Liquidity Derivatives

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ABSTRACT

It is well established that investors price market liquidity risk. Yet, there exists no financial claim contingent on liquidity. Introducing liquidity derivatives in Brunnermeier and Pedersen (2009) improves financial stability by mitigating liquidity spirals. We propose a contract to hedge uncertainty over future transaction costs, detailing potential buy and sell sides. Fitting a stochastic process to their bid-ask spreads, we simulate liquidity option prices for a panel of NYSE stocks spanning 2000 to 2020. These contracts reduce returns exposure to liquidity factors. Their prices provide a novel illiquidity measure reflecting cross-sectional commonalities. Finally, stock returns significantly spread along simulated prices.

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1 Introduction

Market liquidity risk refers to uncertainty over future trade execution costs. Liquidity risk is priced by market participants (Amihud and Mendelson, 1986; Pástor and Stambaugh, 2003) and features persistence over time and commonalities across assets. Nevertheless, there exists no financial instrument contingent on liquidity, albeit many sources of risk are actively exchanged. Prominent examples include interest rates derivatives to hedge against interest rate risk, credit default swaps to trade on default risk, inflation swaps to control the exposure to fluctuations in the price level. Weather derivatives allow to trade on temperature, humidity, rain, sunshine or snowfall (Campbell and Diebold, 2005), regardless of the lack of a replication strategy within financial markets. The absence of contracts conditioning their payoffs on the realization of market liquidity is puzzling.

Notwithstanding the difficulties posed by the multifaceted nature of liquidity, designated market makers (DMMs) routinely agree with firms to quote a maximum bid-ask spread on their stocks and a minimum depth in exchange for an annual fee (Venkataraman and Waisburd, 2007). The benefits of a derivative performing a similar function would be remarkable. Market liquidity deteriorates the most precisely at times of economic downturns, because of firesales sparked by initial losses.[1] Liquidity derivatives have the potential to mitigate systemic risk by allowing investors to derive counterbalancing profits when prices deviate from fundamentals. Ultimately, liquidity derivatives are a natural remedy to amplification dynamics typical of financial crises, which nowadays often require the intervention of Central Banks in their role of lenders of last resort.

This paper offers three contributions. First, it theoretically shows that the lack of a market for liquidity is responsible for the spiral between market and funding liquidity, highlighting the benefits of the proposed class of derivatives for financial stability. In Brunnermeier and Pedersen (2009), an adverse shock to arbitrageurs' wealth reduces their liquidity provision to customers, in turn impacting prices and leading financiers to increase margin

[1] *Illiquidity – the difficulty of selling assets at a reasonable price – is at the heart of all financial crises.* (The Economist).

requirements, which again tighten the capital constraint of arbitrageurs. As prices deviate from fundamentals, arbitrageurs might also come across losses on existing positions. Liquidity derivatives appreciate precisely when such deviations occur, stabilizing markets and correcting both amplifying dynamics. Arbitrageurs disproportionately value payoffs at times of low market liquidity, and have the reciprocated incentive to provide customers with a payment in exchange for insurance against illiquidity. Deviations of prices from fundamentals increase the value of arbitrageurs' liquidity derivatives holdings, whose collateral value also rises exactly at times of margin calls, thus softening the blow of both loss and margin spirals.

Second, we propose a contract to effectively strip liquidity risk from financial instruments in exchange for an upfront payment. The principle of efficient allocation of resources in financial markets suggests that liquidity risk can be optimally distributed among investors by means of contingent claims. We view liquidity derivatives as option contracts based on the evolution of the relative bid-ask spread of a reference asset as recorded by an independent reporting entity, where the payoff accumulates every time the transaction cost per unit of notional exceeds its value at the inception of the contract. Options resonate well with insurance purposes, reflect the volatility of the underlying process, and build on a vastly explored pricing theory. The payoff structure is designed to compensate investors when illiquidity is high or sustained. Path dependence is the appropriate reflection of the "illiquidity dividend" earned by the asset. We detail potential buy and sell sides of the market, which rest on a comparative argument advantage. Clearly, transaction costs are relatively more important for traders with high turnover such as hedge funds than they are for buy-and-hold investors like pension funds, who reap the benefit of higher returns mandated by unmarketable instruments without their actual liquidation. These two categories of investors have opposite downsides with respect to liquidity risk. Long-term investors are therefore naturally eager to trade with arbitrageurs contracts based on liquidity. At the daily frequency, relative bid-ask spreads are positive, mean reverting, and exhibit volatility clusters. Guided by Cox et al. (1985), we price liquidity derivatives in an equilibrium framework where the risk compensation reflects the estimated magnitude and volatility of future transaction costs and their comovement with stock market returns, akin to the liquidity-adjusted CAPM in Acharya and Pedersen (2005).

Third, we use Monte Carlo techniques to simulate a representative panel of model-consistent liquidity option prices for CRSP stocks of firms traded on the NYSE in the period spanning 2000 to 2020, and analyse their empirical properties. The resulting median premium for a three-months horizon is 67 basis points per unit of notional, and the distribution of prices is skewed to the right and follows the recent decline in illiquidity documented in the literature. Liquidity derivatives significantly reduce stock returns exposure to the Pástor and Stambaugh (2003) liquidity factor. Simulated prices provide a novel measure of illiquidity that reflect commonalities across stocks and peak during NBER recessions, in line with other traditional measures such as Amihud (2002) and Pástor and Stambaugh (2003). Differently from benchmark liquidity proxies, the measure we propose embeds a compensation for the forecasted comovement between illiquidity and the market. Equipped with the evidence that simulated prices capture well the illiquidity-induced component of returns, we show that portfolios of stocks sorted on their instrument-level liquidity risk generate significant abnormal returns with respect to classical asset pricing models including Fama and French (2015) factors plus momentum, both at daily and at monthly frequencies, suggesting that liquidity option prices represent a risk dimension yet to be explored. This finding is robust after controlling for the confounding effect of size, volatility, volume, turnover, and relative spread itself.

We organize this paper as follows. The remainder of the Introduction reviews the literature. Section 2 introduces an equilibrium model with heterogeneous agents and liquidity derivatives, and Section 3 proposes a contract contingent on liquidity suggesting a simple pricing algorithm. Section 4 develops testable hypotheses and provides empirical results. Section 5 concludes.

Literature Review Our work builds on the empirical literature that studies the effects of market liquidity on asset prices pioneered by Amihud and Mendelson (1986), who show that expected returns increase in assets' illiquidity or trading costs as approximated by their bid-ask spread. Brennan and Subrahmanyam (1996) and Brennan et al. (1998) corroborate these findings by using alternative measures of illiquidity. Mahanti et al. (2008) measure liquidity as the degree to which assets are held by investors who are expected to trade more frequently. Other notable contributions in this area include Amihud (2002), who measures illiquidity as the price response to trading volume and suggest that expected returns contain an illiquidity premium component and Amihud et al. (2015), who gather international evidence that less liquid assets earn higher returns, arguing that a portfolio of illiquid-minus-liquid stocks (IML) produces significant risk-adjusted returns for 45 different countries. Amihud and Noh (2021) find a time-varying IML premium rising at times of financial distress. Pástor and Stambaugh (2003) show that stocks with higher liquidity risk yield higher expected returns and propose a long-short portfolio based on liquidity betas as a traded proxy for aggregate liquidity risk. Unlike their work, we characterize a financial contract designed to separate the compensation commanded by the instrument-level market liquidity risk from asset returns, analysing its implications for financial stability.

We thus naturally relate to the theoretical asset pricing literature. Among others, Longstaff (1995) provides an upper bound to the discount resulting from the lack of marketability of a security. More recently, Acharya and Pedersen (2005) present a liquidity-adjusted CAPM where a stock's compensation depends on the interplay between its illiquidity and returns with market illiquidity and market returns. Our work is similar in spirit to Bongaerts et al. (2011), who develop an equilibrium framework with heterogeneous agents where illiquidity premia depend on wealth, risk aversion, and trading horizon of short-sellers. We complement this approach by providing a pricing model for financial claims which condition their payoff on market liquidity in exchange for a prespecified cash amount. Indeed, early resolution of uncertainty over future transactions commands a premium in the cross-section (Schlag et al., 2021). Golts and Kritzman (2010) propose a cliquet option on the S&P 500 as a reference process for market-wide illiquidity. Our focus on asset-specific illiquidity risk is different, and beyond proposing a pricing model we simulate liquidity option prices for a representative sample of NYSE firms, show their hedging properties, and introduce a novel measure of liquidity.

Market liquidity is intimately connected to funding liquidity (Gromb and Vayanos, 2002; Garleanu and Pedersen, 2007; Pelizzon et al., 2016) and limits to arbitrage (surveyed in Gromb and Vayanos, 2010). We contribute to this literature by showing that a market for liquidity risk has the potential to improve efficiency and financial stability, by preventing self-fulfilling impairments to orderly markets.

2 Theoretical Properties

This Section builds upon Brunnermeier and Pedersen (2009) (henceforth, B&P), that we enrich with a derivative contingent on the future liquidity of traded assets.[2] Section 2.1 briefly describes the framework. We follow the governing principle of maintaining intact the anatomy of the B&P model, preserving assumptions and notation. We use a well-established, stylized environment to investigate the beneficial implications of a market for liquidity risk. Indeed, our framework includes B&P as a special case with only shares traded. Section 2.2 next shows that a derivative on liquidity is welfare improving and alleviates liquidity spirals in the otherwise identical B&P model. In doing so, we make a clean case that the appetite for liquidity derivatives is a natural result of agents heterogeneity, and that the lack of a market for liquidity is responsible for the feedback loop between firesales and margin calls causing financial instability.

[2] The introduction of a derivative dovetails with Grossman and Miller (1988), that B&P extend, who use a similar structure to address both the futures market and the underlying stock market.

2.1 Model setup

Consider a B&P economy with J risky assets traded at times $t = 0, 1, 2, 3$. At time $t = 3$, each security pays the random variable v^j defined in dollars amount on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The fundamental value of each stock j is $v^j_t = E_t[v^j]$ with ARCH dynamics, so that shocks to fundamentals increase future volatility:

$$v^j_{t+1} = v^j_t + \Delta v^j_{t+1} = v^j_t + \sigma^j_{t+1}\varepsilon^j_{t+1}, \qquad \varepsilon^j_t \overset{i.i.d.}{\sim} \mathcal{N}(0,1) \tag{1}$$
$$\sigma^j_{t+1} = \underline{\sigma}^j + \theta^j |\Delta v^j_t|.$$

for $\underline{\sigma}^j, \theta^j \geq 0$. Customers arrive sequentially to the market, causing imbalance in the order flow. Let Λ^j_t denote the deviation of the price from fundamental value:

$$\Lambda^j_t = p^j_t - v^j_t \tag{2}$$

There are three groups of market participants, namely "customers" and "speculators," who trade assets and are informed about fundamentals, and "financiers" that finance speculators and only observe prices.

Contracts: two contracts, stocks with price p_t and liquidity derivatives. Liquidity derivatives have price λ^j_t and payoff $|\Lambda^j_{t+1}|$. The buyer of the derivative pays a fixed premium at time t in exchange for a payoff contingent on illiquidity in date $t+1$, and the counterparty takes the opposite side of the trade.

Customers: three risk-averse agents are indexed by the time they start trading, $k = 0, 1, 2$. At time 0, costumer k has wealth W^k_0 and zero shares, but finds out that she will experience an endowment shock of $\mathbf{z}^k = \{z^{1,k}, \ldots, z^{J,k}\}$ shares at time $t = 3$, where $\mathbf{z}^k$ are random variables such that the aggregate endowment shock is zero for each stock, $\sum_{k=0}^{2} z^k = 0$.

Denote the vector of total demand shock of customers who have arrived to the market up to time t by $Z_t \equiv \sum_{k=0}^{t} \mathbf{z}^{j,k}$. Customers arrive sequentially to the exchange, causing demand pressure on prices which temporarily deviate from fundamentals as of Eq. (2), until date $t = 2$ when $Z_2 = 0$. Before a customer arrives to the marketplace, her demand for stocks and derivatives is $\mathbf{y}_t^k = 0$, and $\mathbf{c}_t^k = 0$, respectively. After arrival, customers choose their positions in each period to maximize utility over terminal wealth $U(W_3^k) = -\exp\{-\gamma W_3^k\}$. Wealth evolves according to

$$W_{t+1}^k = W_t^k + (\mathbf{p}_{t+1} - \mathbf{p}_t)'(\mathbf{y}_t^k + \mathbf{z}_k) + (\boldsymbol{\lambda}_t - |\boldsymbol{\Lambda}_{t+1}|)'\mathbf{c}_t^k \tag{3}$$

Speculators such as hedge funds provide immediacy to customers, are risk neutral and maximize expected final wealth W_3. Because of their risk neutrality, they are natural counterparties to take the opposite side of customers on liquidity derivatives transactions, which promise a transfer of resources between the two parties contingent on future illiquidity in exchange for a prespecified premium. Each date, speculators select their positions in stocks and derivatives $(\mathbf{x}_t, \mathbf{c}_t)$. Their wealth is affected by an independent shock η_t, and evolves according to

$$W_t = W_{t-1} + (\mathbf{p}_t - \mathbf{p}_{t-1})'\mathbf{x}_{t-1} + (|\boldsymbol{\Lambda}_t| - \boldsymbol{\lambda}_{t-1})'\mathbf{c}_{t-1} + \eta_t \tag{4}$$

The total margin on the positions of speculators cannot exceed their capital W_t. We consider the standard business practice of portfolio margining.[3]

$$\sum_j (x_t^{j+} m_t^{j+} + x_t^{j-} m_t^{j-}) + \sum_j (l_t^{j+} n_t^{j+} + l_t^{j-} n_t^{j-}) \leq W_t \tag{5}$$

where x_t^{j+} and x_t^{j-} are long and short positions in stocks, and l_t^{j+} and l_t^{j-} long and short open positions in liquidity derivatives at time t, respectively. Moreover, m_t^{j+} (m_t^{j-}) indicates the amount of capital borrowed per unit of long (short) stock positions, and similarly n_t^{j+}

[3]See, e.g., https://www.cboe.com/us/options/margin/portfolio_margining_rules/ and https://www.eurex.com/ec-en/services/margining/eurex-clearing-prisma

(n_t^{j-}) denotes how much financing speculators can borrow against each unit of long (short) derivatives position.

Financiers provide capital to speculators, observe only the stock price sequence and set margins to limit counterparty credit risk targeting a value-at-risk π.

$$\pi = Pr(-\Delta p_{t+1}^j > m_t^{j+}) = Pr(\Delta p_{t+1}^j > m_t^{j-}) \tag{6}$$

Financiers accept derivatives as collateral with the same rule.

$$\pi = Pr(|\Lambda_{t+1}^j| < n_t^{j+}) = Pr(-|\Lambda_{t+1}^j| > n_t^{j-}) \tag{7}$$

Since financiers are uninformed about fundamental values, the above probabilities condition on the filtration generated by market prices $\mathcal{F}_t = \sigma\{\mathbf{p}_0, \ldots, \mathbf{p}_t\}$. For example, the VaR specification requires that price drops that exceed margins on long stock positions only happen with probability π. Similarly, the collateral value of derivatives cannot exceed the minimum expected payoff resulting from the position with confidence level $1 - \pi$. Margins m on stocks increase in price volatility and can increase in market illiquidity (Brunnermeier and Pedersen, 2009, Proposition 3). It is immediately seen that margins n on derivatives *decrease* with prices deviations from fundamentals.

Definition: An *equilibrium* is a pair of price processes $(\boldsymbol{p}_t, \boldsymbol{\lambda}_t)$ such that (i) $(\boldsymbol{x}_t, \boldsymbol{c}_t)$ maximize the speculators' expected terminal profits subject to the margin constraint; (ii) each $(\boldsymbol{y}_t^k, \boldsymbol{c}_t^k)$ maximize customer k's expected utility after their arrival to the marketplace and is zero beforehand; (iii) margins are set according to the VaR rule; and (iv) the stock market clears, $\boldsymbol{x}_t^k + \sum_{k=0}^2 \boldsymbol{y}_t^k = 0$, and customers and speculators agree on the amount $\boldsymbol{c}_t = \sum_{k=0}^2 \boldsymbol{c}_t^k$ of derivatives transactions executed.

2.2 Results

Both the model and the equilibrium concept are a generalization of B&P which include derivatives on the illiquidity of stocks. We direct the reader to the Appendix for a rigorous solution of the model, and hereby articulate our main results.

Proposition 1. *Liquidity derivatives are welfare improving.*

Proof. See Appendix A. ■

Comparative advantage explains well the scope for a market on liquidity. In this model, agents differ in their risk attitudes, access to funding, and preferred turnover. Limits to arbitrage arise from order flow imbalance and funding liquidity constraints, which cause the pricing kernel of speculators to reflect capital availability. Speculators are willing to pay more than the expected value of the payoff for an asset whose payoff is high in states where funding is constrained and markets are illiquid, therefore the dollar remuneration per unit of capital is large. Conversely, customers do not have access to funding and are eager to take the other side of a trade earning positive expected return. The equilibrium amount of liquidity insurance provided by customers decreases in risk aversion γ and the variance of future liquidity $\mathrm{Var}_0(|\Lambda_1^j|)$.

Proposition 2. *Liquidity derivatives attenuate both margin and loss spirals, alleviating the effect of shocks to speculators' wealth on market prices.*

Proof. See Appendix B. ■

Importantly, liquidity derivatives stabilize market prices against loss spirals by conditioning the revenues of speculators to future realizations of order flow imbalance. As share prices move adversely and speculators are forced to post more collateral to finance their stock positions, portfolio margining enables to borrow against the increased value of derivatives position.[4]

[4]The results achieved by maintaining preferences' specification as in B&P are conservative. Indeed, real-world arbitrageurs are arguably risk averse about margin calls and losses on previous positions. This feature would lead to higher insurance motive, thus strengthening the stabilizing effects of liquidity derivatives on financial markets.

From a high-level perspective, liquidity spirals are induced by external stakeholders uninformed about fundamental values who require collateral when the two legs of an arbitrage position widen following price swings caused by demand and supply forces (see also Shleifer and Vishny, 1997). Payoffs contingent on market liquidity effectively hedge speculators in these states of the world by establishing a countercyclical connection between market and funding liquidity which prevents self-fulfilling impairments to orderly markets.

3 A Market for Liquidity Risk

We have formally argued that liquidity derivatives improve welfare and financial stability. In the B&P model, both customers and speculators observe the fundamental value of stocks. Real world trading venues are however more complex. In Section 3.1 we describe a suitable proxy for price deviations from fundamental values. Section 3.2 illustrates desirable properties for the payoff structure of derivatives based on liquidity and characterizes potential buy and sell sides of the market. Finally, Section 3.3 develops a pricing technique for liquidity derivatives.

3.1 The Reference Process

Liquidity has many faces. However, the liquidity risk of a position refers to uncertainty about its future transaction costs, and is traditionally identified as a nontraded risk factor (Pástor and Stambaugh, 2003). To fix ideas, suppose an investor has a long (short) stock position and wants to hedge against fluctuations in the bid (ask) price, while willing to retain other risks associated to the asset on the portfolio. The time t immediacy cost is captured by the distance between the market order execution and the midquote m_t,

$$p_t = \frac{1}{2}\Big(a_t + b_t + d_t(a_t - b_t)\Big) = m_t + \frac{1}{2}d_t\Big(a_t - b_t\Big) \tag{8}$$

where a_t and b_t are respectively the best ask and bid quotes, and $d_t = 1$ for buyer initiated trades and −1 otherwise. The parallel with Eq. (2) from B&P is apparent. Eq. (8) decomposes the time t value of a position in its midprice, proxying for fundamental value, and trading costs, capturing the cost of immediate liquidation.[5] Asset returns result from the evolution of both terms (Amihud and Mendelson, 1986). Of course, high prices are bad news for buyers and low prices penalize sellers, so that for each value of d_t the execution costs map to the half bid-ask spread.

The above example clarifies that the bid-ask spread is a reasonable proxy for the instrument-level market liquidity risk of standard-sized positions, i.e., those with negligible price impact, when fundamental values are not observable. To achieve comparability across firms, we consider the time t execution costs c_t as one half times the relative spread.

$$c_t = \frac{1}{2}\frac{a_t - b_t}{m_t} \tag{9}$$

Empirically, relative spreads are a feasible reference process, and do not suffer from estimation issues as other liquidity measures such as price impact. Contemporary data on the relative spread in Eq. (9) are readily available from third-party reporting entities. Market prices are carefully monitored both at the SEC and at the exchange level to prevent and sanction manipulative conducts, and the quotes posted by market makers are tied to the tightest composite bid-ask spread resulting from competition between trading venues.[6]

3.2 Hedging Liquidity Risk

We term `liquidity derivatives` financial claims contingent on the market illiquidity of the underlying asset. Many are however the payoff structures which respond to that crite-

[5] We leave to future research a generalization of round-trip transaction costs to the price impact of large positions, noting that Eq. (8) naturally extends to an arbitrary q-sized position. Indeed, denoting through upper bars weighted averages of best prices at the quantities quoted on the limit order book, $p_t(q) = \frac{1}{2}\left(\overline{a}_t(q) + \overline{b}_t(q) + d_t\left(\overline{a}_t(q) - \overline{b}_t(q)\right)\right)$.

[6] Among other NYSE provisions available at https://nyseguide.srorules.com/rules, rule 104 (a) prescribes that prices entered by DMMs shall be not more than the Designated Percentage away from the then current National Best Bid Offer (NBBO) available across US exchanges. Rule 6140 (d) explicitly forbids exchange members or organizations to participate or have any interest, directly or indirectly, in the profits of a manipulative operation or knowingly manage or finance a manipulative operation.

rion. We have argued that the relative bid-ask spread is a desirable reference process. In order to impose further discipline, we engineer these instruments to separate liquidity risk from fundamental risk over a specified horizon. It is worth noting that returns embed a periodic compensation for both fundamental and liquidity risk, and that modern accounting practices often involve mark-to-market appraisal of financial assets. Because of these reasons, it is appropriate to condition the derivative's payoff on the dynamics of illiquidity of the underlying asset over the holding period.[7] Consider a payoff function of the following form.

$$\mathcal{H}_T = \frac{1}{2} \max \left\{ H_T - K, 0 \right\} \tag{10}$$

H_T accumulates through the investment horizon the relative spreads c_i of the underlying in excess of its value at inception of the contract, and K is a strike price.

$$H_T = \sum_{i=t+1}^{T} \max\{c_i - c_t, 0\} \tag{11}$$

This instrument compensates the holder of an arbitrary asset for large deviations of transaction costs from their level at the beginning of the contract, earning more the higher and longer lasting is illiquidity. As an example, Figure 1 displays the 2000Q1 time-series behavior of the relative spread of Walmart Inc, and the corresponding step-wise option payoff plotted against time.

Together, Eq. (10) and (11) mirror the configuration of weather options actively traded over the counter, where market participants agree on a designed institution to measure the reference process. Importantly, the structure we posit is consistent with the insurance purpose of avoiding large losses resulting from high or sustained illiquidity, and maps the difficult issue of measurement of liquidity risk to the well-understood field of option pricing

[7] The approach resonates well with the Basel III liquidity regulation, which requires high-quality liquid assets to be continuously marketable.

theory.

As the model in Section 2 illustrates, the appetite for a market of derivatives on liquidity results from agents heterogeneity. Clearly, some investors have a comparative advantage in processing information about fundamentals, and other investors are more knowledgeable about the dynamics of the order flow. Both categories of market participants have the incentive to strip the illiquidity compensation from returns. Investors also differ in their behavior. Traders with high turnover such as hedge funds typically hold assets for short investment horizons, which exposes their business model to the risk of large transaction costs and their comovement with the market (see also Bongaerts et al., 2011). On the opposite end of the spectrum, buy-and-hold institutions as pension funds rarely need to trade, view favorably the return compensation resulting from periods of prolonged illiquidity, and are less sensitive to the procyclicality of market liquidity. The scope of liquidity derivatives is to facilitate the transfer liquidity risks between market participants.

3.3 Pricing Liquidity Derivatives

At the core of liquidity risk is the impossibility of a replication in the Black and Scholes (1973) tradition. The lack of stable correlation patterns for liquidity is intriguing, and hinders statistical arbitrage. In these regards, liquidity risk resembles the unspanned risk that weather derivatives channel to active financial markets since two decades. Typically, the pricing of these instruments rests on Monte Carlo techniques, which are based on the specification of a stochastic process for the relevant source of risk (Alaton et al., 2002). At the daily frequency, relative bid-ask spreads are positive, mean reverting, and exhibit volatility clusters.[8] These empirical facts suggest that a stochastic process in the tradition of Cox et al. (1985) (henceforth: CIR) is a reasonable specification to describe the dynamics of the relative bid-ask spreads in Eq. (9). Tractability of the process under both the physical and risk-neutral measure is also appealing. Thus, we specify a CIR process for the relative spread.

[8] Engle and Patton (2004) view spreads as cointegrating relations between the best bid and ask quotes, and Groß-KlußMann and Hautsch (2013) summarize the empirical properties of bid-ask spreads.

Let $(\Omega, \mathcal{F}, \mathbb{P})$ denote a complete probability space with a right continuous and increasingly finer filtration $\{\mathcal{F}_t\}_{t\geq 0}$, with $\mathcal{F}_0$ containing all null sets of $\mathbb{P}$.

$$dc_t = \alpha(\mu - c_t)dt + \sqrt{c_t}\sigma dB_t \tag{12}$$

In Eq. (12), the parameters (α, μ, σ) have the usual interpretation of speed of reversion, long run mean and variance of the square root process, respectively, and are required to conform to the Feller condition to ensure non-negativity of the process.

These parameters can be easily estimated by Maximum Likelihood of the transition density of the CIR process, which is proportional to a noncentral χ^2 distribution, using OLS regressions as initial values (Kladívko, 2007). Under the risk-neutral measure,

$$dc_t = \tilde{\alpha}(\tilde{\mu} - c_t)dt + \sqrt{c_t}\sigma dB_t^{\mathbb{Q}}, \qquad \tilde{\alpha} = \alpha + \varrho,\ \ \tilde{\mu} = \frac{\alpha\mu}{\alpha + \varrho} \tag{13}$$

In incomplete markets, non arbitrage is silent about the market price of risk, which requires an equilibrium argument. Guided by Cox et al. (1985), we let ϱ denote the stock-specific ratio of the covariance between changes in relative bid-ask spreads and percentage changes in optimally invested wealth (approximated through market returns) to the relative spread. Eq. (13) shows that a more negative ϱ implies a slower speed of reversion and a higher long-run mean. Both features increase the standard deviation of the risk-adjusted CIR process (Hördahl and Vestin, 2005). The market price of risk takes the form $\theta = \frac{\varrho\sqrt{c}}{\sigma}$, thus retaining the property of a higher risk compensation for negative comovements of liquidity with the market (remindful of the liquidity-adjusted CAPM in Acharya and Pedersen, 2005). Rephrasing, a liquidity option on a stock whose transactions cost are particularly high at times of negative marketwide returns demands higher premia.

We compute the model-implied prices of liquidity derivatives C_t through Monte Carlo techniques, by simulating N times the underlying process under the $\mathbb{Q}$-measure and averaging the resulting discounted payoffs, as customary with path-dependent instruments.

$$C_{t,T} = e^{-rT}\mathbb{E}_t^{\mathbb{Q}}[\mathcal{H}_T] \approx \frac{1}{2N}e^{-rT}\sum_{n=1}^{N}\max\left\{\left(\sum_{j=t}^{T}\max\{\hat{c}_j^{(n)} - c_t, 0\} - K, 0\right)\right\}, \tag{14}$$

where $c_j^{(n)}$ characterizes the n^{th} simulation of the bid-ask spread at time j. The quality of the approximation increases in the number of paths N.

4 Empirical Evidence

We have shown that liquidity derivatives are beneficial to financial markets and provided a simple pricing method. We now turn to the data and study the empirical properties of simulated liquidity option prices, henceforth LOPs. Section 4.1 illustrates descriptive statistics about option prices for a panel of stocks traded on the NYSE. Section 4.2 develops testable hypotheses illustrating the economic motivation behind them, with empirical findings presented in the subsequent sections. In more detail, Section 4.3 provides evidence that liquidity options effectively strip liquidity risk out of financial assets. Section 4.4 introduces a novel measure of aggregate illiquidity based on option prices. Finally, Section 4.5 documents strong links between LOPs and stock returns which survive risk adjustments and the effect of confounding correlated variables.

4.1 Data Description and Summary Statistics

We apply our pricing formula to CRSP stocks listed on the NYSE during the period January 2000-December 2020. The sample is confined to NYSE-traded stocks to avoid the effect of differences in microstructure (Amihud, 2002; Reinganum, 1990) and in trading algorithms (Korajczyk and Sadka, 2008).[9] Data selection closely follows Amihud (2002) with details reported in Appendix C. Because of better reporting quality regarding bid and ask quotes,

[9]For instance, while trading on Nasdaq takes place mostly through market makers, the majority of trades on the NYSE occurs between buying and selling investors directly.

we use Bloomberg data.

As discussed in Section 3.3, we price liquidity derivatives at the end of each month by fitting a CIR process to relative bid-ask spreads on one-year estimation windows and simulating $N = 100,000$ Monte Carlo paths per firm over a maturity of $T = 3$ months, which is a fairly standard investment horizon, such as the time interval between successive delivery dates of benchmark futures contracts. As a convention, the strike price is set to zero since computational costs prevent us from calculating large panels of option prices for an arbitrary set of strike values. The daily risk-free rate from Kenneth French's website is used for discounting.[10] French's market factor returns proxy for changes in optimally invested wealth when computing their covariance with changes in relative spreads.

Our sample includes 1,755 firms for a total of 192,746 firm-month LOPs. Table 1 shows descriptive statistics for firm characteristics and simulated option prices. The median price required to remove uncertainty about transaction costs over the next three months amounts to 67 bps of the stock price at the contract initiation. Option prices exhibit a substantial degree of variation (the standard deviation is 612 bps) and a skewness of 4.45, with the average price roughly four-fold as big as the median. These and other interesting facts are summarized in Figure 2, which represents cross-sectional option prices deciles every year.[11] Looking at the time-series dimension, liquidity options are particularly expensive during the *dot-com bubble* in 2001 and in the course of the financial crisis peaking in 2009. Unsurprisingly, the premium required to hedge against illiquidity includes also a component related to market-wide conditions. Overall, prices are higher for the first part of the sample and remarkably drop after 2009, in line with the general decline in illiquidity observed in recent years (Amihud and Mendelson, 2015). From a cross-sectional perspective, the distribution is strongly rightly skewed. In other words, liquidity options are cheap for most stocks and thus appealing for investors willing to pay upfront a small amount to engage in

[10] For details, see https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html.

[11] We exclude the year 2000 from the picture as it would represent data about only December, 2000, which is when liquidity option prices are first available using a one-year estimation window.

a hedge-and-forget strategy.

Aware of the fact that the CIR process used in Eq. (12) is an approximation aimed at delivering tractability, we explore how well it captures the behavior of the relative bid-ask spread for pricing purposes with a simple exercise. We compute the realized payoff over a maturity of 3 months that one would have obtained by investing in liquidity options if they were available, discounting it back to the pricing time. Then, we sort options into 10 portfolios based on their price at the end of each month t and regress equal-weighted portfolio payoffs at month $t+3$ on the corresponding time-t equal-weighted portfolio prices using OLS. Simulated option prices predict 53% of the variation in realized payoffs, with a regression slope coefficient of 0.29 and a t-statistic above 51. Figure 3 helps visualizing this performance by showing the time average of portfolios payoffs against the time average of portfolio prices. With the exception of portfolio 10, the two quantities line up nicely with a corresponding R^2 of 0.78. In addition to the pricing algorithm, simulated option prices and realized payoff might diverge because of differences between the physical and the risk-neutral measure. Since prices are smaller than payoffs, these risk adjustments are negative, as it should indeed occur for instruments hedging risks. Intuitively, deviations in the plot increase along the x-axis because more expensive portfolios require higher adjustments. These findings confirm that the pricing procedure is reliable and satisfies fundamental asset pricing restrictions.

4.2 Hypotheses Development

First, we empirically investigate whether the proposed liquidity options effectively strip liquidity risk out of financial assets. Let us assume excess returns follow well-established multifactor models like Fama and French (1993) three-factors model, augmented with a traded risk factor like the one suggested in Pástor and Stambaugh (2003) to account for liquidity risk. If liquidity options provide a valid hedge against liquidity risk, we expect that a portfolio containing one stock and the corresponding option exhibits returns which are less exposed to liquidity factors than non-hedged stock returns.

Hypothesis 1. *The return of a portfolio composed of one stock and one corresponding liquidity option is significantly less exposed to a proxy for liquidity risk than the stock return alone.*

Second, we have reason to believe that LOPs reflect commonalities in the cross-section. Other than being affected by illiquidity at the instrument level, stocks are exposed also to aggregate liquidity risk (Acharya and Pedersen, 2005; Pástor and Stambaugh, 2003) and as such they covary with it. Hence, we expect the price of liquidity derivatives to pick up some of the exposure to common liquidity shocks, such that an aggregated measure of simulated option prices captures market-wide conditions in similar fashion to other well-known measures of illiquidity (Amihud, 2002; Pástor and Stambaugh, 2003). In particular, we expect it to spike during crisis periods.

Hypothesis 2. *A cross-sectionally aggregated measure of liquidity option prices reflects market-wide liquidity conditions, similarly to the aggregate measures in (Amihud, 2002) and Pástor and Stambaugh (2003).*

Third, we explore the link between LOPs and stock returns. If Hypothesis 1 and 2 hold, liquidity options are clearly relevant for financial markets, and as such they are prone to impact the cross-section of returns. Since the payoff of liquidity options accumulates in the realized bid-ask spread, which is known to positively affect stock compensation (Amihud and Mendelson, 1986), we expect stock returns to increase in LOPs. Furthermore, since LOPs reflect asset-specific liquidity risk, they capture a part of returns that is not spanned by traditional factor models lacking an explicit liquidity factor. Thus, we expect that portfolios of stocks sorted according to simulated option prices generate abnormal returns in such a framework.

Hypothesis 3. *Stock returns increase in liquidity option prices. Portfolios sorted on option prices violate the mean-variance efficiency of multifactor models which do not include a liquidity risk factor.*

4.3 Hypothesis 1: Liquidity-hedged Portfolios

Returns of single stocks ("raw" positions) are compared with "hedged" positions obtained financing the purchase of the corresponding liquidity options by selling a part of the shares with the same value, such that the initial investment is equal to the initial stock price in both cases. The return of a liquidity-hedged position between time t and the maturity of the option at $t + 3$ consists of two components, namely the ordinary stock appreciation and the liquidity option payoff, relative to the initial outflow. Dividends and eventual differences between the mid-price and the adjusted price used to calculate returns are accounted for. To reflect the compensation of positions financed by borrowing at the risk-free rate, we focus on excess returns. A thorough explanation of the procedure is provided in the Appendix D.

Next, we test the hypothesis that hedged positions are less exposed to liquidity risk. As a proxy for it, we use the traded liquidity factor provided by Pástor and Stambaugh (2003), which is a long-short portfolio based on stock exposures to aggregate liquidity shocks.[12] We name it LIQ. In order to reduce the noise in the estimation of factor loadings, we first sort stocks into 10 portfolios based on their LOPs, and then we regress raw portfolio returns and hedged portfolio returns on Fama and French (1993) model plus momentum plus LIQ on a rolling basis using 60-months windows. Rolling estimation delivers time series of liquidity factor loadings that allows to formally test Hypothesis 1. Table 2 shows that the absolute mean exposure to LIQ across portfolios almost halves thanks to liquidity options, passing from 0.0235 to 0.0119.[13] Single-portfolio loadings substantially shrink, exhibiting sometimes more than a 10-fold reduction. In three cases the exposure actually increases, but this happens for portfolios starting with low betas (portfolios 3, 4, and 7). We stress that, in addition to sample uncertainty, we cannot expect to observe exactly zero loadings because LIQ is meant to capture price impacts, a dimensions of liquidity only partially overlapping with bid-ask spreads. Importantly, portfolios heavily exposed to LIQ remarkably benefit

[12] Data are collected from Robert Stambaugh's website (http://finance.wharton.upenn.edu/~stambaug/), which provides further details about the construction of this variable.

[13] It is important to focus on absolute mean values to avoid the comparison to be biased due to potentially negative betas.

from liquidity options. A prominent example is portfolio 10, whose liquidity beta is more than 20 times smaller when hedged. This result is of primary importance as portfolio 10 contains the stocks with the highest LOPs and thus those for which liquidity concerns are most urgent.

As an explicit statistical test for Hypothesis 2, we use a t-test for difference in means allowing for different variances by using the loadings obtained with a rolling estimation, following the approach of Savor and Wilson (2013, 2014). The second-to-last column of the table reports p-values for the null hypothesis that the absolute mean liquidity beta of a raw position is less than or equal to the hedged position one. The null is rejected in 7 cases, suggesting that raw positions load significantly more on liquidity risk, as hypothesized. To conclude, we also test the hypothesis that the mean exposure change from raw to hedged position is less than or equal to zero. This is rejected in 9 cases out of 10, as shown by the p-values of the corresponding t-test reported in the last column of the Table: liquidity options significantly alter the liquidity risk profiles of the portfolios considered, in line with the objective of their design. To sum up, these contracts strongly reduce the exposure of portfolios to aggregate liquidity risk, thereby revealing strong potential for the financial industry.

4.4 Hypothesis 2: Commonalities in Liquidity

In the presence of strongly skewed prices, as emerged from Figure 2, a natural cross-sectional measure for LOPs is the median of the distribution in each month, as in Cakici and Zaremba (2021). This constitutes an option-based market-wide proxy for illiquidity capturing transaction costs, that we name OPT. We compare it to two well-known measures. The first is $ILLIQ$ (Amihud, 2002) at the monthly frequency. The second one is the aggregate liquidity from Pástor and Stambaugh (2003).[14] To transform it into an *illiquidity* measure, we flip its sign, and denote it with PS. OPT is also contrasted with a simpler aggregate built

[14] Notice that this corresponds to what is plotted in their Figure 1, which differs from the innovations u_t (Eq. (8) in their paper).

using the cross-sectional median of the relative bid-ask spread, the underlying of liquidity options, that we call $SPREAD$. Standardized time-series of these four measures are plotted in Figure 4, where shaded regions correspond to NBER recession periods.

OPT is reported in red. As other measures, it spikes up in 2001 to reflect investors' concerns about liquidity during the *dot-com bubble*. After a general decrease in the subsequent years, all the four aggregates reach high levels during the sub-prime crisis, but with different timing. PS and $SPREAD$ come to their respective peak of the period first, closely followed by $ILLIQ$. OPT keeps rising during the entire recession, hitting its local highest point some months later. In the remaining years, all measures stay below their mean, except for $ILLIQ$ and PS. Both these two clearly jump upwards towards the end of the sample to reflect the shock induced by the Covid-19 pandemic, but the former remains high also afterwards. OPT and $SPREAD$ are less affected by this phenomenon. The measures are clearly persistent, with a first-order autocorrelation above 0.9, apart from PS, whose distinctive trait is a strong mean reversion (autocorrelation of 0.05).

To a large extent, OPT captures important changes in liquidity at the aggregate level, but exhibits some differences with respect to other measures. This happens because of the complex nature of illiquidity. While $ILLIQ$ and PS reflect price responses associated with every dollar of trading activity and temporary price changes accompanying order flows, respectively, the bid-ask spread is influenced by different aspects of market liquidity. LOPs, in particular, reflect the no-arbitrage compensation required to insure against uncertainty over future transaction costs relative to the spread at the time of pricing, thereby capturing a dimension of liquidity which differ not only from the two we have just mentioned, but also from $SPREAD$, which instead reflects only point-wise deviations from the fundamental price that do not accumulate over the holding period to reflect concerns about cumulative illiquidity. Moreover, OPT includes a risk-adjustment component consistent with an equilibrium model, a feature absent in the other metrics. Finally, it must be stressed that since liquidity options are not actually observed, an option-based illiquidity measure is the result of a simulation exercise. With traded options, OPT would gain a genuine forward-looking behavior, something unprecedented in the liquidity literature. Given these considerations, it is not surprising that $ILLIQ$, OPT and $SPREAD$ span together only 77% of the variation of OPT in a time-series regression, leaving unexplained almost one fourth of the movements in the market-wide illiquidity captured by LOPs. OPT stands therefore as a complementary illiquidity measure to traditional ones that can be employed in future research.

4.5 Hypothesis 3: Abnormal Returns

As a first step to investigate the relation between stock returns and instrument-level liquidity risk, we perform univariate portfolio sorting based on LOPs, which are firm-specific, at the end of each month, grouping stocks into 10 equal-weighted portfolios. Figure 5 shows portfolio daily average excess returns in bps. As expected, there is a clear increasing relation between option prices and returns that is notably strong for stocks whose liquidity options are more expensive. The difference between portfolio 10 and portfolio 1 is 4 bps with a t-statistic of 5.11, which means returns significantly spread along the dimension of liquidity option prices. The results hold also at the monthly frequency, where the top-minus-bottom-decile average excess return is 94 bps (t-statistic=5.20).

Second, we test well-known empirical asset pricing models on the 10 option-prices-sorted portfolios. Table 3 reports the intercept and the R^2 calculated following Kelly et al. (2019) for time-series regressions of the type

$$r_{t,t+1} = \alpha + \beta F_t + \varepsilon_t \tag{15}$$

where α is the intercept, $r_{t,t+1}$ is an $N \times 1$ vector of excess returns from t to $t+1$, β is the loading matrix on the K traded factors in F_t which in turn includes the market excess return (CAPM), Fama and French (1993) factors (FF3), FF3 plus the Momentum factor (Carhart, 1997) (FF4), Fama and French (2015) factors (FF5) and FF5 plus Momentum (FF6), and

ε_t is the error term. t-statistics are computed using Newey and West (1987) standard errors with 5 lags. Overall, traditional factor models struggle to explain portfolios sorted according to LOPs. Despite achieving a high R^2, they consistently fail to clear the asset pricing restriction of zero alpha. In particular, 9 out of 10 are significant for CAPM, FF3 and FF4, and 5 for FF5 and FF6. The GRS statistic (Gibbons et al., 1989) rejects the null hypothesis of zero alphas well below the 1% significance level for all models. Importantly, significant alphas concentrate in portfolios with more expensive liquidity options and increase along this dimension, in line with economic intuition: risk adjustments from well-known factors do not alter the findings of Figure 5. A portfolio going long the top decile and short the bottom decile produces a daily alpha of at least 3.7 bps in all models with a t-statistic never below 4.79. In other words, majority of its mean excess return over the period (4 bps) cannot be explained through the exposure to traditional factors. As a benchmark, the mean excess returns of the market factor, SMB, HML, Momentum, RMA and CMA are 3.36, 1.35, 0.21, 1.41, 0.57 and 0.79 bps, respectively. The abnormal remuneration of the long-short portfolio is therefore not only statistically but also economically significant. Results with monthly data (bottom panel) are very similar. The GRS test hypothesis is rejected in all models and the LOP-long-short portfolio earns a significant alpha of at least 84 basis points.

The reader may be concerned that the positive relation between returns and LOPs could be partially driven by correlation with confounding variables, as often argued in the literature Amihud and Mendelson (2015). Small firms are tipycally more illiquid than larger ones. Moreover, size is well-known in the anomaly literature since it proxies for the exposure to SMB, an important traded risk factor. Volatility and illiquidity are positively correlated (Stoll, 1978), and volatility heavily affects the impact that illiquidity has on stock returns (Spiegel and Wang, 2005). Volume (in dollars), turnover (Datar et al., 1998) and relative bid-ask spread are three alternative measures of illiquidity at the stock level which may partially overlap with LOPs. The correlations between LOPs and these variables have the sign one would expect: -0.21 with size (log market equity); 0.07 with volatility (measured by monthly standard deviation of stock returns following Amihud and Noh (2021)); -0.07 with volume; 0.04 with turnover and 0.30 with relative spread. Table 4 reports the time average of these firm characteristics for the 10 equal-weighted portfolios sorted on LOPs. Mean values are normalized into the $[0, 1]$ interval so that the portfolio with lowest value will display a zero and the one with the highest value will have a 1. Portfolios with high option prices are composed of relatively small and volatile stocks, which are illiquid according to both relative spread and dollar volume. The relation with turnover is instead non-monotonic, increasing at first and then dropping for the last two portfolios.

To account for the influence of these variables on excess returns of option-price-sorted portfolios, we use bivariate conditional sorts as nonparametric tool and carry out a similar analysis to the univariate case. At the end of each month, stocks are first sorted into terciles based on one of the variables just mentioned and *then* they are grouped with respect to LOPs into 5 portfolios within each tercile. We thus end up with 15 control-and-LOP-sorted portfolios in the spirit of Amihud et al. (2015). The patterns of average excess returns of the resulting double-sorted portfolios are summarized in Figure 6, which is the two-dimensional counterpart of Figure 5. It shows that stock returns increase in liquidity option prices within all terciles of the five controls we consider. A more detailed representation is provided in Table 5. This includes also t-statistics for the average return of LOP-long-short portfolios within each conditioning tercile in the second-to-last column, a high hurdle to gauge the robustness of the findings described in the univariate case. As a benchmark, the last column shows that returns follow the patterns documented in the literature for the control variables. The increasing relation between option prices and returns persists after controlling for every variable, i.e. returns are generally larger for higher option prices within all the control-terciles. With the only exception of stocks very frequently traded, the difference between the 5^{th} portfolio ("Expensive") and the 1^{st} one ("Cheap") is always significant. Importantly, this happens even for the first size-tercile and the last volatility-tercile, alleviating concerns that results are driven by micro-caps or by volatility. Returns significantly spread along the

LOP dimension also after conditioning for turnover or the relative bid-ask spread underlying the options. This confirms that the relation between liquidity derivative prices and excess returns goes beyond what traditional liquidity measures capture, thanks to their intrinsic risk-adjustment and their ability to compensate for cumulative illiquidity, similarly to what discussed in Section 4.4. Results are largely confirmed at the monthly frequency, as shown in Table 6.

The previous findings tell us that liquidity option prices capture an "illiquidity dividend" embedded in stock returns which persists at every level of a set of covariates correlated with illiquidity. We now examine whether this conclusion survives the risk-adjustment by testing traditional factor models against the newly built double-sorted portfolios. Results are presented in Table 7, which reports the value of the GRS statistic together with the R^2 for the models listed on the rows. As for the univariate sorting, Fama-French models generate a good fit for the data, yet they do not pass asset pricing tests: the null hypothesis of mean-variance efficiency is rejected for all models and for all the sets of portfolios considered (the 1% critical value for the GRS test is 2.04). To investigate whether the remuneration of the long-short portfolios based LOPs after controlling for size, volatility, volume, turnover and relative spread is due to the exposure to Fama-French factors, the last three columns show their alphas and the relative t-statistic for each characteristic-tercile. For example, the column $1_5 - 1_1$ of the first panel contains the intercept for a long-short portfolio obtained as the difference between the highest-LOP portfolio and the lowest-LOP portfolio within the smallest size-tercile. As economic intuition would suggest, pricing errors on long-short portfolios are somewhat smaller than the univariate case because double-sorting nets out the effect of the conditioning variables, which are known to impact stock returns. The largest alphas occur for small, volatile stocks which are rarely traded with low turnover and high bid-ask spreads. Notably, alphas are strikingly similar to the average excess returns displayed in Table 5, even when 6 factors are considered. Put differently, the spread in stock returns along the LOP dimension cannot be explained as a compensation for traditional risk factor exposures even after controlling for the effect of confounding correlated variables. These findings persist also with monthly data, as reported in Table 8. The GRS statistics in fact always largely exceed the 1% critical value (2.12) and the within-tercile long-short portfolio alphas are almost always significant and very close to the average excess returns of the same portfolios.

5 Conclusion

We show theoretically that the lack of a market for liquidity is responsible for the spiral between market and funding liquidity. Guided by the principle of efficient allocation of resources in financial markets, we propose a novel liquidity derivative to fill this gap and improve financial stability, and suggest a simple pricing algorithm. We view liquidity options as offering a payoff that accumulates every time the relative bid-ask spread of the underlying financial asset exceeds the transaction costs per unit of notional at the beginning of the contract. The hedging property of these instruments is supported empirically. Their pricing delivers a novel measure of liquidity incorporating risk adjustments and expectations of cumulative illiquidity. Stock returns significantly spread along instrument-level liquidity risk and portfolios of stocks sorted their liquidity option price generate anomalies that persist after controlling for variables correlated with illiquidity.

References

Acharya, V. V., and L. H. Pedersen. 2005. Asset pricing with liquidity risk. *Journal of Financial Economics* 77:375–410.

Alaton, P., B. Djehiche, and D. Stillberger. 2002. On modelling and pricing weather derivatives. *Applied Mathematical Finance* 9:1–20.

Amihud, Y. 2002. Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets* 5:31–56.

Amihud, Y., A. Hameed, W. Kang, and H. Zhang. 2015. The illiquidity premium: International evidence. *Journal of Financial economics* 117:350–368.

Amihud, Y., and H. Mendelson. 1986. Asset pricing and the bid-ask spread. *Journal of Financial Economics* 17:223–249.

Amihud, Y., and H. Mendelson. 2015. The pricing of illiquidity as a characteristic and as risk. *Multinational Finance Journal* 19:149–168.

Amihud, Y., and J. Noh. 2021. The pricing of the illiquidity factor's conditional risk with time-varying premium. *Journal of Financial Markets* 56:1–17.

Black, F., and M. Scholes. 1973. The Pricing of Options and Corporate Liabilities. *The Journal of Political Economy* 81:637–654.

Bongaerts, D., F. De Jong, and J. Driessen. 2011. Derivative pricing with liquidity risk: Theory and evidence from the credit default swap market. *The Journal of Finance* 66:203–240.

Brennan, M. J., T. Chordia, and A. Subrahmanyam. 1998. Alternative factor specifications, security characteristics, and the cross-section of expected stock returns. *Journal of Financial Economics* 49:345–373.

Brennan, M. J., and A. Subrahmanyam. 1996. Market microstructure and asset pricing: On the compensation for illiquidity in stock returns. *Journal of Financial Economics* 41:441–464.

Brunnermeier, M. K., and L. H. Pedersen. 2009. Market liquidity and funding liquidity. *The Review of Financial Studies* 22:2201–2238.

Cakici, N., and A. Zaremba. 2021. Liquidity and the cross-section of international stock returns. *Journal of Banking & Finance* 127:106123.

Campbell, S. D., and F. X. Diebold. 2005. Weather forecasting for weather derivatives. *Journal of the American Statistical Association* 100:6–16.

Carhart, M. M. 1997. On persistence in mutual fund performance. *The Journal of Finance* 52:57–82.

Chung, K. H., and H. Zhang. 2014. A simple approximation of intraday spreads using daily data. *Journal of Financial Markets* 17:94–120.

Cox, J. C., J. E. Ingersoll Jr, and S. A. Ross. 1985. An intertemporal general equilibrium model of asset prices. *Econometrica: Journal of the Econometric Society* 53:363–384.

Datar, V. T., N. Y. Naik, and R. Radcliffe. 1998. Liquidity and stock returns: An alternative test. *Journal of Financial Markets* 1:203–219.

Engle, R. F., and A. J. Patton. 2004. Impacts of trades in an error-correction model of quote prices. *Journal of Financial Markets* 7:1–25.

Fama, E. F., and K. R. French. 1993. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics* 33:3–56.

Fama, E. F., and K. R. French. 2015. A five-factor asset pricing model. *Journal of Financial Economics* 116:1–22.

Garleanu, N., and L. H. Pedersen. 2007. Liquidity and risk management. *American Economic Review* 97:193–197.

Gibbons, M. R., S. A. Ross, and J. Shanken. 1989. A test of the efficiency of a given portfolio. *Econometrica: Journal of the Econometric Society* 57:1121–1152.

Golts, M., and M. Kritzman. 2010. Liquidity options. *The Journal of Derivatives* 18:80–89.

Gromb, D., and D. Vayanos. 2002. Equilibrium and welfare in markets with financially constrained arbitrageurs. *Journal of Financial Economics* 66:361–407.

Gromb, D., and D. Vayanos. 2010. Limits of arbitrage. *Annu. Rev. Financ. Econ.* 2:251–275.

Groß-KlußMann, A., and N. Hautsch. 2013. Predicting bid–ask spreads using long-memory autoregressive conditional Poisson models. *Journal of Forecasting* 32:724–742.

Grossman, S. J., and M. H. Miller. 1988. Liquidity and market structure. *the Journal of Finance* 43:617–633.

Hördahl, P., and D. Vestin. 2005. Interpreting implied risk-neutral densities: the role of risk premia. *Review of Finance* 9:97–137.

Kelly, B. T., S. Pruitt, and Y. Su. 2019. Characteristics are covariances: A unified model of risk and return. *Journal of Financial Economics* 134:501–524.

Kladívko, K. 2007. Maximum likelihood estimation of the Cox-Ingersoll-Ross process: the Matlab implementation. *Technical Computing Prague* 7.

Korajczyk, R. A., and R. Sadka. 2008. Pricing the commonality across alternative measures of liquidity. *Journal of Financial Economics* 87:45–72.

Longstaff, F. A. 1995. How much can marketability affect security values? *The Journal of Finance* 50:1767–1774.

Mahanti, S., A. Nashikkar, M. Subrahmanyam, G. Chacko, and G. Mallik. 2008. Latent liquidity: A new measure of liquidity, with an application to corporate bonds. *Journal of Financial Economics* 88:272–298.

Newey, W. K., and K. D. West. 1987. A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica* 55:703–708.

Pástor, L., and R. F. Stambaugh. 2003. Liquidity risk and expected stock returns. *Journal of Political Economy* 111:642–685.

Pelizzon, L., M. G. Subrahmanyam, D. Tomio, and J. Uno. 2016. Sovereign credit risk, liquidity, and European Central Bank intervention: Deus ex machina? *Journal of Financial Economics* 122:86–115.

Reinganum, M. R. 1990. Market microstructure and asset pricing: An empirical investigation of NYSE and NASDAQ securities. *Journal of Financial Economics* 28:127–147.

Savor, P., and M. Wilson. 2013. How much do investors care about macroeconomic risk? Evidence from scheduled economic announcements. *Journal of Financial and Quantitative Analysis* 48:343–375.

Savor, P., and M. Wilson. 2014. Asset pricing: A tale of two days. *Journal of Financial Economics* 113:171–201.

Schlag, C., J. Thimme, and R. Weber. 2021. Implied volatility duration: A measure for the timing of uncertainty resolution. *Journal of Financial Economics* 140:127–144.

Shleifer, A., and R. W. Vishny. 1997. The limits of arbitrage. *The Journal of Finance* 52:35–55.

Spiegel, M. I., and X. Wang. 2005. Cross-sectional variation in stock returns: Liquidity and idiosyncratic risk.

Stoll, H. R. 1978. The supply of dealer services in securities markets. *The Journal of Finance* 33:1133–1151.

Venkataraman, K., and A. C. Waisburd. 2007. The value of the designated market maker. *Journal of Financial and Quantitative Analysis* 42:735–758.

Appendix A: Proof of Proposition I

Proof. In the model, date 3 is a terminal condition for valuing the securities as of time 2 (see also Grossman and Miller, 1988). By backward induction, $p_2 = v_2$ and $\Lambda_2 = 0$, whence $\lambda_1 = 0$ follows, because the payoff associated with this price degenerates to zero. Price deviations from fundamentals arise at time $t = 1$ and $t = 0$, which implies that liquidity derivatives are traded at time $t = 0$ as customer $k = 0$ and speculators populate the marketplace, and settled at time $t = 1$. That is, an equilibrium is such that $\lambda_2 = \lambda_1 = 0$. Intuitively, illiquidity requires two rounds of backward induction to materialize, but the same implications ensue from any finite time model with $T \geq 3$. The solution rests on a recursive optimization argument. Throughout, a customer's value function is denoted Γ and a speculator's value function is denoted J.

Liquidity increases in the wealth of speculators. In fact, when speculators finances are unconstrained, illiquidity is zero. The basic illiquidity problem arises because speculators have funding constraints in the form of the VaR rules in Eq. (6) and (7). Thus, speculators cannot exploit all arbitrage opportunity, and have to cherry pick the most profitable investments. A speculator's shadow cost of capital in $t = 1$, denoted ϕ_1, is one plus the maximum profit per dollar invested.

$$\phi_1 = 1 + \max_j \left\{ \max\left(\frac{-\Lambda_1^j}{m_1^{j+}}, \frac{\Lambda_1^j}{m_1^{j-}} \right) \right\} \tag{A.1}$$

whence,

$$|\Lambda_1^j| = m_1^j(\phi_1 - 1) \tag{A.2}$$

at time $t = 0$, the speculator maximizes

$$\mathbb{E}_0[J_1(W_1, p_1, v_1, p_0, v_0)] = \mathbb{E}_0[W_1 \phi_1] \tag{A.3}$$

subject to the margin constraint. As in Brunnermeier and Pedersen (2009), we consider the case in which speculators are unconstrained at time $t = 0$. The first order condition for speculators' position in security j is $\mathbb{E}_0[\phi_1(p_1^j - p_0^j)] = 0$, and for a position in derivative written on j is $\mathbb{E}_0[\phi_1(\lambda_0^j - |\Lambda_1^j|)] = 0$. Therefore,

$$p_0^j = \frac{\mathbb{E}_0[\phi_1 p_1^j]}{\mathbb{E}_0[\phi_1]} = \mathbb{E}_0[p_1^j] + \frac{\mathrm{Cov}_0[\phi_1, p_1^j]}{\mathbb{E}_0[\phi_1]} \tag{A.4}$$

$$\lambda_0^j = \frac{\mathbb{E}_0[\phi_1 |\Lambda_1^j|]}{\mathbb{E}_0[\phi_1]} = \mathbb{E}_0[\Lambda_1^j] + \frac{\mathrm{Cov}_0[\phi_1, |\Lambda_1^j|]}{\mathbb{E}_0[\phi_1]} \tag{A.5}$$

During funding liquidity crises, ϕ_1 is higher than its expected value and prices deviate from fundamental values because speculators are constrained. Therefore, $|\Lambda_1|$ is linearly increasing in ϕ_1. As a result, arbitrageurs are always willing to pay a premium larger than the expected payoff to obtain insurance against fluctuations in illiquidity $\lambda_0^j \geq |\Lambda_1^j|$. Moreover, in equilibrium both assets are traded. By contradiction, suppose $\mathbb{E}_0[\phi_1(p_1^j - p_0^j)] > \mathbb{E}_0[\phi_1(\lambda_0^j - |\Lambda_1^j|)]$. Arbitrageurs would then buy stocks exerting upward pressure on p_0 until the equality is restored.

We have shown that liquidity derivatives only trade at time $t = 0$, therefore the customers' value function at time $t = 1$ is the same as in B&P, to which we direct the reader for a derivation.

$$\Gamma_1(W_1^k, p_1, v_1) = -\exp\left\{ -\gamma \left[W_1^k + \sum_j \frac{(v_1^j - p_1^j)^2}{2\gamma(\sigma_2^j)^2} \right] \right\} \tag{A.6}$$

At time $t = 0$, customer $k = 0$ arrives to the market and maximizes $\mathbb{E}_0[\Gamma_1(W_1^k, p_1, v_1)]$. Customers are price takers, so that their problem is equivalent to the maximization of expected wealth at $t = 1$,

$$\Gamma_1(W_1^k, p_1, v_1) = -\exp\{-\gamma W_1^k\} + \kappa \tag{A.7}$$

where κ is a constant term in $t = 0$ choice variables (y_0^k, c_0^k).[15] Thus, they solve

$$\begin{aligned} \max_{(y_0^k, c_0^k)_{j \in J}} \quad & -\mathbb{E}_0[\exp\{-\gamma W_1^k\}] \\ & = -\exp\{-\gamma \mathbb{E}_0[W_1^k] - \frac{\gamma}{2}\text{Var}_0[W_1^k]\} \end{aligned} \tag{A.8}$$

replacing Equation (1) into Equation (3), we get

$$\mathbb{E}_0[W_1^k] = W_0^k + (\mathbb{E}_0 \mathbf{v}_1 - \mathbf{p}_0)'(\mathbf{y}_0^k + \mathbf{z}_k) + (\boldsymbol{\lambda}_0 - \mathbb{E}_0|\boldsymbol{\Lambda}_1|)\mathbf{c}_0^k \tag{A.9}$$

$$\text{Var}_0[W_1^k] = (\mathbf{y}_0^k + \mathbf{z}_k)^2 \boldsymbol{\sigma}_1^2 + (\mathbf{c}_0^k)^2 \text{Var}_0(|\Lambda_1^j|) \tag{A.10}$$

where $\boldsymbol{\sigma}_1 = \text{diag}(\sigma_1^1, \ldots, \sigma_1^J)$. From Equation (A.9) it is clear that customers trade the stock based on their expectations of the future fundamental value of the asset relative to current price levels, and the derivative based on their expectations about future deviations of price from fundamentals. We obtain the solution to the customer's problem.

$$y_0^{j,k} = \frac{v_0^j - p_0^j}{\gamma(\sigma_1^j)^2} - z^{j,k} \tag{A.11}$$

$$c_0^{j,k} = \frac{\lambda_0^j - \mathbb{E}_0|\Lambda_1^j|}{\gamma \text{Var}_0(|\Lambda_1^j|)} \tag{A.12}$$

Importantly, the two instruments load on different risk factors. The stock market is a market for the fundamental value, while in the market for liquidity derivatives, the randomness is represented by liquidity risk. The attractiveness of the stock revolves around the endowment shock. Customers view liquidity derivatives as a speculative instrument, and are willing to provide insurance to arbitrageurs as long as they are offered larger premium than the expected future payoff. As is standard, the amount of insurance the customers offer decreases with their risk aversion and the variance of illiquidity.

[15] To see this, either note that prices are state variables of the problem or solve the plain vanilla optimization at $t = 2$.

From the interplay between equilibrium prices and market clearing conditions, the customers and speculators equilibrium holdings are, respectively

$$y_0^{j,k} = -\frac{\mathrm{Cov}_0[\phi_1, p_1^j]/\mathbb{E}_0[\phi_1]}{\gamma(\sigma_1^j)^2} - z^{j,k} \qquad c_0^{j,k} = \frac{\mathrm{Cov}_0[\phi_1, |\Lambda_1^j|]/\mathbb{E}_0[\phi_1]}{\gamma \mathrm{Var}_0(|\Lambda_1|)} \tag{A.13}$$

$$x_0 = -y_0^k \qquad c_0 = c_0^{j,k} \tag{A.14}$$

In terms of welfare, notice that the introduction of liquidity derivatives does not change the value function of speculators. On the other hand, the value function of the customer Γ_0 increases because of the expanded possibility frontier.

$$\Gamma_0(W_0^k, p_0, v_0) = \max_{(y_0^k, c_0^k)_{j\in J}} -\exp\{-\gamma \mathbb{E}_0[W_1^k] - \frac{\gamma}{2}\mathrm{Var}_0[W_1^k]\} + \kappa \tag{A.15}$$

$$= -\exp\Bigg\{-\gamma\Bigg[W_0^k + \frac{1}{2\gamma}\sum_{j\in J}\frac{(v_0^j - \mathbb{E}_0 p_1^j)^2}{(\sigma_1^j)^2} + \frac{1}{2\gamma}\sum_{j\in J}\frac{(\lambda_0^j - \mathbb{E}_0|\Lambda_1^j|)^2}{(\mathrm{Var}_0(|\Lambda_1^j|))^2}\Bigg]\Bigg\} + \kappa$$

The first two terms in square brackets correspond to the value function in the B&P model, whilst the last addendum is non negative and specific to a market for liquidity derivatives. We have formally shown that for all agents contracts on liquidity derivatives do not decrease welfare, and for some agents, namely customers, a market for liquidity improves welfare.

■

Appendix B: Proof of Proposition II

Proof. First, note that in equilibrium the speculator is always long in the derivative $c_0 > 0$. To simplify notation, we prove the statement for the case of $J = 1$ assets. Consider the case $Z_1 > 0$, implying $p_1 \leq v_1$, $\Lambda_1 < 0$, and $x_1 \geq 0$. When the funding constraint binds,

$$m_1^+\left(Z_1 - \frac{2}{\gamma(\sigma_2)^2}(v_1 - p_1)\right) + n_1^+ l_1 = b_0 + p_1 x_0 + c_0|\Lambda_1| + \eta_1 \tag{B.1}$$

Combining the implicit function theorem with the market clearing conditions:

$$\frac{\partial m_1^+}{\partial p_1}\frac{\partial p_1}{\partial \eta_1}x_1 + m_1^+\frac{2}{\gamma(\sigma_2)^2}\frac{\partial p_1}{\partial \eta_1} + \frac{\partial n_1^+}{\partial p_1}\frac{\partial p_1}{\partial \eta_1}c_0 = \frac{\partial p_1}{\partial \eta_1}x_0 + \frac{\partial|\Lambda_1|}{\partial p_1}\frac{\partial p_1}{\partial \eta_1}c_0 + 1 \tag{B.2}$$

It is useful to recall that $\frac{\partial|\Lambda_1|}{\partial p_1} = \frac{|\Lambda_1|}{\Lambda_1}$. After rearranging terms,

$$\frac{\partial p_1}{\partial \eta_1} = \frac{1}{\frac{2}{\gamma(\sigma_2)^2}m_1^+ + \frac{\partial m_1^+}{\partial p_1}x_1 + \frac{\partial n_1^+}{\partial p_1}c_0 - x_0 + c_0} \tag{B.3}$$

Liquidity derivatives counter both margin and loss spirals. Indeed, when financiers are uninformed about the fundamental value of the security the impact of the term $\frac{\partial m_1^+}{\partial p_1}x_1 < 0$ giving rise to a margin spiral is mitigated by that of $\frac{\partial n_1^+}{\partial p_1}c_0 > 0$. Intuitively, while on the one hand financiers require more skin in the game when observing prices moving further below fundamentals, the collateral value of the derivative position profiting from illiquidity increases. Speculators further encounter a loss spiral if x_0 is of the same sign as x_1, because price drops are accompanied by losses on the previous positions. The latter effect is offset by a long position in the liquidity derivative.

The converse case $Z_1 < 0$, implying $p_1 \geq v_1$, $\Lambda_1 > 0$, and $x_1 \leq 0$ is analogous. We have

$$\frac{\partial p_1}{\partial \eta_1} = \frac{-1}{\frac{2}{\gamma(\sigma_2)^2} m_1^- + \frac{\partial m_1^-}{\partial p_1} x_1 + \frac{\partial n_1^+}{\partial p_1} c_0 + x_0 + c_0} \tag{B.4}$$

When financiers are uninformed, the term $\frac{\partial m_1^-}{\partial p_1} x_1 < 0$, which gives rise to a margin spiral, is attenuated by $\frac{\partial n_1^+}{\partial p_1} c_0 > 0$. The speculator who was previously short-selling the stock faces a loss spiral when upward price movements drive prices further away from fundamentals. It is immediate to observe that losses on initial positions are balanced by larger benefits from the derivatives position.

■

Appendix C: Sample Selection

The sample selection closely follows Amihud (2002). We select stocks from CRSP restricting to ordinary common shares (first digit of CRSP code is 1) of non-financial firms (SIC-code excluding the interval $[6000, 6999]$). In every year, only stocks with available closing price for more than 200 days in that year and with price greater than \$5 at the end of the year are included, to avoid that "penny stocks" drive the results. The existence of a lower bound imposed by the SEC to the bid-ask spread would make estimation more noisy for such firms (Amihud, 2002). Finally, all firms must have data on market capitalization at the end of the previous year. This excludes derivatives like American Depositary Receipts of foreign stocks and scores and primes.

Because of better reporting quality regarding bid and ask quotes, we then use data from Bloomberg, including returns and accounting measures. Daily observations where the absolute spread is non-positive or greater than 5\$ are deleted (Chung and Zhang, 2014; Korajczyk and Sadka, 2008). We drop stale data points about bid and ask quotes: if the same ask and bid appear for more than 5 days in a row, only the first 5 observations are kept. If the daily closing price is missing, the midquote is used instead. Volume is winsorized at the 1^{st} and 99^{th} percentile of its cross-sectional distribution in each year.

Liquidity option prices are simulated for firms with less than 100 missing data for the relative spread in each year t. This is a lower bound that ensures reliability of parameter estimates. LOPs equal to zero and prices with negative MLE estimates of σ in Eq. (13) are deleted. Although we impose a high level of precision in our numerical procedure (100,000 Monte Carlo simulations), economically non-meaningful results due to purely computational limits can still occur. Moreover, we drop prices for which the Akaike Information Criterion (AIC) relative to the CIR parameter estimation exceeds the 99^{th} percentile of each year to ensure data are of sound quality. Finally, prices are trimmed at the 99th percentile of the distribution of the corresponding year, and multiplied by 10000 to obtain values in basis points (bps) to improve readability. Whenever a price is lower than 1 bps (8029 observations), it is set to 1 bps to make it a meaningful value.

Appendix D: Liquidity-hedged Positions

We describe here in detail the computation of returns used in Section 4.3. We focus on 3-month returns to match the time to maturity of the 3-months European liquidity options we simulate. For each stock, the raw (non-hedged) return between time t and $t+3$, where t represents months, is simply the percentage change between the dividend-adjusted prices, i.e. $r_{t+3} = p_{t+3}/p_t - 1$. We build a hedged position such that the initial investment is still p_t to make the two strategies comparable. This is achieved by selling a part of the stock with equal value to the price of one liquidity option. Since option prices are expressed in bps of the mid-price at t, m_t, one needs to account for potential differences between p_t and m_t. Let C_t be the price of a 3-month option, now expressed in decimals of m_t. A hedged position ζ_t has initial value

$$\zeta_t = p_t \left(1 - \frac{m_t}{p_t} C_t\right) + p_t \left(\frac{m_t}{p_t} C_t\right) = p_t \tag{D.1}$$

Multiplying C_t by the ratio m_t/p_t converts it into dollars and ensures $\zeta_t = p_t$ even with $p_t \neq m_t$. At time $t+3$, i.e. at the maturity of the option, the investor obtains the corresponding payoff X_{t+3}, which is also expressed in bps per unit of notional. Hence, the value of the position at $t+3$ will be

$$\zeta_{t+3} = p_{t+3} \left(1 - \frac{m_t}{p_t} C_t\right) + m_t X_{t+3} \tag{D.2}$$

The ratio of ζ_{t+3} to p_t minus 1 represents the return of a hedged position, which can be compared with r_{t+3}. Hypothesis 2 is tested after subtracting the risk-free rate such that we compare investments financed by borrowing p_t.

Figures

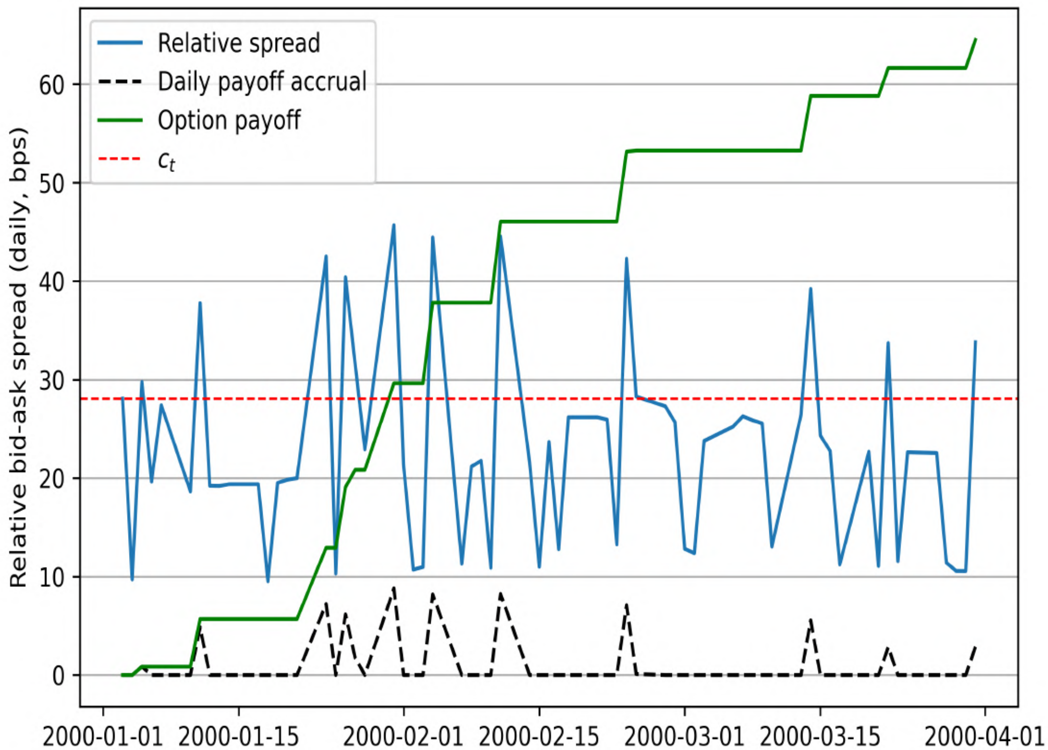

Figure 1: **Example of liquidity option payoff**
This Figure shows the payoff of a liquidity option on the relative spread of Walmart Inc for the first quarter of 2000. c_t represents the relative spread at the initiation of the contract. The daily payoff accrual is one half the difference between the relative spread and c_t, when it is positive, and zero otherwise. The option payoff is the sum of the daily payoff accruals at the maturity.

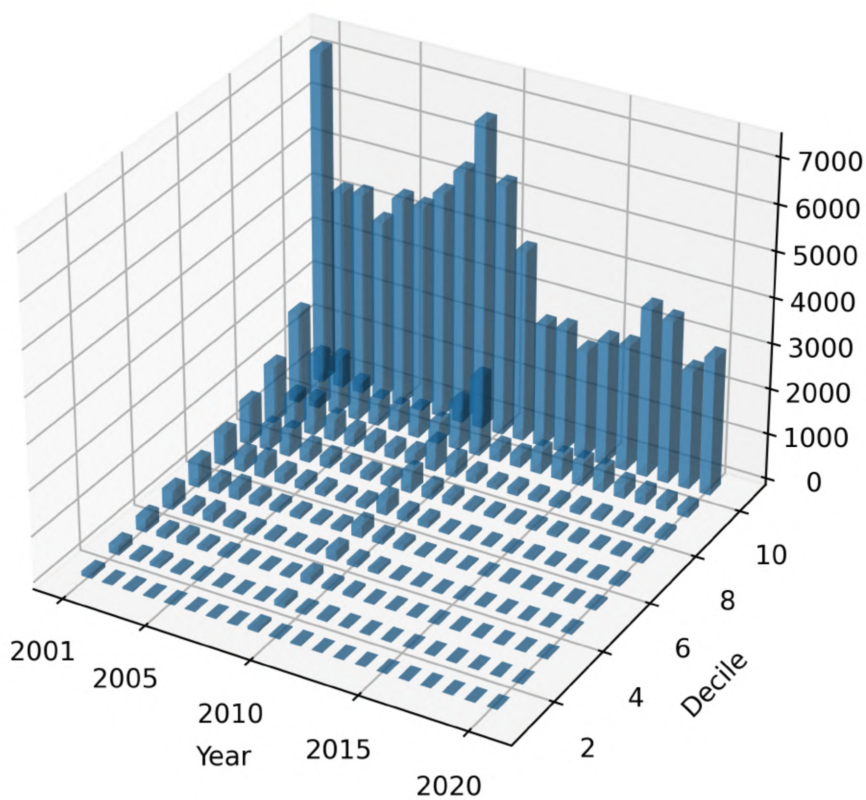

FIGURE 2: **Liquidity option prices distribution**
This Figure shows yearly cross-sectional liquidity option prices deciles (bps) from 2001 to 2020.

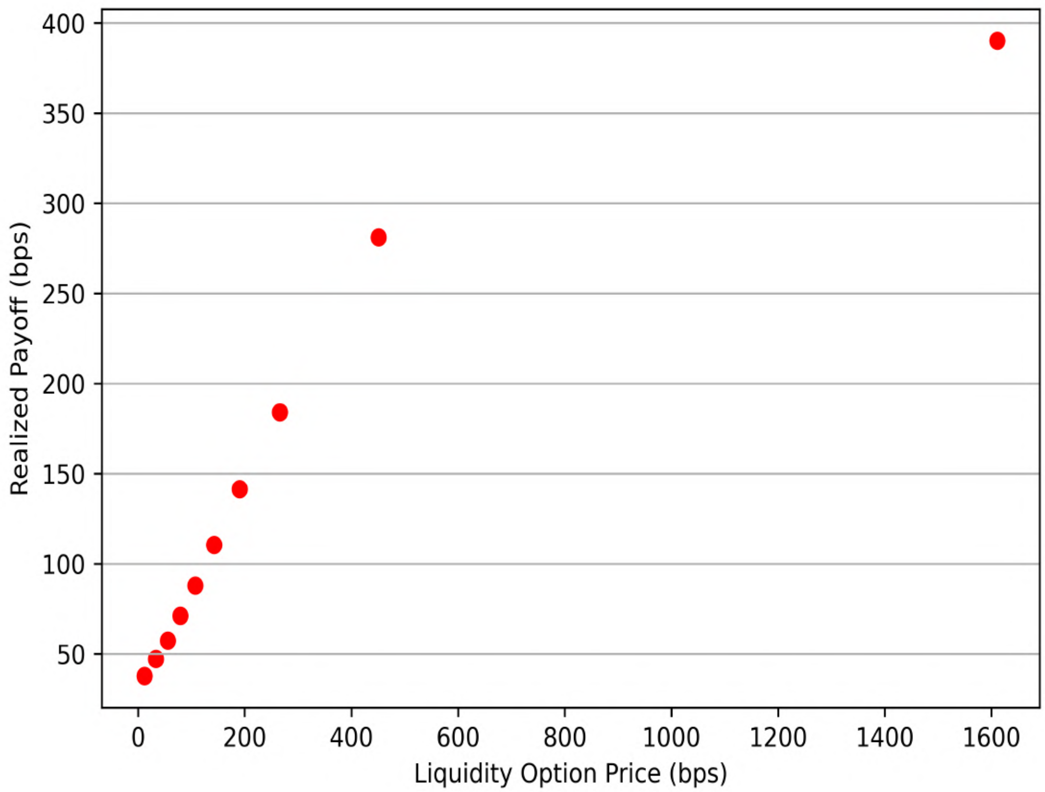

Figure 3: **Liquidity option prices vs Realized payoffs**
This Figure shows average liquidity option prices (bps, x-axis) against average realized payoffs (y-axis) for 10 equal-weighted portfolios sorted on option prices. Data from January 2001 to December 2020.

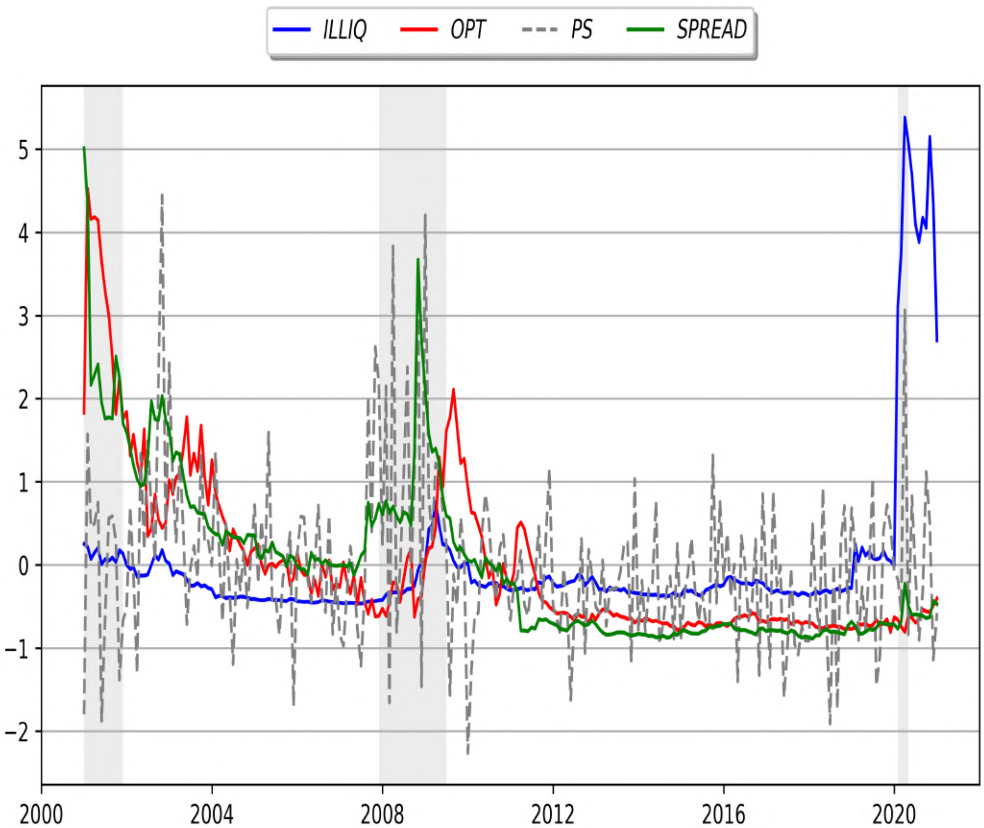

FIGURE 4: **Comparison of liquidity measures**
This Figure shows market-wide illiquidity measures from December 2000 to December 2020. $ILLIQ$ refers to Amihud (2002). OPT is the monthly cross-sectional median option price. PS is the aggregate liquidity measure from Pástor and Stambaugh (2003), with flipped sign. $SPREAD$ is the monthly cross-sectional median relative bid-ask spread. Series are standardized. Grey shaded areas represent NBER recessions.

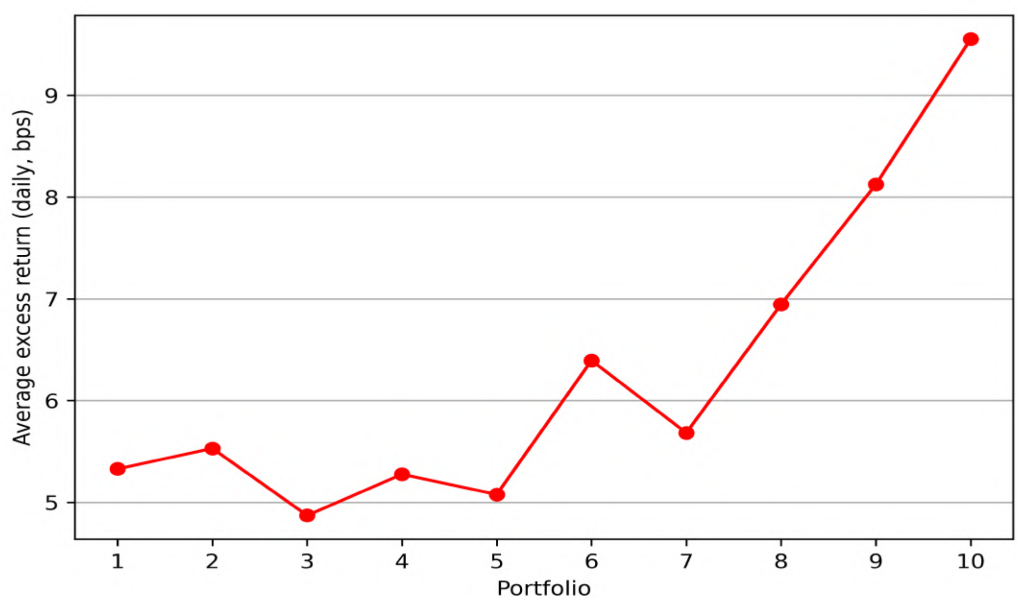

FIGURE 5: **Average excess returns: 10 equal-weighted portfolios**
This Figure shows the average excess returns of 10-equal weighted portfolios of stocks sorted on liquidity option prices on a monthly basis. Daily data from January 2001 to December 2020.

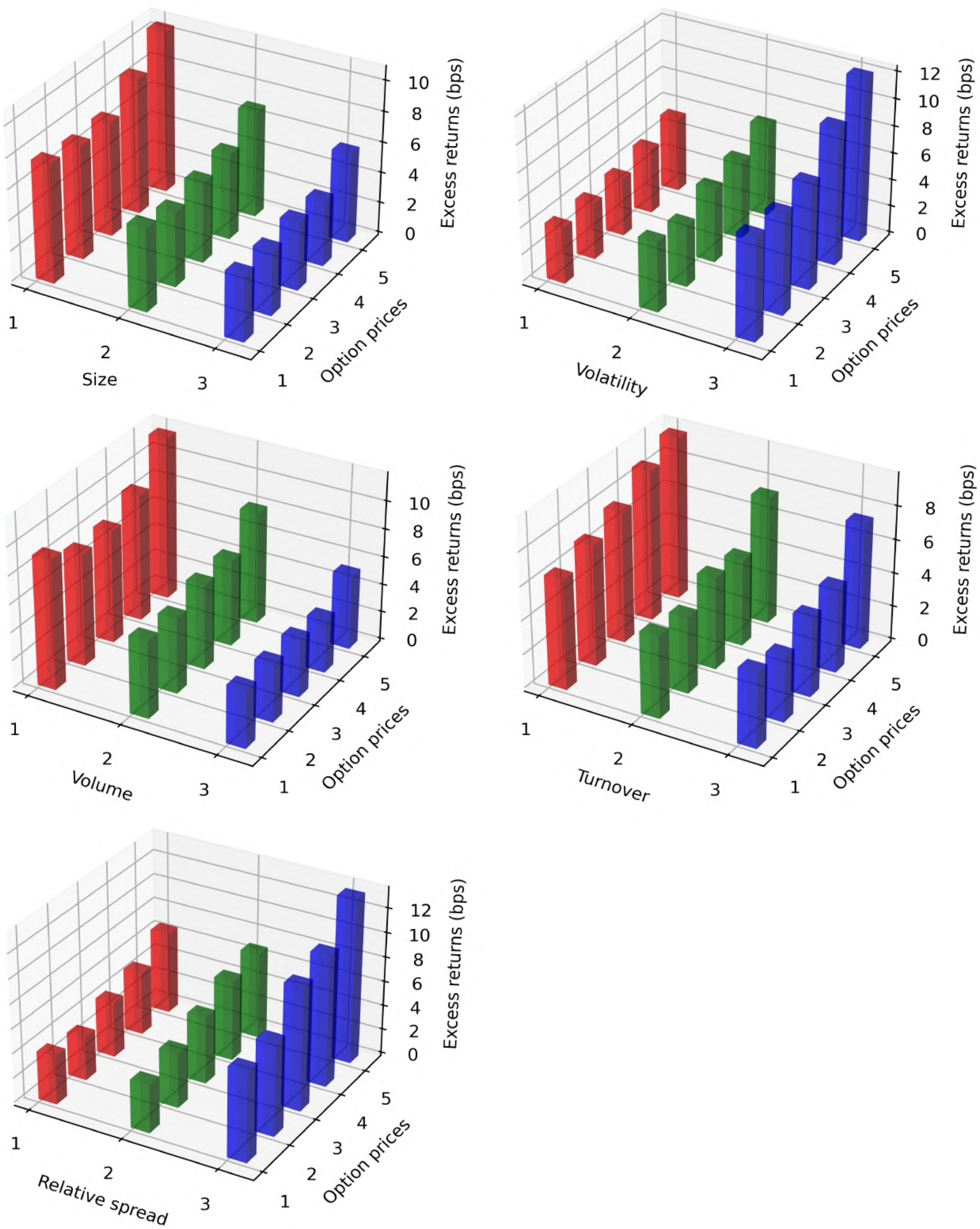

Figure 6: **Average excess returns: conditional double-sorted portfolios**
This Figure shows average excess returns of conditional double-sorted portfolios. Portfolios are formed on a monthly basis by sorting stocks first into terciles based on size (top-left panel), volatility (top-right), dollar volume (middle-left), turnover (middle-right) or relative bid-ask spread (bottom-left) and then into quintiles based on liquidity option prices for each tercile. Daily data from January 2001 to December 2020.

Tables

Table 1: **Summary statistics**

This Table reports summary statistics of the sample consisting of CRSP stocks traded on the NYSE with data from Bloomberg from January, 2000 to December, 2020. Percentages correspond to distribution percentiles for each one of the variables on the columns. *Market Cap.* is market capitalization expressed in million of dollars. *Volume* represents daily traded volume in millions of dollars. *Rel. spread* is the ratio of the difference between the best bid and ask quotes to the midquote, measured in basis points. $ILLIQ_{i,t}$ refers to the Amihud (2002) illiquidity measure at the stock level and monthly frequency. $C_{i,t}$ denotes liquidity option prices expressed in basis points per units of notional for a contract maturity of three months. $m_{i,t}$ is the midquote. *Volatility* is the monthly standard deviation of stock returns. *Turnover* is the ratio of dollar volume to market capitalization (Datar et al., 1998).

							Percentiles						
	N	Mean	SD	Skewness	Min	Max	1%	5%	25%	50%	75%	95%	99%
Market Cap.	241401	10734	29439	6.96	1.07	580934	53.03	160.17	821.91	2444.45	7998.35	43749	160282
Volume	6276160	20.74	48.02	7.71	0.00	5097.77	0.01	0.09	1.29	5.50	19.19	90.16	232.31
Rel. Spread	6277269	34.90	183.90	23.44	0.02	16471	0.75	1.38	3.75	9.11	22.23	105.26	437.96
$ILLIQ_{i,t}$	290608	0.1465	1.6788	20.3975	0.00	45.6963	0.00	0.0001	0.0007	0.0025	0.0122	0.2081	1.5712
$C_{i,t}$	192746	279.72	611.51	4.45	1.00	7974.33	1.00	1.98	20.64	67.38	233.59	1325.58	3318.96
$m_{i,t}$	6277269	41.07	77.96	19.29	0.09	6421.76	2.01	4.94	14.18	25.63	45.57	114.17	269.05
Volatility	289183	0.02	0.26	482.57	0.01	134.29	0.01	0.01	0.01	0.02	0.03	0.05	0.09
Turnover	241261	0.37	0.58	19.41	0.00	55.06	0.01	0.03	0.14	0.24	0.43	1.07	2.25

Table 2: **Liquidity factor loadings: raw and hedged portfolios**

This Table presents absolute mean loadings on Pástor and Stambaugh (2003) liquidity factor in a multifactor model including Fama and French (1993) plus momentum for 10 equal-weighted portfolios sorted on liquidity option prices. Loadings refer to raw returns and returns hedged with liquidity options for each stock in each portfolio. The second-to-last column reports the p-value for the null hypothesis that the mean absolute exposure of raw position is less than or equal to than that of a hedged position, from a t-test for difference in means that allows for unequal variances based on rolling estimates following the approach in Savor and Wilson (2013, 2014). The last columns shows the p-value for the null hypothesis that the mean change in liquidity exposures achieved with liquidity options is less than or equal to zero. Data from January 2001 to December 2020.

Portfolio number	Absolute mean exposure		p-value difference in means	p-value mean change
	Raw	Hedged		
1	0.0223	0.0053	0.004	0.000
2	0.0351	0.0198	0.000	0.000
3	0.0025	0.0214	0.962	0.004
4	0.0004	0.0151	0.991	0.000
5	0.0207	0.0040	0.022	0.000
6	0.0241	0.0010	0.000	0.000
7	0.0013	0.0104	0.799	0.000
8	0.0514	0.0359	0.003	0.000
9	0.0274	0.0037	0.002	0.997
10	0.0494	0.0024	0.001	0.000
Mean	0.0235	0.0119		

Table 3: **Alphas and R^2 of equal-weighted portfolios sorted on liquidity option prices**

This Table shows alphas (bps) and R^2 for CAPM, Fama and French (1993) (FF3), FF3 plus the Momentum factor (Carhart, 1997) (FF4), Fama and French (2015) (FF5) and FF5 plus Momentum (FF6) factor models tested on 10 equal-weighted portfolios sorted on liquidity option prices at the end of each month. The top and the bottom panel present daily and monthly data, respectively, from January 2001 to December 2020. $t-$ statistics in brackets are obtained by using Newey and West (1987) standard errors with 5 lags

	Decile portfolio											
	1	2	3	4	5	6	7	8	9	10	10-1	R^2
	Daily data											
CAPM alpha	1.79	2.13	1.36	1.63	1.67	2.77	2.02	3.36	4.49	5.91	4.12	84.53
	(2.32)	(2.89)	(1.84)	(2.23)	(2.28)	(3.78)	(2.61)	(4.04)	(4.74)	(6.04)	(5.18)	
FF3 alpha	1.42	1.81	1.00	1.28	1.26	2.29	1.41	2.66	3.71	5.31	3.89	92.23
	(2.39)	(3.18)	(1.79)	(2.28)	(2.39)	(4.36)	(2.79)	(5.24)	(6.39)	(7.76)	(5.00)	
FF4 alpha	1.58	1.96	1.08	1.36	1.29	2.29	1.35	2.60	3.58	5.32	3.74	92.40
	(2.75)	(3.49)	(1.90)	(2.41)	(2.41)	(4.34)	(2.67)	(5.14)	(6.27)	(7.71)	(4.79)	
FF5 alpha	0.91	1.22	0.28	0.61	0.64	1.74	0.89	2.23	3.26	4.87	3.96	92.83
	(1.57)	(2.24)	(0.53)	(1.17)	(1.29)	(3.49)	(1.85)	(4.53)	(5.76)	(7.17)	(5.10)	
FF6 alpha	0.95	1.25	0.30	0.64	0.65	1.75	0.89	2.22	3.24	4.88	3.93	93.09
	(1.75)	(2.42)	(0.57)	(1.24)	(1.32)	(3.51)	(1.84)	(4.52)	(5.78)	(7.23)	(5.09)	
	Monthly data											
CAPM alpha	25.57	33.84	19.24	25.29	26.87	50.04	33.65	64.16	85.96	116.89	91.32	80.37
	(1.43)	(2.07)	(1.12)	(1.46)	(1.54)	(2.60)	(1.61)	(3.08)	(3.87)	(3.91)	(5.02)	
FF3 alpha	20.51	29.84	15.14	21.86	21.29	43.30	24.85	52.65	75.52	109.31	88.81	89.68
	(1.58)	(2.83)	(1.37)	(1.67)	(2.07)	(3.38)	(2.05)	(4.75)	(6.18)	(4.96)	(4.14)	
FF4 alpha	27.34	32.75	16.36	23.56	22.87	45.19	28.02	53.60	74.60	111.60	84.26	89.90
	(2.12)	(3.21)	(1.52)	(1.67)	(2.23)	(3.39)	(2.22)	(4.25)	(5.12)	(4.64)	(3.77)	
FF5 alpha	2.30	13.96	-1.56	-2.36	4.06	22.75	11.92	37.06	56.99	86.59	84.28	90.93
	(0.20)	(1.35)	(-0.15)	(-0.19)	(0.41)	(2.01)	(1.00)	(3.65)	(4.83)	(4.52)	(4.16)	
FF6 alpha	2.81	14.23	-1.38	-2.11	4.26	22.99	12.18	37.21	57.06	86.86	84.06	91.58
	(0.27)	(1.49)	(-0.13)	(-0.17)	(0.45)	(2.1)	(1.05)	(3.63)	(4.81)	(4.72)	(4.07)	

Table 4: **Normalized average portfolio characteristics**

This Table presents average firm characteristics for 10 equal-weighted portfolios sorted on liquidity option prices. Average characteristics are normalized into the $[0, 1]$ interval. Size is the log market equity. Volatility is the monthly standard deviation of stock returns. Volume is the trade volume in million of dollars. Turnover is the ratio of dollar volume to market capitalization (Datar et al., 1998). The relative bid-ask spread is the bid-ask spread divided by the midquote. Data from January 2001 to December 2020.

Portfolio number	C_t	Size	Volatility	Volume	Turnover	Relative spread
1	0.000	0.943	0.153	0.921	0.241	0.082
2	0.013	1.000	0.047	1.000	0.268	0.031
3	0.027	0.989	0.000	0.939	0.501	0.006
4	0.042	0.917	0.044	0.805	0.637	0.000
5	0.060	0.781	0.108	0.590	0.749	0.035
6	0.082	0.628	0.220	0.429	0.974	0.091
7	0.112	0.453	0.381	0.250	1.000	0.170
8	0.159	0.225	0.672	0.096	0.920	0.310
9	0.275	0.000	1.000	0.000	0.243	0.624
10	1.000	0.257	0.812	0.267	0.000	1.000

Table 5: **Average excess returns of conditional double-sorted portfolios (daily)**

This Table presents average excess returns for conditional double-sorted portfolios formed at the end of each month by sorting stocks first on the left-hand variable and then on liquidity option prices. Size refers to market capitalization. Volatility is the monthly standard deviation of stock returns. Volume is the trade volume in dollars. Turnover is the ratio of dollar volume to market capitalization (Datar et al., 1998). The relative bid-ask spread is the bid-ask spread divided by the midquote. The second-to-last column reports in brackets the *t*-statistic for the average returns of LOP-long-short portfolio within each tercile of the conditioning variable. Daily data from January 2001 to December 2020.

	Liquidity Option Price						
	Cheap	2	3	4	Expensive	5-1	All
Size							
Small	7.71	7.41	7.47	8.93	10.69	2.98 (2.69)	8.08
Medium	5.41	4.88	5.26	5.62	6.98	1.56 (1.99)	5.77
Large	4.26	4.20	4.44	4.26	5.86	1.60 (2.33)	4.83
Volatility							
Stable	4.14	4.17	4.29	4.81	5.52	1.38 (2.72)	4.77
Medium	5.16	4.55	5.48	5.81	6.82	1.66 (2.35)	5.79
Volatile	7.59	7.41	7.80	10.14	12.12	4.53 (3.58)	8.77
Volume							
Infrequent	9.26	8.03	8.09	9.14	11.76	2.50 (2.31)	8.22
Medium	5.55	5.40	6.11	6.18	8.07	2.52 (2.86)	6.33
Frequent	4.15	4.14	4.12	3.82	5.31	1.16 (1.49)	4.48
Turnover							
Low	6.52	7.16	7.81	9.00	9.75	3.23 (3.46)	7.94
Medium	4.85	4.53	5.46	5.19	7.58	2.72 (3.50)	5.74
High	4.42	3.86	4.71	5.01	7.47	3.05 (2.95)	5.01
Relative bid-ask spread							
Low	3.95	3.55	4.52	5.16	6.69	2.74 (3.21)	4.95
Medium	3.94	4.49	5.45	6.55	6.99	3.04 (3.19)	5.55
High	7.32	7.53	10.06	10.71	13.42	6.11 (5.08)	8.53
All	5.49	5.01	5.74	6.35	8.82	3.32 (4.88)	

Table 6: **Average excess returns of conditional double-sorted portfolios (monthly)**

This Table presents average excess returns for conditional double-sorted portfolios formed at the end of each month by sorting stocks first on the left-hand variable and then on liquidity option prices. Size refers to market capitalization. Volatility is the monthly standard deviation of stock returns. Volume is the trade volume in dollars. Turnover is the ratio of dollar volume to market capitalization (Datar et al., 1998). The relative bid-ask spread is the bid-ask spread divided by the midquote. The second-to-last column reports in brackets the t-statistic for the average returns of LOP-long-short portfolio within each tercile of the conditioning variable. Monthly data from January 2001 to December 2020.

	Liquidity Option Price						
	Cheap	2	3	4	Expensive	5-1	All
Size							
Small	148.29	142.76	154.19	177.12	220.14	71.85 (3.07)	160.91
Medium	104.33	93.64	101.93	112.13	135.68	31.35 (1.98)	112.73
Large	82.78	78.47	85.14	80.48	115.92	33.14 (2.17)	93.26
Volatility							
Stable	80.70	83.14	83.82	96.39	111.76	31.07 (3.09)	95.20
Medium	98.79	87.52	105.46	114.11	133.22	34.44 (2.74)	114.02
Volatile	141.04	141.00	151.23	201.61	253.45	112.41 (3.83)	169.16
Volume							
Infrequent	187.73	162.76	171.92	185.27	252.85	65.12 (2.43)	163.41
Medium	105.21	108.98	121.23	115.98	156.99	51.78 (2.92)	123.15
Frequent	80.06	74.83	75.00	74.16	99.68	19.61 (1.24)	85.36
Turnover							
Low	128.48	141.60	154.62	187.79	198.70	70.22 (3.78)	158.21
Medium	92.27	87.37	104.37	102.36	152.95	60.68 (3.51)	112.77
High	81.99	68.67	87.41	95.83	146.27	64.29 (3.04)	95.86
Relative bid-ask spread							
Low	71.40	79.07	82.53	67.96	97.53	26.14 (2.45)	84.97
Medium	102.30	94.01	107.02	117.87	113.08	10.78 (0.65)	111.79
High	197.44	207.65	201.69	217.38	288.92	91.48 (3.06)	175.13
All	104.90	96.78	111.30	125.47	179.87	74.96 (5.05)	

Table 7: GRS statistic and R^2 of conditional double-sorted portfolios (daily)

This Table reports GRS F-statistic (Gibbons et al., 1989) and R^2 for CAPM, Fama and French (1993) (FF3), FF3 plus the Momentum factor (Carhart, 1997) (FF4), Fama and French (2015) (FF5) and FF5 plus Momentum (FF6) factor models tested on cross-sections of 3x5 equal-weighted (conditionally) double-sorted portfolios. Alphas are reported for long-short portfolios for each tercile of the conditioning variable. $t-$ statistics in brackets are obtained using Newey and West (1987) standard errors with 5 lags. Daily data from January 2001 to December 2020.

	Bivariate portfolios		Long-short portfolio alphas		
	GRS statistic	R^2 on the PFs (%)	$1_5 - 1_1$	$2_5 - 2_1$	$3_5 - 3_1$
		Size x Option price			
CAPM	3.16	79.55	3.27	1.58	1.47
			(3.13)	(2.05)	(2.16)
FF3	6.10	90.05	3.27	1.54	1.41
			(3.13)	(1.99)	(2.09)
FF4	6.09	90.23	2.97	1.35	1.29
			(2.91)	(1.75)	(1.9)
FF5	5.30	90.76	3.43	1.72	1.54
			(3.31)	(2.24)	(2.29)
FF6	5.33	91.05	3.37	1.68	1.52
			(3.37)	(2.23)	(2.27)
		Volatility x Option price			
CAPM	4.47	79.98	1.37	1.85	4.86
			(2.70)	(2.81)	(3.82)
FF3	6.29	88.65	1.14	1.60	4.59
			(2.47)	(2.51)	(3.63)
FF4	6.21	89.55	1.12	1.45	4.08
			(2.41)	(2.30)	(3.39)
FF5	5.59	89.27	1.35	1.80	4.33
			(2.93)	(2.80)	(3.45)
FF6	5.59	90.16	1.34	1.77	4.25
			(2.91)	(2.81)	(3.57)
		Volume x Option price			
CAPM	4.12	79.03	2.73	2.41	0.89
			(2.61)	(2.81)	(1.19)
FF3	8.52	89.64	2.73	2.22	0.73
			(2.61)	(2.62)	(0.99)
FF4	8.74	89.94	2.41	1.94	0.51
			(2.39)	(2.35)	(0.71)
FF5	7.83	90.22	2.81	2.33	1.07
			(2.72)	(2.75)	(1.47)
FF6	8.00	90.62	2.75	2.28	1.03
			(2.76)	(2.8)	(1.48)
		Turnover x Option price			
CAPM	5.23	81.50	3.26	2.52	2.90
			(3.62)	(3.24)	(2.92)
FF3	7.50	88.90	2.92	2.16	2.58
			(3.49)	(2.98)	(2.65)
FF4	7.54	89.09	2.75	2.02	2.28
			(3.28)	(2.83)	(2.41)
FF5	6.75	89.61	2.88	2.23	2.69
			(3.45)	(3.07)	(2.77)
FF6	6.81	89.91	2.85	2.21	2.63
			(3.40)	(3.11)	(2.81)
		Relative spread x Option price			
CAPM	5.58	79.40	2.14	2.59	6.39
			(2.84)	(2.99)	(5.28)
FF3	8.00	89.15	1.62	2.06	6.19
			(2.49)	(2.74)	(5.14)
FF4	8.25	89.54	1.52	1.93	5.96
			(2.32)	(2.56)	(4.91)
FF5	7.40	89.70	1.87	2.26	6.41
			(2.87)	(3.01)	(5.33)
FF6	7.60	90.20	1.85	2.24	6.37
			(2.89)	(3.00)	(5.29)

Table 8: **GRS statistic and R^2 of conditional double-sorted portfolios (monthly)**

This Table reports GRS F-statistic (Gibbons et al., 1989) and R^2 for CAPM, Fama and French (1993) (FF3), FF3 plus the Momentum factor (Carhart, 1997) (FF4), Fama and French (2015) (FF5) and FF5 plus Momentum (FF6) factor models tested on cross-sections of 3x5 equal-weighted (conditionally) double-sorted portfolios. Alphas are reported for long-short portfolios for each tercile of the conditioning variable. $t-$ statistics in brackets are obtained using Newey and West (1987) standard errors with 5 lags. Monthly data from January 2001 to December 2020.

	Bivariate portfolios		Long-short portfolios alpha		
	GRS statistic	R^2 on the PFs (%)	$1_5 - 1_1$	$2_5 - 2_1$	$3_5 - 3_1$
			Size x Option price		
CAPM	2.95	75.01	83.70	35.05	28.33
			(3.88)	(2.36)	(1.92)
FF3	4.48	87.45	85.33	35.85	25.47
			(4.00)	(2.35)	(1.75)
FF4	4.59	87.79	73.94	34.21	27.12
			(3.48)	(1.99)	(1.53)
FF5	3.27	88.60	91.12	42.69	33.42
			(3.77)	(2.78)	(2.22)
FF6	3.34	89.43	90.45	42.55	33.46
			(3.69)	(2.75)	(2.20)
			Volatility x Option price		
CAPM	3.12	75.37	26.58	39.27	122.43
			(2.58)	(3.83)	(3.65)
FF3	4.41	85.38	21.68	36.39	121.42
			(2.66)	(3.79)	(3.60)
FF4	4.28	86.96	20.13	32.60	105.20
			(2.58)	(3.49)	(3.16)
FF5	3.10	86.46	22.45	35.14	108.95
			(2.69)	(3.77)	(3.37)
FF6	3.10	88.52	22.36	34.93	108.12
			(2.79)	(3.85)	(3.27)
			Volume x Option price		
CAPM	2.93	74.54	73.88	53.83	15.03
			(3.15)	(3.06)	(1.00)
FF3	4.32	85.81	75.36	51.67	11.73
			(3.07)	(2.94)	(0.80)
FF4	4.74	86.49	61.67	46.52	8.92
			(2.58)	(2.38)	(0.55)
FF5	3.18	87.20	72.71	57.76	18.62
			(2.91)	(3.36)	(1.26)
FF6	3.43	88.52	71.96	57.43	18.42
			(2.73)	(3.31)	(1.29)
			Turnover x Option price		
CAPM	4.25	75.83	66.81	55.25	66.09
			(3.28)	(2.98)	(3.39)
FF3	4.97	84.89	61.72	49.96	62.99
			(3.54)	(3.33)	(3.43)
FF4	5.54	85.73	58.51	46.70	56.22
			(3.06)	(2.8)	(2.87)
FF5	4.22	85.96	59.82	50.06	61.41
			(3.49)	(3.30)	(3.02)
FF6	4.46	87.32	59.65	49.88	61.04
			(3.45)	(3.28)	(3.03)
			Relative spread x Option price		
CAPM	3.93	72.44	21.40	19.78	106.45
			(2.09)	(1.37)	(3.62)
FF3	4.60	83.77	18.05	18.85	106.14
			(1.77)	(1.41)	(3.59)
FF4	4.77	85.17	14.89	8.35	91.53
			(1.46)	(0.69)	(3.28)
FF5	3.40	85.11	20.17	16.73	102.24
			(2.00)	(1.03)	(3.56)
FF6	3.53	87.30	19.98	16.15	101.44
			(2.08)	(1.2)	(3.53)