

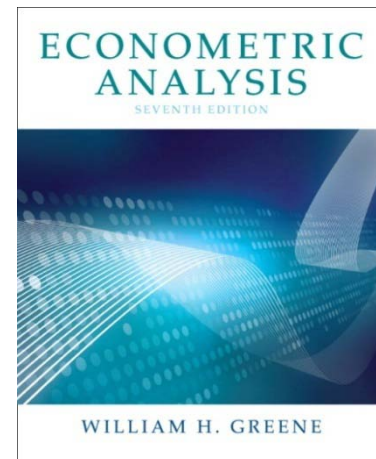
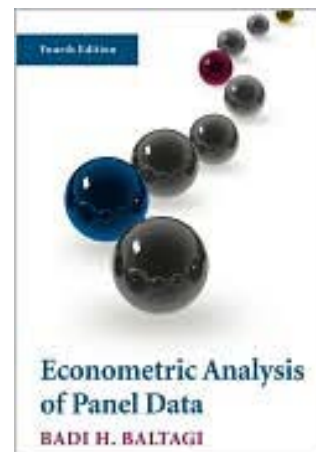
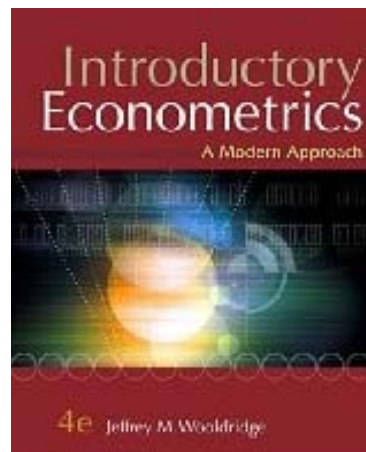
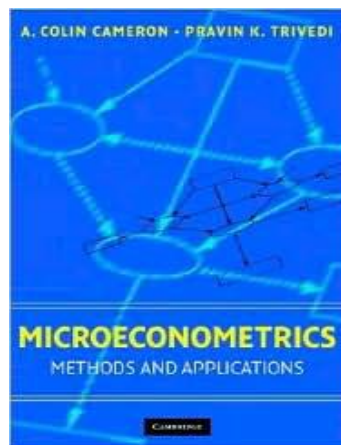


Topics in Microeconometrics

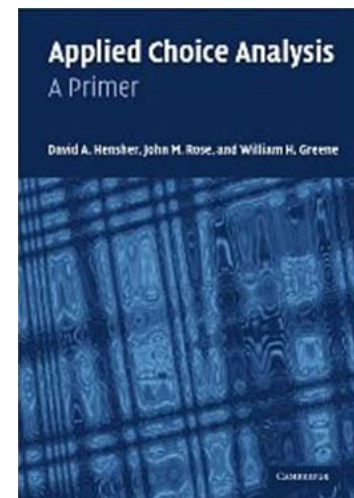
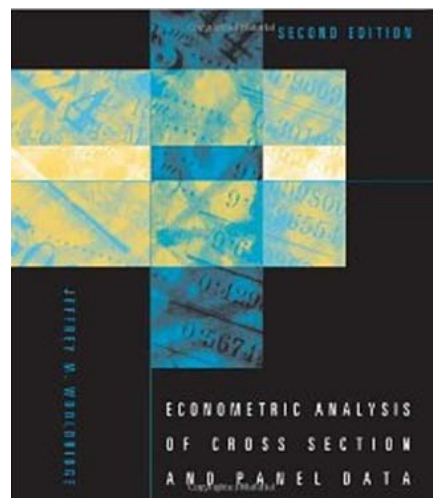
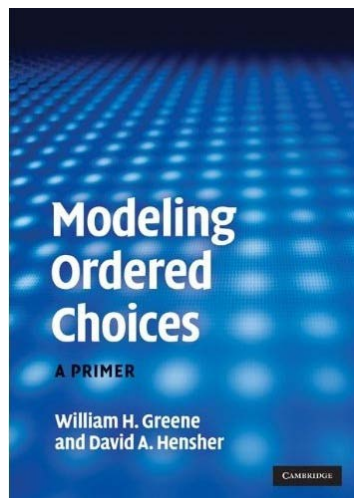
William Greene

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Part 2: Endogenous Variables in Linear Regression






Endogeneity

- $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$,
- Definition: $E[\boldsymbol{\varepsilon}|\mathbf{x}] \neq 0$
- Why not?
 - Omitted variables
 - Unobserved heterogeneity (equivalent to omitted variables)
 - Measurement error on the RHS (equivalent to omitted variables)
 - Structural aspects of the model
 - Endogenous sampling and attrition
 - Simultaneity (?)



Instrumental Variable Estimation

- One “problem” variable – the “last” one
- $y_{it} = \beta_1 x_{1it} + \beta_2 x_{2it} + \dots + \beta_K x_{Kit} + \varepsilon_{it}$
- $E[\varepsilon_{it} | x_{Kit}] \neq 0$. (0 for all others)
- There exists a variable z_{it} such that 
 - $E[x_{Kit} | x_{1it}, x_{2it}, \dots, x_{K-1,it}, z_{it}] = g(x_{1it}, x_{2it}, \dots, x_{K-1,it}, z_{it})$
In the presence of the other variables, z_{it} “explains” x_{it}
 - $E[\varepsilon_{it} | x_{1it}, x_{2it}, \dots, x_{K-1,it}, z_{it}] = 0$
In the presence of the other variables, z_{it} and ε_{it} are uncorrelated.
- A projection interpretation: In the projection
$$x_{Kt} = \theta_1 x_{1it} + \theta_2 x_{2it} + \dots + \theta_{K-1} x_{K-1,it} + \theta_K z_{it},$$
$$\theta_K \neq 0.$$



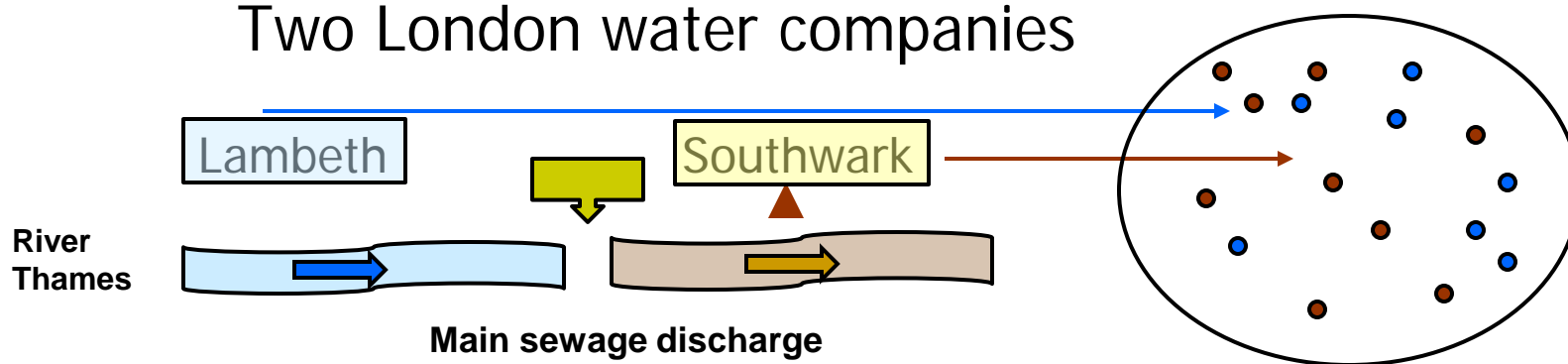
The First IV Study: Natural Experiment

(Snow, J., On the Mode of Communication of Cholera, 1855)

<http://www.ph.ucla.edu/epi/snow/snowbook3.html>

- London Cholera epidemic, ca 1853-4
- Cholera = $f(\text{Water Purity}, u) + \varepsilon$.
 - 'Causal' effect of water purity on cholera?
 - Purity = $f(\text{cholera prone environment (poor, garbage in streets, rodents, etc.)})$. Regression does not work.

Two London water companies



Paul Grootendorst: A Review of Instrumental Variables Estimation of Treatment Effects...

http://individual.utoronto.ca/grootendorst/pdf/IV_Paper_Sept6_2007.pdf



IV Estimation

- $\text{Cholera} = f(\text{Purity}, u) + \varepsilon$
- $Z = \text{water company}$
- $\text{Cov}(\text{Cholera}, Z) = \delta \text{Cov}(\text{Purity}, Z)$
- Z is randomly mixed in the population (two full sets of pipes) and uncorrelated with behavioral unobservables, u)
- $\text{Cholera} = \alpha + \delta \text{Purity} + u + \varepsilon$
 - $\text{Purity} = \text{Mean} + \text{random variation} + \lambda u$
 - $\text{Cov}(\text{Cholera}, Z) = \delta \text{Cov}(\text{Purity}, Z)$



Cornwell and Rupert Data

Cornwell and Rupert Returns to Schooling Data, 595 Individuals, 7 Years
Variables in the file are

EXP = work experience
WKS = weeks worked
OCC = occupation, 1 if blue collar,
IND = 1 if manufacturing industry
SOUTH = 1 if resides in south
SMSA = 1 if resides in a city (SMSA)
MS = 1 if married
FEM = 1 if female
UNION = 1 if wage set by union contract

ED = years of education

LWAGE = log of wage = dependent variable in regressions

These data were analyzed in Cornwell, C. and Rupert, P., "Efficient Estimation with Panel Data: An Empirical Comparison of Instrumental Variable Estimators," *Journal of Applied Econometrics*, 3, 1988, pp. 149-155. See Baltagi, page 122 for further analysis. The data were downloaded from the website for Baltagi's text.

Specification: Quadratic Effect of Experience

```

-----
Ordinary least squares regression -----
LHS=LWAGE Mean = 6.67635
Standard deviation = .46151
-----
No. of observations = 4165 DegFreedom Mean square
Regression Sum of Squares = 370.955 10 37.09546
Residual Sum of Squares = 515.950 4154 .12421
Total Sum of Squares = 886.905 4164 .21299
-----
Standard error of e = .35243 Root MSE .35196
Fit R-squared = .41826 R-bar squared .41686
Model test F[ 10, 4154] = 298.66153 Prob F > F* .00000
  
```

LWAGE	Coefficient	Standard Error	z	Prob. z >Z*	95% Confidence Interval	
Constant	5.24547***	.07170	73.15	.0000	5.10493	5.38600
ED	.05654***	.00261	21.64	.0000	.05142	.06166
EXP	.04045***	.00217	18.61	.0000	.03619	.04471
EXP*EXP	-.00068***	.4783D-04	-14.24	.0000	-.00077	-.00059
WKS	.00449***	.00109	4.12	.0000	.00235	.00662
OCC	-.14053***	.01472	-9.54	.0000	-.16939	-.11167
SOUTH	-.07210***	.01249	-5.77	.0000	-.09658	-.04762
SMSA	.13901***	.01207	11.51	.0000	.11534	.16267
MS	.06736***	.02063	3.26	.0011	.02692	.10779
FEM	-.38922***	.02518	-15.46	.0000	-.43857	-.33987
UNION	.09015***	.01289	6.99	.0000	.06488	.11542

nnnnn.D-xx or D+xx => multiply by 10 to -xx or +xx.

***, **, * ==> Significance at 1%, 5%, 10% level.



The Effect of Education on LWAGE

$$\text{LWAGE} = \beta_1 + \beta_2 \text{EDUC} + \beta_3 \text{EXP} + \beta_4 \text{EXP}^2 + \dots \varepsilon$$

What is ε ? **Ability, Motivation**, ... + everything else

$$\text{EDUC} = f(\text{GENDER}, \text{SMSA}, \text{SOUTH}, \text{Ability, Motivation}, \dots)$$



What Influences LWAGE?

$$\begin{aligned}\text{LWAGE} = & \beta_1 + \beta_2 \text{EDUC}(\mathbf{X}, \text{Ability}, \text{Motivation}, \dots) \\ & + \beta_3 \text{EXP} + \beta_4 \text{EXP}^2 + \dots \\ & \varepsilon(\text{Ability}, \text{Motivation})\end{aligned}$$

Increased **Ability** is associated with increases in **EDUC**(\mathbf{X} , **Ability**, **Motivation**, ...) and $\varepsilon(\text{Ability}, \text{Motivation})$

What looks like an effect due to increase in **EDUC** may be an increase in **Ability**. The estimate of β_2 picks up the effect of **EDUC** and the hidden effect of **Ability**.



An Exogenous Influence

$$\begin{aligned}\text{LWAGE} = & \beta_1 + \beta_2 \text{EDUC}(\text{X}, \text{Z}, \text{Ability}, \text{Motivation}, \dots) \\ & + \beta_3 \text{EXP} + \beta_4 \text{EXP}^2 + \dots \\ & \varepsilon(\text{Ability}, \text{Motivation})\end{aligned}$$

Increased **Z** is associated with increases in

EDUC(**X**, **Z**, **Ability**, **Motivation**, ...) and not $\varepsilon(\text{Ability}, \text{Motivation})$

An effect due to the effect of an increase **Z** on **EDUC** will only be an increase in **EDUC**. The estimate of β_2 picks up the effect of **EDUC** only.

Z is an Instrumental Variable



Instrumental Variables

- Structure
 - LWAGE (**ED**,**EXP**,**EXPSQ**,**WKS**,**OCC**,**SOUTH**,**SMSA**,**UNION**)
 - ED (**MS**, **FEM**)
- Reduced Form:
LWAGE[**ED** (**MS**, **FEM**),
EXP,**EXPSQ**,**WKS**,**OCC**,
SOUTH,**SMSA**,**UNION**]



Two Stage Least Squares Strategy

- Reduced Form:

LWAGE[**ED** (**MS**, **FEM**, **X**),
EXP,EXPSQ,WKS,OCC,
SOUTH,SMSA,UNION]

- Strategy
 - (1) Purge ED of the influence of everything but MS, FEM (and the other variables). Predict ED using all exogenous information in the sample (**X** and **Z**).
 - (2) Regress LWAGE on this prediction of ED and everything else.
 - Standard errors must be adjusted for the predicted ED

OLS

```

-----
Ordinary least squares regression
LHS=LWAGE
Mean = 6.67635
Standard deviation = .46151
-----
No. of observations = 4165
DegFreedom 8
Mean square 36.38019
Regression Sum of Squares = 291.042
Residual Sum of Squares = 595.863
Total Sum of Squares = 886.905
-----
Standard error of e = .37865
Root MSE .37824
Fit R-squared = .32815
R-bar squared .32686
Model test F[ 8, 4156] = 253.74283
Prob F > F* .00000
  
```

LWAGE	Coefficient	Standard Error	z	Prob. z >Z*	95% Confidence Interval	
Constant	4.97986***	.07430	67.02	.0000	4.83424	5.12549
EXP	.04308***	.00232	18.54	.0000	.03853	.04764
EXPSQ	-.00070***	.5128D-04	-13.68	.0000	-.00080	-.00060
WKS	.00760***	.00116	6.53	.0000	.00532	.00988
OCC	-.11578***	.01578	-7.34	.0000	-.14672	-.08485
SOUTH	-.08207***	.01341	-6.12	.0000	-.10835	-.05578
SMSA	.09885***	.01285	7.69	.0000	.07367	.12403
UNION	.12891***	.01374	9.38	.0000	.10197	.15584
ED	.06365***	.00279	22.82	.0000	.05818	.06911

Note: nnnnn.D-xx or D+xx => multiply by 10 to -xx or +xx.
 Note: ***, **, * ==> Significance at 1%, 5%, 10% level.



```

Two stage least squares regression .....
LHS=LWAGE  Mean          =      6.67635
           Standard deviation =      .46151
           Number of observs. =      4165
Model size Parameters    =          9
           Degrees of freedom =      4156
Residuals  Sum of squares =     6921.67
           Standard error of e =      1.29053
Fit         R-squared      =     -6.82120
           Adjusted R-squared =     -6.83625
Not using OLS or no constant. Rsqrd & F may be < 0

```

Instrumental Variables:

ONE	MS	FEM	EXP	Intrct01	WKS
OCC	SOUTH	SMSA	UNION		

LWAGE	Coefficient	Standard Error	z	Prob. z >Z*	95% Confidence Interval	
Constant	-4.38670***	1.40197	-3.13	.0018	-7.13451	-1.63889
EXP	.06447***	.00852	7.56	.0000	.04777	.08117
EXP*EXP	-.00058***	.00018	-3.32	.0009	-.00093	-.00024
WKS	.01533***	.00413	3.72	.0002	.00725	.02342
OCC	1.71424***	.27473	6.24	.0000	1.17578	2.25270
SOUTH	.31274***	.07394	4.23	.0000	.16782	.45767
SMSA	-.13695**	.05588	-2.45	.0142	-.24647	-.02744
UNION	.37025***	.05879	6.30	.0000	.25502	.48548
ED	.65029***	.00689	7.48	.0000	.40000	.82059

***, **, * ==> Significance at 1%, 5%, 10% level.

The weird results for the coefficient on ED happened because the instruments, MS and FEM are dummy variables. There is not enough variation in these variables.

4.97986***
.04308***
-.00070***
.00760***
-.11578***
-.08207***
.09885***
.12891***
.06365***





Source of Endogeneity

- **LWAGE** = $f(\text{ED}, \text{EXP}, \text{EXPSQ}, \text{WKS}, \text{OCC}, \text{SOUTH}, \text{SMSA}, \text{UNION}) + \varepsilon$
 - **ED** = $f(\text{MS}, \text{FEM}, \text{EXP}, \text{EXPSQ}, \text{WKS}, \text{OCC}, \text{SOUTH}, \text{SMSA}, \text{UNION}) + u$
-



Remove the Endogeneity

- **LWAGE** = $f(\text{ED}, \text{EXP}, \text{EXPSQ}, \text{WKS}, \text{OCC}, \text{SOUTH}, \text{SMSA}, \text{UNION}) + \boxed{u + \varepsilon}$

- **LWAGE** = $f(\text{ED}, \text{EXP}, \text{EXPSQ}, \text{WKS}, \text{OCC}, \text{SOUTH}, \text{SMSA}, \text{UNION}) + u + \varepsilon$

- Strategy
 - Estimate u
 - Add u to the equation. ED is uncorrelated with ε when u is in the equation.

Auxiliary Regression for ED to Obtain Residuals

```

Ordinary least squares regression .....
LHS=ED Mean = 12.84538
Standard deviation = 2.78800
-----
No. of observations = 4165 DegFreedom Mean square
Regression Sum of Squares = 14162.8 9 1573.64724
Residual Sum of Squares = 18203.6 4155 4.38113
Total Sum of Squares = 32366.4 4164 7.77292
-----
Standard error of e = 2.09312 Root MSE 2.09060
Fit R-squared = .43758 R-bar squared .43636
Model test F[ 9, 4155] = 359.18746 Prob F > F* .00000
  
```

ED	Coefficient	Standard Error	z	Prob. z >Z*	95% Confidence Interval	
Constant	16.0756***	.34520	46.57	.0000	15.3990	16.7521
MS	.27698**	.12245	2.26	.0237	.03698	.51698
FEM	-.46653***	.14937	-3.12	.0018	-.75929	-.17376
EXP	-.04189***	.01290	-3.25	.0012	-.06716	-.01661
EXP*EXP	-.00014	.00028	-.50	.6181	-.00070	.00042
WKS	-.01810***	.00647	-2.80	.0051	-.03078	-.00543
OCC	-3.12102***	.07282	-42.86	.0000	-3.26376	-2.97829
SOUTH	-.65003***	.07349	-8.85	.0000	-.79407	-.50599
SMSA	.46655***	.07134	6.54	.0000	.32672	.60638
UNION	-.47323***	.07621	-6.21	.0000	-.62260	-.32385

***, **, * ==> Significance at 1%, 5%, 10% level.



OLS with Residual (Control Function) Added

```

-----
Ordinary least squares regression
LHS=LWAGE
Mean = 6.67635
Standard deviation = .46151
No. of observations = 4165
DegFreedom 9
Mean square 40.87643
Regression Sum of Squares = 367.888
Residual Sum of Squares = 519.017
Total Sum of Squares = 886.905
Standard error of e = .35343
Root MSE .35301
Fit R-squared = .41480
R-bar squared .41353
Model test F[ 9, 4155] = 327.23700
Prob F > F* .00000
  
```

LWAGE	Coefficient	Standard Error	z	Prob. z >Z*	95% Confidence Interval	
Constant	-4.38670***	.38395	-11.43	.0000	-5.13923	-3.63417
EXP	.06447***	.00233	27.62	.0000	.05990	.06904
EXP*EXP	-.00058***	.4810D-04	-12.13	.0000	-.00068	-.00049
WKS	.01533***	.00113	13.57	.0000	.01312	.01755
OCC	1.71424***	.07524	22.78	.0000	1.56678	1.86171
SOUTH	.31274***	.02025	15.44	.0000	.27305	.35243
SMSA	-.13695***	.01530	-8.95	.0000	-.16695	-.10696
UNION	.37025***	.01610	23.00	.0000	.33869	.40180
ED	.65029***	.02380	27.33	.0000	.60366	.69693
U	-.59376***	.02394	-24.80	.0000	-.64068	-.54684

nnnnn.D-xx or D+xx => multiply by 10 to -xx or +xx.
 ***, **, * ==> Significance at 1%, 5%, 10% level.

2SLS

```

-4.38670***
.06447***
-.00058***
.01533***
1.71424***
.31274***
-.13695**
.37025***
.65029***
  
```

A Warning About Control Functions

Two stage least squares regression						
Standard error of e = 1.29053						
LWAGE	Coefficient	Standard Error	z	Prob. z >Z*	95% Confidence Interval	
Constant	-4.38670***	1.40197	-3.13	.0018	-7.13451	-1.63889
EXP	.06447***	.00852	7.56	.0000	.04777	.08117
EXP*EXP	-.00058***	.00018	-3.32	.0009	-.00093	-.00024
WKS	.01533***	.00413	3.72	.0002	.00725	.02342
OCC	1.71424***	.27473	6.24	.0000	1.17578	2.25270
SOUTH	.31274***	.07394	4.23	.0000	.16782	.45767
SMSA	-.13695**	.05588	-2.45	.0142	-.24647	-.02744
UNION	.37025***	.05879	6.30	.0000	.25502	.48548
ED	.65029***	.08689	7.48	.0000	.48000	.82059
Residual augmented least squares regression						
Standard error of e = .35343						
LWAGE	Coefficient	Standard Error	z	Prob. z >Z*	95% Confidence Interval	
Constant	-4.38670***	.38395	-11.43	.0000	-5.13923	-3.63417
EXP	.06447***	.00233	27.62	.0000	.05990	.06904
EXP*EXP	-.00058***	.4810D-04	-12.13	.0000	-.00068	-.00049
WKS	.01533***	.00113	13.57	.0000	.01312	.01755
OCC	1.71424***	.07524	22.78	.0000	1.56678	1.86171
SOUTH	.31274***	.02025	15.44	.0000	.27305	.35243
SMSA	-.13695***	.01530	-8.95	.0000	-.16695	-.10696
UNION	.37025***	.01610	23.00	.0000	.33869	.40180
ED	.65029***	.02380	27.33	.0000	.60366	.69693
U	-.59376***	.02394	-24.80	.0000	-.64068	-.54684

Sum of squares is not computed correctly because U is in the regression. A general result. Control function estimators usually require a fix to the estimated covariance matrix for the estimator.



Endogenous Dummy Variable

- $Y = x\beta + \delta T + \varepsilon$ (**unobservable factors**)
- T = a dummy variable (treatment)
- $T = 0/1$ depending on:
 - x and z
 - The same **unobservable factors**
- T is endogenous – same as ED



Application: Health Care Panel Data

German Health Care Usage Data, 7,293 Individuals, Varying Numbers of Periods

Variables in the file are

Data downloaded from Journal of Applied Econometrics Archive. This is an unbalanced panel with 7,293 individuals. They can be used for regression, count models, binary choice, ordered choice, and bivariate binary choice. **This is a large data set. There are altogether 27,326 observations. The number of observations ranges from 1 to 7. (Frequencies are: 1=1525, 2=2158, 3=825, 4=926, 5=1051, 6=1000, 7=987).** Note, the variable NUMOBS below tells how many observations there are for each person. This variable is repeated in each row of the data for the person. (Downloaded from the JAE Archive)

- ➔ **DOCTOR** = 1(Number of doctor visits > 0)
- HOSPITAL** = 1(Number of hospital visits > 0)
- HSAT** = health satisfaction, coded 0 (low) - 10 (high)
- DOCVIS** = number of doctor visits in last three months
- HOSPVIS** = number of hospital visits in last calendar year
- ➔ **PUBLIC** = insured in public health insurance = 1; otherwise = 0
- ➔ **ADDON** = insured by add-on insurance = 1; otherwise = 0
- HHNINC** = household nominal monthly net income in German marks / 10000.
(4 observations with income=0 were dropped)
- HHKIDS** = children under age 16 in the household = 1; otherwise = 0
- EDUC** = years of schooling
- AGE** = age in years
- MARRIED** = marital status
- EDUC** = years of education



A study of moral hazard

Riphahn, Wambach, Million: "Incentive Effects in the Demand for Healthcare"

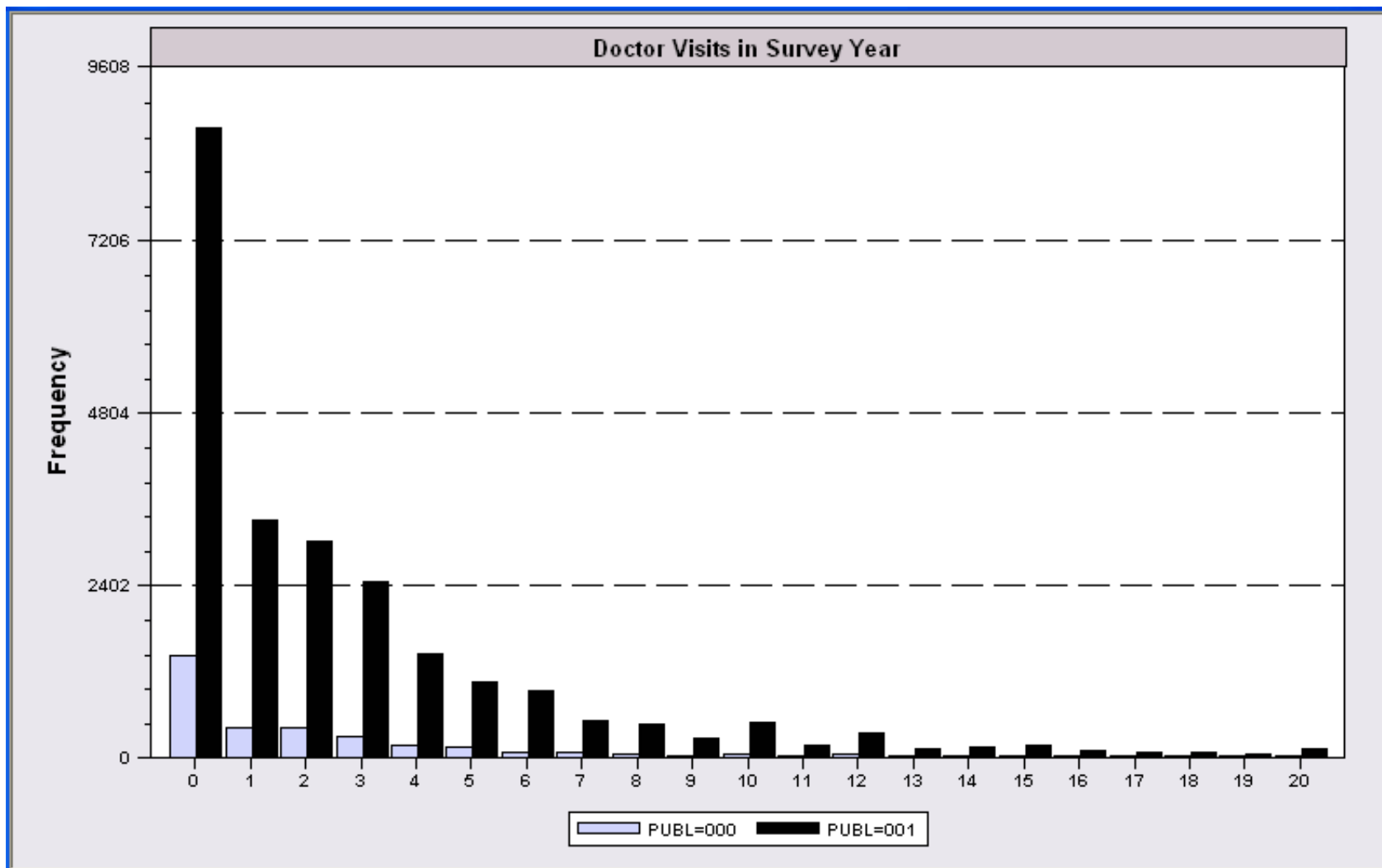
Journal of Applied Econometrics, 2003

Did the presence of the ADDON insurance influence the demand for health care – doctor visits and hospital visits?

For a simple example, we examine the PUBLIC insurance (89%) instead of ADDON insurance (2%).



Evidence of Moral Hazard?



Regression Study

```

-----
Ordinary      least squares regression .....
LHS=DOCVIS   Mean                =          3.18352
              Standard deviation =          5.68969
              Number of observs. =          27326
Model size    Parameters          =           6
              Degrees of freedom =          27320
Residuals     Sum of squares      =      853326.41135
              Standard error of e =          5.58878
Fit           R-squared           =          .03533
              Adjusted R-squared  =          .03516
Model test    F[  5, 27320] (prob) =      200.1(.0000)
-----
  
```

DOCVIS	Coefficient	Standard Error	z	Prob. z> Z	Mean of X
Constant	.43660	.29014	1.50	.1324	
AGE	.06754***	.00304	22.25	.0000	43.5257
HHNINC	-1.54898***	.19956	-7.76	.0000	.35208
FEMALE	.94128***	.06895	13.65	.0000	.47877
EDUC	-.05549***	.01624	-3.42	.0006	11.3206
PUBLIC	.59843***	.11370	5.26	.0000	.88571

Note: ***, **, * ==> Significance at 1%, 5%, 10% level.



Endogenous Dummy Variable

- Doctor Visits = $f(\text{Age, Educ, Health, Presence of Insurance, Other unobservables})$
 - Insurance = $f(\text{Expected Doctor Visits, Other unobservables})$
- A purple arrow points from the 'Other unobservables' term in the first equation to the 'Other unobservables' term in the second equation, illustrating the endogeneity between the two variables.



Approaches

- (Parametric) Control Function: Build a structural model for the two variables (Heckman)
- (Semiparametric) Instrumental Variable: Create an instrumental variable for the dummy variable (Barnow/Cain/ Goldberger, Angrist, Current generation of researchers)
- (?) Propensity Score Matching (Heckman et al., Becker/Ichino, Many recent researchers)



Heckman's Control Function Approach

- $Y = x\beta + \delta T + E[\varepsilon|T] + \{\varepsilon - E[\varepsilon|T]\}$
- $\lambda = E[\varepsilon|T]$, computed from a model for whether $T = 0$ or 1

```
-----
Sample Selection Model.....
Two step      least squares regression .....
LHS=DOCVIS   Mean                      =          3.18352
Correlation of disturbance in regression
and Selection Criterion (Rho).....      -.88169
-----+-----
```

DOCVIS	Coefficient	Standard Error	z	Prob. z> Z	Mean of X
Constant	-14.8749***	1.01175	-14.70	.0000	
AGE	.07062***	.00348	20.28	.0000	43.5257
HHNINC	.58241**	.26463	2.20	.0277	.35208
FEMALE	1.00046***	.06885	14.53	.0000	.47877
EDUC	.39321***	.03360	11.70	.0000	11.3206
PUBLIC	11.1200***	.66997	16.60	.0000	.88571
LAMBDA	-5.64728***	.35142	-16.07	.0000	.497D-09

```
-----+-----
Note: ***, **, * ==> Significance at 1%, 5%, 10% level.
-----
```

Magnitude = 11.1200 is nonsensical in this context.



Instrumental Variable Approach

- Construct a prediction for T using only the exogenous information
- Use 2SLS using this instrumental variable.

Two stage least squares regression					
LHS=DOCVIS	Mean	= 3.18352			
ONE	AGE	HHNINC	FEMALE	EDUC	TFIT

DOCVIS	Coefficient	Standard Error	z	Prob. z> Z	Mean of X

Constant	-33.1176***	2.56970	-12.89	.0000	
AGE	.07535***	.00487	15.47	.0000	43.5257
HHNINC	3.17825***	.47734	6.66	.0000	.35208
FEMALE	.62839***	.11232	5.59	.0000	.47877
EDUC	.92150***	.07802	11.81	.0000	11.3206
PUBLIC	23.9012***	1.76483	13.54	.0000	.88571

Note: ***, **, * ==> Significance at 1%, 5%, 10% level.					

Magnitude = 23.9012 is also nonsensical in this context.



Propensity Score Matching

- Create a model for T that produces probabilities for T=1: "Propensity Scores"
- Find people with the same propensity score – some with T=1, some with T=0
- Compare number of doctor visits of those with T=1 to those with T=0.

```
+-----+
| Estimated Average Treatment Effect (PUBLIC ) Outcome is DOCVIS |
| Nearest Neighbor Using average of 1 closest neighbors          |
| Note, controls may be reused in defining matches.              |
| Number of bootstrap replications used to obtain variance      = 25 |
+-----+
| Estimated average treatment effect = .258108                   |
| Begin bootstrap iterations *****                           |
| End bootstrap iterations *****                               |
+-----+
| Number of Treated observations = 24203 Number of controls = 2568 |
| Estimated Average Treatment Effect = .258108                    |
| Estimated Asymptotic Standard Error = .163314                   |
| t statistic (ATT/Est.S.E.) = 1.580447                           |
| Confidence Interval for ATT = ( -.061986 to .578203) 95%         |
| Average Bootstrap estimate of ATT = .315962                     |
| ATT - Average bootstrap estimate = -.057853                     |
+-----+
```



Difference in Differences

With two periods,

$$\Delta y_{it} = y_{i2} - y_{i1} = \delta_0 + (\mathbf{x}'_{i2} \boldsymbol{\beta}_{i1}) + u_i$$

Consider a "treatment, D_i ," that takes place between time 1 and time 2 for some of the individuals

$$\Delta y_i = \delta_0 + (\Delta \mathbf{x} \boldsymbol{\beta}' + \delta_1 D_i + u_i$$

D_i = the "treatment dummy"

This is a linear regression model. If there are no regressors,

$$\hat{\delta}_1 = \overline{\Delta y} \mid \text{treatment} - \overline{\Delta y} \mid \text{control}$$

= "difference in differences" estimator.

$\hat{\delta}_0$ = Average change in y_i for the "treated"



Difference-in-Differences Model

With two periods and strict exogeneity of D and T ,

$$y_{it} = \beta_0 + \beta_1 D_{it} + \beta_2 T_t + \beta_3 T_t D_{it} + \varepsilon_{it}$$

D_{it} = dummy variable for a treatment that takes place
between time 1 and time 2 for some of the individuals,

T_t = a time period dummy variable, 0 in period 1,
1 in period 2.

This is a linear regression model. If there are no regressors,

Using least squares,

$$b_3 = (\bar{y}_2 - \bar{y}_1)_{D=1} - (\bar{y}_2 - \bar{y}_1)_{D=0}$$



Difference in Differences

$$y_{it} = \beta_0 + \beta_1 D_{it} + \beta_2 T_t + \beta_3 D_{it} T_t + \mathbf{\beta}' \mathbf{x}_{it} + \varepsilon_{it}, t = 1, 2$$

$$\Delta y_{it} = \beta_2 + \beta_3 D_{i2} + \Delta(\mathbf{\beta}' \mathbf{x}_{it}) + \Delta \varepsilon_{it}$$

$$= \beta_2 + \beta_3 D_{i2} + \mathbf{\beta}'(\Delta \mathbf{x}_{it}) + u_i$$

$$(\Delta y_{it} \mid D = 1) - (\Delta y_{it} \mid D = 0)$$

$$= \beta_3 + \mathbf{\beta}' [(\Delta \mathbf{x}_{it} \mid D = 1) - (\Delta \mathbf{x}_{it} \mid D = 0)]$$

If the same individual is observed in both states, the second term is zero. If the effect is estimated by averaging individuals with $D = 1$ and different individuals with $D=0$, then part of the 'effect' is explained by change in the covariates, not the treatment.