregated volatility, as measured by the realized variance,* $RV^{x|y} = \sum_{i=1}^{I} \left(\Delta p_i^{x|y}\right)^2$*, or by the power variation of Barndorff-Nielsen and Shephard, 2003,* $RPV^{x|y} = \sum_{i=1}^{I} |\Delta p_i^{x|y}|$*, carry information about the investor disagreement on a given FX rate.*

Proof in Appendix A.1.

The extension to a continuous-time framework allows us to precisely measure the variability of the FX rates components in the limit for $I \to \infty$, and to relate it to the level of disagreement among investors leading to the observed trading volume. It should be stressed that the asymptotic results behind Proposition 1 are derived by abstracting from microstructural frictions (namely *microstructure noise*), like transaction costs in the form of bid-ask spread, clearing fees or price discreteness, which are intimately related and endogenous to the trading process; see the recent works of Darolles et al. (2015, 2017) for an extension of reduced-form version of the MHD with liquidity frictions. From a statistical point of view, as $I \to \infty$, the microstructure noise dominates over the volatility signal, thus leading to distorted measurements of the variance. However, over moderate sampling frequencies, e.g. 5-minute intervals over 24 hours ($I = 288$), the prices and quantities determined in equilibrium in each sub-interval can be considered (almost) free of

[9]For ease of exposition, we assume that trades happen on an equally spaced and uniform grid, $i = 1, 2, \ldots I$. This assumption can be relaxed allowing for random trading times.

microstructure noise contamination, and representative of new equilibria on the aggregated supply/demand functions. Rather than microstructural features, this setting outlines the aggregated disagreement on fundamentals leading to the price discovery process in which each trader participates to the equilibrium price variations in proportion to the information contained in her new reservation prices. As it is common in the literature on volatility measurement, see Bandi and Russell (2008) and Liu et al. (2015), in the following analysis we will work under the maintained assumption that sampling at 5-minute intervals is sufficient to guarantee that a new equilibrium price is determined. The latter is representative of the aggregated information contained on the demand and supply sides of the market. Furthermore, the assumption of independence between ϕ_i and $\psi_{i,j}$ and across traders does not allow for reversal or spill-over effects such as those studied in Grossman and Miller (1988) to investigate the mechanics of liquidity provision. The same type of sequential trading behavior has been recently proved to be responsible for crash episodes in Christensen et al. (2016) and associated with changes in the level of investors' disagreement around important news announcements, see Bollerslev et al. (2016). Despite the stylized set of assumptions, the next section shows how the theory outlined above can be successfully adopted as an encompassing framework to characterize the illiquidity and the commonalities in volatility and volume on the global FX markets.

2.1 Measuring FX Illiquidity

In light of Proposition 1 and analogously to the price impact illiquidity proxy in Amihud (2002), we can define a continuous-time version of the illiquidity index as

$$A^{x|y} := \frac{RPV^{x|y}}{\nu^{x|y}}, \tag{4}$$

which measures the price impact of a given trade, that is the amount of volatility of the FX rate associated with a unit of trading volume. The following proposition highlights the main determinants of market illiquidity.

Proposition 2. *Consider the illiquidity measure defined in (4). In the limit for $I \to \infty$ and under* *homogeneity of traders, i.e.* $\sigma_j^{x|y} = \sigma^{x|y} \quad \forall j = 1, 2, \ldots, J$,

$$p \lim_{I \to \infty} A^{x|y} = \frac{2}{\xi^{x|y} J \sqrt{(J-1)}}. \tag{5}$$

Proof in Appendix A.2.

Proposition 2 shows that on a period of unit length, $A^{x|y}$ is inversely related to the slope, $\xi^{x|y}$, of the equilibrium function in (1). That is, for a given difference between the reservation price and the market price, $A^{x|y}$ decreases as this slope increases. In particular, for large values of $\xi^{x|y}$ large volume would be associated with small variations between the prevailing price and the reservation price for each trader, thus signaling market depth and liquidity. Instead, when $\xi^{x|y} \to 0^+$, i.e. in the limiting case of a flat equilibrium function in (1), the liquidity is minimal (and $A^{x|y}$ diverges), since no actual trade takes place. Under the assumption of homogeneity of the traders, i.e $\sigma_j^2(t) = \sigma^2(t)\ \forall j = 1, \ldots, J$, Proposition 2 also highlights the inverse relationship between the number of active traders on the market and illiquidity.[10]

In the extreme case of only one observation per trading period $I = \Delta = 1$, the illiquidity measure in (4) reduces to the original Amihud index (up to the rescaling by $\sqrt{2/\pi}$),

$$A^{x|y,*} = \frac{|r|^{x|y}}{v^{x|y}}, \tag{6}$$

for which it is not trivial to obtain an expression as a function of the structural parameters analogous to the one in (5). For instance, the expected value of $|r|^{x|y}$ under Gaussianity is proportional to the daily (constant) volatility parameter, i.e. $E\left(|r|^{x|y}\right) = \sigma\sqrt{\frac{2}{\pi}}$, where $\sigma = \sqrt{Var(\phi) + Var(\psi)/J}$ in the original MDH theory. In the classic framework, inference on the structural parameters is performed through GMM by relying on the unconditional moments of the observable quantities which depend on the underlying (unobservable) information flow, see Richardson and Smith (1994) and Andersen (1996). The availability of high-frequency data coupled with the theory of quadratic variation makes the volatility and consequently the information flow *measurable* quantities. This means that inference on the structural parameters becomes more precise as we adopt moment conditions based on high-frequency data, see Li and

[10] Relaxing the assumption of homogeneity would result in the ratio of two aggregated volatility measures, each estimating the weighted average of the variance carried by each trader, see equation (30) in Appendix A.2.

Xiu (2016).

2.2 Commonalities in FX volume and volatility

In this section, we derive equilibrium relations between returns, trading volumes and volatilities *across* different FX rates. These relations are instrumental to the interpretation of commonalities in trading volumes and volatilities as well as information processing in global FX markets. Let's therefore consider a world with three currencies, x , y and z. The market for the currency pairs $x|y$, $x|z$ and $z|y$ also passes through a sequence of $i = 1, \ldots I$ equilibria and the evolution of the equilibrium price of each currency pair is motivated by the arrival of new information to the market. By the triangular no-arbitrage parity it must hold that

$$p_i^{x|y} = p_i^{x|z} + p_i^{z|y}, \tag{7}$$

where $p_i^{x|z} = \sum_{j=1}^{J} p_i^{x|z,*}$ and $p_i^{z|y} = \sum_{j=1}^{J} p_i^{z|y,*}$. By imposing that $\Delta p_{i,j}^{x|z,*} = \phi_i^{x|z} + \psi_{i,j}^{x|z}$, and $\Delta p_{i,j}^{z|y,*} = \phi_i^{z|y} + \psi_{i,j}^{z|y}$, the *synthetic* return on $x|y$ results to be

$$\widetilde{r}_i^{x|y} = \phi_i^{x|z} + \phi_i^{z|y} + \frac{1}{J}\sum_{j=1}^{J} \psi_{i,j}^{x|z} + \frac{1}{J}\sum_{j=1}^{J} \psi_{i,j}^{z|y}. \tag{8}$$

Assuming that the common information component on the rate $x|y$, can be disentangled into two currency-specific terms ϕ_i^x and ϕ_i^y, with $\phi_i^{x|y} = \phi_i^x - \phi_i^y$,[11] it follows that

$$\widetilde{r}_i^{x|y} = \phi_i^x - \phi_i^y + \frac{1}{J}\sum_{j=1}^{J} \psi_{i,j}^{x|z} + \frac{1}{J}\sum_{j=1}^{J} \psi_{i,j}^{z|y},$$

where the common information part of $\widetilde{r}_i^{x|y}$ is the same as for $r_i^{x|y}$, that is $\phi_i^x - \phi_i^y$. It follows that the MDH coupled with the triangular no-arbitrage relation on the FX rates, i.e. $r_i^{x|y} = \widetilde{r}_i^{x|y}$, prescribes that

$$\frac{1}{J}\sum_{j=1}^{J} \psi_{i,j}^{x|y} = \frac{1}{J}\sum_{j=1}^{J} \psi_{i,j}^{x|z} + \frac{1}{J}\sum_{j=1}^{J} \psi_{i,j}^{z|y}, \tag{9}$$

[11] In Section 4.1 we discuss a strategy to separately identify ϕ_i^x and ϕ_i^y based on a cross section of FX rates and provide an empirical validation of such an assumption.

which means that the average of the traders' specific terms on $x|y$ must be equal to the sum of the average traders' specific terms of $z|y$ and $x|z$. This means that each trader can take a direct position on $x|y$ or operate on the synthetic rate by forming independent beliefs on $x|z$ and $z|y$, thus generating trading volume on each individual FX market.

Proposition 3. *Trading volume, volatility and liquidity across FX rates are linked by no-arbitrage constraints, which lead to the commonalities across FX rates. The synthetic volatility, as measured by* $\widetilde{RV}^{x|y} = \sum_{i=1}^{I} (\tilde{r}_i^{x|y})^2$, *and synthetic volume, denoted as*

$$\tilde{v}_i^{x|y} = \frac{\xi^{x|y}}{2} \sum_{j=1}^{J} |\Delta p_{i,j}^{x|z,*} - \Delta p_i^{x|z} + \Delta p_{i,j}^{z|y,*} - \Delta p_i^{z|y}|, \tag{10}$$

reveal the strength of the correlation across FX rates.

Proof in Appendix A.3.

Proposition 3 introduces the concept of *synthetic* volatility and volume, which are associated with the no-arbitrage equilibrium constraints and depend on the extent of the individual disagreement on the FX rates of $x|z$ and $z|y$. Furthermore, both synthetic volatility and volume are functions of the aggregated correlation in beliefs between $x|z$ and $z|y$, and hence expression of the commonalities in the global FX rates. For a given level of traders' disagreement on $x|y$ (leading to trading volume on $x|y$), we can measure the associated synthetic volume on $x|z$ and $z|y$, which is proportional to the correlation between the aggregated reservation prices on $x|z$ and $z|y$. The same holds true for the synthetic volatility, as measured by the realized variance of the synthetic return.

3 Data and Preliminary Analysis

3.1 Data Sets

Our empirical analysis relies on two data sets covering 29 currency pairs (15 currencies) over the period from November 2011 to November 2016.[12] First, trading volume data come from CLS,

[12]The full dataset contains data for 18 major currencies and 33 currency pairs. To maintain a balanced panel, we exclude the Hungarian forint (HUF), which enters the dataset only on 07 November 2015. Moreover, we discard USDILS and USDKRW due to very infrequent trades. We obtain very similar results by including them. The remaining

which is the largest payment system for the settlement of foreign exchange transactions launched in 2002. By means of a payment-versus-payment mechanism, this infrastructure supports FX trading by removing settlement risk and supporting market efficiency. For each hour of our sample period and each currency pair, we observe the settlement value and number of settlement instructions. Following the literature (e.g. Mancini et al., 2013), we exclude observations between Friday 10PM and Sunday 10PM since only minimal trading activity is observed during these nonstandard hours.[13] In 2017, the core of CLS was composed of 60 settlement members including the top ten FX global dealers, and thousands of third parties (other banks, non-bank financial institutions, multinational corporations and funds), which are customers of settlement members. The total average daily traded volume submitted to CLS was more than USD 1.5 trillion, which is around 30% of the total daily volume recorded in the last available BIS triennial survey (Bank of International Settlements 2016). However, after adjusting for the large fraction of BIS volume originated from interbank trading across desks and double-counted prime brokered "give-up" trades, the CLS data should cover about 50% of the FX market (Gargano et al., 2019 & Hasbrouck and Levich, 2017). In our study, we focus on FX spot transactions. Except for some exceptions such as the Renminbi, the CLS spot FX rates in our sample are highly representative of the entire FX market. For instance, the currency pairs involving the USD and EUR cover more than 85% (94%) of the total trading volume of the BIS triennial survey.

To the best of our knowledge, only few papers have analyzed CLS volume data so far. First, Fischer and Ranaldo (2011) study five aggregated currencies (e.g. all CLS-eligible currencies against the U.S. dollar, Euro, Yen, Sterling, and Swiss franc) rather than currency pairs. Hasbrouck and Levich (2017) analyze every CLS settlement instruction during April 2013. Gargano et al. (2019) use the same dataset to perform an asset pricing analysis. Ranaldo and Somogyi (2019) analyze the heterogeneous price impact of CLS order flows decomposed by market participants .

The second data set is obtained from Olsen Financial Technologies, which is the standard source for academic research on intraday FX rates. By compiling historical tick data from the

29 currency pairs are: AUDJPY, AUDNZD, AUDUSD, CADJPY, EURAUD, EURCAD, EURCHF, EURDKK, EURGBP, EURJPY, EURNOK, EURSEK, EURUSD, GBPAUD, GBPCAD, GBPCHF, GBPJPY, GBPUSD, NZDUSD, USDCAD, USDCHF, USDDKK, USDHKD, USDJPY, USDMXN, USDNOK, USDSEK, USDSGD, and USDZAR.

[13]In this paper, times are expressed in GMT

main consolidators such as Reuters, Knight Ridder, GTIS and Tenfore, Olsen data are representative of the entire FX spot market rather than specific segments such as the interdealer FX market dominated by two electronic limit order markets: EBS and Reuters. For each minute of our sample period and each currency pair, we observe the following quotes: bid, ask, high, low, and midquotes. With these data at hand, we can analyze at least four aspects of FX rates: (i) the FX rate movements at one minute or lower frequencies; (ii) the realized volatility or other measures of return dispersion; (iii) the quoted bid-ask spread as a measure of transaction cost; and (iv) violations of triangular arbitrage conditions.

3.2 Descriptive Analysis

In this subsection, we highlight some (new) stylized facts characterizing the times series of volume, volatilities and illiquidity measures associated with the 29 FX rates under investigation. First, we look into intraday patterns and then we study the daily time series of FX volume, volatility, and illiquidity.

To start with the intraday analysis, Figure 2 displays the total hourly volume series, denoted as $v_t^{tot} = \sum_{l=1}^{L} v_t^l$, where v_t^l is the hourly volume on the l-th FX rate. This plot highlights the size

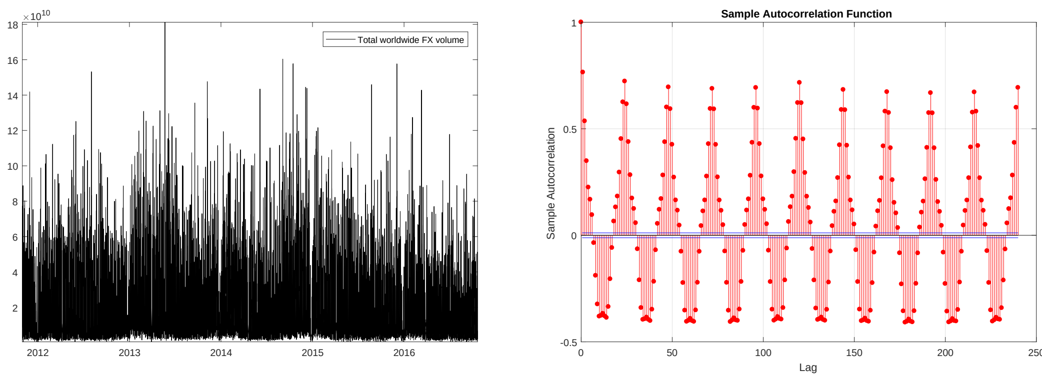


Figure 2: Time series and auto-correlation function of the total global volume.

and deepness of the FX market, with an average of around 20 billions USD dollar traded every hour. Moreover, the series of total volume is rather persistent and it clearly displays cyclical patterns, which can be associated with strong intradaily seasonality. We explicitly model the intradaily patterns by estimating the following model with OLS

$$\log(v_t^l) = \delta_t \beta + \epsilon_t, \tag{11}$$

where δ_t contains hourly and day-of the-week dummies. We can also obtain the *filtered* volume as $v_t^l = \frac{v_t^l}{e^{\delta_t \hat{\beta}}}$. The hourly average of the total global volume is reported in Figure 3. The plot highlights that the average total volume is higher during the opening hours of the European and American stock markets, while it is very low between 10PM and 12AM as most of the largest stock markets are closed, while it has a relative peak associated with the opening of Tokyo (2 AM). Moreover, the total volume is the largest on average between 3PM and 4PM, i.e. before the WMR Fix, for which there is a well documented literature about the large traders submitting a rush of orders before the setting of the daily benchmarks for FX prices, see e.g. Marsh et al. (2017) and Evans (2018). Finally, Figure 3 shows that the filtering successfully removes the largest part of the seasonal pattern and that the filtered volume displays significant autocorrelation after many periods.

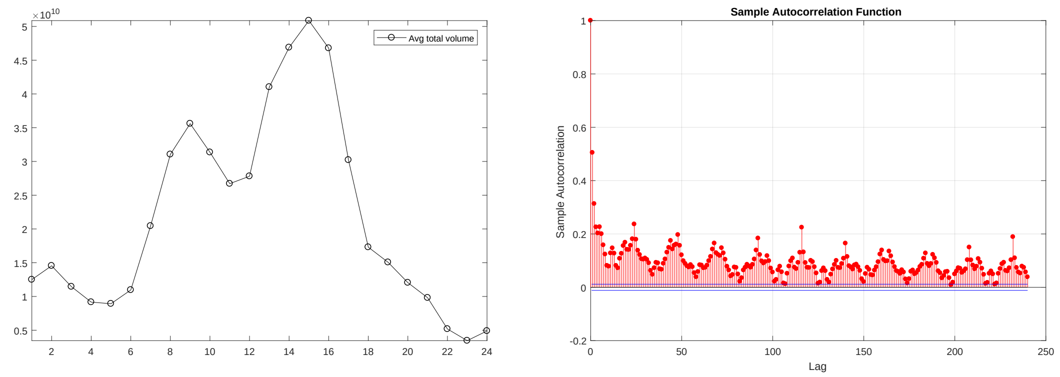


Figure 3: Hourly average of the total global volume and ACF of the *filtered* volume.

Turning the attention to individual FX rates, Figure 4 reports the hourly average share of the total volume of the five most liquid FX rates (by volume size). Firstly, as expected all the most liquid FX rates involve the USD as either base or quote currency. As for the total volume, the trading volume of the most liquid FX rates displays clear (intraday) seasonal patterns. For the individual FX rates, these patterns are suggestive of local effects in given geographical areas, coherent with the OTC segmented nature of FX markets. For instance, USDJPY covers around 30% of the total FX volume between 12PM and 4PM, that are the hours in which Far East markets

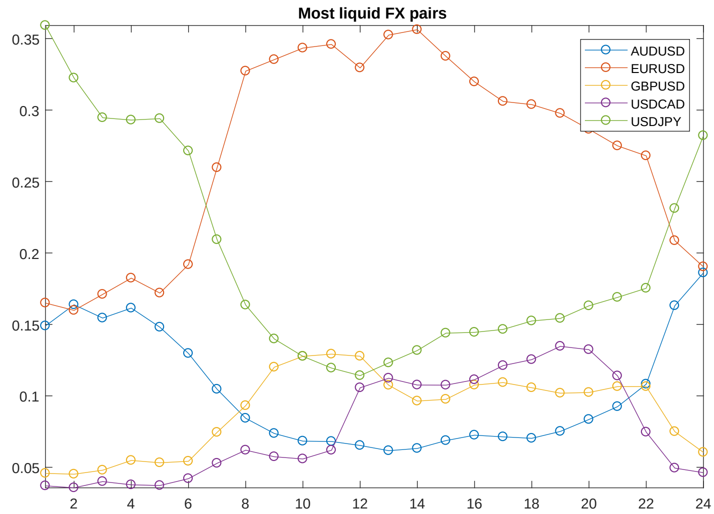


Figure 4: Hourly volume averages of the five most liquid FX rates, which are (in order) USDEUR, USDJPY, USDGBP, USDAUD, USDCAD.

are open. AUDUSD contributes with a 15% in the same hours, while its market share strongly declines to 7% during the central hours of the day. EURUSD is by far the most traded FX rate, with a share above 30% between 7AM to 6PM. A similar pattern characterizes also GBPUSD with an average share ranging between 5% and 10%. Finally, USDCAD is mostly traded at the opening of the business hours in North America, i.e. between 12PM and 10PM, with approximately 10% share on the total volume. These five FX rates amount for a share of more than 70% of the total global volume in every hour. Summarizing, the seasonal patterns are clearly discernible in two dimensions. First, on an intraday scale the trading volume follows the working time in each country or jurisdiction defining the currency pair. This means that round-the-clock, the trading volume of New Zeeland dollar is the first to increase, followed by Asian, European, and American currencies. Second, official banking holidays clearly reduce the trading activity. The seasonalities and calendar effects will be carefully considered in our empirical analysis.

Concerning the relationship between volatility and volume, Figure 5 shows that the hourly averages of realized volatility and volume for USDEUR and USDJPY follow the same patterns. At the intradaily level when the volatility on the FX rates is high, also the volume is high, which points to a wider variation of traders' reservation prices. Thus, Figure 5 provides *prima facie* evidence to Proposition 1 in our theoretical setting, that is, volatility and volume are mostly governed by a common latent factor, which seen through the lenses of the MDH represents the

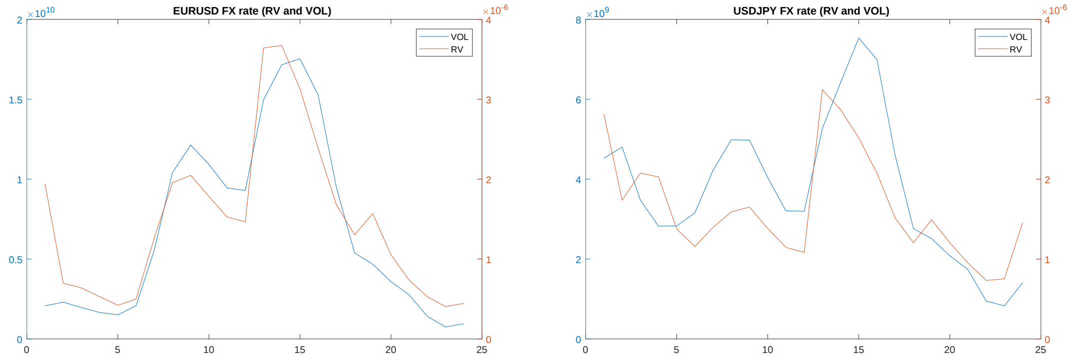


Figure 5: Hourly Averages of RV and VOL. In Panel a) USDEUR, in Panel b) USDJPY.

information flow proportional to the level of heterogeneous beliefs (disagreement) between the agents.

Before performing the empirical analysis and test our model predictions, we examine how daily changes in trading volume correlate with daily changes in realized volatility and other factors that proved to explain FX liquidity in the previous literature (e.g. Mancini et al., 2013 and Karnaukh et al., 2015) and trading activity in stock markets (e.g. Chordia et al., 2001).

Some of these variables are likely to determine each other endogenously. Rather than causation, the purpose of this analysis is to document some novel correlation patterns pertaining to FX trading volume. More specifically, we perform a panel regression of all currency pairs, in which the daily FX volume is explained by daily (realized) volatility, (average intraday) relative bid-ask spread (BAS), a dummy variable for the dollar appreciation, two common proxies of market stress such as the TED spread (the yield spread between the U.S. three-month Libor and T-bills) and FX VIX (i.e. the JP Morgan Global FX volatility index), and four weekday dummy variables equal to one if the trading day is on Monday, Tuesday, Thursday, and Friday, respectively. All variables except the dummy variables are taken in logs and changes and all regressions include the lagged dependent variable as additional regressor. For sake of comparison, we repeat similar regressions using dependent variables the realized volatility, the relative bid-ask spread, as well as the Amihud illiquidity measure, which will be studied in more details later.

Some novel patterns emerge from the analysis reported in the Table 1. On the one hand, FX trading volume increases with realized and implied FX volatility as well as TED spread, whereas it decreases with the relative bid-ask spread. Trading volume follows an inverted U-shape across

	(1) Δ Volume	(2) Δ RPV	(3) Δ Amihud	(4) Δ Relative BAS
Δ Volume	-	0.1374^a	-	-0.04395^a
		(98.778)		(-43.356)
Δ RPV	1.301^a	-	-	0.3995^a
	(92.12)			(148.69)
Δ Relative BAS	-0.9197^a	0.9049^a	0.7961^a	-
	(-40.87)	(154.33)	(42.501)	
USD	0.00679	-0.00852^a	-0.00866^a	-0.00856^a
	(1.6193)	(-6.232)	(-1.9637)	(-9.4226)
Δ TED	0.1174^b	0.0301^c	-0.1746^a	-0.0754^a
	(2.347)	(1.8444)	(-3.3154)	(-6.9454)
Δ VXY	0.1956^b	0.6120^a	-0.5974^a	0.4271^a
	(2.338)	(22.477)	(-6.8248)	(23.521)
Monday	-0.3508^a	0.0324^a	0.3285^a	-0.0858^a
	(-50.68)	(14.572)	(45.258)	(-60.692)
Tuesday	0.0206^a	-0.0176^a	-0.1089^a	-0.0362^a
	(3.01)	(-8.1402)	(-15.509)	(-24.383)
Thursday	-0.1072^a	0.0005	0.1300^a	0.0054^a
	(-16.65)	(0.2354)	(19.102)	(3.9088)
Friday	-0.0766^a	-0.1072^a	0.1932^a	0.0643^a
	(-11.34)	(-51.62)	(28.41)	(46.22)
Lagged Dep.	-0.3494^a	-0.0458^a	-0.4211^a	-0.2114^a
	(-81.065)	(-35.95)	(-91.694)	(-55.475)
Constant	0.0986^a	0.0231^a	-0.1034^a	0.0146^a
	(19.167)	(14.227)	(-19.324)	(13.516)
R^2	0.418	0.586	0.404	0.573
N	37055	37054	37054	37055

Table 1: Regressions of volume, volatility, illiquidity, and bid-ask spread. Volume and RPV are the daily trading volume and realized variance respectively, Amihud is the ratio between daily RPV and daily volume, and the bid-ask spread is the daily average of one-minute spreads. The t-statistics are in parentheses and the error variance are robust to heteroskedasticity and autocorrelation in the residuals. Except for dummy variables, all variables are taken in logs and changes. The superscripts a, b and c indicate significance at 1%, 5% and 10% significance level respectivelyy.

weekdays, that is, larger trading volumes tend to occur in the middle of the week. On the other hand, realized volatility increases with bid-ask spreads and tends to be lower when the U.S. dollar appreciates, possibly due to its status as international currency reserve and safe haven against several currencies (e.g. Ranaldo and Söderlind, 2010 and Maggiori, 2017). In addition to FX volume, (negative) autocorrelation and weekdays effects are discernible for FX volatility, illiquidity, and relative bid-ask spread.

4 Empirical Analysis

Our theoretical setup in Section 2 offers three main propositions. For each of them we provide an in-depth empirical analysis in a separate subsection.

4.1 Determinants of FX trading volume and volatility

The first theoretical proposition postulates that volatility and trading volume are proportional to the level of heterogeneous beliefs between agents, that is traders' disagreement about the fundamental value of the FX rates. This proposition delivers two main empirical predictions: On the one hand, both volume and volatility should increase with disagreement. On the other hand, common news leading to a currency appreciation or depreciation with no or little disagreement can generates above-average volatility but no extraordinary trading volume.

To test the first empirical prediction, we follow Beber et al. (2010) and measure disagreement as heterogeneity in beliefs of market participants by using a detailed data set of currency forecasts made by a large cross-section of professional market participants. More specifically, we collect all Thomson Reuters surveys recorded at the beginning of every month during our sample period and compute measures of cross-sectional dispersion such as the (standardized) standard deviation of FX forecast and the high-low range from the distribution of FX forecasts of on average about 50 market participants.[14] This measure of heterogeneity in beliefs that we call *disagreement* is the main regressor in two panel regressions in which total trading volume and realized volatility are the dependent variables. In addition to our measure of disagreement, we include a constant, the lagged dependent variable, and FX illiquidity proposed in Karnaukh et al., 2015 as a control. All variables are taken in logs and changes.[15] As shown in column (1) and (2) of Table 2, both

[14] The total number of monthly observations included in the regression is 940, which includes the following 26 currency pairs: AUDJPY, AUDNZD, CADJPY, EURAUD, EURCAD, EURCHF, EURGBP, EURJPY, EURNOK, EURSEK, GBPCAD, GBPCHF, GBPJPY, USDAUD, USDCAD, USDCHF, USDEUR, USDGBP, USDHKD, USDJPY, USDMXP, USDNOK, USDNZD, USDSEK, USDSGD and USDZAR. Not for all currency pairs, forecasts are available from November 2011 onwards. The exact number of market participants depends on the currency pair. We report results using standard deviations of FX forecast. Using ranges, we obtain very similar results.

[15] We perform additional analyses including further regressors such as the TED and VXY and the results remain qualitatively the same.

	(1) Δ Volume	(2) Δ RV	(3) Δ Amihud	(4) Δ Relative BAS
Δ Disagreement	0.0503^{b}	0.1736^{a}	0.0397^{b}	0.0373^{a}
	(2.25)	(3.41)	(1.96)	(3.62)
Δ Illiquidity	0.0779^{a}	0.5431^{a}	0.1961^{a}	0.1405^{a}
	(5.29)	(8.12)	(5.66)	(10.65)
Lagged Dep.	-0.3499^{a}	-0.2625^{a}	-0.2389^{a}	0.0289
	(-10.03)	(-5.34)	(-4.38)	(0.97)
Constant	-0.0097	-0.0113	0.0041	-0.0067^{c}
	(-1.50)	(-0.81)	(0.61)	(-1.91)
R^2	0.146	0.367	0.254	0.345

Table 2: Monthly regression analysis - disagreement. The t-statistics are in parentheses and the error variance are robust to heteroskedasticity and autocorrelation in the residuals. Disagreement is the standardized standard deviations of Thomson Reuters forecasts, which are available on a monthly basis. Volume and RV are the daily trading volume and realized variance respectively, Amihud is the ratio between daily RPV and daily volume, bid-ask spread is the daily average bid-ask spread, and illiquidity is taken from Karnaukh, Ranaldo and Söderlind (2015). Except for illiquidity, all variables are taken in logs. The superscripts a, b and c indicate significance at 1%, 5% and 10% significance level respectively.

trading volume and volatility increase with disagreement providing evidence in support to our first empirical prediction. Moreover, both trading volume and volatility tend to increase with FX illiquidity, consistent with dealers' inventory imbalances and hot potato effects Lyons (1997).

The next empirical prediction is that common news or informational events sparking little disagreement across traders should not generate any extraordinary trading volume but it might result in above-average volatility. More specifically, the model prescribes that if new information is common as for macroeconomic announcements, then traders would promptly revise their reservation prices in the same manner and nearly no additional transaction volume should be generated. To avoid confounders and overlapping occurrences, the detection of such informational events needs an accurate identification econometric technique and granular (intraday) data. The recent advances in the literature on *jump* processes come to the aid of this analysis. Similarly to Bollerslev et al. (2016), we rely on a simple setup for the common news component, i.e. the "jumps", to separately identify it from the component of the variations in the FX rates due to the disagreement among traders.[16] For instance, $\phi_i^{x|y}$ can be modeled as compound Poisson

[16] Other studies associating large price jumps with news announcements are in Andersen et al. (2007) and Lee (2011).

processes as

$$\phi_i^{x|y} = \sum_{l=1}^{N_i^{x|y}} Z_l^{x|y}, \tag{12}$$

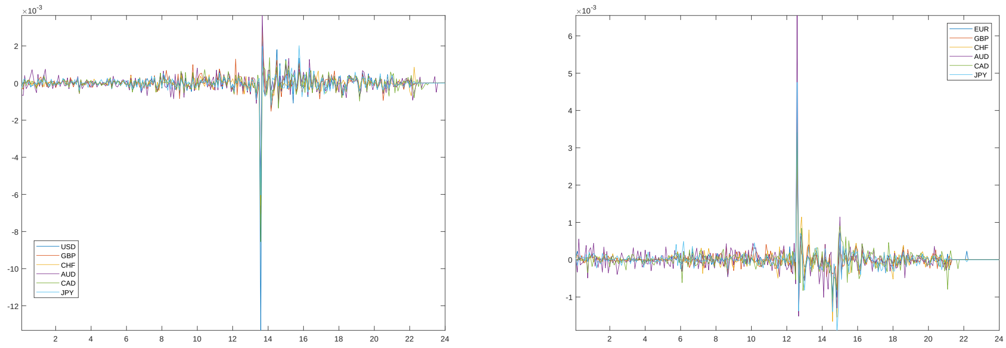
where $N_i^{x|y}$ is an independent Poisson random variable with intensity $\lambda_{x|y}\Delta$, where $\lambda_{x|y}$ is expressed with respect to the unit scale (e.g. daily). $Z_l^{x|y} \overset{iid}{\sim} D_{x|y}(\theta_{x|y}) \in \mathbb{R}$ where $\theta_{x|y}$ are the parameters associated with the distribution $D_{x|y}$. Furthermore, we assume that $\phi_i^{x|y}$ can be further decomposed into currency specific variations, that is $\phi_i^{x|y} = \phi_i^x - \phi_i^y$. For instance, we can assume that $\phi_i^x = \sum_{l=1}^{N_i^x} Z_l^x$ and $\phi_i^y = \sum_{l=1}^{N_i^y} Z_l^y$. The terms ϕ_i^x and ϕ_i^y cannot be uniquely identified by looking at a single FX rate since a large variation in the FX rate might be due to good (bad) news on x or bad (good) news on y. Therefore, we rely on the theory of *co-jumps*, as developed in Caporin et al. (2017), to identify ϕ_i^x given a cross section of FX rates with the same base currency x. In other words, the simultaneous occurrence of a jump in all the FX rates trading with a given base currency x allows us to identify episodes characterized by the ex-post realization of a currency-specific news common to all traders. In turns, this enables us to identify large and sudden directional appreciations or depreciations of one currency against the other currencies associated with no or little disagreement. The test for co-jumps proposed by Caporin et al. (2017) takes the form

$$\mathcal{CJ} = \frac{1}{\zeta} \sum_{j=1}^{N} \frac{\left(SRV_j - \widetilde{SRV}_j\right)^2}{SQ_j}, \tag{13}$$

where N denotes the number of FX rates, ζ is a design parameter, SRV is the smoothed randomized realized variance of Podolskij and Ziggel (2010), $\widetilde{SRV}$ is the smoothed version of the truncated realized variance estimator of Mancini (2009) which is robust to jumps, while SQ is a smoothed estimator of the quarticity. Under the null hypothesis of absence of co-jumps, $\mathcal{CJ}$ converges to a chi-square distribution with N degrees of freedom. Under the alternative hypothesis of at least one co-jump across all N series, $\mathcal{CJ}$ diverges.

Figure 6 illustrates two representative episodes detected with the test for co-jumps developed in Caporin et al. (2017).[17] The left panel reports the log-returns of the FX rates of EUR against the six major currencies, USD, GBP, CHF, AUD, CAD and JPY on November 6, 2015. The sudden depreciation of the Euro occurred in reaction to a speech by the President of ECB, Mario Draghi

[17]We thank the authors for sharing with us their MATLAB code to detect co-jumps.



(a) EUR

(b) USD

Figure 6: Co-jumps analysis. The figures reports the five-minute returns on six FX rates on days when the test of co-jumps of Caporin et al. (2017) has detected significant jumps at 0.01% significance level. The left plot reports the returns of the FX rates of USD, GBP, CHF, AUD, CAD and JPY against EUR on November 6, 2015. The right plot reports the returns of the FX rates of EUR, GBP, CHF, AUD, CAD and JPY against USD on May 1, 2014.

reinforcing traders' belief about the continuation of the Eurosystem's bond purchases (Quantitative Easing) as a stabilization tool to resolve the crisis situations in the financial market. The FX rate reacted with a sudden depreciation of EUR against all other currencies by approximately 1% on an interval of five minutes. The magnitude of such a variation is several times larger than the variation under *normal* market conditions, where the changes in the reservation prices of each individual trader is averaged over J traders. An analogous evidence arises for the appreciation of the USD against all major currencies on May 1, 2014, following the rumors on the beginning of a tapering policy by the Federal reserve.

To test our second empirical prediction, we examine whether trading volume significantly increases when the FX rates are hit by large and directional news. Using hourly time series, we perform the following panel regression with fixed effects

$$V_{i,t} = \alpha_i + \beta CJ_t + \delta BA_{i,t} + \gamma_h h_t + \gamma_w w_t + \rho V_{i,t-1} + \varepsilon_{i,t}, \tag{14}$$

where $V_{i,t}$ is the log-volume on the i-th FX rate trading against a given base currency, CJ is a dummy variable for a significant co-jump on the base currency. We control for illiquidity by including $BA_{i,t}$, i.e. the relative bid-ask spread on the i-th FX rate, and seasonal effects with h_t and w_t that are hourly and day-of-the-week dummies. The coefficient β captures the sudden increase/reduction in the average trading volume associated with co-jumps. To guarantee enough counterparts to each currency, we analyze the four main currencies, i.e. co-jumps of USD, EUR, JPY, and GBP. Regression (14) can be considered the multiple-jumps analogous in the panel setting of the jump regression formalized in Li et al. (2017) and applied in Bollerslev et al. (2016) in the context of macroeconomic announcements. We replicate this analysis for realized volatility.

Table 3 reports the estimation results for four different base currencies, EUR, GBP, USD and JPY and 6 FX rates each (including also CHF, AUD and CAD). For the trading volume, the coefficient β is almost never significant at 5% level supporting the hypothesis that despite a sizable currency movement, common news with little disagreement does not induce abnormal trading volume. On the other hand, (realized) volatility is positively affected by the arrival of large directional news in almost all cases. To sum up, as prescribed by the theory common news that is similarly interpreted by all market participants induces price variation but no abnormal trading volume. On the other hand, both volume and volatility tend to increase with disagreement.

	EUR		**GBP**		**USD**		**JPY**	
Volume	FE	PO	FE	PO	FE	PO	FE	PO
Baseline	0.0172	0.0170	-1.2007^b	0.7844	0.1082	0.1083	-0.2252^b	-0.2256^b
Controls	0.0412	0.0147	-0.0683	-0.1257	0.0609	0.0346	-0.0168	-0.0799
	EUR		**GBP**		**USD**		**JPY**	
Volatility	FE	PO	FE	PO	FE	PO	FE	PO
Baseline	0.6586^a	0.6579^a	0.0511	0.0510	0.3966^a	0.3960^a	0.4233^a	0.4229^a
Controls	0.2757^a	0.3127^a	0.0643^b	-0.0545	0.0973^a	0.1217^a	0.2818^a	0.2798^a

Table 3: Common news, volume and volatility. Panel regression estimates with fixed effect (FE) and pooling (PO) of the parameter β in (14). The dependent variable are logarithm of the hourly trading volume and RV for six FX rates with different base currency EUR, GBP, USD and JPY. The regressors are the dummy variable for the co-jump (CJ) on the base currency (baseline specification), and a number of controls: the average relative bid-ask spread (BA), and hourly and day-of-the-week dummies and an AR(1) term. The superscripts a, b and c indicate significance at 1%, 5% and 10% significance level respectively.

4.2 FX Illiquidity

Proposition 2 in Section 2 provides a closed-form expression for illiquidity in the spirit of Amihud (2002), i.e. the ratio between volatility and trading volume. The empirical prediction is that illiquidity decreases with market depth and the number of active traders. It is difficult to accurately measure these quantities. However, the visual inspection of Figure 7 representing the intraday development of our EURUSD Amihud measure suggests that illiquidity tends to decrease when international financial centers are open, that is, when the FX market is deep and populated by active traders. More precisely, it is discernible that FX illiquidity abruptly decreases at the opening of the European markets and it is minimal when both the European and the American markets are jointly open. After 8PM the illiquidity grows again and it is maximal during the night hours. A consistent pattern also holds for USDJPY (the right-hand side figure 7): market illiquidity reduces at the opening of the main financial markets Tokyo, London and New York and it sensibly increases again after 4PM. To shed further light on the measurement ability of our illiquidity

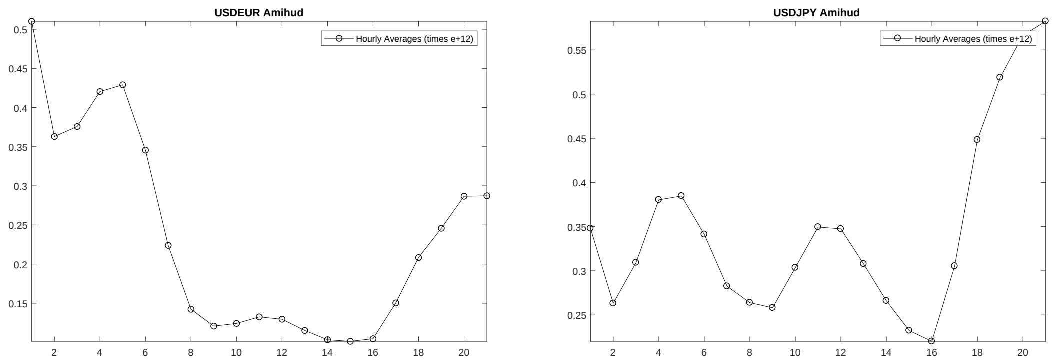


Figure 7: Hourly Averages of FX Amihud measures. In Panel a) USDEUR, in Panel b) USDJPY.

indicator, we perform various regressions similar to those shown in Table 1 and in Table 2. First, we regress changes in our daily illiquidity indicator on daily changes of bid-ask spreads. The results are exhibited in column (3) of in Table 1. Second, we regress monthly changes of our illiquidity indicator on a comprehensive measure of FX illiquidity proposed in Karnaukh et al., 2015 that proved to be highly correlated with precise high-frequency (intraday) data from Electronic Broking Services, which is the major interdealer trading platform for many currencies. The results are presented in column (3) of in Table 2. In both regressions, we include control variables.[18] Overall, we find that our illiquidity measure in the spirit of the Amihud indicator increases with other well-accepted measures of FX illiquidity.

So far, we have analyzed FX illiquidity on a global scale. Now, we ask the question whether our FX illiquidity measure is highly correlated with other illiquidity proxies in the FX interdealer segment. To do this, we obtain intraday data from Electronic Broking Services (EBS), the leading

[18] In addition to daily and monthly time intervals, we have performed the same regressions with weekly data and obtained consistent results.

platform for spot FX interdealer trading for various FX rates including the EURUSD. For the entire 2016, we access the depth of book at ten levels on both sides (bid and offer quotes) snapped every 100 milliseconds, the exact identification whether the deal is given or paid, transaction prices and amounts. We focus on EURUSD, which is primarily traded on this interdealer trading platform.[19] In the same spirit of Hasbrouck (2009), we analyze correlations between illiquidity measures. More specifically, we compute the following proxies: quoted spread (i.e. ask minus bid quotes), relative quoted spread (i.e. quoted spread divided by midquote), effective cost (i.e. the absolute value of the difference between transaction price and midquote), traditional Amihud measure (i.e. absolute return over trading volume), cost estimates implied by the Roll model (Roll, 1984), order flow price impacts (i.e. at five-minute intervals and trade-by-trade).

	A_t	BAS_t	Rel-BAS_t	EC_t	R_t	γ_1	γ_2	γ_3	Daily-A_t
Pearson correlation									
A_t	1.0000	0.5176	0.5628	0.8958	0.9000	0.6220	0.6945	0.7393	0.8527
BAS_t	0.5176	1.0000	0.9890	0.6092	0.5474	0.2520	0.2484	0.4524	0.4175
Rel-BAS_t	0.5628	0.9890	1.0000	0.6534	0.5933	0.2917	0.2965	0.4995	0.4685
EC_t	0.8958	0.6092	0.6534	1.0000	0.9329	0.5332	0.5426	0.6718	0.8132
R_t	0.9000	0.5474	0.5933	0.9329	1.0000	0.5741	0.6329	0.5883	0.8995
γ_1	0.6220	0.2520	0.2917	0.5332	0.5741	1.0000	0.7936	0.4199	0.6189
γ_2	0.6945	0.2484	0.2965	0.5426	0.6329	0.7936	1.0000	0.4363	0.6331
γ_3	0.7393	0.4524	0.4995	0.6718	0.5883	0.4199	0.4363	1.0000	0.5234
Daily-A_t	0.8527	0.4175	0.4685	0.8132	0.8995	0.6189	0.6331	0.5234	1.0000
Spearman rank correlation									
A_t	1.0000	0.8867	0.8617	0.8103	0.8266	0.3182	0.2208	0.2722	0.9403
BAS_t	0.8867	1.0000	0.9831	0.8938	0.8274	0.2225	0.1256	0.3557	0.8190
Rel-BAS_t	0.8617	0.9831	1.0000	0.8995	0.8343	0.2175	0.1066	0.3552	0.7988
EC_t	0.8103	0.8938	0.8995	1.0000	0.9253	0.2202	0.0690	0.4219	0.8174
R_t	0.8266	0.8274	0.8343	0.9253	1.0000	0.2727	0.1327	0.3633	0.8746
γ_1	0.3182	0.2225	0.2175	0.2202	0.2727	1.0000	0.7452	0.1628	0.3481
γ_2	0.2208	0.1256	0.1066	0.0690	0.1327	0.7452	1.0000	0.1032	0.2122
γ_3	0.2722	0.3557	0.3552	0.4219	0.3633	0.1628	0.1032	1.0000	0.2613
Daily-A_t	0.9403	0.8190	0.7988	0.8174	0.8746	0.3481	0.2122	0.2613	1.0000

Table 4: Correlation matrix for illiquidity measures on a daily basis. Sample: EBS data from 01-Jan-2016 to 17-Jul-2016. A_t: High-frequency Amihud measure, BAS_t: Bid-ask spread, Rel-BAS_t: Relative bid-ask spread, EC_t: effective cost, R_t: Roll measure, γ_1: 5min price impact coefficient,γ_2: order flow price impact coefficient,γ_3: trade-by-trade price impact coefficient, Daily-A_t; classic Amihud measure computed with the absolute value of daily log-return.

Table 4 delivers two main messages: First, it clearly shows that our FX illiquidity measure is

[19] The other main interdealer platform is Thomson Reuters. Some FX rates e.g. involving the British pound are mainly traded on it.

highly correlated with intraday illiquidity proxies based on EBS data, in particular the effective cost and order flow price impact. Second, it is also highly correlated with the traditional Amihud indicator suggesting that even approximating volatility with daily absolute returns (as in the traditional Amihud indicator) rather than gauging it with more accurate high-frequency measures realized power variation, as in our proxy), one can obtain a fairly accurate proxy of FX illiquidity. Spearman rank correlations confirm these results. Overall, we find that our illiquidity measure in the spirit of the Amihud indicator increases with high-frequency and well-accepted measures of FX illiquidity.

4.2.1 A natural experiment

Another method to assess the validity of our illiquidity measure is by means of a meaningful natural experiment. Through the lens of the theory developed in Section 2, the announcement of the cap removal of the Swiss franc by the Swiss National Bank (SNB) on January 15, 2015 represents an ideal natural experiment. Indeed, starting from September 6, 2011, the SNB set a minimum exchange rate of 1.20 francs to the euro (capping franc's appreciation) saying "the value of the franc is a threat to the economy", and that it was "prepared to buy foreign currency in unlimited quantities"'. This means that the SNB had a declared binding *cap* on the transaction price that was removed on January 15, 2015.[20]

In terms of our model, the SNB can be considered as the $(J+1)$-th trader. The SNB intervention strategy of selling CHF for EUR in potentially unlimited quantities is implemented if the average of the reservation prices of the J traders falls below the cap, that is if $\frac{1}{J}\sum_{j=1}^{J} p^*_{i,j} < \log(1.2)$. Indeed, despite the cap on the transaction price, the reservation prices of each individual trader might well be below the 1.20 threshold. For instance, a trader with a reservation price of 1.15, which observes a market price above 1.20, will sell EUR for CHF expecting the cap to be removed at some point in the future.[21] In other words, SNB buys (sells) foreign (domestic) currency to guarantee

[20]The SNB announcement was mostly unanticipated by market participants, see e.g. Jermann, 2017 and Mirkov et al., 2016

[21]The Thomson Reuters survey indicates dispersion of the beliefs of professional market participants around 1.20 along most of the capping period.

that the transaction price is above the threshold, that is

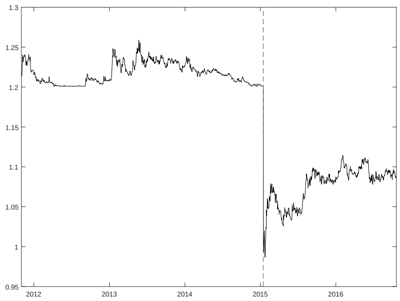
$$p_i = \frac{1}{J+1} \sum_{j=1}^{J+1} p_{i,j}^* \geq \log(1.2), \tag{15}$$

where $p_{i,J+1}^* = (\log(1.2) - \frac{1}{J}\sum_{j=1}^{J} p_{i,j}^*)\mathcal{I}(\sum_{j=1}^{J} p_{i,j}^* < 1.2)$, where $\mathcal{I}(\cdot)$ is the indicator function. The enforcement of the capping regime by SNB generates extra trading volume. In particular, the trading volume is

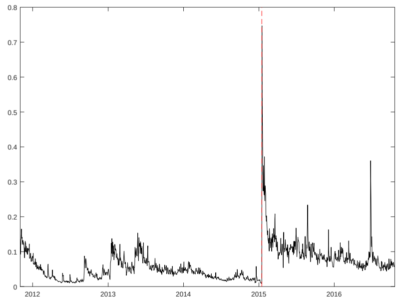
$$v_i = \frac{\xi^{x|y}}{2} \sum_{j=1}^{J} |\psi_{i,j} - \bar{\psi}_{i,j}| + v_i^{SNB}, \tag{16}$$

where v_i^{SNB} is the trading volume generated by the central bank to maintain the cap on the FX rate. Hence, the model prescribes a low volatility of the observed returns due to the implicit constraint given by the capping and a larger volume due to FX interventions. This implies that the Amihud illiquidity index is lower (higher) before (after) the removal of the FX capping regime.

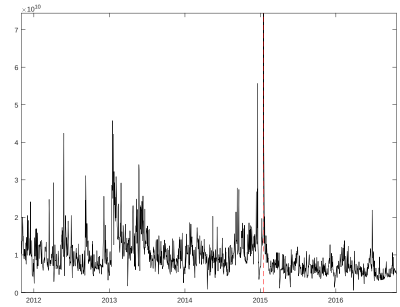
Figure 8 provides graphical support for the prescriptions of the theoretical model. Indeed, volatility (realized power variation) is relatively low until January 15, 2015, it spikes on the day of the announcement of the un-capping and it remains high until the end of 2016. The trading volume has the opposite behavior, being relatively high during the capping period and reverting to a lower value after January 15, 2015. Finally, our FX Amihud measure displays a clear upward shift after the removal of the Swiss franc cap. To provide a statistical support, Table 5 reports the sample average of the main market variables before and after the cap removal. After the announcement, FX volatility significantly increases, trading volume decreases, and liquidity dries up (even discarding the announcement day). Furthermore, the average trading volume size significantly decreases, suggesting a reduction in market depth. The lack of statistical significance in the change of the dispersion (standard deviations and high-low ranges) in Thomson Reuters survey of forecasts before and after the announcement suggests that market participants do not disagree more (less) after (before) the currency cap removal. This also suggests that the liquidity dry-up after the cap removal cannot be explained by a stronger consensus regarding agents' reservation prices. All in all, the analysis of this natural experiment corroborates the empirical predictions of the theory, that is, the central bank's enforcement of its reservation price leads to lower volatility, larger trading volume, and higher liquidity. By abandoning this regime, opposite



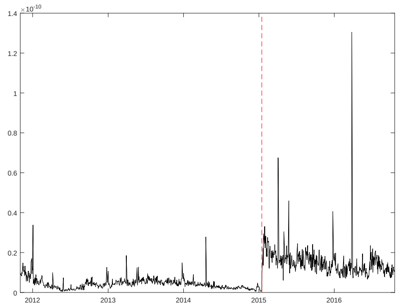
(a) Price



(b) RPV



(c) Volume



(d) Amihud

Figure 8: FX Rate (Figure a), Realized Power Variation (RPV, b), Trading Volume (c) and FX Amihud measure (d) of the EUR/CHF currency pair from 2012 to 2016 with the announcement of the cap removal of the Swiss franc by SNB on January 15, 2015 (red-dashed line).

patterns arise.

	Before	After	Test	p-value
RPV	4.548	9.7531	-12.71	0.000
RV	0.0825	0.3508	-5.842	0.000
VOL	1148.75	682.80	14.34	0.000
AMIHUD	0.4255	1.5002	-24.21	0.000
SIZE	264.84	207.65	27.55	0.000
DIS_1	0.0146	0.0166	-0.1704	0.865
DIS_2	0.0898	0.1015	-0.1252	0.908

Table 5: Sample averages of realized power variation (RPV), realized volatility (RV), trading volume (Volume), FX illiquidity measure (Amihud), and average trade size (Size) before (from Nov 1, 2011 to Jan 14, 2015) and after (from Jan 16, 2015 to Nov 30, 2016) announcement of un-capping (on Jan 15, 2015) - daily frequency. To proxy disagreement, we compute the average of the standard deviations (DIS_1) and high-low range (DIS_2) of monthly Thomson Reuters survey of forecasts on the EUR-CHF rate. The variables have been rescaled. Table also reports a test for the equality of the averages in the two sub-samples, $z = \frac{m_1 - m_2}{\sqrt{v1/n1 + v2/n2}}$, and associated p-values (one-tail) calculated accounting for the auto-correlation in the data.

4.3 Commonalities

Proposition 3 in Section 2 is about commonalities in FX trading volume, volatility and liquidity arising from the no-arbitrage condition. The purpose of this subsection is to empirically assess this idea. More precisely, we proceed in two steps: First, we analyze commonalities by means of three methods: (i) the factor analysis, (ii) the construction of a FX connectedness index, and (iii) the regression analysis. Second, we study the pricing implications stemming from arbitrage deviations and commonalities.

4.3.1 Factor Analysis

By means of the triangular no-arbitrage relation, Section 2.2 provides a theoretical underpinning that trading volume across FX rates are driven by common factors, which are function of the aggregated traders' specific components on different currency pairs. Notice that the FX-rate triangular condition can be extended to more than three FX rates. Actually, it is generalizable to any numbers of FX rates tied by triangular relationships. For instance, with four currencies, x, w, z, and y, the log-price is $p_i^{x|y} = p_i^{x|z} + p_i^{z|w} + p_i^{w|y}$, and the synthetic volume becomes $\widetilde{v}_i^{x|y} = \frac{\xi^{x|y}}{2} \sum_{j=1}^{J} |\psi_{i,j}^{x|z} - \bar{\psi}_i^{x|z} + \psi_{i,j}^{z|w} - \bar{\psi}_i^{z|w} + \psi_{i,j}^{w|y} - \bar{\psi}_i^{w|y}|$. This provides support for the existence of a factor structure in cross sections of FX rates of any order.

To the purpose of studying the commonality in volume, volatility and liquidity across multiple FX rates, we follow the common approach in the literature (e.g. Hasbrouck and Seppi, 2001) and apply the principal component analysis (PCA) to the panel of 29 FX rates introduced in Section 3.1. The goal is to identify a common factor structure across the volume, volatility, and illiquidity series of the FX rates and to study the exposure of each rate to it. Table 6 shows of these quantities for each individual FX rate load positively on the first principal component in all cases. Notably, the first component explains a large portion of the overall variation of volume, volatility and illiquidity measures of the panel of FX rates, being above 50% in many cases. Moreover, the weight associated with the volume and illiquidity measure of USDEUR is the highest signaling the leading role of the information on the USDEUR rate in determining the global FX volume. Instead, the loading on RPV for EURDKK is the smallest across all currencies, signaling that the volatility on EURDKK is strongly influenced by the pegging of DKK to EUR. These findings

	Hourly			Hourly Seasonally Adjusted			Daily		
	Volume	RPV	Amihud	Volume	RPV	Amihud	Volume	RPV	Amihud
AUDJPY	0.1555	0.1884	0.1526	0.2031	0.2143	0.1963	0.1830	0.1986	0.2230
AUDNZD	0.1288	0.1461	0.1418	0.1539	0.1774	0.1559	0.1781	0.1895	0.2195
CADJPY	0.1327	0.1966	0.1045	0.1528	0.2019	0.1133	0.1387	0.1884	0.1153
EURAUD	0.1854	0.1992	0.1852	0.1934	0.2144	0.1823	0.1968	0.2083	0.2277
EURCAD	0.1829	0.2138	0.1709	0.1711	0.2180	0.1627	0.1873	0.2111	0.1835
EURCHF	0.2173	0.1520	0.2065	0.1997	0.1542	0.1852	0.1677	0.1510	0.1871
EURDKK	0.1841	0.0863	0.1819	0.0910	0.0495	0.0879	0.1500	0.0623	0.0111
EURGBP	0.2285	0.2128	0.2419	0.2284	0.2139	0.2355	0.2155	0.2069	0.2508
EURJPY	0.1971	0.1954	0.2110	0.2112	0.1959	0.2367	0.1617	0.1677	0.2401
EURNOK	0.2142	0.1472	0.2217	0.1805	0.1203	0.1729	0.2118	0.1472	0.0612
EURSEK	0.2131	0.1393	0.2179	0.1764	0.1001	0.1571	0.2041	0.1314	0.0535
GBPAUD	0.1599	0.2034	0.1592	0.1672	0.2170	0.1821	0.1865	0.2163	0.2165
GBPCAD	0.1279	0.2045	0.1098	0.1284	0.2047	0.1398	0.1603	0.2067	0.1729
GBPCHF	0.1692	0.2148	0.1595	0.1488	0.2163	0.1648	0.1664	0.2094	0.1882
GBPJPY	0.1823	0.1984	0.1690	0.1754	0.1980	0.1724	0.1463	0.1774	0.1789
USDAUD	0.1839	0.1965	0.1880	0.2256	0.2129	0.2361	0.1951	0.2115	0.2417
USDCAD	0.2070	0.2066	0.2099	0.2144	0.2049	0.2158	0.2106	0.2079	0.2538
USDCHF	0.2236	0.2140	0.2227	0.2301	0.2131	0.2321	0.2184	0.2014	0.2516
USDDKK	0.1573	0.2127	0.1499	0.0979	0.2129	0.1089	0.1394	0.1979	0.1035
USDEUR	0.2320	0.2142	0.2482	0.2461	0.2155	0.2633	0.2011	0.1966	0.2690
USDGBP	0.2291	0.2131	0.2363	0.2384	0.2095	0.2575	0.2235	0.2050	0.2733
USDHKD	0.1555	0.0673	0.1650	0.1266	0.0917	0.1502	0.1515	0.1244	0.0891
USDJPY	0.1748	0.1676	0.1739	0.2263	0.1701	0.2116	0.1912	0.1487	0.2067
USDMXP	0.1459	0.1656	0.1442	0.1881	0.1538	0.1850	0.2211	0.1841	0.1138
USDNOK	0.1909	0.2026	0.1892	0.1518	0.1852	0.1484	0.1611	0.2014	0.0920
USDNZD	0.1669	0.1914	0.1606	0.1908	0.1966	0.1922	0.2003	0.2041	0.2365
USDSEK	0.1939	0.1974	0.1887	0.1585	0.1755	0.1457	0.1737	0.1861	0.0842
USDSGD	0.1528	0.1633	0.1403	0.1887	0.1677	0.1580	0.1671	0.1846	0.0927
USDZAR	0.2179	0.1681	0.2246	0.1989	0.1313	0.1915	0.2233	0.1705	0.0953
EXPL	0.5514	0.6135	0.4519	0.3440	0.5236	0.3066	0.4756	0.6494	0.3771

Table 6: PCA Analysis. The table reports the loadings for each currency pair for trading volume (Volume), volatility (realized power variation, RPV) and illiquidity (Amihud) to the first principal component. The bottom line reports the percentage of explained variance of the first principal component.

remain qualitatively the same for daily and hourly (seasonally un- or adjusted) time series.

Another way to analyze commonalities is by studying the dynamic interplay between the FX rates across currencies by means of the total connectedness index of Diebold and Yilmaz (2014).[22] The TCI is defined as

$$TCI = \frac{1}{N} \sum_{i,j=1 i \neq j}^{N} \tilde{d}_{i,j}, \tag{17}$$

[22]See also Greenwood-Nimmo et al. (2016) for an application of the connecteness measure in the context of returns and option-implied moments of FX rates.

where N denotes the number of variables in the system, and $\tilde{d}_{i,j}$ is the i, j entry of the standardized connectedness matrix $\tilde{D}$. The matrix $\tilde{D}$ is defined as

$$\tilde{d}_{i,j} = \frac{d_{i,j}}{\sum_{j=1}^{N} d_{i,j}}, \tag{18}$$

with

$$d_{i,j} = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{H} (e_i A_h \Sigma e_j)^2}{\sum_{h=0}^{H} (e_i' A_h \Sigma A_h' e_i)}, \tag{19}$$

where A_h is the impulse-response matrix at horizon h associated with a VAR(p) model, Σ is the covariance matrix of the errors, and e_i, e_j are $N \times 1$ selection vectors. By construction, $\sum_{j=1}^{N} \tilde{d}_{i,j} = 1$ and $\sum_{i,j=1}^{N} \tilde{d}_{i,j} = N$. Equation (19) defines the generalized forecast error decomposition, as introduced by Pesaran and Shin (1998). In other words, the TCI measures the average portion over N variables of the forecast error variation of variable i coming from shocks arising from the other $j = 1, \ldots, N-1$ variables of the system. Although less standard in the literature on liquidity commonalities, the TCI approach provides an informative characterization of the connectedness of a system that is richer than the one obtained with a simple linear correlation coefficient. Indeed, the TCI combines information coming from both the contemporaneous and the dynamic dependence structure of the system trough Σ and A_h, respectively. Moreover, by estimating the VAR model over rolling windows, it is possible to characterize the evolution of the dependence structure between two or more variables by looking at the variations of the TCI over time.

As showed in Table 7, the connectedness analysis delivers two main findings: First, the overall level of connectedness of volume and volatility is very high and constant over time, being close to 90% for both volatility and volume at hourly and daily level. The connectedness remains very high also when volume and RPV are filtered from intradaily seasonality, being around 70%-80%. This picture corroborates the previous findings obtained from the Factor Analysis, that is, there is a strong commonality across FX volumes and volatilities. Second, the comparison between the most and least liquid FX rates indicates that a stronger connectedness of volume and volatility for the former set of currencies. Indeed, the connectedness on the most liquid FX rates is above 85% and it remains relatively high for hourly seasonally adjusted series. On the other hand, the connectedness level sensibly reduces when focusing on the least liquid FX rates. This result is

	Hourly				Hourly Seasonally Adjusted				Daily			
	Full	11/14	12/15	13/16	Full	11/14	12/15	13/16	Full	11/14	12/15	13/16
All FX rates												
Volume	0.884	0.880	0.885	0.890	0.726	0.719	0.730	0.731	0.891	0.889	0.891	0.898
RPV	0.910	0.907	0.910	0.916	0.846	0.844	0.850	0.856	0.921	0.920	0.928	0.930
10 Most Liquid												
Volume	0.875	0.873	0.883	0.880	0.709	0.702	0.714	0.722	0.862	0.864	0.863	0.870
RPV	0.893	0.890	0.893	0.904	0.814	0.815	0.818	0.838	0.919	0.920	0.922	0.935
10 Least Liquid												
Volume	0.621	0.607	0.634	0.649	0.275	0.270	0.284	0.289	0.623	0.608	0.617	0.659
RPV	0.808	0.810	0.821	0.819	0.718	0.710	0.728	0.732	0.846	0.829	0.861	0.873

Table 7: Connectedness. The table reports the value of the connectedness index of Diebold and Yilmaz (2014) of trading volume (Volume) and volatility (in terms of realized power variation, RPV) for different sampling periods (Full sample, 2011/2014, 2012/2015, and 2013/2016) and for different sets of FX rates. "10 Most Liquid" and "10 Least Liquid" refer to the ten most and least liquid FX rates in terms of total trading volume.

fully consistent the adage that "liquidity begets liquidity" (e.g. Foucault et al., 2013), in the sense that higher liquidity goes with stronger commonality. It is also consistent with the Proposition 3 in Section 2 in which FX rates are connected by arbitrage trading volume and this connection is stronger for liquid currencies (captured by the term $\xi^{x|y}$ in (10)). This result squares well with the idea that illiquid currency pairs are less (more) exposed to the common (specific) FX-factors as it emerges from the magnitude of the loadings of the first principal component in Table 6. In sum, liquid currencies appear to have stronger cross-currency commonalities than illiquid ones.

4.3.2 Measuring Commonalities

Common measures of liquidity commonalities are statistical measures such as R^2 or estimated slope coefficient when regressing liquidity of an asset on market liquidity (e.g. Chordia et al., 2000). Following the same reasoning but to be consistent with the arbitrage framework theorized in Section 2.2, we measure the strength of the pairwise commonality in volume between $x|y$, $x|z$ and $z|y$ through the following reduced-form model,

$$\log(v_t^{x|y}) = \beta_0 + \beta_1 \log\left(v_t^{x|z} + v_t^{z|y}\right) + \varepsilon_t, \quad t = 1, \ldots, T \tag{20}$$

where $v_t^{x|y}$, $v_t^{x|z}$ and $v_t^{z|y}$ are the log-volume on period t on the FX rates $x|y$, $x|z$ and $z|y$, respectively. In this regression, β_0 reflects the differential in the resiliency levels in the three markets, while β_1 measures the magnitude of commonality in the volume of the three FX rates. The MDH theory outlined in Section 3 prescribes that $\beta_1 > 0$. The term ε_t can also be interpreted as the deviation from the long-run equilibrium between FX volume. Table 8 reports the estimates of regression (20) for the EUR/USD rate, where the aggregate volume combining $v_t^{x|z}$ and $v_t^{z|y}$ (*synthetic* volume) supports the triangular arbitrage with CHF, GBP, DKK, JPY, AUD, CAD, NOK, SEK.[23]

	Hourly			Daily			Daily (interacted)			
	β_0^H	β_1^H	R_H^2	β_0^D	β_1^D	R_D^2	β_0^i	β_1^i	β_2^i	R_i^2
Volume										
CHF	6.2791^a	0.7861^a	0.8068	9.0985^a	0.6973^a	0.5305	9.2331^a	0.6908^a	0.0246	0.5329
GBP	3.8371^a	0.8625^a	0.8195	7.8218^a	0.7209^a	0.4134	8.2372^a	0.6966^a	0.2057^a	0.4341
DKK	15.592^a	0.2962^a	0.5526	16.073^a	0.4452^a	0.3751	16.248^a	0.4293^a	0.2755^a	0.4054
JPY	2.8344^a	0.8799^a	0.4632	15.7280^a	0.3963^a	0.2099	17.185^a	0.3277^a	0.3040^a	0.2467
AUD	0.7486^a	1.0096^a	0.5051	7.1182^a	0.7599^a	0.5789	7.6327^a	0.7304^a	0.1926^a	0.6189
CAD	5.6124^a	0.7936^a	0.7248	10.634^a	0.6176^a	0.3875	11.372^a	0.5724^a	0.3874^a	0.4756
NOK	13.577^a	0.4635^a	0.6739	14.887^a	0.4782^a	0.2482	14.891^a	0.4781^a	-0.0014	0.2482
SEK	13.388^a	0.4700^a	0.6778	14.220^a	0.5051^a	0.2580	14.226^a	0.5050^a	-0.0029	0.2581
RPV										
CHF	-0.9108^a	0.9189^a	0.7752	-0.9071^a	0.7412^a	0.7091	-0.9387^a	0.7482^a	-0.8456^a	0.7130
GBP	-0.9777^a	0.9303^a	0.7552	-0.7973^a	0.8884^a	0.6257	-1.0890^a	0.9725^a	-8.0796^a	0.6892
DKK	0.2941^a	1.094^a	0.9320	0.0646^a	1.1223^a	0.9620	0.1065^a	1.1145^a	1.0515^b	0.9646
JPY	-1.2987^a	0.9156^a	0.5717	-1.2604^a	0.6790^a	0.3995	-1.6528^a	0.8527^a	-14.235^a	0.5317
AUD	-0.8686^a	1.0431^a	0.5542	-1.0562^a	0.9334^a	0.6033	-1.3224^a	1.0314^a	-8.0224^a	0.7045
CAD	-0.7850^a	0.9968^a	0.7238	-0.7546^a	0.9889^a	0.6680	-1.1531^a	1.0427^a	-9.3480^a	0.7634
NOK	-1.7326^a	0.8126^a	0.5366	-1.0991^a	0.8241^a	0.4756	-0.9015^a	0.7945^a	3.7473^a	0.4988
SEK	-1.3851^a	0.8718^a	0.5678	-0.7150^a	1.0555^a	0.5693	-0.5284^a	1.0459^a	3.2424^c	0.5845
Amihud										
CHF	-14.066^a	0.5552^a	0.6956	-14.4250^a	0.5417^a	0.7234	-14.0150^a	0.5570^a	0.0345^c	0.7277
GBP	-12.749^a	0.5985^a	0.7241	-8.7892^a	0.7538^a	0.7489	-9.9900^a	0.7117^a	-0.1495^a	0.7584
DKK	-24.583^a	0.1506^a	0.2566	-23.235^a	0.2086^a	0.2375	-23.244^a	0.2083^a	-0.0069	0.2375
JPY	-8.8037^a	0.7518^a	0.5280	-11.483^a	0.6518^a	0.5832	-12.621^a	0.6167^a	-0.2805^a	0.6191
AUD	-14.852^a	0.5419^a	0.3830	-13.826^a	0.5931^a	0.5401	-15.229^a	0.5418^a	-0.1812^a	0.5721
CAD	-18.857^a	0.3836^a	0.3733	-13.235^a	0.6319^a	0.5111	-14.574^a	0.5829^a	-0.2284^a	0.5382
NOK	-22.516^a	0.2321^a	0.4465	-17.387^a	0.4554^a	0.3539	-18.241^a	0.4210^a	-0.0485^b	0.3602
SEK	-22.151^a	0.2452^a	0.4510	-19.111^a	0.3772^a	0.2138	-20.320^a	0.3295^a	-0.0885^b	0.2304

Table 8: Commonalities in volume, volatility (realized power variation, RPV) and illiquidity (Amihud index, Amihud). For each currency, the table reports the intercept, slope and R^2 of the regression of the log volume/volatility/Amihud of EURUSD on the log of the sum of volume/volatility/Amihud index on the FX rate of the currency indicated in the first column against USD and EUR. The superscripts a, b and c indicate significance at 1%, 5% and 10% significance level, respectively.

[23] Besides these 8 FX rates providing triangular constructions with the EUR/USD rate, in our sample the following synthetic FX rates exist: (a) for the USDGBP, via AUD, CAD, CHF, EUR, and JPY; (b) for USDAUD, via EUR, GBP, JPY, and NZD; and (c) for EURCHF, via GBP and USD. We have analyzed all of them obtaining consistent results.

Overall, it emerges that regression (20) is able to explain a large portion of variability of $v^{EUR/USD}$, and this can be attributed to the portions of common information in $\psi_j^{USD/\cdot}$ and $\psi_j^{EUR/\cdot}$, which determine the synthetic volume in (35). At the hourly level, the estimated parameter β_0 reflects the average liquidity differential across currencies, with DKK, SEK and NOK being consistently less liquid than JPY, AUD and GBP. Notably, the parameter β_1 is positive in all cases and it is closer to 1 for the most liquid rates corroborating the idea that liquidity begets commonality. As expected, higher β_1 are associated with higher R^2. When removing the intradaily seasonality in volume or aggregating at the daily level, the R^2 slightly decreases but the result is qualitatively the same as for the raw hourly volume. The residuals display significant autocorrelation, suggesting that volume imbalances across FX markets are stationary but persistent. These long-lasting disequilibria in volume might be explained by the fragmented OTC structure of the FX market and prolonged time to incorporate agents' heterogeneous priors and (public and private) information into prices, as for conditional volatility (Engle et al., 1990).

When replacing volume with volatility (RPV) in (20), we note that also volatility displays a large degree of commonality across currencies. The R^2 is generally very well above 50% at both hourly and daily level. Interestingly, the R^2 and the slope coefficient of DKK are almost 1 consistent with the Danish Central Bank policy to keep EUR/DKK within a very narrow corridor (0.133-0.1346), thus the $Cov(p^{USD/DKK}, p^{USD/EUR}) \approx 1$. Consistent with the theory, the Danish central bank's intervention to fix the EUR/DKK rate reduces the commonality in volume and liquidity with the other currencies. Not surprisingly, the Amihud illiquidity measure, which combines information on both volatility and volume, also displays an analogous amount of commonality across currencies, being the highest for the most liquid ones.

The theory outlined in Section 2.2 suggests that the commonalities in trading volume across FX rates are driven by the level of correlation among the FX rates, where the synthetic volume is a function of the correlation of the aggregated traders' specific components on different currency pairs, see the the right-hand side of (35) in Appendix A. In other words, our theory predicts that the synthetic volume reveals the strength of the correlation across FX rates. To test this empirical prediction, we consider the following regression

$$\log(\tilde{v}_t^{x|y}) = \gamma_0 + \gamma_1 \log(\zeta_t) + \gamma_2 \tilde{v}_{t-1}^{x|y} + \varepsilon_t, \quad t = 1, \ldots, T, \tag{21}$$

where (log) $\tilde{v}_t^{x|y}$ is the synthetic volume as measured by the fitted volume in regression (20), while $\zeta_t = \log(1 + |\rho_t|)$ and ρ_t is the realized correlation between $x|z$ and $z|y$. Hence, the term ζ_t measures the strength of the correlation in the FX rates $x|z$ and $z|y$, and the parameter γ_1 is expected to be positive. Table 9 contains the estimates of γ_1 based on regression (21) and on the extended version which controls for liquidity as measured by the bid-ask spreads on $x|z$ and $z|y$. At hourly frequency, the estimates of γ_1 are positive and highly significant in most cases, with the

	Hourly		Daily		Weekly	
	γ_0	γ_1	γ_0	γ_1	γ_0	γ_1
Baseline Regression						
CHF	4.2444	0.2141^a	13.9536	0.1697^a	8.4427	0.0993^c
GBP	3.7079	0.2310^a	18.2764	0.0897^c	12.1883	0.0485
DKK	6.8630	-0.1447^a	18.5609	-0.1159	11.2898	0.0615
JPY	6.1447	0.2810^a	10.9029	0.1511^a	7.0136	0.0331
AUD	6.8131	0.0642^a	12.5726	0.0609	8.4234	-0.0696
CAD	4.3665	-0.0764^a	22.5847	-0.0203	18.2935	0.0079
NOK	6.0960	0.7085^a	17.2974	0.4519^a	17.6356	0.1196^c
SEK	5.6549	0.5923^a	18.6241	0.1409^a	15.6949	-0.0160
Control for Liquidity						
CHF	5.3883	0.3172^a	16.6392	0.4082^a	12.1837	0.2538^b
GBP	4.8816	0.2439^a	18.7933	0.1463^a	12.9884	0.0658
DKK	7.0838	-0.1499^a	19.4777	-0.1202^a	13.8504	0.0317
JPY	7.1983	0.1077^a	14.6026	-0.0455	10.9355	-0.0353
AUD	7.6219	0.1614^a	17.4960	0.2028^a	16.9214	0.0394
CAD	4.5109	-0.0853^a	22.7906	0.0288	19.6981	0.0878
NOK	8.0024	0.7449^a	17.3807	0.4529^a	17.6188	0.1140^c
SEK	8.2093	0.6576^a	18.9913	0.1571^a	16.0763	-0.0129

Table 9: Synthetic volume and correlation. For each currency, the table reports the intercept and the slope of the regression of the log synthetic volume of EURUSD on the log of the correlation of the FX rates with USD and EUR.

notable exception of DKK. Again, the results suggest that the intervention of the central bank to peg DKK to EUR prevents the trading activity on EUR/DKK and DKK/USD from fully revealing the correlation structure of the investors' beliefs on EUR and USD. When aggregating over days and weeks, we still obtain generally positive estimates of γ_1 but they are often not significantly different from zero.

4.3.3 Commonality and Pricing Implications

One of our previous results is that liquidity begets liquidity across currencies. As the last step of our study, we address the question whether liquidity begets price efficiency as well. The rationale of this relationship is again our third theoretical proposition implying that arbitrage keeping FX rates tied to equilibrium relations passes through the trading activity (volume), which in turn it is sustained by liquidity. To do this, we build a simple measure of pricing errors to investigate whether high liquidity is associated with smaller *mispricing* errors. Specifically, $pe_{i,t}$ is the hourly cumulative no-arbitrage error at time time t for the i-th synthetic relation defined as

$$pe_{i,t} = \sum_{l=1}^{60} |r_{l,t}^{x|y} - \tilde{r}_{l,t}^{z_{n_i}}|,$$

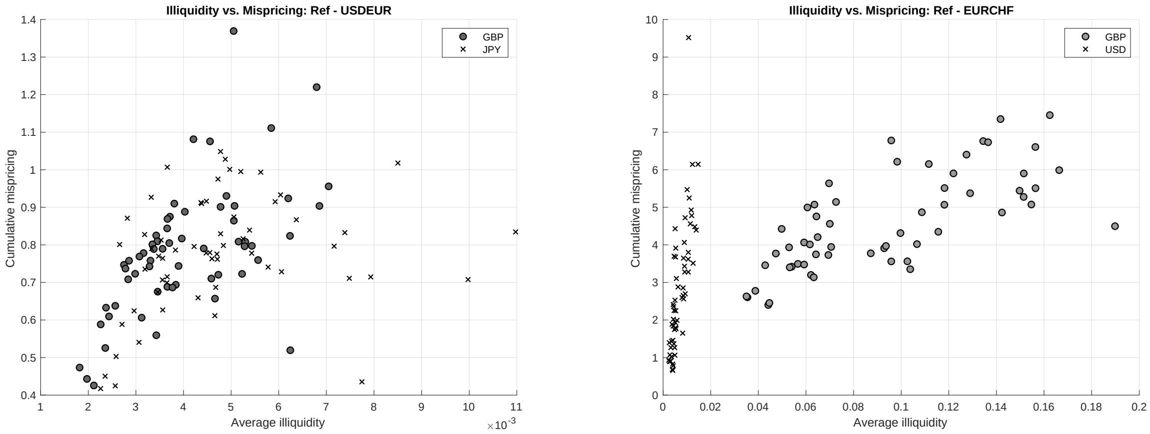
where $r_{l,t}^{x|y}$ is the *direct* one-minute midquote log-return on the FX rate between the currency x and y, while $\tilde{r}_{l,t}^{z}$ is the *synthetic* one-minute log-return on the FX rate $x|y$ using the currency z_{n_i}. Empirically, we test the price-liquidity relation in two ways: First, by looking at the systematic relationship between arbitrage deviations and illiquidity; Second, by inspecting whether more liquidity facilitates the price adjustment process.

To analyze the systematic price-liquidity relationships, we apply two methods: First, we extend the previous commonality analysis in (20) by interacting synthetic volume and pricing error as follows:

$$\log(v_t^{x|y}) = \beta_0 + \beta_1 \log\left(v_t^{x|z} + v_t^{z|y}\right) + \beta_2 \log\left(v_t^{x|z} + v_t^{z|y}\right) pe_{i,t} + \varepsilon_t, \quad t = 1, \ldots, T \tag{22}$$

The results are showed on the right-hand side of Table 8. As predicted by our theory, we find a positive β_2 indicating that arbitrage deviations attract more trading volume to reestablish price equilibrium. We extend the analysis to volatility and illiquidity, for which we do not have clear empirical predictions. In both cases, we find a negative β_2 suggesting that the departure from arbitrage conditions goes with divergent liquidity and volatility patterns across currencies, consistent with the idea that illiquidity hinders the restoration of equilibrium prices.

The second method to study the systematic price-liquidity relationship is to compute the monthly average mispricing errors and *synthetic* illiquidity for each FX rate allowing for a triangular FX construction. For instance, for EUR/USD, EUR/GBP, and GBP/USD we calculate the average deviations between direct (EUR/USD) and synthetic rate (via EUR/GBP, and GBP/USD) and the average Amihud measures of the two FX rates to operate triangular arbitrage (EUR/GBP and GBP/USD).



(a) Cumulative mispricing against illiquidity: USDEUR (b) Cumulative mispricing against illiquidity: EURCHF

Figure 9: Monthly cumulative mispricing ($pe_t^{x|y}$) against synthetic illiquidity, $\widetilde{A}_t^{x|y}$.

Figure 9 clearly shows a positive relationship between mispricing and illiquidity. Also, more liquid currencies have steeper curves suggesting that the same amount of additional liquidity is more effective in reducing arbitrage deviations in liquid currencies. We also carry out a statistical analysis to validate these findings and consider the following regression

$$pe_t^{x|y} = \alpha + \delta \widetilde{A}_t^{x|y} + \gamma \widetilde{BAS}_t + \varepsilon_t, \tag{23}$$

where $\tilde{Am}_t^{x|y}$ denotes the *synthetic* illiquidity on the FX rate $x|y$ computed with the same currency used to calculate $pe_t^{x|y}$. We expect the parameter δ to be positive and significant, signaling a positive relation between illiquidity and pricing errors. Analogously, the *synthetic* bid-ask spread, denoted as $\widetilde{BAS}$, is also computed in a similar way and it is added to the regression to control for deviations from the pricing equilibrium due another dimension of illiquidity that is the bid-ask spread. The results of regression (23) are reported Table 10, and the parameter estimates validate the findings observed in the scatter plots in Figure 9. In particular, by regressing monthly mispricing on (synthetic) FX illiquidity, we find compelling evidence that liquidity begets price efficiency, i.e. limiting arbitrage deviations. This holds true also when controlling for bid-ask spread differentials, although the significance is reduced for EURUSD when combining with the least liquid currencies (e.g. NOK and SEK).

	EURUSD								EURCHF	
	CHF	**GBP**	**DKK**	**JPY**	**AUD**	**CAD**	**NOK**	**SEK**	**USD**	**GBP**
α	0.36^a	0.50^a	0.49^a	0.62^a	0.51^a	0.26^a	0.51^a	0.72^a	0.25	2.51^a
δ	54.77^a	65.12^a	1.01^a	34.06^b	21.54^a	17.37^a	16.15^a	9.07^a	357.9^a	-22.13^a
R^2	0.54	0.32	0.14	0.11	0.34	0.40	0.36	0.13	0.65	0.49
α	-0.30^a	0.04	0.59^a	0.14^b	0.13	-0.27^b	-0.10	-0.18^b	-2.48^a	-0.78^c
δ	23.21^a	11.15	1.27^a	30.77^a	10.18^b	2.95	2.92	-1.52	227.7^a	16.21^a
γ	30.05^a	31.25^a	-0.98	0.20^a	14.55^a	26.27^a	2.97^a	3.51^a	124.0^a	93.64^a
R^2	0.88	0.64	0.18	0.47	0.46	0.69	0.76	0.71	0.82	0.69

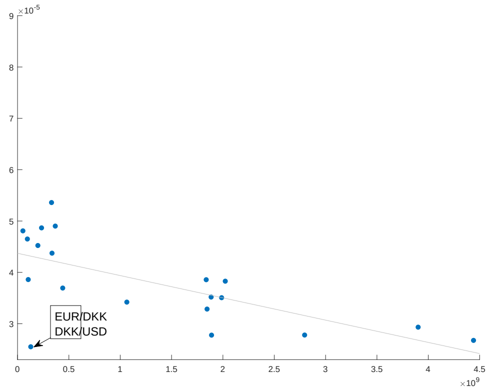
Table 10: Mispricing vs. Liquidity regression estimation. Table reports the estimates of the linear regression (23) for the FX rates EURUSD and EURCHF, when the triangular no-arbitrage condition is computed with of a third currency, that is CHF, GBP DKK, JPY, AUD, NOK and SEK for EURUSD; USD and GBP for EURCHF. The sample size is $N = 58$ months. The top (bottom) panel reports the estimates when BAS is excluded (included) among regressors in (23). The superscripts a, b and c indicate significance at 1%, 5% and 10% significance level respectively.

To study whether liquidity facilitates price adjustments, we benefit again from the identification of large price co-movements captured by the co-jumps. More specifically, we test whether the chances of mispricing are higher for less liquid currencies in reaction to directional FX movements measured by co-jumps. To carry out this test, we consider the following panel regression with fixed effects

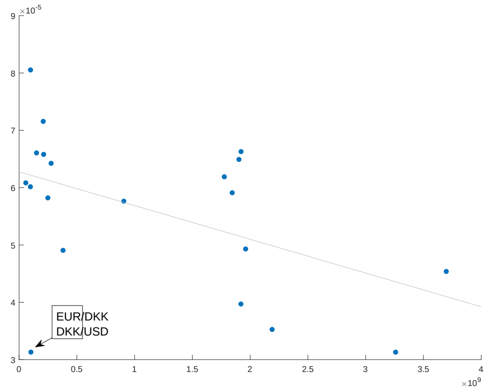
$$pe_{i,t} = \alpha_i + \beta \bar{V}_{i,t}(1 + \zeta \; CJ_t) + \theta CJ_t + \delta BA_{i,t} + \gamma_h h_t + \gamma_w w_t + \varepsilon_{i,t}, \tag{24}$$

The term $\bar{V}_{i,t}$ is the aggregate or synthetic volume from the FX rates $x|z_{n_i}$ and $z_{n_i}|y$. Our sample consists of $n = 10$ currencies and allows us to consider $I = 20$ combinations of x, y and z_{n_i}.[24] The relation between the average volume $\bar{V}_{i,t}$ and the average no-arbitrage error $pe_{i,t}$ is depicted in Figure 10. The figure clearly displays a cross-sectional negative relation between the trading volume on the FX rates and the no-arbitrage pricing error. In other words, the pricing errors are higher for less liquid currencies, such as SEK and NOK. A notable exception is given by

[24] The combinations are: USDAUD/EURAUD, USDSEK/EURSEK, USDNOK/EURNOK, USDCHF/EURCHF, USDCAD/EURCAD, USDJPY/EURJPY, USDGBP/EURGBP, USDDKK/EURDKK, USDAUD/GBPAUD, USDCAD/GBPCAD, USDJPY/GBPJPY, USDCAD/JPYCAD, USDAUD/JPYAUD, EURCAD/GBPCAD, EURJPY/GBPJPY, EURCHF/GBPCHF, EURCAD/JPYCAD, USDAUD/JPYAUD, GBPAUD/JPYAUD, GBPCAD/JPYCAD.



(a) Unconditional



(b) Co-Jump

Figure 10: Trading Volume and pricing errors. The figures show the scatter of the average volume (x-axis) versus the average triangular pricing error (y-axis) for 20 combination of currencies x, y and z. The left panel reports the unconditional relation, while the right panel is conditional to the event of a co-jump on the individual currencies, EUR, JPY, USD and GBP. The line represents the least squares fit.

DKK, which again it can be explained by the fixed exchange rate policy. When conditioning on the arrival of a large common news on the main individual currencies EUR, JPY, USD and GBP (right panel), the average mispricing error on the y-axis increases relatively to the left panel, suggesting that big news arrivals prompt price adjustment processes on individual currencies that can generate larger price dispersion and mispricing errors. However, the negative relation between magnitude of the mispricing and trading volume is maintained.

Table 11 reports the parameter estimates of (24) based on the sample of $I = 20$ combination of FX rates and for a sample of $T = 30720$ hours (24×1280 days). The results confirm our empirical prediction, that is, a negative relation between mispricing errors and volume, which is robust to the inclusion of the relative bid-ask spread as a control for transaction costs (where parameter δ is found significantly positive in all cases). As it also emerges from Figure 10, the co-jumps events are associated with a significant increase in the average level of mispricing ($\theta > 0$), and also with a significantly negative slope of volume ($\zeta < 0$). In sum, our results support the idea that liquidity begets price efficiency by reducing pricing errors, systematically and facilitating the information processing.

	FE	PO	FE	PO	FE	PO	FE	PO	FE	PO
Volume	-0.018^{a}	-0.028^{a}	-0.024^{a}	-0.028^{a}	-0.024^{a}	-0.028^{a}	0.004^{a}	-0.007^{a}	0.004^{a}	-0.007^{a}
Bid-Ask	–	–	0.142^{a}	0.210^{a}	0.141^{a}	0.209^{a}	-0.002	0.073^{b}	-0.002	0.073^{b}
CJ	–	–	–	–	0.001^{a}	0.001^{a}	0.001^{a}	0.001^{a}	0.001^{a}	0.001^{a}
CJ-Volume	–	–	–	–	–	–	–	–	-0.041^{a}	-0.046^{a}
Daily	no	no	no	no	no	no	yes	yes	yes	yes
Weekly	no	no	no	no	no	no	yes	yes	yes	yes
AR(1)	no	no	no	no	no	no	yes	yes	yes	yes

Table 11: No-arbitrage pricing errors and volume. Panel regression with fixed effect (FE) and pooling (PO). The dependent variable is the triangular pricing error accumulated at the hourly horizon for 20 combinations of FX rates. The regressors are the hourly aggregate (synthetic) trading volume of the two indirect FX rates (Volume) of the triangular arbitrage, the average relative bid-ask spread (Bid-Ask) of the direct FX rate, the dummy variable of the co-jump index on its own (CJ) and interacted with (synthetic) trading volume (CJ-Volume) as well as hourly and weekly dummies. The superscripts a, b and c indicate significance at 1%, 5% and 10% significance level respectively. The standard errors are computed with the White (1980) sandwich estimator for panel data models.

5 Conclusion

We provide a unified model for asset prices, trading volume, and volatility. The model is built in continuous-time and allows for multi-asset framework. We apply it to currency markets in which foreign exchange (FX) rates are tied by arbitrage conditions. Our model outlines new properties of the FX market including the relationships between trading volume and volatility of direct and arbitrage-related (or synthetic) FX rates. It also provides a theoretical foundation for common patterns (commonality) of trading volume, volatility, and illiquidity across currencies and time, and an intuitive closed-form solution for measuring illiquidity in the spirit of Amihud (2002).

We test the empirical predictions from our model using new and unique (intraday) data representative of the global FX spot market. A distinguishing characteristic of our data set is that it includes granular and intraday data on global FX trading volume. As predicted by our model, three main empirical findings arise: First, the difference in market participants' beliefs (disagreement) is the common source of trading volume and volatility. Second, our FX Amihud measure is effective in gauging FX illiquidity. Third, we find strong commonalities in FX volume, volatility, and illiquidity across time and FX rates. Consistent with the adage that "liquidity begets liquidity", we find that more liquid currencies reveal stronger commonality in liquidity. Furthermore, we find that liquidity begets price efficiency, in the sense that more liquid currencies obey more to the triangular arbitrage condition.

Several implications emerge from our study. First, by shedding light on the intricate interrelations between FX rates, volume, and volatility, our work should support an integrated analysis of FX rate evolution and risk. Our work also offers a straightforward method to measure FX illiquidity and commonality. For investors, these insights should increase the efficiency of trading and risk analysis. For policy makers, our work highlights the developments of FX global volume, volatility, and illiquidity across time and currencies, which can be important for the implementation of monetary policy and financial stability.

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A Proofs

A.1 Proof of Proposition 1

The log-return and volume at trade i are given by

$$r_i^{x|y} = \Delta p_i^{x|y} = \phi_i^{x|y} + \frac{1}{J}\sum_{j=1}^{J} \psi_{i,j}^{x|y}, \tag{25}$$

and the volume at i-th trade is

$$v_i^{x|y} = \frac{\xi^{x|y}}{2}\sum_{j=1}^{J} |\psi_{i,j}^{x|y} - \bar{\psi}_i^{x|y}|, \tag{26}$$

where $\bar{\psi}_i^{x|y} = \frac{1}{J}\sum_{j=1}^{J} \psi_{i,j}^{x|y}$. We assume for the moment that the common news term is zero, i.e. $\phi_i^{x|y} = 0$. Based on the return on the i-th interval, we can consider the realized variance, defined as $RV^{x|y} = \sum_{i=1}^{I} (r_i^{x|y})^2$ with $\Delta = 1/I > 0$, as introduced by Andersen and Bollerslev (1998). Following Barndorff-Nielsen and Shephard (2002b,a), taking the limit for $\Delta \longrightarrow 0$ (that is $I \longrightarrow \infty$), we get

$$p \lim_{I\to\infty} RV^{x|y} = \frac{1}{J^2}\mathcal{V}_{\psi^{x|y}}, \tag{27}$$

where $\mathcal{V}_{\psi^{x|y}} = \sum_{j=1}^{J} V_{\psi^{x|y},j}$ is the variation of the FX rate on the unit interval generated by the aggregated individual components of $r^{x|y}$. The term $V_{\psi^{x|y},j} = \int_0^1 \left(\sigma_j^{x|y}(s)\right)^2 ds$ is the *integrated variance* associated with the j-th trader's specific component. The term $\mu_j(t)$ does not enter in the expression of $V_{\psi^{x|y},j}$ since the magnitude of the drift, when measured over infinitesimal intervals, is dominated by the diffusive component of $\psi_{i,j}$ that is driven by the Brownian motion. Following Barndorff-Nielsen and Shephard (2003), for a given $\Delta > 0$ we can also define the realized power variation of order one (or realized absolute variation) as $RPV^{x|y} = \sum_{i=1}^{I} |r_i|$. By the properties of the super-position of independent SV processes,[25] the limit for $\Delta \longrightarrow 0$ of $RPV^{x|y}$ is

$$p \lim_{I\to\infty} \Delta^{1/2} RPV^{x|y} = \sqrt{\frac{2}{\pi}}\mathcal{S}_{\psi^{x|y}}, \tag{28}$$

[25] Similarly to Barndorff-Nielsen and Shephard (2002b), $\bar{\psi}_i^{x|y}(t) = \frac{1}{J}\sum_{j=1}^{J} \psi_{i,j}^{x|y}$ is equivalent in law to $\bar{\psi}_i^{x|y,*} = \int_{\Delta(i-1)}^{\Delta i} \bar{\sigma}^{x|y}(t) dW^{x|y,*}(t)$, where $\bar{\sigma}^{x|y}(t) = \frac{1}{J}\sqrt{\sum_{j=1}^{J} {\sigma_j^{x|y}}^2(t)}$.

where $\mathcal{S}_{\psi^{x|y},j} = \int_0^1 \bar{\sigma}^{x|y}(s)ds$ is the integrated average standard-deviation, where the latter is defined as $\bar{\sigma}^{x|y}(t) = \frac{1}{J}\sqrt{\sum_{j=1}^{J} {\sigma_j^{x|y}}^2(t)}$. Given equation (26), the aggregated volume of $x|y$ on a unit (daily) interval is $v^{x|y} = \sum_{i=1}^{I} v_i^{x|y}$, and letting $I \to \infty$, we get

$$p \lim_{I\to\infty} \Delta^{1/2} v^{x|y} = \frac{\xi^{x|y}}{2}\sqrt{\frac{2}{\pi}}\bar{\mathcal{S}}_{\psi_{x|y}}, \tag{29}$$

with $\bar{\mathcal{S}}_{\psi_{x|y}} = \frac{1}{J}\sum_{j=1}^{J}\int_0^1 \tilde{\sigma}_j^{x|y}(s)ds$, where $\tilde{\sigma}_j^{x|y}(t) = \sqrt{(J-1)^2{\sigma_j^{x|y}}^2(t) + \sum_{s\neq j}{\sigma_s^{x|y}}^2(t)}$.

A.2 Proof of Proposition 2

Given Proposition 1, we get that

$$p \lim_{I\to\infty} A^{x|y} = \frac{2\mathcal{S}_{\psi^{x|y}}}{\xi^{x|y}\bar{\mathcal{S}}_{\psi_{x|y}}}, \tag{30}$$

which reflects the ratio of the total average standard deviation carried by each trader. Under homogeneity of the traders, we get that

$$\bar{\mathcal{S}}_{\psi_{x|y}} = J\sqrt{J-1}\mathcal{S}_{\psi^{x|y}}, \tag{31}$$

and Proposition 2 follows directly.

A.3 Proof of Proposition 3

By imposing the no-arbitrage restriction as in Brandt and Diebold (2006), it follows from (8) that the squares of the synthetic returns at the i-th trade can be written as

$$(\widetilde{r}_i^{x|y})^2 = (r_i^{x|z} + r_i^{z|y})^2 = (r_i^{x|z})^2 + (r_i^{z|y})^2 + 2r_i^{x|z}r_i^{z|y}.$$

Under the maintained assumption that $\phi_i^{x|y} = 0$, the synthetic return can be expressed as $\widetilde{r}_i^{x|y} = \bar{\psi}_i^{x|z} + \bar{\psi}_i^{z|y}$, so that we can define the *synthetic* realized variance as $\widetilde{RV}^{x|y} = \sum_{i=1}^{I}(\widetilde{r}_i^{x|y})^2$. By letting $I \to \infty$, we get

$$p \lim_{I\to\infty} \widetilde{RV}^{x|y} = \frac{\mathcal{V}_{\psi^{x|z}} + \mathcal{V}_{\psi^{z|y}} + 2\mathcal{CV}_{\psi^{x|z},\psi^{z|y}}}{J^2}, \tag{32}$$

where $\mathcal{V}_{\psi^{x|z}} = \sum_{j=1}^{J} \int_0^1 \left(\sigma_j^{x|z}(s)\right)^2 ds$ and $\mathcal{V}_{\psi^{z|y}} = \sum_{j=1}^{J} \int_0^1 \left(\sigma_j^{z|y}(s)\right)^2 ds$ are the components of the return variation generated by the cumulative individual variations of the reservation prices on $x|z$ and $z|y$. The term $C\mathcal{V}_{\psi^{x|z},\psi^{z|y}}$ is given by

$$C\mathcal{V}_{\psi^{x|z},\psi^{z|y}} = \sum_{i=1}^{I} \left(\sum_{j=1}^{J} \int_{\Delta(i-1)}^{i\Delta} \sigma_j^{x|z}(s)\sigma_j^{z|y}(s)\rho_j^{x|z,z|y}(s)ds \right),$$

where $\rho_j^{x|z,z|y}(t) = Corr\left(dW_j^{x|z}(t), dW_j^{z|y}(t)\right)$ is the correlation between the individual components on $x|z$ and $z|y$. All the other covariance terms are zero due to independence. For what concerns the trading volume, for the i-th trade on $x|y$ and $x|y$ we have

$$v_i^{x|z} = \frac{\xi^{x|z}}{2} \sum_{j=1}^{J} |\Delta p_{i,j}^{x|z,*} - \Delta p_i^{x|z}|, \quad v_i^{z|y} = \frac{\xi^{z|y}}{2} \sum_{j=1}^{J} |\Delta p_{i,j}^{z|y,*} - \Delta p_i^{z|y}|.$$

Moreover, by the triangular no-arbitrage, $\Delta\tilde{p}_{i,j}^{x|z,*} = \phi_i^x - \phi_i^y + \psi_{i,j}^{x|z} + \psi_{i,j}^{z|y}$ and $\Delta\tilde{p}_i^{x|z} = \phi_i^x - \phi_i^y + \bar{\psi}_{i,j}^{x|z} + \bar{\psi}_{i,j}^{z|y}$, so that the *synthetic volume* of $x|y$ is given by

$$\widetilde{v}_i^{x|y} = \frac{\xi^{x|y}}{2} \sum_{j=1}^{J} |\psi_{i,j}^{x|z} - \bar{\psi}_i^{x|z} + \psi_{i,j}^{z|y} - \bar{\psi}_i^{z|y}|, \tag{33}$$

which involves quantities that cannot be directly observed. However, by letting $I \longrightarrow \infty$, we get

$$p \lim_{I\to\infty} \Delta^{1/2}\tilde{v}^{x|y} = \frac{\xi^{x|y}}{2}\sqrt{\frac{2}{\pi}}\bar{\mathcal{S}}_{\psi_{x|z,z|x}}, \tag{34}$$

where $\bar{\mathcal{S}}_{\psi_{x|z,z|y}} = \frac{1}{J}\sum_{j=1}^{J} \int_0^1 \tilde{\sigma}_j^{x|z,z|y}(s)ds$, and

$$\tilde{\sigma}_j^{x|z,z|y}(t) = \sqrt{\tilde{\sigma}_j^{x|z^2}(t) + \tilde{\sigma}_j^{z|y^2}(t) + 2\tilde{\sigma}_j^{x|z}(t)\tilde{\sigma}_j^{z|y}(t)\rho_j^{x|z,z|y}(t)}. \tag{35}$$

Equation (35) highlights that the synthetic volume reflects the aggregated trader-specific components on the individual FX rates, $x|z$ and $z|y$, as well as their aggregated correlation as measured by $\rho^{x|z,z|y}$, which reflects the correlation between $\psi_j^{x|z}$ and $\psi_j^{z|y}$.