THE BOND MARKET'S q^*

Thomas Philippon

I propose an implementation of the q-theory of investment using bond prices instead of equity prices. Credit risk makes corporate bond prices sensitive to future asset values, and q can be inferred from bond prices. With aggregate U.S. data, the bond market's q fits the investment equation six times better than the usual measure of q, it drives out cash flows, and it reduces the implied adjustment costs by more than an order of magnitude. Theoretical interpretations for these results are discussed.

I. Introduction

In his 1969 article, James Tobin argued that "the rate of investment—the speed at which investors wish to increase the capital stock—should be related, if to anything, to q, the value of capital relative to its replacement cost" (Tobin 1969, p. 21). Tobin also recognized, however, that q must depend on "expectations, estimates of risk, attitudes towards risk, and a host of other factors," and he concluded that "it is not to be expected that the essential impact of [. . .] financial events will be easy to measure in the absence of direct observation of the relevant variables (q in the models)." The quest for an observable proxy for q was therefore recognized as a crucial objective from the very beginning.

Subsequent research succeeded in integrating Tobin's approach with the neoclassical investment theory of Jorgenson (1963). Lucas and Prescott (1971) proposed a dynamic model of investment with convex adjustment costs, and Abel (1979) showed that the rate of investment is optimal when the marginal cost of installment is equal to $q - 1$. Finally, Hayashi (1982) showed that, under perfect competition and constant returns to scale, marginal q (the market value of an *additional unit* of capital divided by its replacement cost) is equal to average q (the market value of *existing* capital divided by its replacement cost). Because average q is observable, the theory became empirically relevant.

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Unfortunately, its implementation proved disappointing. The investment equation fits poorly, leaves large unexplained residuals correlated with cash flows, and implies implausible parameters for the adjustment cost function (see Summers [1981] for an early contribution, and Hassett and Hubbard [1997] and Caballero [1999] for recent literature reviews).

Several theories have been proposed to explain this failure. Firms could have market power, and might not operate under constant returns to scale. Adjustment costs might not be convex (Dixit and Pindyck 1994; Caballero and Engle 1999). Firms might be credit-constrained (Fazzari, Hubbard, and Petersen 1988; Bernanke and Gertler 1989). Finally, there could be measurement errors and aggregation biases in the capital stock or the rate of investment. None of these explanations is fully satisfactory, however. The evidence for constant returns and price-taking seems quite strong (Hall 2003). Adjustment costs are certainly not convex at the plant level, but it is not clear that it really matters in the aggregate (Thomas 2002; Hall 2004), although this is still a controversial issue (Bachmann, Caballero, and Engel 2006). Gomes (2001) shows that Tobin's q should capture most of investment dynamics even when there are credit constraints. Heterogeneity and aggregation do not seem to create strong biases (Hall 2004).

In fact, an intriguing message comes out of the more recent empirical research: the market value of equity seems to be the culprit for the empirical failure of the investment equation. Gilchrist and Himmelberg (1995), following Abel and Blanchard (1986), use VARs to forecast cash flows and to construct q, and they find that it performs better than the traditional measure based on equity prices. Cumins, Hasset, and Oliner (2006) use analysts' forecasts instead of VAR forecasts and reach similar conclusions. Erickson and Whited (2000, 2006) use GMM estimators to purge q from measurement errors. They find that only 40% of observed variations are due to fundamental changes, and, once again, that market values contain large "measurement errors."

Applied research has therefore reached an uncomfortable situation, where the benchmark investment equation appears to be successful only when market prices are not used to construct q. This is unfortunate, because Tobin's insight was precisely to link observed quantities and market prices. The contribution of this paper is to show that a market-based measure of q can be constructed from corporate bond prices and that this measure performs much better than the traditional one.

Why would the bond market's q perform better than the usual measure? There are several possible explanations, two of which are discussed in details in this paper. The first explanation is that total firm value includes the value of growth options, that is, opportunities to expand into new areas and new technologies. With enough skewness, these growth options end up affecting equity prices much more than bond prices. If, in addition, these growth options are unrelated to existing operations, they do not affect current capital expenditures. As a result, bond prices are more closely related to the existing technology's q, while equity prices reflect organizational rents.

A second possible explanation is that the bond market is less susceptible to bubbles than the equity market. In fact, there is empirical and theoretical support for the idea that mispricing is more likely to happen when returns are positively skewed. Barberis and Huang (2007) show that cumulative prospect theory can explain how a positively skewed security becomes overpriced. Brunnermeier, Gollier, and Parker (2007) argue that preference for skewness arises endogenously because investors choose to be optimistic about the states associated with the most skewed Arrow–Debreu securities. Empirically, Mitton and Vorkink (2007) document that underdiversification is largely explained by the fact that investors sacrifice mean–variance efficiency for higher skewness exposure. These insights, combined with the work of Stein (1996) and Gilchrist, Himmelberg, and Huberman (2005) showing why rational managers might not react (or, at least, not much) to asset bubbles, provide another class of explanations.[1]

Of course, even if we accept the idea that bond prices are somehow more reliable than equity prices, it is far from obvious that it is actually *possible* to use bond prices to construct q. The contribution of this paper is precisely to show how one can do so, by combining the insights of Black and Scholes (1973) and Merton (1974) with the approach of Abel (1979) and Hayashi (1982). In the Black–Scholes–Merton model, debt and equity are seen as derivatives of the underlying assets. In the simplest case, the market value of corporate debt is a function of its face value, asset

1. Other rational explanations can also be proposed. These explanations typically involve different degrees of asymmetric information, market segmentation, and heterogeneity in adjustment costs and stochastic processes. For instance, firms might be reluctant to use equity to finance capital expenditures, because of adverse selection, in which case the bond market might provide a better measure of investment opportunities (Myers 1984). It is much too early at this stage to take a stand on which explanations are most relevant.

volatility, and asset value. But one can also invert the function, so that, given asset volatility and the face value of debt, one can construct an estimate of asset value from observed bond prices. I extend this logic to the case where asset value is endogenously determined by capital expenditures decisions.

As in Hayashi (1982), I assume constant returns to scale, perfect competition, and convex adjustment costs. There are no taxes and no bankruptcy costs, so the Modigliani–Miller theorem holds, and real investment decisions are independent from capital structure decisions.[2] Firms issue long-term, coupon-paying bonds as in Leland (1998), and the default boundary is endogenously determined to maximize equity value, as in Leland and Toft (1996). There are two crucial differences between my model and the usual asset pricing models. First, physical assets change over time. Under constant returns to scale, however, I obtain tractable pricing formulas, where the usual variables are simply scaled by the book value of assets. Thus, book leverage plays the role of the face value of principal outstanding, and q plays the role of total asset value. The second difference is that cash flows are endogenous, because they depend on adjustment costs and investment decisions.

I model an economy with a continuum of firms hit by aggregate and idiosyncratic shocks. Even though default is a discrete event at the firm level, the aggregate default rate is a continuous function of the state of the economy. To build economic intuition, I consider first a simple example with one-period debt, constant risk-free rates, and i.i.d. firm-level shocks. I find that, to first order (i.e., for small aggregate shocks), Tobin's q is a linear function of the spread of corporate bonds over government bonds. The sensitivity of q to bond spreads depends on the risk-neutral default rate, just like the delta of an option in the Black–Scholes formula. In the general case, I choose the parameters of the model to match aggregate and firm level dynamics, estimated with postwar U.S. data. Given book leverage and idiosyncratic volatility, the model produces a nonlinear mapping from bond prices to q.

I then use the theoretical mapping to construct a time series for q based on the relative prices of corporate and government bonds, taking into account trends in book leverage and

2. One could introduce taxes and bankruptcy costs if one wanted to derive an optimal capital structure, but this is not the focus of this paper. See Hackbarth, Miao, and Morellec (2006) for such an analysis, with a focus on macroeconomic risk.

idiosyncractic risk, as well as changes in real risk-free rates. This bond market's q fits the investment equation quite well with postwar aggregate U.S. data. The R^2 is around 60%, cash flows become insignificant, and the implied adjustment costs are more than an order of magnitude smaller than with the usual measure of q. The fit is as good in levels as in differences. The theoretical predictions for the roles of leverage and volatility are supported by the data, as well as the nonlinearities implied by the model.

Using simulations, I find that the predictions of the model are robust to specification errors, as well as to taxes and bankruptcy costs. The theoretical predictions for firm level dynamics are consistent with the empirical results of Gilchrist and Zakrajsek (2007), who show that firm-specific interest rates forecast firm-level investment.

The remainder of the paper is organized as follows. Section II presents the setup of the model. Section III uses a simple example to build economic intuition. Section IV presents the numerical solution for the general case. Section V presents the evidence for aggregate U.S. data. Section VI discusses the theoretical interpretations of the results. Section VII discusses the robustness of the results to various changes in the specification of the model. Section VIII concludes.

II. Model

II.A. Firm Value and Investment

Time is discrete and runs from $t = 0$ to ∞. The production technology has constant returns to scale and all markets are perfectly competitive. All factors of production, except physical capital, can be freely adjusted within each period. Physical capital is predetermined in period t and, to make this clear, I denote it by k_{t-1}. Once other inputs have been chosen optimally, the firm's profits are therefore equal to $p_t k_{t-1}$, where p_t is the exogenous profit rate in period t. Let the function $\Gamma(k_{t-1}, k_t)$ capture the total cost of adjusting the level of capital from k_{t-1} to k_t. For convenience, I include depreciation in the function Γ, and I assume that it is homogeneous of degree one, as in Hayashi (1982).[3]

3. For instance, the often-used case of quadratic adjustment costs corresponds to $\Gamma(k_t, k_{t+1}) = k_{t+1} - (1-d)k_t + 0.5\gamma_2(k_{t+1} - k_t)^2/k_t$, where d is the depreciation rate, and γ_2 is a constant that pins down the curvature of the adjustment cost function.

Let r_t be the one-period real interest rate, and let $E^{\pi}[.]$ denote expectations under the risk-neutral probability measure π.[4] The state of the firm at time t is characterized by the endogenous state variable k_t and a vector of exogenous state variables ω_t, which follows a Markov process under π. The profit rate and the risk-free rate are functions of ω_t. The value of the firm solves the Bellman equation,

$$V(k_{t-1}, \omega_t) = \max_{k_t \geq 0} \left\{ p(\omega_t) k_{t-1} - \Gamma(k_{t-1}, k_t) + \frac{E^{\pi}[V(k_t, \omega_{t+1}) | \omega_t]}{1 + r(\omega_t)} \right\}. \tag{1}$$

Because the technology exhibits constant returns to scale, it is convenient to work with the scaled value function,

$$v_t \equiv \frac{V_t}{k_{t-1}}. \tag{2}$$

Similarly, define the growth of k as $x_t \equiv k_t / k_{t-1}$. After dividing both sides of equation (1) by k_{t-1}, and using the shortcut notation ω' for ω_{t+1}, we obtain

$$v(\omega) = \max_{x \geq 0} \left\{ p(\omega) - \gamma(x) + \frac{x}{1 + r(\omega)} E^{\pi}[v(\omega') | \omega] \right\}, \tag{3}$$

where γ is the renormalized version of Γ. The function γ is assumed to be convex and to satisfy $\lim_{x \to 0} \gamma(x) = \infty$ and $\lim_{x \to \infty} \gamma(x) = \infty$. The optimal investment rate $x(\omega)$ solves

$$\frac{\partial \gamma}{\partial x}(x(\omega)) = q(\omega) \equiv \frac{E^{\pi}[v(\omega') | \omega]}{1 + r(\omega)}. \tag{4}$$

Equation (4) defines the q-theory of investment: it says that the marginal cost of investment is equal to the expected discounted marginal product of capital. The most important practical issue is the construction of the right-hand side of equation (4).

II.B. Measuring q

The value of the firm is the value of its debt plus the value of its equity. Let B_t be the market values of the bonds outstanding

4. This is equivalent to using a pricing kernel, but it simplifies the notations and the algebra. If m' is the pricing kernel between states ω and ω', then for any random variable z', $E[m'z' | \omega] = E^{\pi}[z' | \omega] / (1 + r(\omega))$. It is crucial to account for risk premia in any case. Berndt et al. (2005) show that objective probabilities of default are much smaller than risk-adjusted probabilities of default. Lettau and Ludvigson (2002) also emphasize the role of time-varying risk premia.

at the end of period t, and define b_t as the value scaled by end-of-period physical assets

$$b_t \equiv \frac{B_t}{k_t}. \tag{5}$$

Similarly, let $e(\omega)$ be the ex-dividend value of equity, scaled by end-of-period assets. Then q is simply

$$q(\omega) = e(\omega) + b(\omega). \tag{6}$$

The most natural way to test the q-theory of investment is therefore to use equation (6) to construct the right-hand side of equation (4). Unfortunately, it fits poorly in practice (Summers 1981; Hassett and Hubbard 1997; Caballero 1999). Equation (6) has been estimated using aggregate and firm-level data, in levels or in first differences, with or without debt on the right-hand side. It leaves large unexplained residuals correlated with cash flows, and it implies implausible values for the adjustment cost function $\gamma(x)$.

As argued in the Introduction, there are potential explanations for this empirical failure, but none is really satisfactory. Moreover, a common finding of the recent research is that "measurement errors" in equity seem to be responsible for the failure of q-theory (Gilchrist and Himmelberg 1995; Erickson and Whited 2000, 2006; Cumins, Hasset, and Oliner 2006). I do not attempt in this paper to explain the meaning of these "measurement errors." I simply argue that, even if equity prices do not provide a good measure of q, it is still possible to construct another one using observed bond prices.

II.C. Corporate Debt

I assume that there are no taxes and no deadweight losses from financial distress. The Modigliani–Miller theorem implies that leverage policy does not affect firm value or investment. Leverage does affect bond prices, however, and I must specify debt dynamics before I can use bond prices to estimate q. The model used here belongs to the class of structural models of debt with endogenous default boundary. In this class of models, default is chosen endogenously to maximize equity value (see Leland [2004] for an illuminating discussion).

There are many different types of long-term liabilities, and my goal here is not to study all of them, but rather to focus on a tractable model of long-term debt. To do so, I use a version of the exponential model introduced in Leland (1994), and used by Leland (1998) and Hackbarth, Miao, and Morellec (2006), among others. In this model, the firm continuously issues and retires bonds. Specifically, a fraction ϕ of the remaining principal is called at par every period. The retired bonds are replaced by new ones. To understand the timing of cash flows, consider a bond with coupon c and principal normalized to 1, issued at the end of period t. The promised cash flows for this particular bond are as follows:

$$\begin{array}{llllll} t+1 & t+2 & \ldots & \tau & & \ldots \\ c+\phi & (1-\phi)(c+\phi) & \ldots & (1-\phi)^{\tau-t-1}(c+\phi) & & \ldots \end{array}$$

Let $\Psi_{\tau-1}$ be the sum of the face values of all the bonds outstanding at the beginning of period τ. I use the index $\tau-1$ to make clear that this variable, just like physical capital, is predetermined at the beginning of each period. The timing of events in each period is the following:

1. The firm enters period τ with capital $k_{\tau-1}$ and total face value of outstanding bonds $\Psi_{\tau-1}$.
2. The state variable ω_τ is realized. The value of the firm is then $V_\tau = v_\tau k_{\tau-1}$, defined in equations (1) and (3).
   (a) If equity value falls to zero, the firm defaults and the bond holders recover V_τ.
   (b) Otherwise, the bond holders receive cash flows $(c+\phi)\,\Psi_{\tau-1}$.
3. At the end of period τ, the capital stock is k_τ, the face value of the bonds (including newly issued ones) is Ψ_τ, and their market value is $B_\tau = b_\tau k_\tau$. New issuances represent a principal of $\Psi_\tau - (1-\phi)\,\Psi_{\tau-1}$.

In Leland (1994) and Leland (1998), book assets are constant, because there is no physical investment, and the firm simply chooses a constant face value Ψ. In my setup, the corresponding assumption is that the firm chooses a constant book leverage ratio. In the theoretical analysis, I therefore maintain the following assumption:

ASSUMPTION. Firms keep a constant book leverage ratio: $\psi \equiv \Psi_t/k_t$.

A bond issued at the end of period t has a remaining face value of $(1-\phi)^{\tau-t-1}$ at the beginning of period τ. In case of default during period τ, all bonds are treated similarly and the bond issued at time t receives $(1-\phi)^{\tau-t-1}V_\tau/\Psi_{\tau-1}$. Because all outstanding bonds are treated similarly in case of default, we can characterize the price without specifying when this principal was issued. The following proposition characterizes the debt pricing function.

PROPOSITION 1. The scaled value of corporate debt solves the equation

$$b(\omega) = \frac{1}{1+r(\omega)} E^\pi[\min\{(c+\phi)\psi + (1-\phi)b(\omega'); v(\omega')\}|\omega]. \tag{7}$$

Proof. See Appendix. ∎

The intuition behind equation (7) is relatively simple. Default happens when equity value falls to zero, that is, when $v-(c+\phi)\psi-(1-\phi)b=0$. There are no deadweight losses and bondholders simply recover the value of the company. When there is no default, bondholders receive the cash flows $(c+\phi)\psi$ and they own $(1-\phi)$ remaining bonds. A few special cases are worth pointing out. Short-term debt corresponds to $\phi=1$ and $c=0$, and the pricing function is simply

$$b^{\text{short}}(\omega) = \frac{1}{1+r(\omega)} E^\pi[\min(\psi; v(\omega'))|\omega]. \tag{8}$$

The main difference between short- and long-term debt is the presence of the pricing function b on both sides of equation (7), whereas it appears only on the left-hand side in equation (8). A perpetuity corresponds to $\phi=0$, and, more generally, $1/\phi$ is the average maturity of the debt. The value of a default-free bond with the same coupon and maturity structure would be

$$b^{\text{free}}(\omega) = \frac{(c+\phi)\psi + (1-\phi)E^\pi[b^{\text{free}}(\omega')|\omega]}{1+r(\omega)}. \tag{9}$$

With a constant risk-free rate, b^{free} is simply equal to $(c+\phi)\psi/(\phi+r)$.

III. SIMPLE EXAMPLE

This section presents a simple example in order to build intuition for the more general case. The specific assumptions made in this section, and relaxed later, are that the risk-free rate is constant; firms issue only short-term debt; and idiosyncratic shocks are i.i.d. Let us first decompose the state ω into its aggregate component s, and its idiosyncratic component η. The aggregate state follows a discrete Markov chain over the set $[1, 2, \ldots, S]$, and it pins down the aggregate profit rate $a(s)$, as well as the conditional risk-neutral expectations. The profit rate of the firm depends on the aggregate state and on the idiosyncratic shock:

$$p(s, \eta) = a(s) + \eta. \tag{10}$$

The shocks η are independent over time, and distributed according to the density function $\zeta(.)$. Because idiosyncratic profitability shocks are i.i.d., the value function is additive and can be written $v(s, \eta) = v(s) + \eta$. I assume that s and η are such that $v(s, \eta)$ is always positive, so that firms never exit. Tobin's q is the same for all firms, and I normalize the mean of η to zero; therefore,

$$q(s) = \frac{E^{\pi}[v(s')|s]}{1+r}. \tag{11}$$

Let $\bar{v} \equiv E^{\pi}[v(s)]$ be the unconditional risk-neutral average asset value, and define $q \equiv \bar{v}/(1+r)$. All the firms choose the same investment rate in this simple example. This will not be true in the general model with persistent idiosyncratic shocks.

We can write the value of the aggregate portfolio of corporate bonds by integrating (8) over idiosyncratic shocks:

$$b(s) = \frac{1}{1+r} E^{\pi}\left[\psi + \int_{-\infty}^{\psi - v(s')} (v(s') + \eta' - \psi)\zeta(\eta')\, d\eta' | s\right]. \tag{12}$$

In equation (12), ψ is the promised payment, and the integral measures credit losses. Let δ be the default rate estimated at the risk-neutral average value

$$\delta \equiv \int_{-\infty}^{\psi - \bar{v}} \zeta(\eta')\, d\eta'. \tag{13}$$

Let $\bar{b} \equiv (\psi + \int_{-\infty}^{\psi - \bar{v}} (\bar{v} + \eta' - \psi)\zeta(\eta')\, d\eta')/(1+r)$ be the corresponding price for the aggregate bond portfolio. Using (13) and (11), we can write (12) as

$$b(s) - \bar{b} = \delta(q(s) - q) + \frac{E^{\pi}[o(v')]}{1+r}, \tag{14}$$

where $o(v') \equiv \int_{\psi - \bar{v}}^{\psi - v'} (v' + \eta' - \psi)\zeta(\eta')\, d\eta'$ is first-order small, in the sense that $o(\bar{v}) = 0$ and $\partial o / \partial v' = 0$ when evaluated at $\bar{v}$. When aggregate shocks are small, so that v stays relatively close to $\bar{v}$, $E^{\pi}[o(v')]$ is negligible.

Equation (14) is the equivalent of the Black–Scholes–Merton formula, applied to Tobin's q. The value of the option (debt) depends on the value of the underlying (q), and the delta of the option is the probability of default. If this probability is exactly zero, bond prices do not contain information about q. The fact that the sensitivity of b to q is given by δ is intuitive. Indeed, b responds to q precisely because a fraction δ of firms default on average each period. A one-unit move in aggregate q therefore translates into a δ move in the price of a diversified portfolio of bonds.

To make equation (14) empirically relevant, we need to express it in terms of bond yields. All the prices we have discussed so far are in real terms, but, in practice, we observe nominal yields. Let $r^{\$}$ be the nominal risk-free rate, and let $y^{\$}$ be the nominal yield on corporate bonds. With short-term debt, the market value is equal to the nominal face value divided by $1 + y^{\$}$. Under the assumption we have made in this section, and neglecting the terms that are first-order small, a simple manipulation of equation (14) leads to the following proposition.

PROPOSITION 2. To a first-order approximation, Tobin's q is a linear function of the relative yields of corporate and government bonds,

$$q_t \approx \frac{\psi}{\delta(1+r)} \frac{1+r_t^{\$}}{1+y_t^{\$}} + \text{constant}, \tag{15}$$

where r is the real risk-free rate, ψ is average book leverage, and δ is the risk-neutral default rate.

The proposition sheds light on existing empirical studies, such as Bernanke (1983), Stock and Watson (1989), and Lettau and Ludvigson (2002), showing that the spread of corporate bonds over government bonds predicts future output.[5] This finding is consistent with q-theory, because the proposition shows that corporate bond spreads are, to first order, *proportional* to Tobin's q.

5. In the proposition, I use the relative bond price (the ratio) instead of the spread (the difference) because this is more accurate when inflation is high. The approximation of small aggregate shocks made in this section refers to real shocks, but does not require average inflation to be small.

IV. Long-Term Debt and Persistent Idiosyncratic Shocks

I now consider the case of long-term debt and persistent firm-level shocks. The goal is to obtain a mapping from bond yields to Tobin's q that extends the simple case presented above. As in the previous section, let s denote the aggregate state and let η denote the idiosyncratic component of the profit rate, defined in equation (10). With persistent idiosyncratic shocks, Tobin's q and the investment rate depend on both s and η, and the value function is no longer additively separable. There is no closed-form solution for bond prices, and the approximation of Proposition 2 is cumbersome because of the fixed point problem in equation (7). I therefore turn directly to numerical simulations. I maintain for now the assumptions of a constant risk-free rate r and of constant book leverage ψ. I use a quadratic adjustment cost function:

$$\gamma(x) = \gamma_1 x + 0.5\gamma_2 x^2. \tag{16}$$

With this functional form, the investment equation is simply $x = (q - \gamma_1)/\gamma_2$. Idiosyncratic profitability is assumed to follow an AR(1) process:

$$\eta_t = \rho_\eta \eta_{t-1} + \sigma_\eta \varepsilon_t^\eta. \tag{17}$$

Similarly, I specify aggregate dynamics as

$$a_t - \bar{a} = \rho_a(a_{t-1} - \bar{a}) + \sigma_a \varepsilon_t^a. \tag{18}$$

The shocks $\{\varepsilon_t^\eta\}_{\eta\in[0,1]}$ and ε_t^a follow independent normal distributions with zero mean and unit variance. The results discussed below are based on the following parameters:

r	ψ	ϕ	γ_1	γ_2	ρ_η	σ_η	ρ_a	σ_a	$\bar{a}/r$	c
3%	0.45	0.1	1	10	0.47	14%	0.7	4.5%	0.925	4.3%

Book leverage is set to 0.45 and average debt maturity to ten years ($\phi = 0.1$), based on Leland (2004), who uses these values as benchmarks for Baa bonds. The parameter γ_1 is irrelevant and is normalized to one in this section. There is much disagreement about the parameter γ_2 in the literature. Shapiro (1986) estimates a value of around 2.2 years, and Hall (2004) finds even smaller adjustment costs.[6] On the other hand, Gilchrist and Himmelberg

6. Shapiro (1986) estimates between 8 and 9 using quarterly data, which corresponds to 2 to 2.2 at annual frequencies.

(1995) find values of around twenty years, and estimates from macro data are often implausibly high (Summers 1981). I pick a value of $\gamma_2 = 10$ years, which is in the middle of the set of existing estimates. It turns out, however, that the mapping from bond yields to q is not very sensitive to this parameter. The parameters of equations (17) and (18) are calibrated using U.S. firm and aggregate data, as explained in Section V. Finally, the coupon rate c is chosen so that bonds are issued at par value, as in Leland (1998).

We can now use the model to understand the relationship between bond prices and Tobin's q. The main idea of the paper is to use the price of corporate bonds relative to Treasury to construct a measure of q. The model is simulated with the parameters just described. The processes (17) and (18) are approximated with discrete-state Markov chains using the method in Tauchen (1986). The investment rate $x(s, \eta)$ and the value of the firm value $v(s, \eta)$ are obtained by solving the dynamic programming problem in equation (3). Equation (7) is then used to compute the bond pricing function $b(s, \eta)$. The aggregate bond price $b(s)$ and the aggregate corporate yield $y(s)$ are obtained by integrating over the ergodic distribution of η.

Figure I presents the main result. It shows the model-implied aggregate $q(s)$ as a function of the model-implied average relative bond price $(\phi + r)/(\phi + y(s))$. Figure I is generated by considering all the possible values of the aggregate state variable s. Tobin's q is an increasing and convex function of the relative price of corporate bonds. Figure I therefore extends Proposition 2 to the case of long-term debt, persistent firm-level shocks, and large aggregate shocks.

The mapping from bond yields to Tobin's q is conditional on the calibrated parameters, in particular on book leverage and idiosyncratic volatility. Figure II shows the comparative statics with respect to book leverage (ψ) and firm volatility (σ_η). The comparative statics is intuitive. For a given value of q, an increase in leverage leads to more credit risk and lower bond prices, so the mapping shifts left when leverage increases. Similarly, for a given value of q, an increase in idiosyncratic volatility increases credit risk, and the mapping shifts left when volatility increases. In this case, the slope and the curvature of the mapping also change, and the intuition is given by Proposition 2: idiosyncratic volatility increases the delta of the bond with respect to q.

In the next section, mappings like the ones displayed in Figure II are used to construct a new measure of q from observed

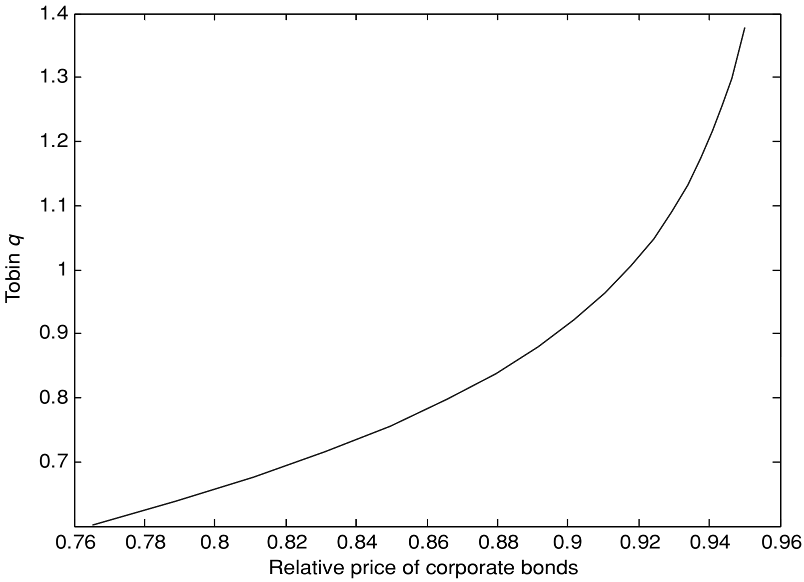

FIGURE I

Aggregate Tobin's q and the Relative Price of Corporate Bonds

The figure shows the implicit mapping between average bond prices and q across aggregate states (with different aggregate profit rates). The price of corporate bonds relative to risk-free bonds is defined as $(0.1 + r)/(0.1 + y)$, where r is the risk-free rate and y is the average yield on corporate bonds. The factor 0.1 reflects the average maturity of 10 years. The mapping is for benchmark values of book leverage, idiosyncratic volatility, and a constant risk-free rate of 3% (see Table II).

bond yields, leverage, and volatility. With respect to leverage, it is important to emphasize the role played by the maintained assumptions of no taxes and no bankruptcy costs. These assumptions imply that capital structure is irrelevant for real decisions (i.e., investment) and for firm value (Modigliani and Miller 1958). Leverage is relevant for bond pricing, however. Bond prices depend on leverage in the same way that they do in the model of Merton (1974): higher leverage increases default risk and therefore decreases the relative price of corporate bonds. Thus, it is crucial to use a mapping that is conditional on leverage to recover the correct value of q. To see why, imagine a world where firms choose their leverage to stabilize their credit spreads. In this case, the correlation between spreads and investment could be arbitrarily small. This would not invalidate the construction of q, however, because the explanatory power would then come from observed changes in leverage. In terms of Figure II, firms would

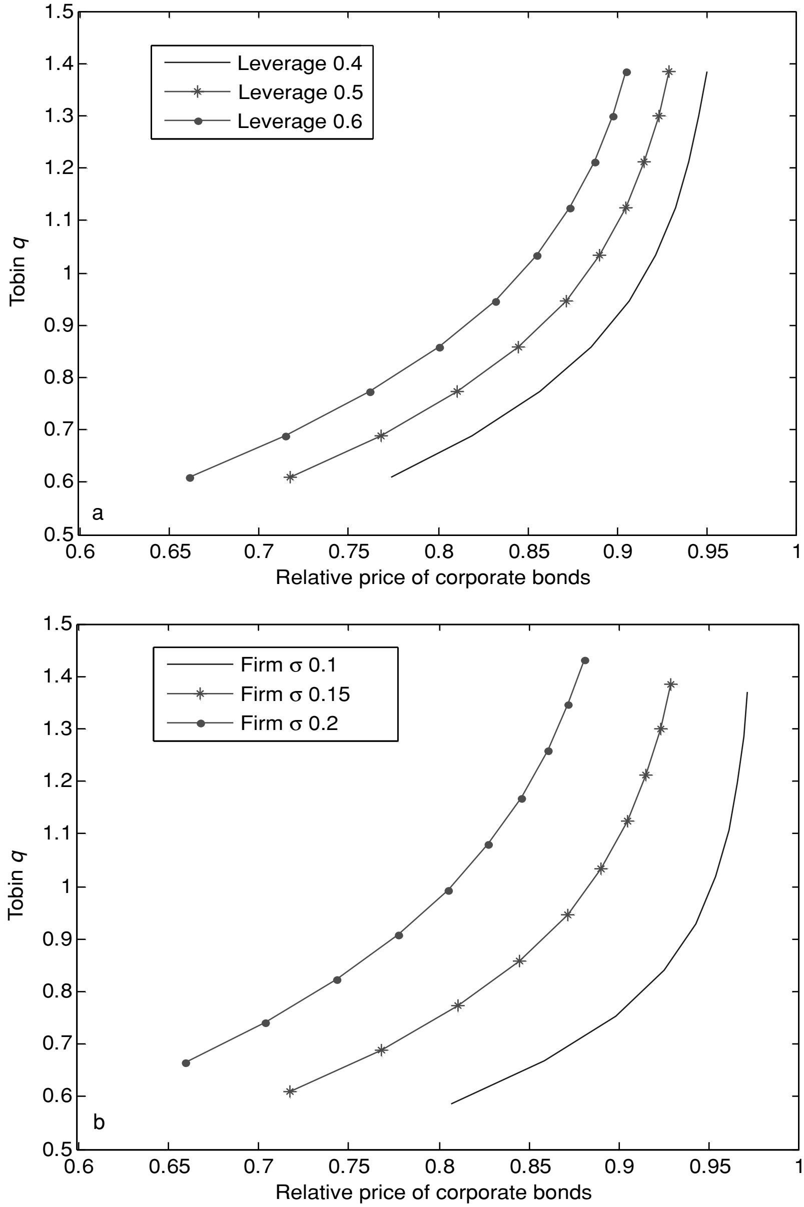

FIGURE II
Impact of Leverage and Firm Volatility

Calibration a in Figure I, except for book leverage in the top panel, and firm volatility in the bottom panel. (a) Mapping for different values of book leverage; (b) mapping for different volatilities of idiosyncratic shocks.

TABLE I
SUMMARY STATISTICS: QUARTERLY AGGREGATE DATA, 1953:2–2007:2

	Obs.	Mean	St. dev.	Min	Max
I/K	217	0.105	0.010	0.082	0.125
E(inflation)	217	0.037	0.025	−0.016	0.113
y^{Baa}	217	0.082	0.030	0.035	0.170
r^{10}	217	0.065	0.027	0.023	0.148
$(0.1 + r^{10})/(0.1 + y^{\text{Baa}})$	217	0.908	0.033	0.796	0.974
Classic Tobin's q	217	2.029	0.845	0.821	4.989
Bond market's q	217	1.500	0.117	1.154	1.720

Notes. Investment and replacement cost of capital are from NIPA. Expected inflation is from the Livingston survey. Yields on 10-year Treasuries and Moody's Baa index are from FRED. Classic Tobin's q is computed from the flow of funds, following Hall (2001). Bond market's q is computed using the structural model, and its mean is normalized to 1.5.

maintain a constant relative price, but their leverage would jump from one mapping to another. I return to this issue in Section VII.

V. EMPIRICAL EVIDENCE

In this section, I construct a new measure of q using only data from the bond market. I then compare this measure to the usual measure of q, and I assess their respective performances in the aggregate investment equation. The data used in the calibration are summarized in Table I. All the parameters used in the calibration, and the empirical moments used to infer them, are presented in Table II.

V.A. Data and Estimation of the Parameters

I now describe the data used to estimate the parameters of equations (17) and (18) and the construction of q.

Leverage. In the baseline case, book leverage is set to 0.45 based on Leland (2004). Using Compustat, I find a slow increase in average book leverage from 0.4 to 0.55 over the postwar period (Figure IIIa). The sample includes nonfinancial firms, with at least five years of nonmissing values for assets, stock price, operating income, debt, capital expenditures, and property, plants, and equipment.

Idiosyncratic Risk. Equation (17) is estimated with firm-level data from Compustat. The profit rate is operating income divided by the net stock of property, plants, and equipment, and η is the idiosyncractic component of this profit rate. Firms in finance and

TABLE II
PARAMETERS OF BENCHMARK MODEL

		Data	Model
Parameters chosen exogenously			
Real risk-free rate	r	3%	
Curvature of adjustment cost function	γ_2	10 years	
Average maturity	$1/\Phi$	10 years	
Book leverage	Ψ	0.45	
Parameters directly observed in the data			
Persistence of idiosyncratic profit rate	ρ_η	0.47	0.47
Volatility of idiosyncratic innovations	σ_η	0.14	0.14
Persitence of aggregate profit rate	ρ_{a}	0.7	0.7
Moments matched			
Relative bond price (mean)	$(0.1+r)/(0.1+y)$	0.908	0.908
Relative bond price (volatility of detrended series)	$(0.1+r)/(0.1+y)$	0.027	0.027
Average bond issued at par value	$E[b]/f$	1	1
Implied parameters			
Average profit rate	a/r	0.925	
Volatility of aggregate innovations	σ_{a}	0.045	
Coupon rate	c	0.043	

real estate are excluded. The panel regression includes firm fixed effects to remove permanent differences in average profitability across firms or industries due to accounting and technological differences. The estimated baseline parameters, $\rho_\eta = 0.47$ and $\sigma_\eta = 14\%$, are consistent with many previous studies.[7]

An important issue is that the idiosyncratic volatility of publicly traded companies is not constant. Campbell and Taksler (2003) show that changes in idiosyncratic risk have contributed to changes in yield spreads. The frequency of accounting data is too low to estimate quarterly changes in volatility. In addition, we need a forward-looking measure of idiosyncratic risk to capture market expectations. For all these reasons, the best measure should be based on idiosyncratic stock returns. Following the standard practice in the literature, I use a six-month moving average

7. For instance, Gomes (2001) uses a volatility of 15% and a persistence of 0.62 for the technology shocks. Hennessy, Levy, and Whited (2007) report a persistence of the profit rate of 0.51 and a volatility of 11.85%, which they match with a persistence of 0.684 and a volatility of 11.8% for the technology shocks. Note that in both of these papers, firms operate a technology with decreasing returns. Here, by contrast, the technology has constant returns to scale. This explains why some details of the calibration are different.

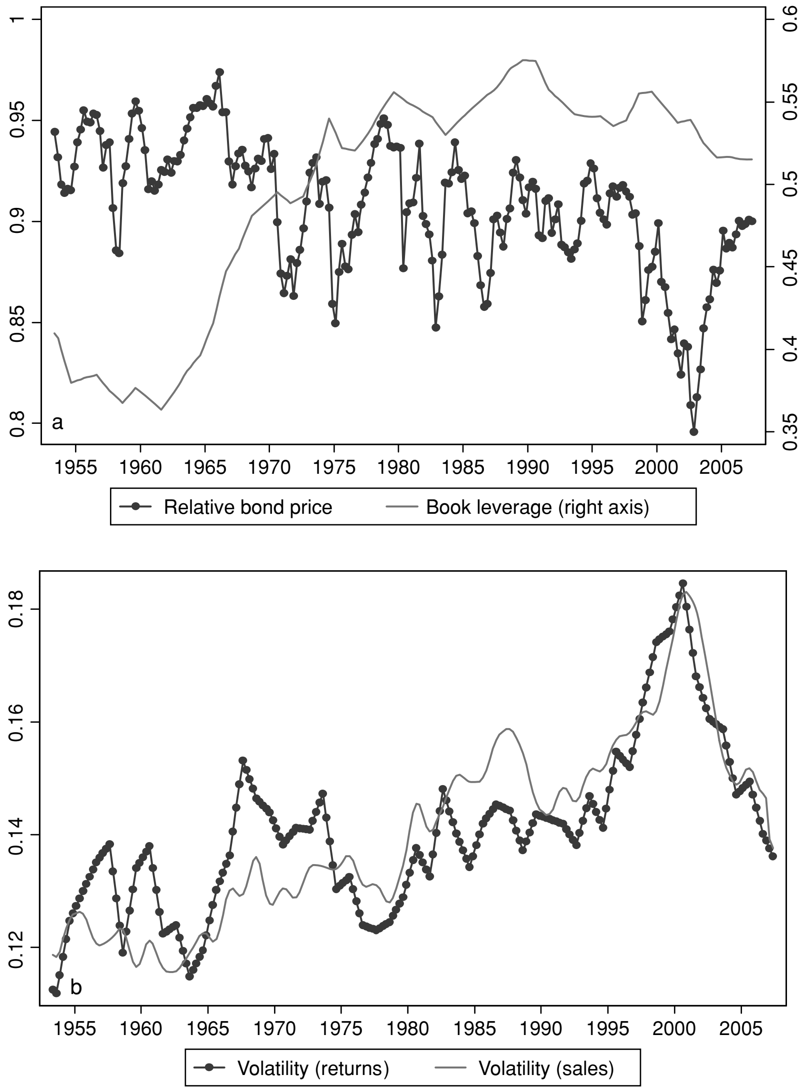

FIGURE III

The Components of Bond Market q

Leverage is average book leverage among nonfinancial firms in Compustat. Idiosyncratic volatility is estimated either from idiosyncratic stock returns or from the dispersion of sales growth. Both measures are then translated into the parameter σ_η of the model. Relative bond price is the relative price of corporate and government bonds, defined as $(0.1 + r)/(0.1 + y)$, using Moody's Baa and 10-year Treasury yields. (a) Bond prices and leverage; (b) two measures of idiosyncratic risk.

of the monthly cross-sectional standard deviation of individual stock returns. I scale this new measure to have a sample mean of 14% to obtain $\hat{\sigma}_t^{\eta}$, a time-varying estimate of idiosyncratic risk. As a robustness check, I also consider the cross-sectional standard deviation of the growth rate of sales, measured from Compustat, as a measure of volatility that avoids using stock returns.[8] The two measures of $\hat{\sigma}_t^{\eta}$ are presented in Figure IIIb.

Aggregate Bond Prices. Moody's Baa index, denoted y_t^{Baa}, is the main measure of the yield on risky corporate debt. Moody's index is the equal weighted average of yields on Baa-rated bonds issued by large nonfinancial corporations.[9] Following the literature, the 10-year treasury yield is used as the benchmark risk-free rate. Both r_t^{10} and y_t^{Baa} are obtained from FRED.[10]

For equation (18), using annual NIPA data on corporate profits and the stock of nonresidential capital over the postwar period, I estimate $\rho_a = 0.7$. The parameters $\bar{a}$ and σ_a cannot be calibrated with historical aggregate profit rates because they must capture risk-adjusted values, not historical ones.[11] Instead, the model must be consistent with observed bond prices. Three parameters are thus not directly observed in the data: these are c (the coupon rate), $\bar{a}$, and σ_a. Their values are inferred by matching empirical and simulated moments. The empirical moments are the mean and standard deviation of the price of Baa bonds

8. The dispersion of sales growth is not a perfect measure either, because permanent differences in growth rates would make dispersion positive even if there is no risk. There are other ways to define idiosyncratic risk at the firm level, but they produce similar trends. See Comin and Philippon (2005) for a comparison of various measures of firm volatility. See also Campbell et al. (2001) and Davis et al. (2006) for evidence on privately held companies.

9. To be included in the index, a bond must have a face value of at least 100 million, an *initial* maturity of at least 20 years, and most importantly, a liquid secondary market. Beyond these characteristics, Moody's has some discretion on the selection of the bonds. The number of bonds included in the index varies from 75 to 100 in any given year. The main advantages of Moody's measure are that it is available since 1919, and that it is broadly representative of the U.S. nonfinancial sector, because Baa is close to the median among rated companies.

10. Federal Reserve Economic Data: http://research.stlouisfed.org/fred2/. The issue with using the ten-year treasury bond is that it incorporates a liquidity premium relative to corporate bonds. To adjust for this, it is customary to use the LIBOR/swap rate instead of the treasury rate as a measure of risk-free rate (see Duffie and Singleton [2003] and Lando [2004]), but these rates are only available for relatively recent years. I add 30 basis points to the risk-free rate to adjust for liquidity (see Almeida and Philippon [2007] for a discussion of this issue).

11. Note that, in theory, the same applies to ρ_a, because persistence under the risk-neutral measure can be different from persistence under the physical measure. In practice, however, the difference for ρ_a is much smaller than for $\bar{a}$ or σ_a. I therefore take the historical persistence to be a good approximation of the risk neutral persistence. Section VII shows that the model is robust to various assumptions regarding aggregate dynamics.

relative to Treasuries, defined as $(\phi + r_t^{\$})/(\phi + y_t^{\$})$, where $y^{\$}$ is the yield on Baa corporate bonds and $r^{\$}$ is the yield on government bonds. The final requirement is that the average bond be issued at par. The three parameters c, $\bar{a}$, and σ_a are chosen simultaneously to match the par-value requirement and the two empirical moments. The parameters inferred from the simulated moments are $c = 4.3\%$, $\bar{a}/r = 0.925$, and $\sigma_a = 4.5\%$.

Expected Inflation and Real Rate. The Livingston survey is used to construct expected inflation, and the yield on the ten-year treasury to construct the *ex ante* real interest rate, $\hat{r}_t^{\text{real}}$.

Creating q^{bond}. The model described in Section IV constructs q from the relative price of corporate bonds, conditional on the baseline values for the risk-free rate, book leverage, and idiosyncratic risk. As I have just explained, the risk-free rate, book leverage, and idiosyncratic volatility move over time. Therefore, q^{bond} is a function of four observed inputs: average book leverage $\hat{\psi}_t$, average idiosyncratic volatility $\hat{\sigma}_t^{\eta}$, the *ex ante* real rate $\hat{r}_t^{\text{real}}$, and the relative price of corporate bonds

$$q_t^{\text{bond}} = F\left(\frac{\phi + r_t^{10}}{\phi + y_t^{\text{Baa}}}; \hat{\sigma}_t^{\eta}; \hat{\psi}_t; \hat{r}_t^{\text{real}}\right). \tag{19}$$

Figure III displays the three main components: leverage, volatility, and the relative price. In theory, the dynamics of the four inputs must be jointly specified to construct the mapping of equation (19). Quantitatively, however, it turns out that one can estimate mappings with respect to $(\phi + r_t^{10})/(\phi + y_t^{\text{Baa}})$ assuming constant values for the other three parameters, as I did in Figure II. For the risk-free rate, this follows from a well-known fact in the bond pricing literature: risk-free rate dynamics plays a negligible role in fitting corporate spreads. For $\hat{\sigma}_t^{\eta}$ and $\hat{\psi}_t$, the historical series are so persistent that there is little difference between the mapping assuming a constant value and the mapping conditional on the same value in the time-varying model.[12]

Classic Measure of Tobin's q. The usual measure of Tobin's q is constructed from the flow of funds as in Hall (2001). The usual

12. To check this, I construct an extended Markov model where all the parameters follow AR(1) processes calibrated from the data. I then create mappings conditional on each realization of the parameters and I compare them to the mappings from Figure II. I find that the discrepancies are small for volatility and invisible for book leverage and the risk-free rate. Detailed results and figures are available upon request.

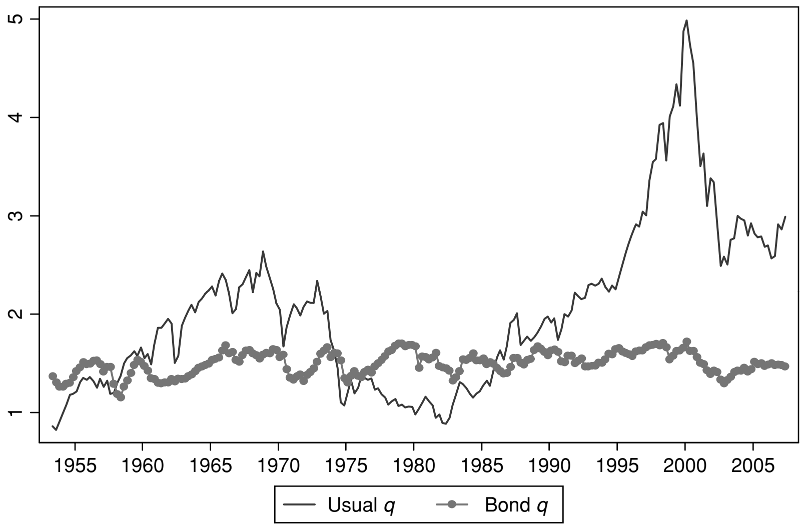

FIGURE IV
Usual Measure of q and Bond Market's q

Tobin's q is constructed from the flow of funds, as in Hall (2001). Bond q is constructed from Moody's yield on Baa bonds, using the structural model calibrated to the observed evolutions of book leverage and firm volatility, expected inflation from the Livingston survey, and the yield on 10-year Treasury bonds.

measure is the ratio of the value of ownership claims on the firm less the book value of inventories to the reproduction cost of plant and equipment. All the details on the construction of this measure can be found in Hall (2001).

Investment and Capital Stock. I use the series on private nonresidential fixed investment and the corresponding current stock of capital from the Bureau of Economic Analysis. Table I displays the summary statistics.

V.B. Investment Equations

Figure IV shows the two measures of q: q^{usual}, constructed from the flow of funds as in Hall (2001), q^{bond} constructed using bond yields, leverage, idiosyncratic volatility, expected inflation, and the theoretical mappings described in the previous sections. The average value of q^{bond} is arbitrary, because γ_1 is a free parameter, and I normalize it to 1.5. Figure IV shows that q^{usual} is approximately seven times more volatile than q^{bond}. The standard deviation of q^{usual} is 0.845, whereas the standard deviation

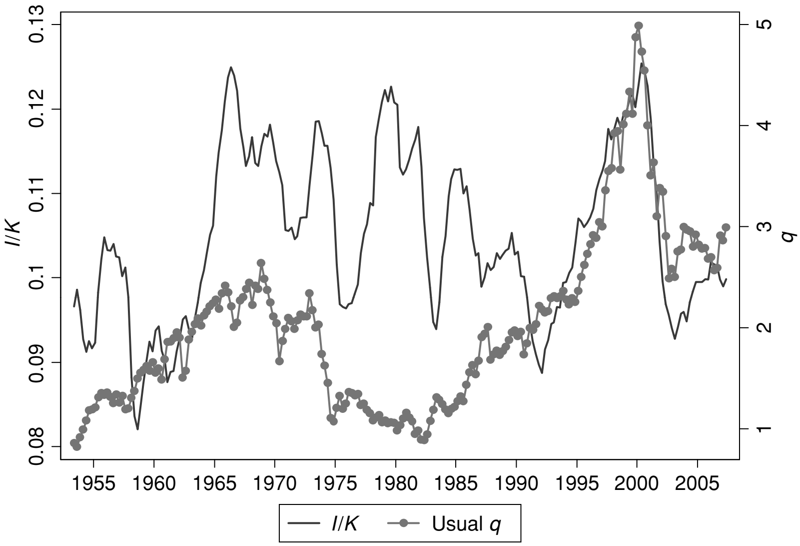

FIGURE V
Usual Measure of q and Investment Rate

I/K is corporate fixed investment over the replacement cost of equipment and structure. Usual q is constructed from the flow of funds, as in Hall (2001).

of q^{bond} is only 0.117, as reported in Table I. It is also interesting to note that q^{bond} is approximately stationary, because the mappings take into account the evolution of idiosyncratic volatility and book leverage, as explained above. In the short run, q^{bond} depends mostly on the relative price component. Year-to-year changes in $(\phi + r_t^{10})/(\phi + y_t^{\text{Baa}})$ account for 85% of the year-to-year changes in q^{bond}. In the long run, leverage and, especially, idiosyncratic risk are also important.

Figure V shows q^{usual} and the investment rate in structure and equipment. Figure VI shows q^{bond} and the same investment rate. The corresponding regressions are reported in the upper panel of Table III. They are based on quarterly data. The investment rate in structure and equipment is regressed on the two measures of q, measured at the end of the previous quarter:

$$x_t = \alpha + \beta^b q_{t-1}^{\text{bond}} + \beta^e q_{t-1}^{\text{usual}} + \varepsilon_t.$$

The standard errors control for autocorrelation in the error terms up to four quarters. q^{bond} alone accounts for almost 60% of aggregate variations in the investment rate. q^{usual} accounts for only

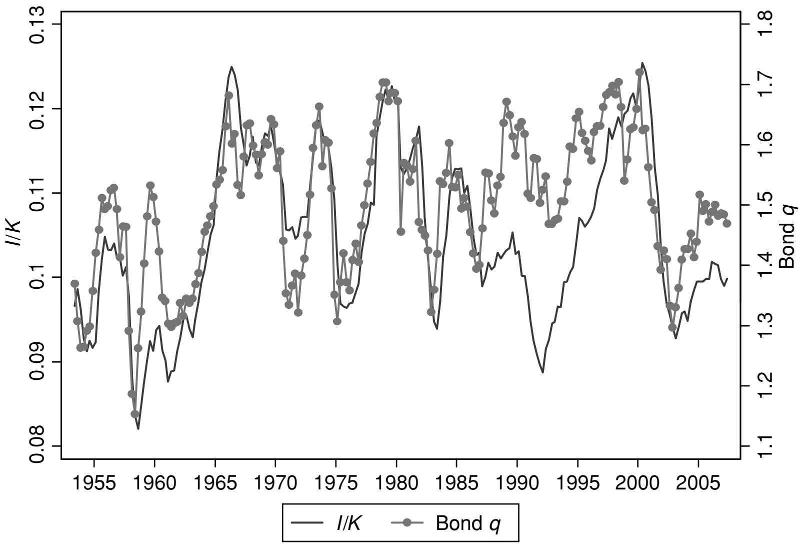

FIGURE VI
Bond Market's q and Investment Rate

I/K is corporate fixed investment over the replacement cost of equipment and structure. Bond q is constructed from Moody's yield on Baa bonds, using the structural model calibrated to the observed evolutions of book leverage and firm volatility, expected inflation from the Livingston survey, and the yield on 10-year Treasury bonds.

10% of aggregate variations. Moreover, once q^{bond} is included, the standard measure has no additional explanatory power. Looking at Figure V, the fit of the investment equation is uniformly good, except in the late 1980s and early 1990s, where, even though the series remain correlated in changes (see below), there is a persistent discrepancy in levels.

q^{bond} is more correlated with the investment rate, hence the better fit of the estimated equation, but it is also less volatile than q^{usual}. As a result, the elasticity of investment to q is almost eighteen times higher with this new measure, which is an encouraging result because the low elasticity of investment with respect to q has long been a puzzle in the academic literature. The estimated coefficient still implies adjustment costs that are too high, around 15 years, but, as Erickson and Whited (2000) point out, there are many theoretical and empirical reasons that the inverse of the estimated coefficient is likely to underestimate the true elasticity.[13]

13. Note that the mapping is calibrated assuming $\gamma_2 = 10$, so in theory the coefficient should be 0.1. In Table III, it is 0.065. I have also solved for the model

TABLE III
BENCHMARK REGRESSIONS

	Equation in levels: $I/K(t)$			
Bond q $(t-1)$	0.0650***		0.0629***	
S.e.	(0.00594)		(0.00642)	
Classic q $(t-1)$		0.00366**	0.000928	
S.e.		(0.00155)	(0.000970)	
Bond q $(t-1)$, alt. measure				0.0521***
S.e.				(0.00706)
Observations	216	216	216	216
OLS R^2	.574	.095	.580	.432
	Estimation in changes: $I/K(t) - I/K(t-4)$			
Δ[bond q] $(t-5, t-1)$	0.0515***		0.0471***	
S.e.	(0.00495)		(0.00584)	
Δ[classic q] $(t-5, t-1)$		0.00700***	0.00240*	
S.e.		(0.00187)	(0.00133)	
Δ[profit rate] $(t-5, t-1)$			0.0530	
S.e.			(0.0514)	
Δ[bond q] $(t-5, t-1)$, alt. measure				0.0517***
S.e.				(0.00500)
Observations	212	212	212	212
OLS R^2	.613	.102	.628	.561

Notes. Fixed private nonresidential capital and investment series are from the BEA. Quarterly data, 1953:3 to 2007:2. Classic q is constructed from the flow of funds, as in Hall (2001). Bond q is constructed by applying the structural model to Corporate and Treasury yields, expected inflation, book leverage, and firm volatility measured with idiosyncratic stock returns. The alternate measure of Bond q uses idiosyncratic sales growth volatility as an input. Newey–West standard errors with autocorrelation up to four quarters are reported in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels. Constant terms are omitted.

Figure VII shows the four-quarter difference in the investment rate, a measure used by Hassett and Hubbard (1997), among others, because of the high autocorrelation of the series in levels. The corresponding regressions are presented in the bottom panel of Table III. The fit of the equation in difference is even better than the fit in levels, with an R^2 above 60%. In the third regression, the change in corporate cash flows over capital is added to the right-hand side of the equation, but it is insignificant and does not improve the fit of the equation.

The construction of q^{bond} uses idiosyncratic stock returns to measure firm volatility. Note that using *idiosyncratic* return volatility is justified even when the *aggregate* stock market is

assuming $\gamma = 15$. This makes the theoretical and actual coefficients similar, but does not change anything to the rest of the results. See also Section VII for a discussion of biases.

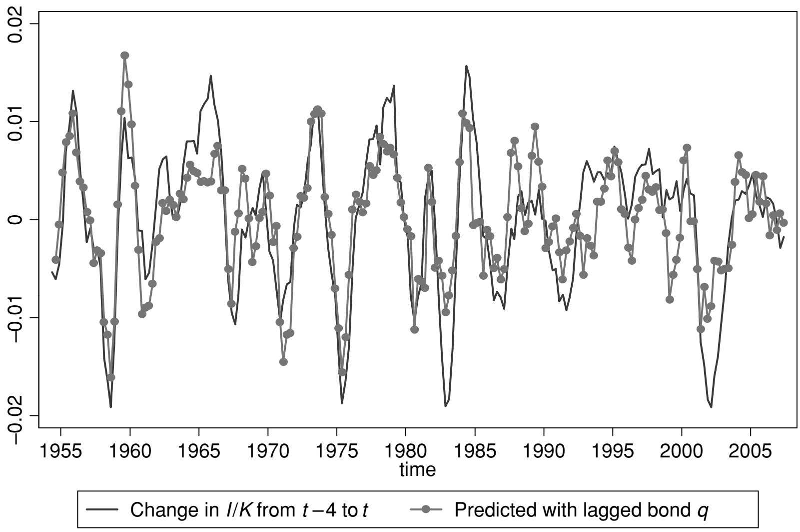

FIGURE VII

Four-Quarter Changes in Investment Rate, Actual and Predicted

I/K is corporate fixed investment over the replacement cost of equipment and structure. Bond q is constructed from Moody's yield on Baa bonds, using the structural model calibrated to the observed evolutions of book leverage and firm volatility, expected inflation from the Livingston survey, and the yield on 10-year Treasury bonds.

potentially mispriced. Mispricing across firms is limited by the possibility of arbitrage. In the aggregate, however, arbitrage is much more difficult. There is therefore no inconsistency in using the *idiosyncratic* component of stock returns to measure idiosyncratic risk, although acknowledging that the aggregate stock market can sometimes be over valued.

Nonetheless, one might be concerned about the use of equity returns here, and I have repeated the calibration using the standard deviation of sales growth as a measure of volatility. The results, in the last column of Table III, are somewhat weaker than with the benchmark model. The reason is that sales volatility is a lagging indicator of idiosyncratic risk. Hilscher (2007) shows that the bond market is actually forward-looking for volatility. As a result, using a measure of volatility that lags the true information—and all accounting measures do—creates a specification error. This matters less for the equation in changes because of the smaller role of volatility in that equation.

The conclusions from this empirical section are the following:

- With aggregate U.S. data, q^{bond} fits the investment equation well, both in levels and in differences.
- The estimated elasticity of investment to q^{bond} is 18 times higher than the one estimated with q^{usual}.
- Corporate cash flows do not have significant explanatory power once q^{bond} is included in the regression.

V.C. Further Evidence

The evidence presented above is based on the construction of q^{bond} in equation (19). In this section, I provide evidence on the explanatory power of the components separately, and on the role of nonlinearities in the model. I also test the predictive power of the model. The results are in Tables IV and V.

Explanatory Power of Individual Components. The econometric literature has studied the predictive power of default spreads for real economic activity.[14] Table IV shows the explanatory power of the components of q^{bond}, in levels and in four-quarter differences.

Consider first the top part of Table IV, for the regressions in level. Column (1) shows that the Baa spread, by itself, has no explanatory power for investment. The explanatory power appears only when idiosyncratic volatility and leverage are also included, in Column (3). These factors have not been used in the empirical literature. Their empirical importance provides support for the theory developed in this paper.

In addition, notice that even when all the components are entered linearly, the explanatory power is only 45%. By contrast, the q^{bond} has an R^2 of 57.4% with one degree of freedom instead of four. This shows that the nonlinearities are important in the level equation, as explained below.

For the equation in four-quarter differences, the spread by itself has significant explanatory power. This is what one would expect, because the low-frequency movements in leverage and volatility matter less in these regressions. Nonetheless, leverage and volatility are still highly significant. The unrestricted linear

14. Bernanke (1983) notes that the spread of Baa over treasury went "from 2.5 percent during 1929–30 to nearly 8 percent in the mid-1932" and shows that the spread was a useful predictor of industrial production growth. Using monthly data from 1959 to 1988, Stock and Watson (1989) find that the spread between commercial paper and Treasury bills predicts output growth. Some of these relationships are unstable over time (see Stock and Watson [2003] for a survey).

TABLE IV
DECOMPOSING BOND q

	Equation in levels: $I/K(t)$				
$y^{\text{Baa}} - r^{10}(t-1)$	−0.166	−0.152	−1.051***		
S.e.	(0.189)	(0.189)	(0.177)		
Real risk-free rate $(t-1)$		−0.0700	−0.0781		
S.e.		(0.0796)	(0.0744)		
Idiosyncratic volatility $(t-1)$			0.278***	0.424***	0.407***
S.e.			(0.0759)	(0.0672)	(0.0690)
Book leverage $(t-1)$			0.0910***	0.0633***	0.0727***
S.e.			(0.0214)	(0.0172)	(0.0159)
$[0.1 + r^{10}]/[0.1 + y^{\text{Baa}}]$ $(t-1)$				0.252***	0.263***
S.e.				(0.0268)	(0.0291)
Real discount factor $(t-1)$				0.203***	0.187***
S.e.				(0.0673)	(0.0649)
Quadratic term $(t-1)$					1.069**
S.e.					(0.522)
N	216	216	216	216	216
OLS R^2	.013	.023	.451	.582	.604
	Estimation in changes: $I/K(t) - I/K(t-4)$				
$\Delta[y^{\text{Baa}} - r^{10}](t-5, t-1)$	−0.942***	−0.953***	−0.997***		
S.e.	(0.103)	(0.108)	(0.0910)		
Δ[real riskfree rate] $(t-5, t-1)$		−0.0237	−0.00297		
S.e.		(0.0355)	(0.0338)		
Δ[idiosyncratic volatility] $(t-5, t-1)$			0.283***	0.265***	0.270***
S.e.			(0.0885)	(0.0896)	(0.0912)
Δ[book leverage] $(t-5, t-1)$			0.172***	0.142***	0.133***
S.e.			(0.0516)	(0.0515)	(0.0502)
$\Delta[(0.1 + r^{10})/(0.1 + y^{\text{Baa}})](t-5, t-1)$				0.199***	0.201***
S.e.				(0.0195)	(0.0195)
Δ[real discount factor] $(t-5, t-1)$				0.0787*	0.0765*
S.e.				(0.0419)	(0.0412)
Δ[quadratic term] $(t-5, t-1)$					0.398
S.e.					(0.293)
Observations	212	212	212	212	212
OLS R^2	.478	.479	.618	.618	.622

Notes. Fixed private nonresidential capital and investment series are from the BEA. Quarterly data, 1953:3 to 2007:2. The *ex ante* real rate is the nominal rate minus expected inflation from the Livingston survey. The real discount factor is $(1 + E[\text{inflation}])/(1+\text{nominal rate})$. Firm volatility is measured with idiosyncratic stock returns. The nominal rates are r^{10} for 10-year Treasury bonds, and y^{Baa} for Moody's index of Baa bonds. Quadratic term is the square of the relative price of Baa bonds minus its mean: $[(0.1 + r^{10})/(0.1 + y^{\text{Baa}}) - 0.9]^2$. Newey–West standard errors with autocorrelation up to 4 quarters are reported in parentheses. *, **, and *** denote statistical significance at the 10%, 5% and 1% levels. Constant terms are omitted.

TABLE V
PREDICTIVE REGRESSIONS, ONE QUARTER AHEAD: MAXIMUM LIKELIHOOD ESTIMATION OF AUTOREGRESSIVE MODEL

	Growth rate of private nonresidential fixed investment				Growth rate of consumption	Growth rate of residential investment
Δ[bond q] $(t-1)$	0.148***		0.160***	0.161***	0.0119	0.0279
S.e.	(0.0231)		(0.0251)	(0.0247)	(0.00917)	(0.0417)
Δlog[real GDP] $(t-1)$		0.710***	0.760***	0.797***	0.169***	−0.162
S.e.		(0.182)	(0.199)	(0.200)	(0.0585)	(0.346)
Δ[classic Q] $(t-1)$				0.0130	0.00976***	0.0401**
S.e.				(0.00955)	(0.00279)	(0.0184)
AR(1)	0.396***	0.337***	0.131	0.0996	0.0206	0.511***
S.e.	(0.0778)	(0.0832)	(0.0935)	(0.0928)	(0.0832)	(0.0543)
Observations	215	215	215	215	215	215
R^2 of OLS	.398	.341	.430	.443	.149	.335

Notes. Maximum likelihood estimation of coefficients and standard errors, assuming AR(1) model. The R^2 is for the corresponding OLS regression with lagged dependent variable on the right-hand side. Fixed private nonresidential investment series is from the BEA. Quarterly data, 1953:3 to 2007:2. Bond q is constructed by applying the structural model to Corporate and Treasury yields, expected inflation, book leverage, and firm volatility measured with idiosyncratic stock returns. Constant terms are omitted.

model has an R^2 of 61.6%, compared to 61.3% for the bond q model. This suggests that, also as expected, nonlinear effects are not crucial for the specification in changes.

Nonlinear Effects. There are several nonlinear effects in the model. Consider equation (4): Tobin's q has two components, the real discount factor and the expected risk-neutral value of capital, $E^{\pi}[v(\omega')|\omega]$. This letter item is a function of the relative price of corporate bonds, as shown in Figures I and II. Thus, the model suggests the use of the relative price $(\phi + r_t^{10})/(\phi + y_t^{\mathrm{Baa}})$ instead of the spread $y_t^{\mathrm{Baa}} - r_t^{10}$. When rates are stable, the difference between the spread and the relative price is negligible. In the data, however, the level of nominal rates changes a lot. A given change in the spread has a larger impact on the relative price when rates are low than when they are high.

Column (4) provides strong support for this first nonlinearity. The relative price does much better than the spread in the level regression.[15] The R^2 increases from 45.1% to 58.2% because of the nonlinear correction.

A second nonlinearity comes from the mapping of Figure I. Tobin's q is a convex function of the relative bond price. Column (5) shows that this effect is significant, but it only increases the R^2 by 2 percentage points.

The last column of Table IV can also be compared to the first column of Table III. In level, the structural model has a fit of 57.4%. The unrestricted nonlinear model has a fit of 60.4%. In a statistical sense, the difference is significant, but in an economic sense, it does not appear very important. In differences, the respective performances are 61.3% and 62.2%. These results support the restrictions imposed by the theory.

Predictive Regressions. Table V reports the results from predictive regressions of the growth rate of three macroeconomic variables: real corporate investment, real consumption expenditures, and real residential investment. In each case, I run two separate regressions. I estimate an AR(1) model by maximum likelihood to obtain the correct coefficients and standard errors. I also run an OLS regression with the lagged dependent variable on the RHS to get a sense of the R^2 of the simple linear regression.

15. Note that, in theory, this could also apply to the real discount factor: $(1 + E[\text{inflation}])/(1 + r^{\$})$ is not the same as $E[\text{inflation}] - r^{\$}$ when nominal shocks are large. Empirically, this nonlinearity seems to matter much less, probably because the real rate is not as volatile as the Baa yield.

The first column shows that q^{bond} is a very significant predictor of corporate investment growth. It predicts better than the "accelerator" model based on lagged output growth (column (2)). While lagged output growth still has significant marginal forecasting power, it increases the R^2 by only 3 percentage points (column (3)). In addition, the coefficient on q^{bond} actually goes up. Column (4) shows that q^{usual} has no predictive power for corporate investment.[16]

The last two columns focus on consumption and residential investment. Although q^{bond} is the best predictor of corporate investment, it does not predict housing or consumption. q^{usual}, on the other hand, does not predict corporate investment, but it does predict housing and (to some extent) consumption. These results are suggestive of wealth effects from the equity market. They are consistent with the results of Hassett and Hubbard (1997) but clearly inconsistent with the usual implementation of the q-theory.

The conclusions from this empirical section are the following:

- All the components identified by the theory (bond spreads, volatility, leverage, risk-free rate) are statistically and economically significant.
- The fit of the restricted structural model is almost as good as the fit of the unrestricted regressions.
- The nonlinearities of the model (relative price instead of spreads, convexity of mapping) are important for the level regressions.
- The bond market predicts future corporate investment well, whereas the equity market has no marginal predictive power.

VI. Theoretical Explanations

The results so far show that it is possible to link corporate investment and asset prices, using the corporate bond market and modern asset pricing theory. They do not explain why the usual approach fails, however. This section sheds some light on this complex question.

16. Fama (1981) shows that stock prices have little forecasting power for output. Cochrane (1996) finds a significant correlation between stock returns and the growth rate of the aggregate capital stock, but Hassett and Hubbard (1997) argue that it is driven by the correlation with residential investment, not corporate investment. In any case, I find that the bond market's q outperforms the usual measure both in differences and in levels.

It is important to recognize that a satisfactory explanation must address two related but distinct issues:

1. Why is q^{usual} more volatile than q^{bond}?
2. Why does q^{bond} fit the investment equation better?

I consider two explanations.[17] The first explanation is based on growth options and the distinction between average and marginal q. The second explanation is based on mispricing in the equity market. I chose these explanations because they provide useful benchmarks. They are not mutually exclusive, and they are not the only possible explanations.

VI.A. Growth Option Interpretation

Suppose that, in addition to the value process in equation (3), the firm also has a growth option of value G_t. Total firm value is then

$$V_t = v_t k_{t-1} + G_t. \tag{20}$$

Consider for simplicity the example of Section III, with short-term debt and a constant risk-free rate. The value of short-term debt is

$$B_t = \frac{1}{1+r} E_t^{\pi} [\min(\Psi_t; v_{t+1} k_t + G_{t+1})]. \tag{21}$$

Let G_t be a binary variable. $G_t = G^{\mathrm{H}}$, with risk-neutral probability λ_{t-1} and G_{L} otherwise. The following proposition states that a growth option with enough skewness can explain why q^{bond} fits better than q^{usual}.

PROPOSITION 3. Consider the model of equations (20) and (21). By choosing λ_t and G^{L} small enough, and G^{H} large enough, the fit of the investment equation can be arbitrarily good for q^{bond}, and arbitrarily poor for q^{usual}.

Proof. See the Appendix. ■

The intuition behind Proposition 3 is straightforward. A small probability of a large positive shock has a large impact on equity prices, and almost no impact on bond prices. Because growth options do not depend on the capital stock, news about the likelihood of these future shocks does not affect investment. In essence,

17. For a investigation of whether the same pricing kernel can price bonds and stocks, see Chen, Collin-Dufresne, and Goldstein (forthcoming).

growth options drive wedges between bond and equity prices, and between marginal and average q.

What are the possible interpretations of these shocks? The simplest one is that firms earn organizational rents. Think of a large industrial corporation with outstanding organizational capital. This firm will be able to seize new opportunities if and when they arrive. This might happen through mergers and acquisitions or through internal development of new lines of business. Investing more in the current business and current technology does not improve this option value.[18]

To summarize, the rational interpretation proposes the following answers to the two questions posed at the beginning of this section:

1. Why is q^{usual} more volatile than q^{bond}? Because growth options affect stocks much more than bonds.
2. Why does q^{bond} fit the investment equation better? Because growth options are unrelated to *current* capital expenditures.

The example given is obviously extreme, but the lesson is a general one. It is not difficult to come up with a story where current capital expenditures are well explained by the bond market, whereas firm creation, IPOs, and perhaps R&D, are better explained by the equity market. A complete understanding of these joint dynamics is an important topic for future research.

VI.B. Mispricing Interpretation

Stein (1996) analyzes capital budgeting in the presence of systematic pricing errors by investors, assuming that managers have rational expectations. He emphasizes three crucial aspects of capital budgeting in such a world: (i) the true NPV of investment, (ii) the gains from trading mispriced securities, and (iii) the costs of deviating from an optimal capital structure in order to achieve (i) and (ii). For the purpose of my paper, the most important result is that *when capital structure is not a constraint, and when managers have long horizons, real investment decisions are not influenced by mispricing* (Stein 1996, Proposition 3).

18. Some other expenditures could be complement with the option value. These could include R&D and reorganizations. At the aggregate level, one might think that new options were realized by new firms. This would explain why IPOs are correlated with the equity market (Jovanovic and Rousseau 2001). For a model of growth option at the firm level, see Abel and Eberly (2005).

Gilchrist, Himmelberg, and Huberman (2005) consider a model where mispricing comes from heterogeneous beliefs and short sales constraints. They show that increases in dispersion of investor opinion cause stock prices to rise above their fundamental values. This leads to an increase in q, share issues, and real investment. The main difference from Stein (1996) is that they assume that investors do not overvalue cash held in the firm. This assumption rules out the separation of real and financial decisions: managers who seek to exploit mispricing must alter their investment decisions and Proposition 3 in Stein (1996) does not hold. However, Gilchrist, Himmelberg, and Huberman (2005) show that even large pricing errors need not have large effects on investment. Thus, it is possible to explain the fact that investment does not react much to equity mispricing, even when the strict dichotomy of Stein (1996)'s Proposition 3 fails.

Neither Stein (1996) nor Gilchrist, Himmelberg, and Huberman (2005) consider the role of bonds and stocks separately, so it appears that the story is still incomplete. It turns out, however, that recent work in behavioral finance has shown that skewed assets are more likely to be mispriced (Barberis and Huang 2007; Brunnermeier, Gollier, and Parker 2007; Mitton and Vorkink 2007). A direct implication is that mispricing is more likely to appear in the equity market than in the bond market. Of course, mispricing can also happen in the bond market. Piazzesi and Schneider (2006), for instance, analyze the consequences for asset prices of disagreement about inflation expectations.

To summarize, the behavioral interpretation proposes the following answers to the two questions posed at the beginning of this section:

1. Why is q^{usual} more volatile than q^{bond}? Because mispricing is more likely in the equity market than in the bond market.
2. Why does q^{bond} fit the investment equation better? Because managers do not react (much) to mispricing.

The growth option and mispricing interpretations are not mutually exclusive. In fact, the term G_t in equation (20) is the most likely to be mispriced. The rational and behavioral explanations simply rely on different critical assumptions. In the rational case, G_t must not depend on k, otherwise investment would respond. In the behavioral story, it is important that managers have long horizons.

VII. Theoretical Robustness

The beauty of the standard q theory is its parsimony. Beyond the assumptions of constant returns and convex costs, it is extremely versatile. In equation (4), the sources of variations in $q(\omega)$ include changes in the term structure of risk-free rates, cash flow news that has aggregate, industry, and firm components, and changes in risk premia that separate the market value $E^{\pi}[v']$ from the objective expectation $E[v']$. These multidimensional shocks can be combined in arbitrary ways, and yet their joint impact on investment can be summarized by one real number. Unfortunately, the standard approach fails. The previous section has presented two explanations for this failure, as well as for the (relative) success of the new approach.

The new approach, however, is not as model-free as the standard approach. The mappings of Figures I and II are constructed under specific assumptions regarding firm and aggregate dynamics. The goal of this section is to study the *theoretical robustness* of the new approach. To do so, I focus on three issues:

- Is there an exact mapping at the firm level, similar to the one in Figure I, for aggregate q? The answer turns out to be no, but q^{bond} is still a useful measure.
- Suppose that aggregate dynamics does not follow a simple autoregressive process under the risk neutral measure. Would the misspecified mapping of Figure I still deliver a good fit? Yes.
- What happens when the Modigliani–Miller assumptions do not hold? If anything, the model seems to work better in this case.

VII.A. Firm-Level Mappings

I first study the extent to which the aggregate mapping of Figure II applies at the firm level. Figure VIII and the left part of Table VI report the results based on a simulated panel of fifty years and 100 firms, using the benchmark model with the parameters in Table II.

To get an exact mapping, there must be a monotonic relationship between asset value and bond prices. This is typically the case when there is only one dimension of heterogeneity. In the top left panel of Table VI, the R^2 for the aggregate regression is 1 and the estimated elasticity is exactly equal to $1/\gamma_2$ (0.1, because γ_2 is calibrated to 10 years). At the firm level, there are two sources of

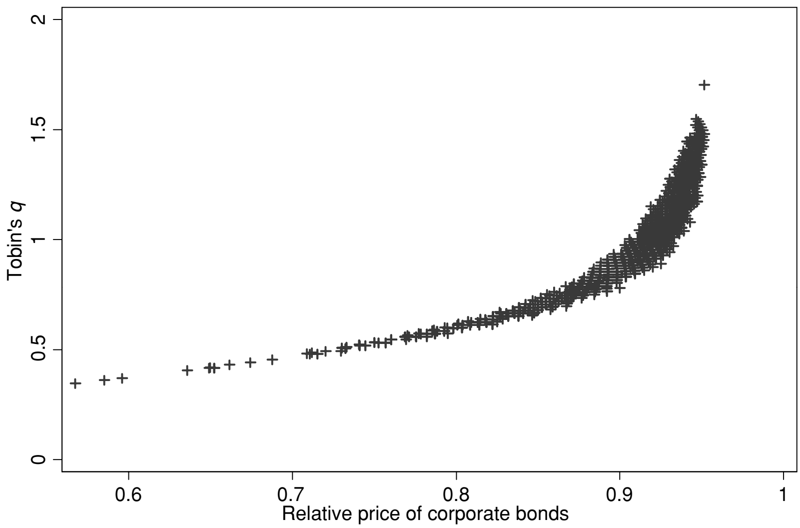

FIGURE VIII
Simulations of a Panel of Firms

5,000 firm–year observations of relative bond price and Tobin's q. Simulation with constant real rate of 3%, constant idiosyncratic volatility, and constant book leverage.

variation, aggregate and idiosyncratic. Conditional on one shock, there is an exact mapping,[19] but there is no guarantee that the ranking would be preserved across several types of shocks. In fact, Figure VIII shows that they are not.[20]

The bottom left panel of Table VI shows that R^2 for firm-level regressions is less than one. In the univariate regression, this does not bias the point estimate. In the multivariate regression, firm-level cash flows are significant, R^2 increases, but the point estimate of q^{bond} becomes unreliable.

The conclusion is that, at the firm level, the bond q should be significant but cash flows are likely to remain significant as well. These predictions are consistent with the results obtained

19. For instance, fix the aggregate state, and look at the cross section. Then firms with good earnings shocks have high value and high bond prices. Or fix the firm-level shock, and then states with high values have high bond prices.

20. The intuition is the following. Suppose a firm–year observation has a true q of 1.1 based on a good firm shock in a bad aggregate state. Suppose another firm–year observation has a true q of 1.1 based on a medium firm shock in a medium aggregate state. There is no reason to expect them to have the same bond price (for instance, because persistence and volatility are not the same for idiosyncratic and aggregate shocks). As a result, the same value of q for one firm–year observation is associated with several relative prices of bonds. This is what Figure VIII shows.

TABLE VI
INVESTMENT REGRESSIONS IN SIMULATED DATA

	Aggregate regressions: Dependent variable is aggregate I/K					
	Benchmark model			Model B: binary aggregate cash flows		
Bond q	0.100***		0.100***	0.0799***		0.0796***
S.e.	(0.00000111)		(0.000148)	(0.000623)		(0.000648)
Profit rate		0.210***	−0.000318		0.0394*	0.00176
S.e.		(0.000159)	(0.000312)		(0.0222)	(0.00158)
Observations	100	100	100	100	100	100
R^2	1.000	.98	1.000	.994	.031	.995
	Firm-level regressions: Dependent variable is firm I/K					
	Benchmark model			Model B: binary aggregate cash flows		
Bond q	0.1000***		0.0610***	0.0791***		0.0703***
S.e.	(0.000525)		(0.000638)	(0.000408)		(0.000276)
Profit rate		0.0993***	0.0482***		0.0898***	0.0434***
S.e.		(0.000619)	(0.000649)		(0.00173)	(0.000498)
Observations	5,000	5,000	5,000	5,000	5,000	5,000
R^2	.879	.838	.943	.883	.349	.953

Notes. Simulated annual data. The benchmark model is the one used in the main part of the paper, calibrated in Table II. In model B, the aggregate component of the profit rate is either high or low, and the probability of the high state follows a Markov chain under the risk-neutral measure. In both cases, the risk-free rate is constant at 3%, book leverage is constant at 0.45, and idiosyncratic shocks follow the benchmark model. Bond q is constructed using the benchmark mapping of Figure I (it is thus deliberately misspecified for model B). The simulation is for fifty firms and 100 years. OLS standard errors are in parentheses.

by Gilchrist and Zakrajsek (2007) with a large panel data set of firm-level bond prices. They regress the investment rate on a firm-specific measure of the cost of capital, based on firm-level bond yields and industry-specific prices for capital. They find a strong negative relationship between the investment rate and the corporate yields, and they also find that q^{usual} and cash flows remain significant.

There are other explanations for the discrepancy between micro and macro results. Returns to scale might be decreasing at the level of an individual firm, even though they are constant for the economy as a whole. This could explain why cash flows are significant in the micro data but not in the macro data. Finally, to the extent that mispricing explains some of the discrepancy between q^{usual} and q^{bond}, the results are consistent with the argument in Lamont and Stein (2006) that there is more mispricing at the aggregate level than at the firm level.

VII.B. Robustness to Model Misspecifications

I now turn to the issue of the specification of aggregate dynamics. The mapping in Figure I assumes that aggregate dynamics follow an AR(1) process. This is a restrictive assumption, especially under the risk-neutral measure.[21] A second model is therefore used to check the robustness of the results. Model B (described in the Appendix) is meant to be the polar opposite to the benchmark model as far as aggregate dynamics are concerned (idiosyncratic shocks are unchanged). This model captures two important ideas. First, cash flows might have a short- and a long-run component, the long-run one being more relevant for valuation and investment. Second, holding constant the objective distribution of cash flows, changes in the market price of risk (due to changes in risk aversion or conditional volatilities) affect the risk-neutral likelihood of good and bad states. In both cases, aggregate cash flows would not summarize the aggregate state. This model is empirically relevant because Vuolteenaho (2002) shows that much of the volatility at the firm level reflects cash-flow news, whereas discount rate shocks are much more important in the aggregate.

I simulate a panel similar to the one discussed above (fifty years, 100 firms). I then use the mapping of Figure I to construct

21. Moreover, this assumption implies that the current aggregate profit rate is a sufficient statistic for the current aggregate state, which is clearly unrealistic. This can be seen in the simulated aggregate regressions where aggregate cash flows have an R^2 of .98 (Table VI, column (2)).

q. The model is therefore misspecified because the benchmark mapping is used to construct q^{bond} in a world where aggregate dynamics are substantially different from the benchmark.

The first result in Table VI is that the model retains most of its explanatory power. q^{bond} is a reliable predictor even when the model is misspecified. The R^2 is still close to one, at 99.4%. The only issue is the bias in the estimated coefficient, which overestimates adjustment costs by about 20%.

The second important message of Table VI is that cash flows are not reliable in the aggregate regression. In model B, cash flows have no explanatory power for aggregate investment. At the firm level, cash flows remain significant, as expected, because firm-level dynamics is the same as in the benchmark model.

VII.C. Bankruptcy Costs and Leverage

The benchmark model is built under the assumptions of no taxes and no bankruptcy costs (Modigliani and Miller 1958).[22] I now study how q^{bond} performs if there *are* taxes and bankruptcy costs. To focus on the crucial issues and to avoid heavy notations, I consider here a simple one-period example. Investment takes place at the beginning of the period, and returns are realized at the end. The risk-free rate is normalized to zero. Profits are taxed at a flat rate, payments to bondholders are deductible, and there are bankruptcy costs. The details of the model are described in the Appendix.

Figure IX shows the mappings for different values of distress costs (in the range of values consistent with empirical estimates). Distress costs do not appear to invalidate the approach taken in this paper. The shape of the mapping is similar across the various models.[23] If anything, higher bankruptcy costs make the mapping from relative bond prices to q more linear, and thus easier to estimate empirically.

VIII. Conclusion

This paper has shown that it is possible to construct Tobin's q using bond prices, by bringing the insights of Black and Scholes

22. In such a world, capital structure is irrelevant, and arbitrary changes in leverage are possible without affecting investment. This issue was discussed at the end of Section IV.

23. The fact that one mapping is higher than another on average is irrelevant because it relates only to the average value of q.

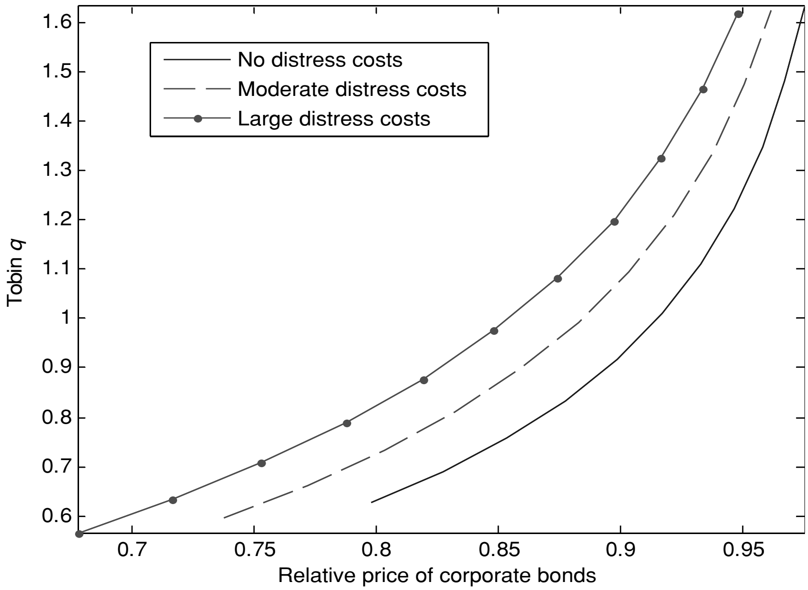

FIGURE IX

Mappings with Taxes and Bankruptcy Costs

Computations for the simple one-period model described in the Appendix, assuming that asset values are lognormally distributed. Moderate distress costs are consistent with the estimates of Andrade and Kaplan (1998).

(1973) and Merton (1974) to the investment models of Abel (1979) and Hayashi (1982). The bond market's q performs much better than the usual measure of q when used to fit the investment equation using postwar U.S. data. The explanatory power is good (both in level and in differences), cash flows are no longer significant, and the inferred adjustment costs are almost twenty times smaller.

Two interpretations of these results are possible. The first is that the equity market is subject to severe mispricing, whereas the bond market is not, or at least not as much. This interpretation is consistent with the arguments in Shiller (2000) and the work of Stein (1996), Gilchrist, Himmelberg, and Huberman (2005), Barberis and Huang (2007), and Brunnermeier, Gollier, and Parker (2007).

Another interpretation is that the stock market is mostly right, but that it measures something other than the value of the existing stock of physical capital. This is the view pushed by Hall (2001) and McGrattan and Prescott (2007). According to this view, firms accumulate and decumulate large stocks of intangible capital. If the payoffs from intangible capital were highly skewed, then they could affect equity prices more than bond prices, and this could explain the results presented in this paper. The difficulty of this theory, of course, is that it rests on a stock of intangible capital that we cannot readily measure (see Atkeson and Kehoe [2005] for a plant-level analysis).

Looking back at Figure IV, it is difficult to imagine a satisfactory answer that does not mix the two theories. Moreover, these theories are not as contradictory as they appear, because the fact that intangible capital is hard to measure increases the scope for disagreement and mispricing. One can hope that future research will be able to reconcile the two explanations.

Appendix

A. Proof of Proposition 1

Let θ_τ be the marginal default rate during period τ. Let $\Theta_{t,\tau}$ be the cumulative default rate in periods $t+1$ up to $\tau-1$. In other words, if a bond has not defaulted at time t, the probability that it enters time $\tau > t$ is $1-\Theta_{t,\tau}$. Thus, by definition, $\Theta_{t,t+1}=0$ and the default rates satisfy the recursive structure: $1-\Theta_{t,\tau} = (1-\theta_{t+1})\left(1-\Theta_{t+1,\tau}\right)$. The value at the end of period t of *one unit of outstanding principal* is

$$b_t^1 = E_t^\pi\Bigg[\sum_{\tau=t+1}^{\infty}(1-\Theta_{t,\tau})\frac{(1-\phi)^{\tau-t-1}}{(1+r_{t,\tau})^{\tau-t}}((1-\theta_\tau)(c+\phi) + \theta_\tau V_\tau/\Psi_{\tau-1})\Bigg]. \tag{22}$$

Similarly, and just to be clear, the price of one unit of principal at the end of $t+1$ is

$$b_{t+1}^1 = E_{t+1}^\pi\Bigg[\sum_{\tau=t+2}^{\infty}(1-\Theta_{t+1,\tau})\frac{(1-\phi)^{\tau-t-2}}{(1+r_{t+1,\tau})^{\tau-t-1}}((1-\theta_\tau)(c+\phi) + \theta_\tau V_\tau/\Psi_{\tau-1})\Bigg]. \tag{23}$$

Using the recursive structure of Θ and the law of iterated expectations, we can substitute (23) into (22) and obtain

$$b_t^1 = \frac{1}{1+r_t} E_t^{\pi}[(1-\theta_{t+1})(c+\phi) + \theta_{t+1} V_{t+1}/\Psi_t] + \frac{1-\phi}{1+r_t} E_t^{\pi}\left[(1-\theta_{t+1}) b_{t+1}^1\right]. \tag{24}$$

Default happens when equity value reaches zero, that is, when

$$V_t < \Psi_{t-1}\left(\phi + c + (1-\phi) b_t^1\right).$$

Therefore, the pricing function satisfies

$$b_t^1 = \frac{1}{1+r_t} E_t^{\pi}\left[\min\left\{\phi + c + (1-\phi) b_{t+1}^1;\ V_{t+1}/\Psi_t\right\}\right]. \tag{25}$$

Now recall that b_t^1 is the price of one unit of outstanding capital. Let us define b_t as the value of bonds outstanding at the end of time t, scaled by end-of-period physical assets,

$$b_t \equiv \psi b_t^1, \tag{26}$$

where book leverage was defined in the main text as $\psi \equiv \Psi_t / k_t$, and assumed to be constant. Multiplying both sides of (25) by ψ, we obtain

$$b_t = \frac{1}{1+r_t} E_t^{\pi}[\min\{(\phi + c)\psi + (1-\phi) b_{t+1};\ v_{t+1}\}].$$

In recursive form, and with constant book leverage, this leads to equation (7). ■

Note that if book leverage were state-contingent, the first term in the min function would simply be $(\phi + c)\psi_t + (1-\phi) b_{t+1} \frac{\psi_t}{\psi_{t+1}}$.

B. Proof of Proposition 3

Assume that $G^H > \Psi$. We can then write the debt pricing formula (21) as

$$B_t = \frac{1}{1+r}\left\{(1-\lambda_t) E_t^{\pi}\left[\min\left(\Psi_t;\ v_{t+1} k_t + G^L\right)\right] + \lambda_t \Psi_t\right\}. \tag{27}$$

Taking the limit in equation (27) as $\lambda_t \to 0$ and $G^L \to 0$, it is clear that

$$B_t = \frac{1}{1+r} E_t^{\pi}[\min(\Psi_t;\ v_{t+1}k_t)].$$

This is the same pricing formula we used earlier, and we have already seen that one can construct a sufficient statistic for investment in this case. On the other hand, the market value of equity moves when it is revealed that $G_t = G^H$. It is always possible to increase the variance of the shocks by increasing G^H. Because these shocks are uncorrelated with investment, the explanatory power of the traditional measure can become arbitrarily small. ■

Note that in a growing economy, it would make sense to index the growth option on aggregate TFP to obtain a model with a balanced growth path.

C. Model B

In this model, the conditional distribution of cash flows follows a Markov process. Aggregate cash flows can be either high, a^H, or low, a^L, and the risk-neutral probability of observing a high cash flow is state-dependent:

$$\Pr(a = a^H) = f(s).$$

State s follows a four-state Markov process under the risk-neutral measure. The complete aggregate state is (s, a). There are therefore eight possible aggregate states: four states for s and two for a. The persistence in the aggregate time series comes from the persistence in s. Conditional on s, aggregate cash flows are i.i.d. The transition matrix of s is chosen to match the empirical moments in Table II. Firm-level dynamics η are given by equation (17) as in the benchmark model.

D. Distress Costs

This is a one-period model. Without taxes or bankruptcy costs, the program of the firm is

$$\max_k E^{\pi}[vk] - k - \gamma k^2/2, \tag{28}$$

where k is investment and v is a random variable. Optimal investment is

$$k = (q-1)/\gamma, \tag{29}$$

where $q \equiv E^{\pi}[v]$. Now assume that profits are taxed at rate τ, that payments to bondholders are deductible, and that there are proportional bankruptcy costs φ. In case of default, a value φvk is lost. The firm is financed with debt and equity, and let ψ be book leverage. It is then straightforward to see that (29) still holds, but the definition of q must be adjusted to

$$q \equiv (1-\tau)E^{\pi}[v] + \tau E^{\pi}[\min(\psi, v)] - \varphi E^{\pi}[v 1_{v<\psi}]. \tag{30}$$

The first term is the unlevered q. The second term captures the tax benefits of debt. The last term captures bankruptcy costs. The value of debt (relative to book assets) is

$$b = E^{\pi}[\min(\psi, v)] - \varphi E^{\pi}[v 1_{v<\psi}]. \tag{31}$$

Equity is $e = E[(1-\tau)(v-\psi)1_{v>\psi}]$. Finally, optimal leverage solves the program

$$\max_{\psi} \tau \int_0^{\infty} \min(\psi, v)\, dH(v) - \varphi \int_0^{\psi} v dH(v), \tag{32}$$

where $H(.)$ is the cumulative distribution of v, and $h(.)$ the associated density. The first term measures the tax benefits of debt, whereas the second term measures the deadweight losses from financial distress. The first-order condition for optimal leverage is

$$\tau \int_{\psi}^{\infty} dH(v) = \varphi \psi h(\psi). \tag{33}$$

I assume that v is lognormally distributed with volatility 0.75. In the benchmark case, I use $\varphi = 0.2$ and a lognormal mean of -0.2. These parameters yield values consistent with the evidence in Andrade and Kaplan (1998) and Almeida and Philippon (2007). Andrade and Kaplan (1998) estimate losses around 10%–15% of firm value one-year before bankruptcy. The parameter φ applies to *ex post* losses, and these happen when v turns out to be low. A value of 20% implies that the deadweight losses relative to initial firm value are around 10%. To be consistent with a book leverage of 0.5, equation (33) implies that $\tau = 11.5\%$. This is consistent with Graham (2000).

The benchmark case is therefore chosen so that leverage is optimal (on average, not state by state) and distress costs are consistent with empirical estimates. To create Figure IX, I simulate the model with different values of the mean of the lognormal distribution, from −0.7 to +0.3. Finally, I repeat the exercise for each value of the distress cost parameter: $\varphi \in \{0, 0.2, 0.4\}$.

STERN SCHOOL OF BUSINESS, NEW YORK UNIVERSITY, NBER, AND CEPR

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