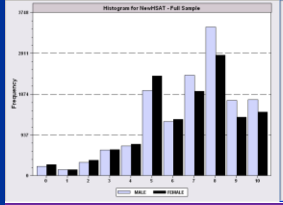
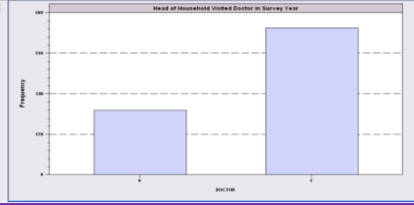
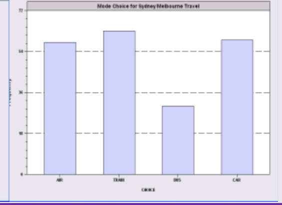
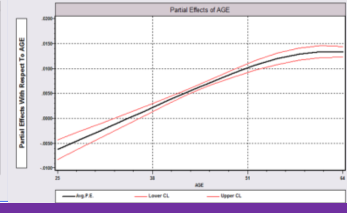


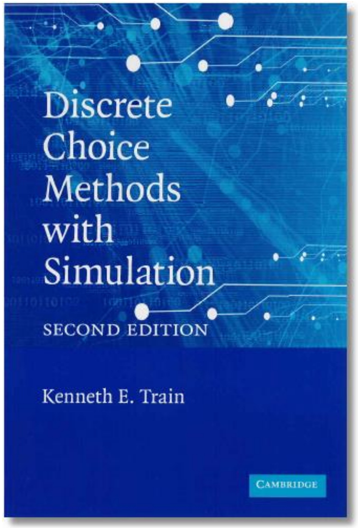


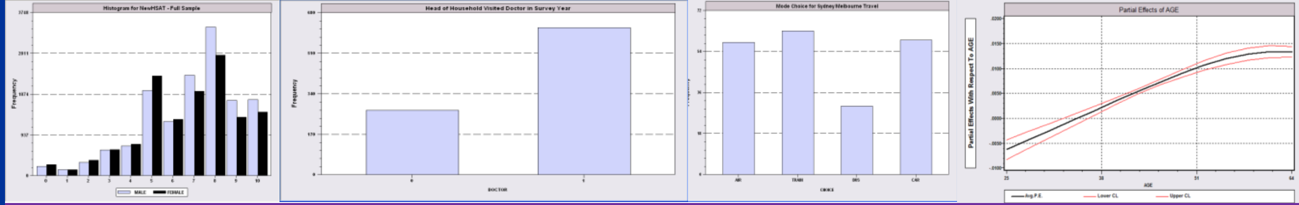




Discrete Choice Modeling

William Greene
Stern School of Business
New York University



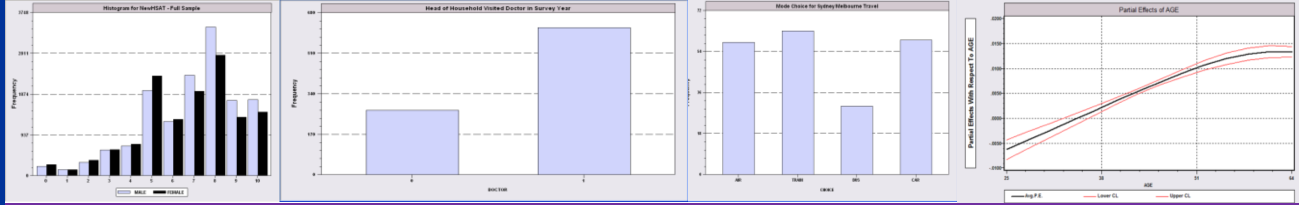


Random Parameters Model

- Allow model parameters as well as constants to be random
- Allow multiple observations with persistent effects
- Allow a hierarchical structure for parameters – not completely random

$$U_{itj} = \boldsymbol{\beta}_1'\mathbf{x}_{i1tj} + \boldsymbol{\beta}_{2i}'\mathbf{x}_{i2tj} + \boldsymbol{\gamma}_i'\mathbf{z}_{it} + \varepsilon_{ijt}$$

- Random parameters in multinomial logit model
  - β_1 = nonrandom (fixed) parameters
  - β_{2i} = random parameters that may vary across individuals and across time
- Maintain I.I.D. assumption for ε_{ijt} (given β)



Continuous Random Variation in Preference Weights

Heterogeneity arises from continuous variation in $\boldsymbol{\beta}_i$ across individuals. (Note Classical and Bayesian)

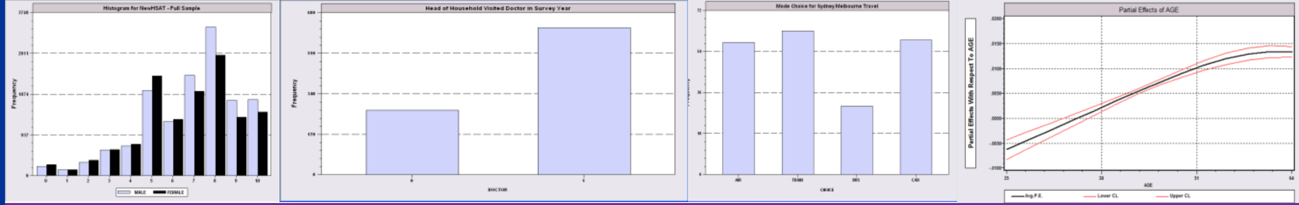
$$U_{ijt} = \alpha_j + \boldsymbol{\beta}_i'\mathbf{x}_{itj} + \boldsymbol{\gamma}_j'\mathbf{z}_{it} + \varepsilon_{ijt}$$

$$\boldsymbol{\beta}_i = \boldsymbol{\beta} + \boldsymbol{\Delta}\mathbf{h}_i + \mathbf{w}_i$$

$$\beta_{i,k} = \beta_k + \boldsymbol{\delta}_k'\mathbf{h}_i + w_{i,k}$$

Most treatments set $\boldsymbol{\Delta} = \mathbf{0}$, $\quad \boldsymbol{\beta}_i = \boldsymbol{\beta} + \mathbf{w}_i$

$$\text{Prob}[\text{choice } j \mid i, t, \boldsymbol{\beta}_i] = \frac{\exp(\alpha_j + \boldsymbol{\beta}_i'\mathbf{x}_{itj} + \boldsymbol{\gamma}_j'\mathbf{z}_{it})}{\sum_{j=1}^{J_t(i)} \exp(\alpha_j + \boldsymbol{\beta}_i'\mathbf{x}_{itj} + \boldsymbol{\gamma}_j'\mathbf{z}_{it})}$$

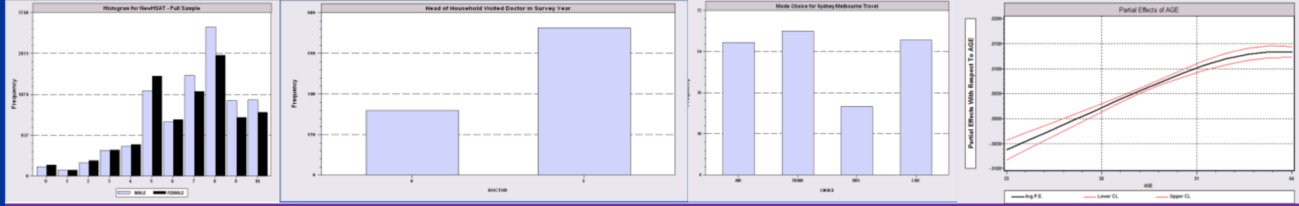


The Random Parameters Logit Model

$$\text{Prob}[\text{choice } j \,|\, i,t,\boldsymbol{\beta}_i] = \frac{\exp(\alpha_j + \boldsymbol{\beta}_i'\mathbf{x}_{itj} + \boldsymbol{\gamma}_{j,i}'\mathbf{z}_{it})}{\sum_{j=1}^{J_t(i)} \exp(\alpha_j + \boldsymbol{\beta}_i'\mathbf{x}_{itj} + \boldsymbol{\gamma}_{j.i}'\mathbf{z}_{it})}$$

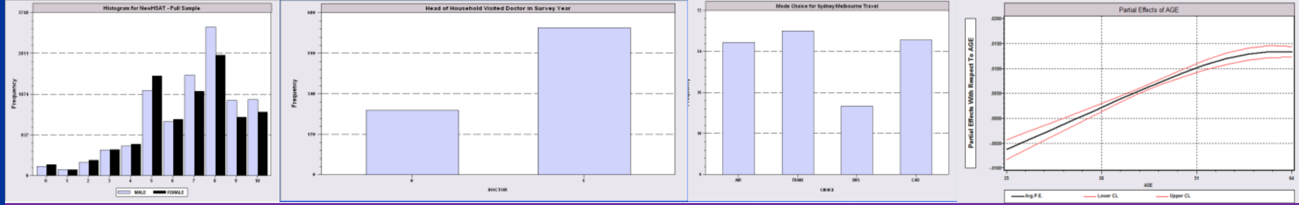
Multiple choice situations: Independent conditioned on the individual specific parameters

$$\text{Prob}[\text{choice } j \,|\, i,t=1,\ldots,T_i,\boldsymbol{\beta}_i] = \prod_{t=1}^{T(i)} \frac{\exp(\alpha_j + \boldsymbol{\beta}_i'\mathbf{x}_{itj} + \boldsymbol{\gamma}_{j,i}'\mathbf{z}_{it})}{\sum_{j=1}^{J_t(i)} \exp(\alpha_j + \boldsymbol{\beta}_i'\mathbf{x}_{itj} + \boldsymbol{\gamma}_{j,i}'\mathbf{z}_{it})}$$



Modeling Variations

- Parameter specification
  - "Nonrandom" – variance = 0
  - Correlation across parameters – random parts correlated
  - Fixed mean – not to be estimated. Free variance
  - Fixed range – mean estimated, triangular from 0 to 2β
  - Hierarchical structure - $\beta_{ik} = \beta_k + \delta_k'\mathbf{z}_i$
- Stochastic specification
  - Normal, uniform, triangular (tent) distributions
  - Strictly positive – lognormal parameters (e.g., on income)
  - Autoregressive: v(i,t,k) = u(i,t,k) + r(k)v(i,t-1,k) [this picks up time effects in multiple choice situations, e.g., fatigue.]



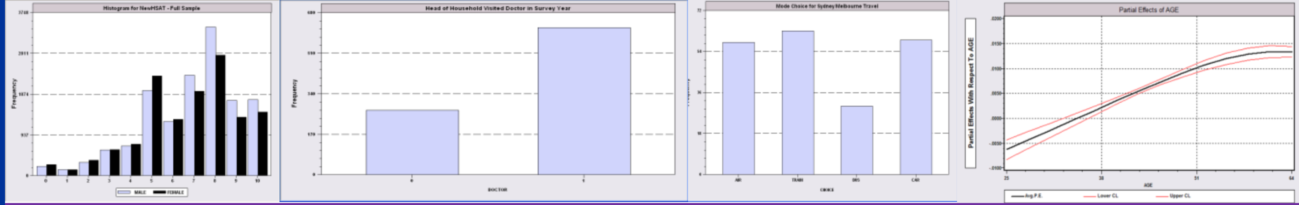
Estimating the Model

$$\boxed{P[\text{choice } j \mid i,t] = \frac{\exp(\alpha_{j,i} + \boldsymbol{\beta}_i'\mathbf{x}_{itj} + \boldsymbol{\gamma}_{j,i}'\mathbf{z}_{it})}{\sum_{j=1}^{J(i)} \exp(\alpha_{j,i} + \boldsymbol{\beta}_i'\mathbf{x}_{itj} + \boldsymbol{\gamma}_{j,i}'\mathbf{z}_{it})}}$$

$$[\alpha_i, \boldsymbol{\beta}_i, \boldsymbol{\gamma}_i] = \text{functions of underlying } [\alpha, \boldsymbol{\beta}, \boldsymbol{\Delta}, \boldsymbol{\Gamma}, \boldsymbol{\rho}, \mathbf{z}_i, \mathbf{v}_i]$$

Denote by β_1 all “fixed” parameters in the model

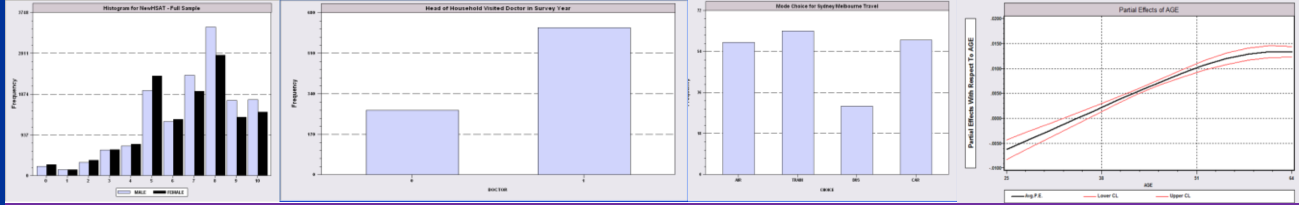
Denote by $\beta_{2i,t}$ all random and hierarchical parameters in the model



Customers' Choice of Energy Supplier

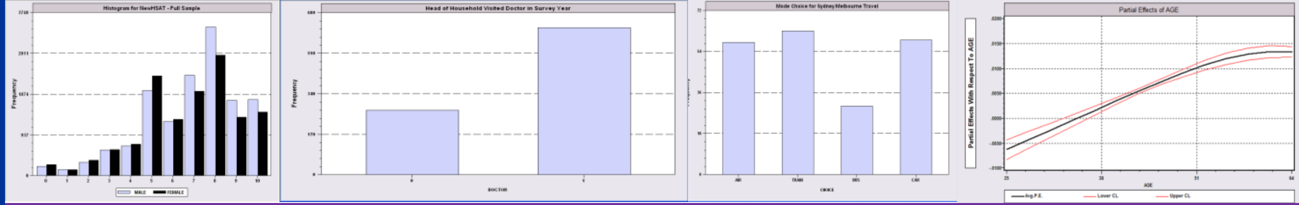
- California, Stated Preference Survey
- 361 customers presented with 8-12 choice situations each
- Supplier attributes:
  - Fixed price: cents per kWh
  - Length of contract
  - Local utility
  - Well-known company
  - Time-of-day rates (11¢ in day, 5¢ at night)
  - Seasonal rates (10¢ in summer, 8¢ in winter, 6¢ in spring/fall)

(TrainCalUtilitySurvey.lpj)



Population Distributions

- Normal for:
  - Contract length
  - Local utility
  - Well-known company
- Log-normal for:
  - Time-of-day rates
  - Seasonal rates
- Price coefficient held fixed

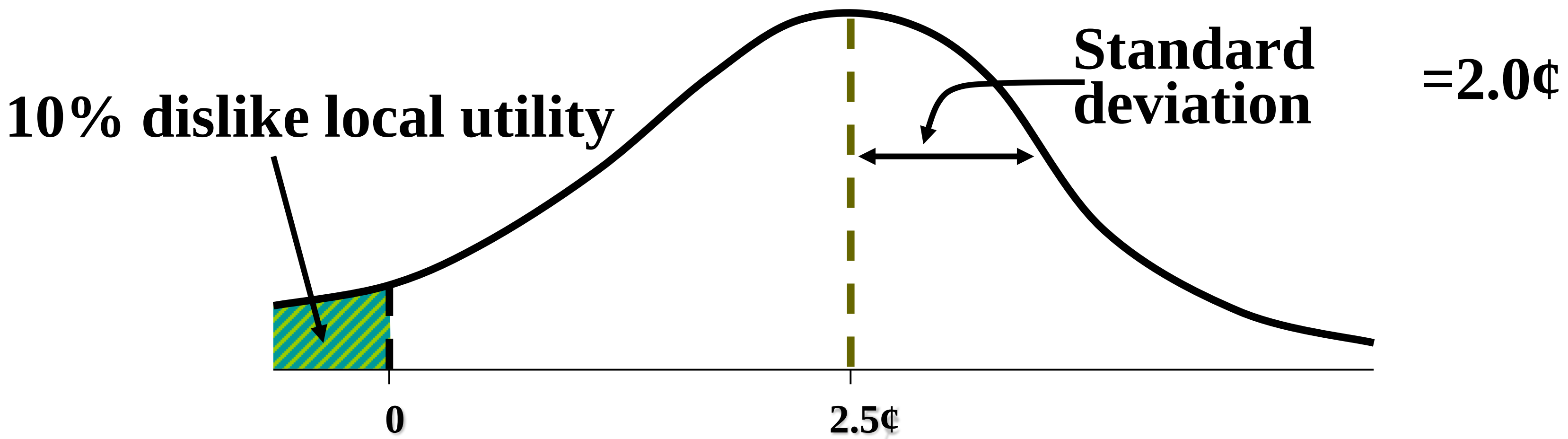


Estimated Model

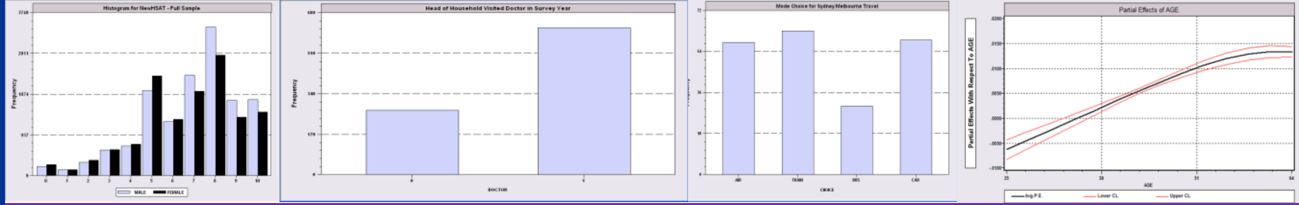
		Estimate	Std error
Price		-.883	0.050
Contract	mean	-.213	0.026
	std dev	.386	0.028
Local	mean	2.23	0.127
	std dev	1.75	0.137
Known	mean	1.59	0.100
	std dev	.962	0.098
TOD	mean*	2.13	0.054
	std dev*	.411	0.040
Seasonal	mean*	2.16	0.051
	std dev*	.281	0.022

*Parameters of underlying normal.

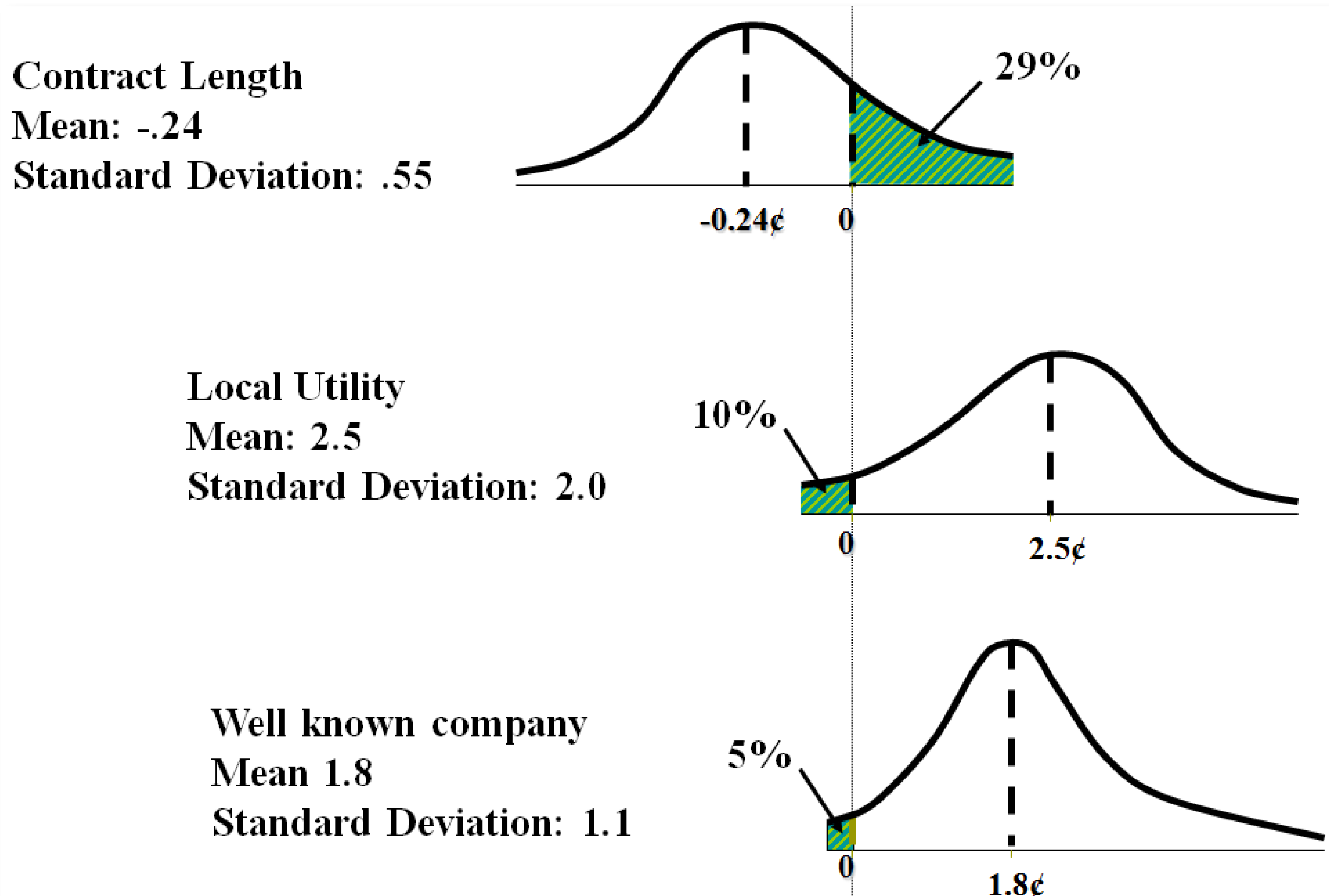
Distribution of Brand Value

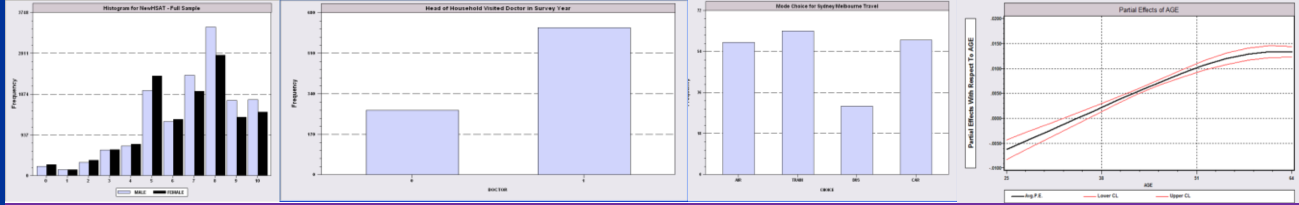


Brand value of local utility



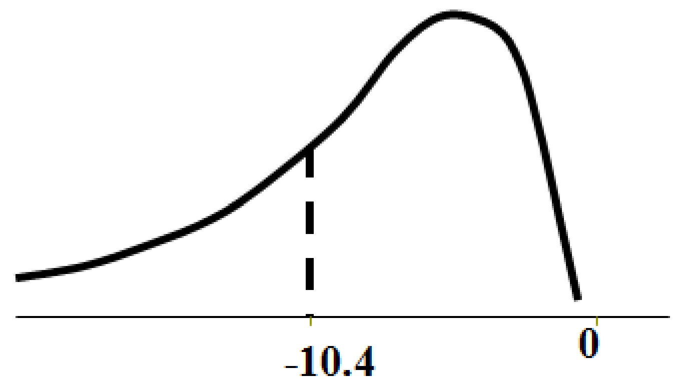
Random Parameter Distributions



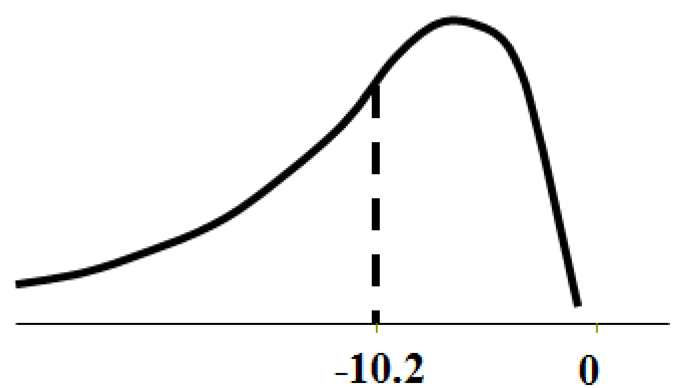


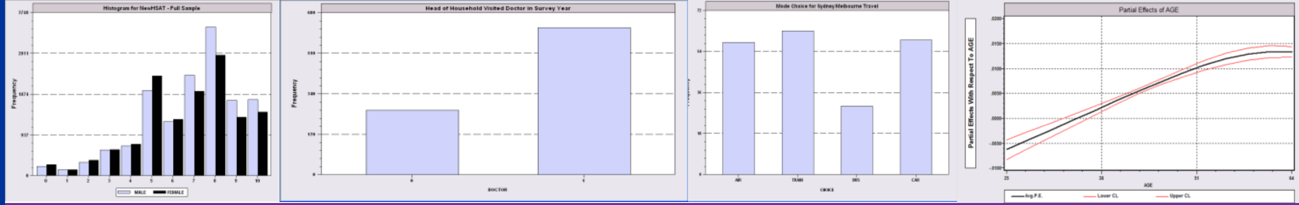
Time of Day Rates (Customers do not like – lognormal coefficient. Multiply variable by -1.)

Time-of-day Rates



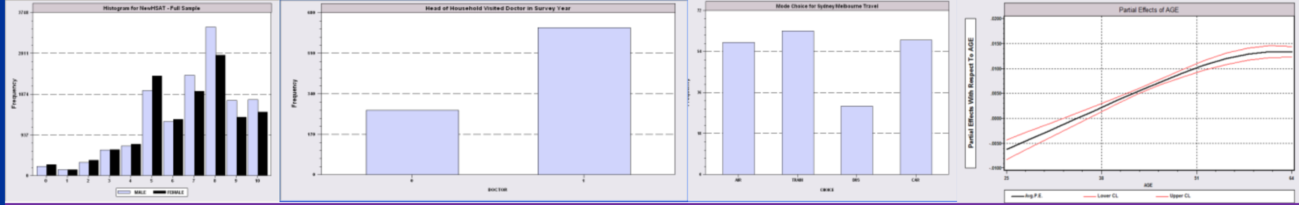
Seasonal Rates





Estimating Individual Parameters

- Model estimates = structural parameters, **α**, **β**, **ρ**, **Δ**, **Σ**, **Γ**
- Objective, a model of individual specific parameters, $\boldsymbol{\beta}_i$
- Can individual specific parameters be estimated?
  - Not quite – $\boldsymbol{\beta}_i$ is a single realization of a random process; one random draw.
  - We estimate E[$\boldsymbol{\beta}_i$ | all information about i]
  - (This is also true of Bayesian treatments, despite claims to the contrary.)



Posterior Estimation of β_i

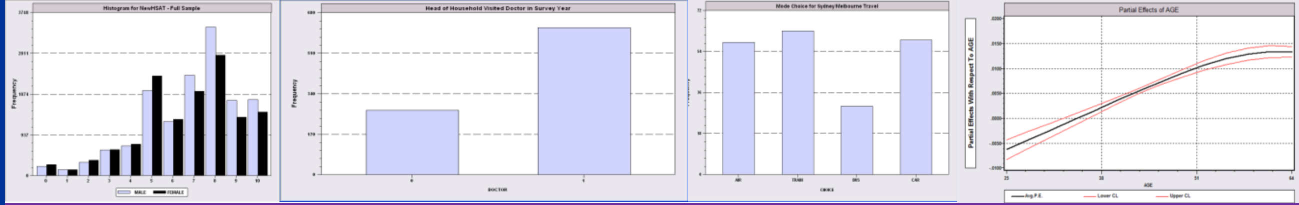
$$\hat{\boldsymbol{\beta}}_i = E\left[\boldsymbol{\beta}_i \mid \boldsymbol{\beta}, \boldsymbol{\Delta}, \boldsymbol{\Gamma}, \mathbf{y}_i, \mathbf{X}_i, z_i\right]$$

$$= \frac{\int_{\boldsymbol{\beta}_i} \boldsymbol{\beta}_i \left[\prod_{t=1}^{T} P(\text{choice } j \mid X_i, \boldsymbol{\beta}_i) g(\boldsymbol{\beta}_i \mid \boldsymbol{\beta}, \boldsymbol{\Delta}, \boldsymbol{\Gamma}, \text{etc.}, \mathbf{z}_i)\right] d\boldsymbol{\beta}_i}{\int_{\boldsymbol{\beta}_i} \left[\prod_{t=1}^{T} P(\text{choice } j \mid X_i, \boldsymbol{\beta}_i) g(\beta_i \mid \boldsymbol{\beta}, \boldsymbol{\Delta}, \boldsymbol{\Gamma}, \text{etc.}, \mathbf{z}_i)\right] d\boldsymbol{\beta}_i}$$

Estimate by simulation

$$\hat{\boldsymbol{\beta}}_i = \frac{\frac{1}{R}\sum_{r=1}^{R} \hat{\beta}_{ir} \left[\prod_{t=1}^{T} P(\text{choice } j \mid X_i, \hat{\boldsymbol{\beta}}_i) g(\hat{\boldsymbol{\beta}}_i \mid \hat{\boldsymbol{\beta}}, \hat{\boldsymbol{\Delta}}, \hat{\boldsymbol{\Gamma}}, \text{etc.}, z_i)\right]}{\frac{1}{R}\sum_{r=1}^{R} \left[\prod_{t=1}^{T} P(\text{choice } j \mid X_i, \hat{\boldsymbol{\beta}}_i) g(\hat{\boldsymbol{\beta}}_i \mid \hat{\boldsymbol{\beta}}, \hat{\boldsymbol{\Delta}}, \hat{\boldsymbol{\Gamma}}, \text{etc.}, z_i)\right]},$$

$$\hat{\boldsymbol{\beta}}_{ir} = \hat{\boldsymbol{\beta}} + \hat{\boldsymbol{\Delta}} z_i + \hat{\boldsymbol{\Gamma}} w_{ir}$$

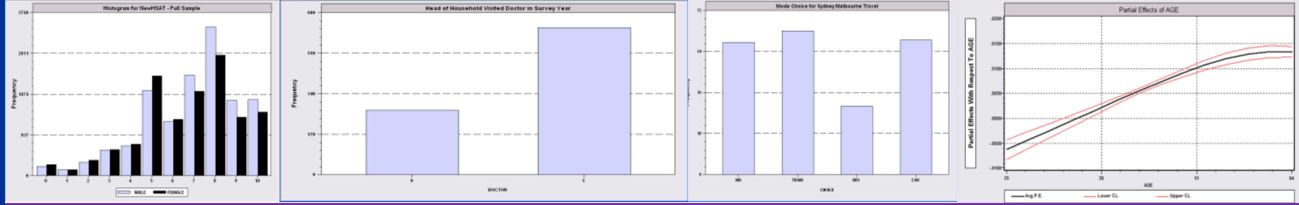


Expected Preferences of Each Customer

	Population Mean	Customer A's Conditional Mean
Contract length	-0.24	2.20
Local utility	2.50	3.30
Well-known company	1.80	2.00
Time-of-day rates	-10.40	-6.30
Seasonal rates	-10.20	-6.60

Customer likes long-term contract, local utility, and non-fixed rates.

Local utility can retain and make profit from this customer by offering a long-term contract with time-of-day or seasonal rates.



Model Extensions

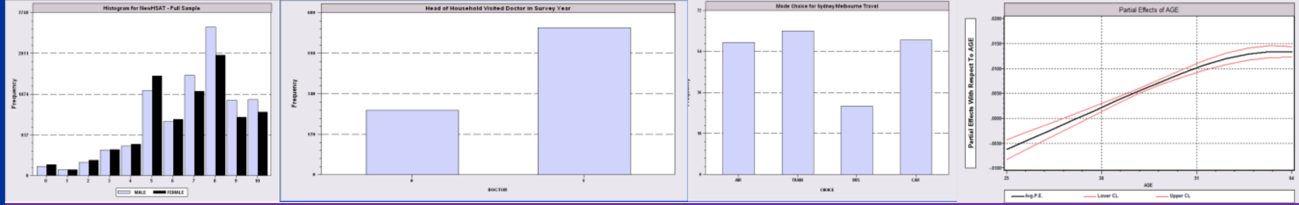
$$\boxed{\boldsymbol{\beta}_i = \boldsymbol{\beta} + \boldsymbol{\Delta}\mathbf{z}_i + \boldsymbol{\Gamma}\mathbf{w}_i}$$

- AR(1): $w_{i,k,t} = \rho_k w_{i,k,t-1} + v_{i,k,t}$
  Dynamic effects in the model
- Restricting sign – lognormal distribution:

$$\boxed{\beta_{i,k} = \exp(\mu_k + \boldsymbol{\delta}_k'\mathbf{z}_i + \boldsymbol{\gamma}_k'\mathbf{w}_i)}$$

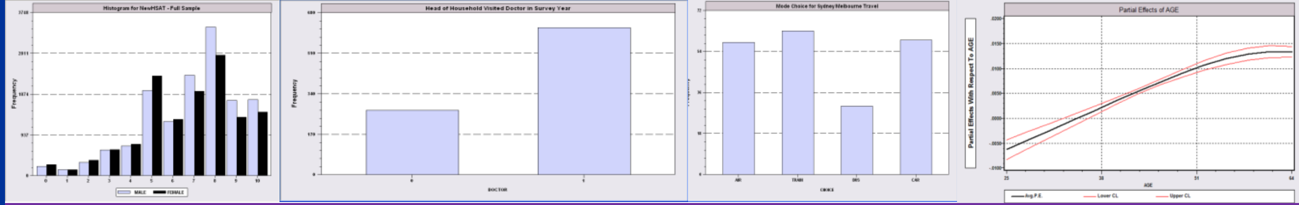
- Restricting Range and Sign: Using triangular distribution and range = 0 to 2β.
- Heteroscedasticity and heterogeneity

$$\boxed{\sigma_{k,i} = \sigma_k \exp(\boldsymbol{\theta}'\mathbf{h}_i)}$$



Application: Shoe Brand Choice

- Simulated Data: Stated Choice,
  - 400 respondents,
  - 8 choice situations, 3,200 observations
- 3 choice/attributes + NONE
  - Fashion = High / Low
  - Quality = High / Low
  - Price = 25/50/75,100 coded 1,2,3,4
- Heterogeneity: Sex (Male=1), Age (<25, 25-39, 40+)
- Underlying data generated by a 3 class latent class process (100, 200, 100 in classes)



Stated Choice Experiment: Unlabeled Alternatives, One Observation

	ID	BRAND	CHOICE	FASH	QUAL	PRICE	PRICESQ	ASC4
1 »	1	1	0	0	0	0.12	0.0144	0
2 »	1	2	1	1	0	0.12	0.0144	0
3 »	1	3	0	0	1	0.08	0.0064	0
4 »	1	4	0	0	0	0	0	1
5 »	1	1	1	1	1	0.12	0.0144	0
6 »	1	2	0	0	1	0.12	0.0144	0
7 »	1	3	0	1	0	0.12	0.0144	0
8 »	1	4	0	0	0	0	0	1
9 »	1	1	0	0	1	0.08	0.0064	0
10 »	1	2	0	1	1	0.2	0.04	0
11 »	1	3	1	1	0	0.08	0.0064	0
12 »	1	4	0	0	0	0	0	1
13 »	1	1	0	0	0	0.08	0.0064	0
14 »	1	2	1	0	1	0.16	0.0256	0
15 »	1	3	0	1	1	0.2	0.04	0
16 »	1	4	0	0	0	0	0	1
17 »	1	1	1	0	0	0.04	0.0016	0
18 »	1	2	0	1	0	0.12	0.0144	0
19 »	1	3	0	1	0	0.08	0.0064	0
20 »	1	4	0	0	0	0	0	1
21 »	1	1	0	0	0	0.08	0.0064	0
22 »	1	2	0	0	1	0.12	0.0144	0
23 »	1	3	1	1	0	0.08	0.0064	0
24 »	1	4	0	0	0	0	0	1
25 »	1	1	0	1	1	0.2	0.04	0
26 »	1	2	1	0	0	0.08	0.0064	0
27 »	1	3	0	0	1	0.08	0.0064	0
28 »	1	4	0	0	0	0	0	1
29 »	1	1	0	0	1	0.08	0.0064	0
30 »	1	2	1	1	0	0.12	0.0144	0
31 »	1	3	0	0	0	0.04	0.0016	0
32 »	1	4	0	0	0	0	0	1

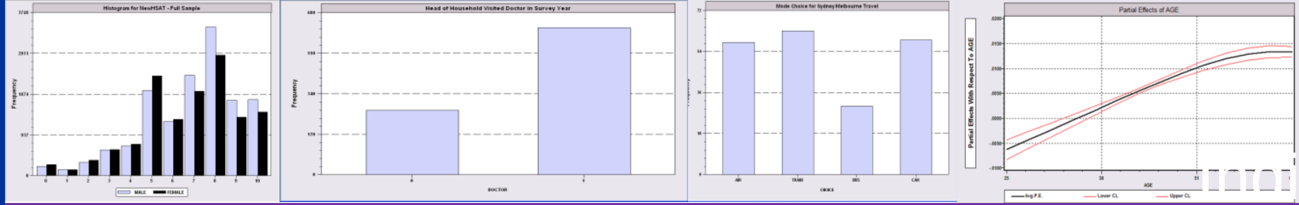
Brand 1
Brand 2
Brand 3
None

Brand 1
Brand 2
Brand 3
None

Brand 1
Brand 2
Brand 3
None

Brand 1
Brand 2
Brand 3
None

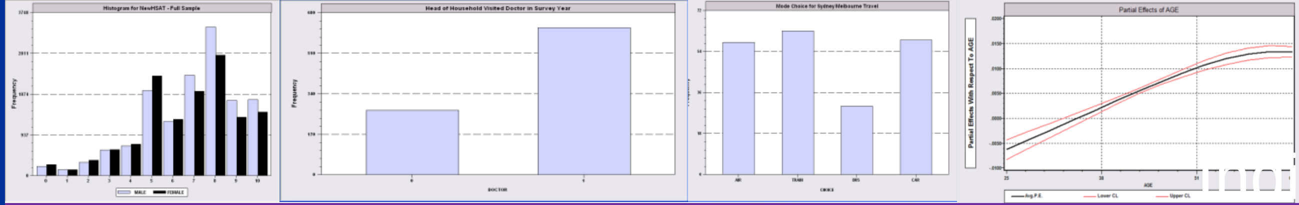
t=1
t=2
t=3
t=4
t=5
t=6
t=7
t=8



Random Parameters Logit Model

$$
\begin{aligned}
U(\text{brand1})_{n,s} &= \beta_{1,n}\text{Fashion}_{1,n,s} + \beta_2\text{Quality}_{1,n,s} + \beta_3\text{Price}_{1,n,s} + \varepsilon_{\text{Brand1},n,s} \\
U(\text{brand2})_{n,s} &= \beta_{1,n}\text{Fashion}_{2,n,s} + \beta_2\text{Quality}_{2,n,s} + \beta_3\text{Price}_{2,n,s} + \varepsilon_{\text{Brand2},n,s} \\
U(\text{brand3})_{n,s} &= \beta_{1,n}\text{Fashion}_{3,n,s} + \beta_2\text{Quality}_{3,n,s} + \beta_3\text{Price}_{3,n,s} + \varepsilon_{\text{Brand3},n,s} \\
U(\text{None})_{n,s} &= \beta_4 + \varepsilon_{\text{No Brand},n,s}
\end{aligned}
$$

$$
\beta_{1,n} = \overline{\beta}_1 + \delta_{11}\text{Sex} + \delta_{12}\text{Age2539} + \delta_{13}\text{Age40} + \eta_1 z_{n1}
$$



```
----------------------------------------------------------------------
Random Parameters Logit Model
Dependent variable               CHOICE
Log likelihood function     -4076.25831
Restricted log likelihood   -4436.14196
Chi squared [   8](P= .000)   719.76728
Significance level               .00000
McFadden Pseudo R-squared      .0811254
Estimation based on N =    3200, K =   8
Inf.Cr.AIC  =    8168.5 AIC/N =    2.553
R2=1-LogL/LogL* Log-L fncn R-sqrd R2Adj
No coefficients -4436.1420  .0811  .0804
Constants only  -4391.1804  .0717  .0709
At start values -4158.5029  .0198  .0190
Response data are given as ind. choices
Replications for simulated probs. = 100
Used Halton sequences in simulations.
RPL model with panel has     400 groups
Fixed number of obsrvs./group=         8
Number of obs.=  3200, skipped     0 obs
--------+-------------------------------------------------------------
        |                Standard            Prob.       95% Confidence
  CHOICE|  Coefficient     Error       z    |z|>Z*         Interval
--------+-------------------------------------------------------------
        |Random parameters in utility functions
      B1|    1.76473***    .11992    14.72  .0000     1.52969   1.99977
        |Nonrandom parameters in utility functions
      B2|    1.04936***    .06590    15.92  .0000      .92020   1.17852
      B3|   -12.5002***    .83163   -15.03  .0000   -14.1302  -10.8702
   BNONE|    -.00789       .07330    -.11   .9142    -.15157    .13578
        |Heterogeneity in mean, Parameter:Variable
  B1:MAL|     .15926       .10932     1.46  .1452    -.05501    .37352
  B1:AGE|    -.48572***    .14057    -3.46  .0005    -.76124   -.21020
 B10:AGE|    -.80576***    .12465    -6.46  .0000   -1.05006   -.56145
        |Distns. of RPs. Std.Devs or limits of triangular
    NsB1|     .73143***    .06263    11.68  .0000     .60867    .85418
--------+-------------------------------------------------------------
***, **, * ==>  Significance at 1%, 5%, 10% level.
----------------------------------------------------------------------
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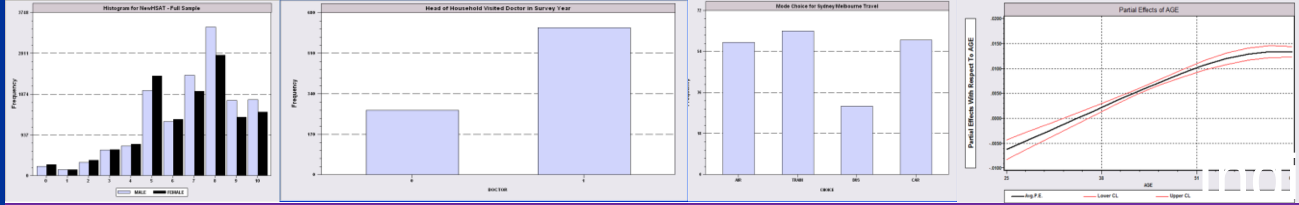
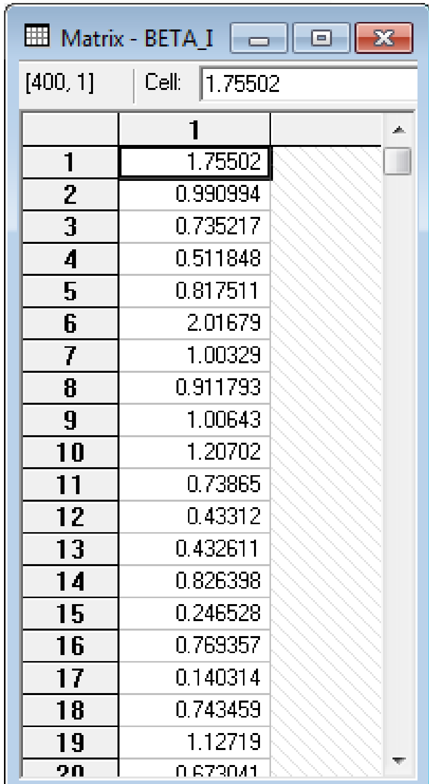
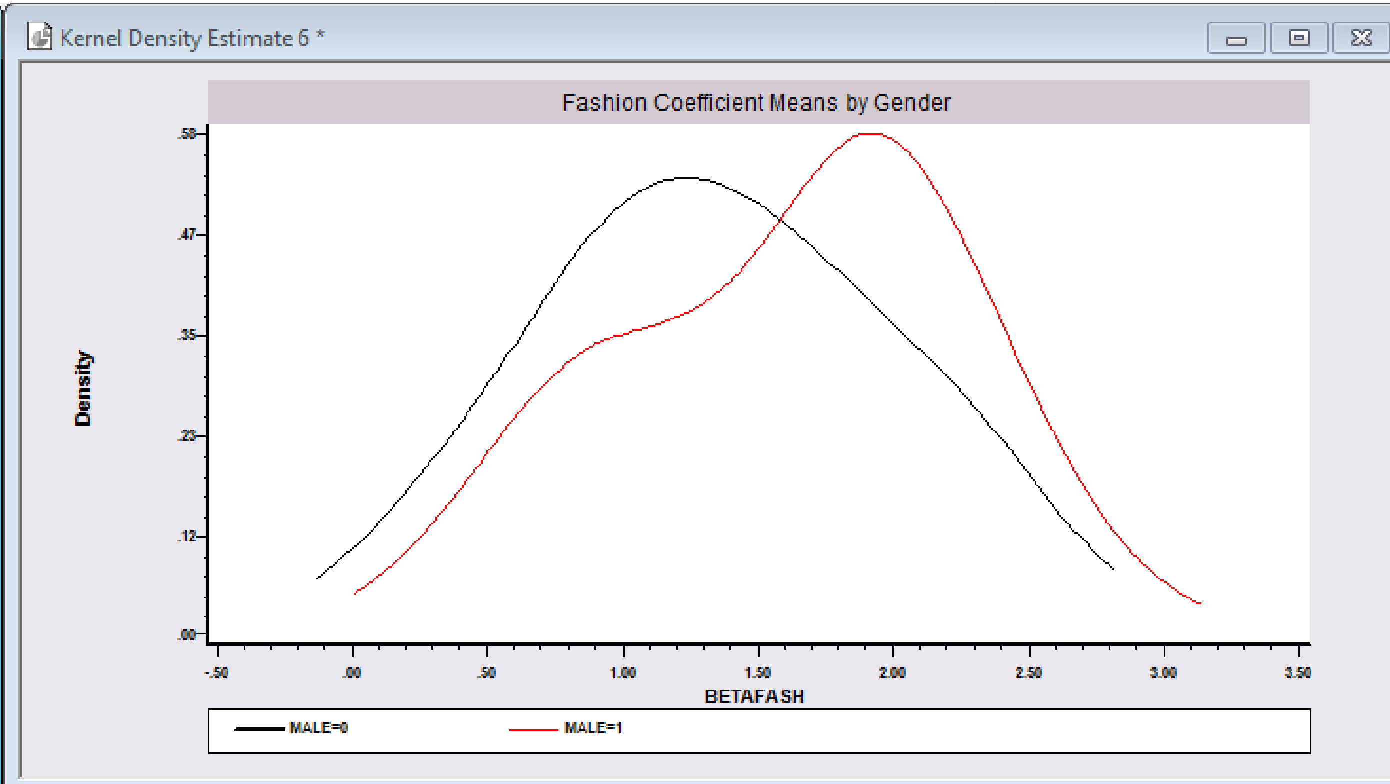


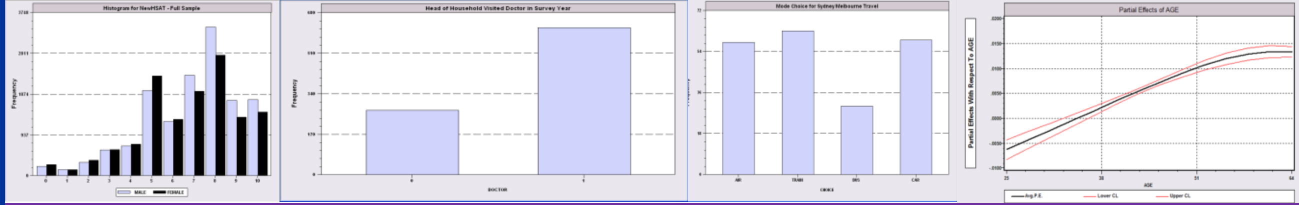
Descriptive Statistics for BETAFASH
Stratification is based on MALE

Subsample	Mean	Std.Dev.	Cases	Sum of wts
MALE = 0	1.354027	.592887	5472	5472.00
MALE = 1	1.588455	.619941	7328	7328.00
Full Sample	1.488237	.619453	12800	12800.00



	1
1	1.75502
2	0.990994
3	0.735217
4	0.511848
5	0.817511
6	2.01679
7	1.00329
8	0.911793
9	1.00643
10	1.20702
11	0.73865
12	0.43312
13	0.432611
14	0.826398
15	0.246528
16	0.769357
17	0.140314
18	0.743459
19	1.12719
20	0.673041

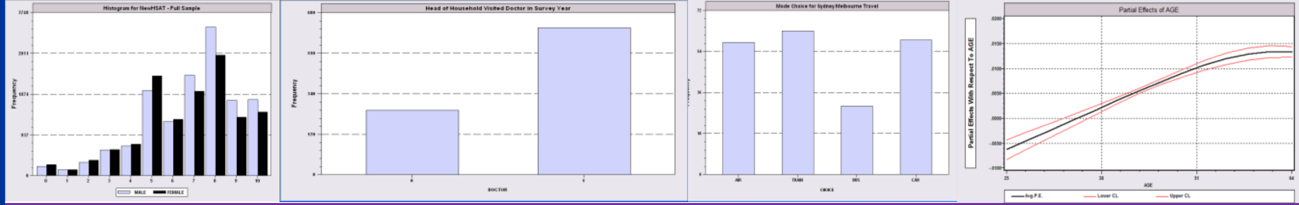




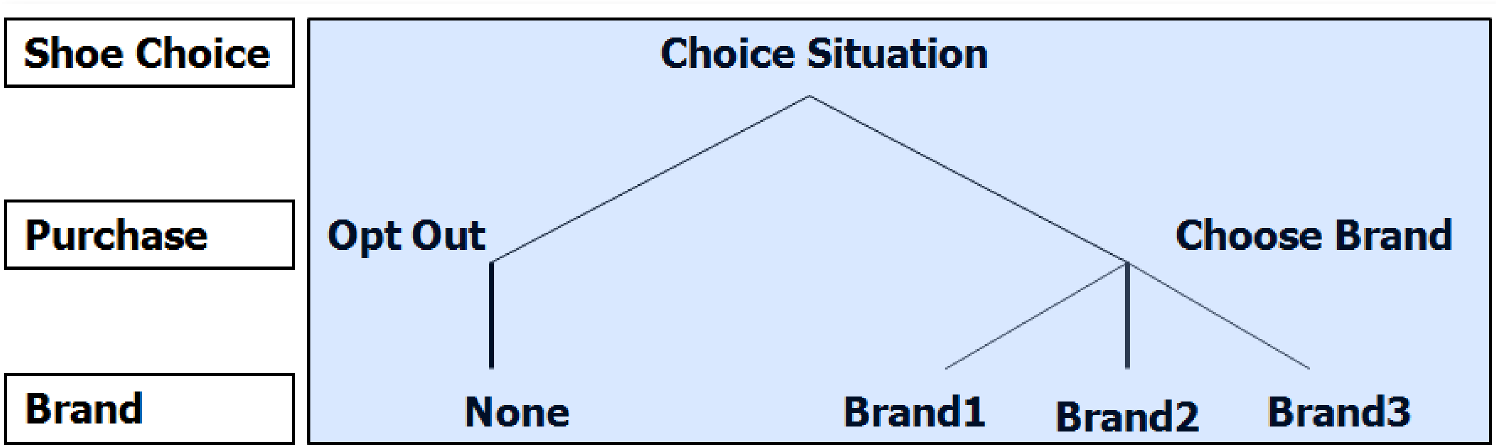
Error Components Logit Modeling

- Alternative approach to building cross choice correlation
- Common 'effects.' W_i is a 'random individual effect.'

$$
\begin{array}{lll}
U(\text{brand1})_i = \beta_1\text{Fashion}_{1,i} + \beta_2\text{Quality}_{1,i} + \beta_3\text{Price}_{1,i} + \varepsilon_{\text{Brand1},i} + & \boxed{\sigma\ W_i} \\
U(\text{brand2})_i = \beta_1\text{Fashion}_{2,i} + \beta_2\text{Quality}_{2,i} + \beta_3\text{Price}_{2,i} + \varepsilon_{\text{Brand2},i} + & \boxed{\sigma\ W_i} \\
U(\text{brand3})_i = \beta_1\text{Fashion}_{3,i} + \beta_2\text{Quality}_{3,i} + \beta_3\text{Price}_{3,i} + \varepsilon_{\text{Brand3},i} + & \boxed{\sigma\ W_i} \\
U(\text{None}) \quad = \beta_4 \qquad\qquad\qquad\qquad\qquad + \varepsilon_{\text{No Brand},i} &
\end{array}
$$



Implied Covariance Matrix
Nested Logit Formulation

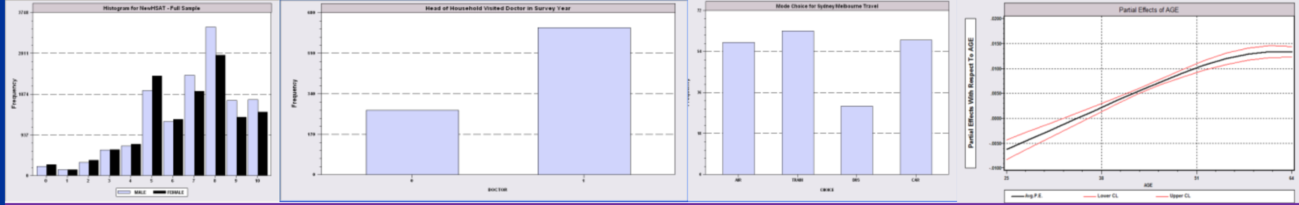


$$\text{Var}[\varepsilon] = \pi^2 / 6 = 1.6449$$

$$\text{Var}[W] = 1$$

$$\Sigma = \text{Var}\begin{bmatrix} \varepsilon_{\text{Brand1}} + \sigma W \\ \varepsilon_{\text{Brand2}} + \sigma W \\ \varepsilon_{\text{Brand3}} + \sigma W \\ \varepsilon_{\text{NONE}} \end{bmatrix} = \begin{bmatrix} 1.6449+\sigma^2 & \sigma^2 & \sigma^2 & 0 \\ \sigma^2 & 1.6449+\sigma^2 & \sigma^2 & 0 \\ \sigma^2 & \sigma^2 & 1.6449+\sigma^2 & 0 \\ 0 & 0 & 0 & 1.6449 \end{bmatrix}$$

$$\text{Cross Brand Correlation} = \sigma^2 / [1.6449 + \sigma^2]$$

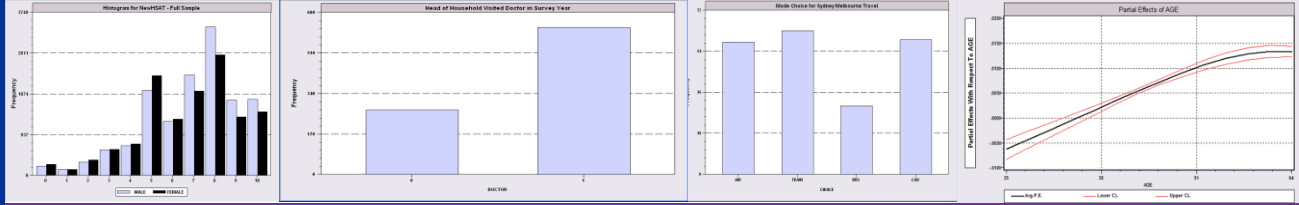


Error Components Logit Model

```
-----------------------------------------------------------
Error Components (Random Effects) model
Dependent variable               CHOICE
Log likelihood function     -4158.45044
Estimation based on N =    3200, K =   5
Response data are given as ind. choices
Replications for simulated probs. =  50
Halton sequences used for simulations
ECM model with panel has     400 groups
Fixed number of obsrvs./group=         8
Number of obs.=  3200, skipped    0 obs
--------+--------------------------------------------------
Variable| Coefficient    Standard Error  b/St.Er. P[|Z|>z]
--------+--------------------------------------------------
        |Nonrandom parameters in utility functions
    FASH|    1.47913***      .06971       21.218   .0000
    QUAL|    1.01385***      .06580       15.409   .0000
   PRICE|   -11.8052***      .86019      -13.724   .0000
    ASC4|     .03363         .07441         .452   .6513
SigmaE01|     .09585***      .02529        3.791   .0002
--------+--------------------------------------------------
```

```
Random Effects Logit Model
Appearance of Latent Random
Effects in Utilities
 Alternative    E01
+--------------+---+
|  BRAND1      | * |
+--------------+---+
|  BRAND2      | * |
+--------------+---+
|  BRAND3      | * |
+--------------+---+
|  NONE        |   |
+--------------+---+
```

Correlation = $\{0.0959^2 / [1.6449 + 0.0959^2]\}^{1/2}$ = 0.0954



Extended MNL Model

Utility Functions

$$U_{i,1,t} = \beta_{F,i}\,\text{Fashion}_{i,1,t} + \beta_Q \text{Quality}_{i,1,t} + \beta_{P,i}\,\text{Price}_{i,1,t} + \lambda_{Brand} W_{i,Brand} + \varepsilon_{i,1,t}$$

$$U_{i,2,t} = \beta_{F,i}\,\text{Fashion}_{i,2,t} + \beta_Q \text{Quality}_{i,2,t} + \beta_{P,i}\,\text{Price}_{i,2,t} + \lambda_{Brand} W_{i,Brand} + \varepsilon_{i,2,t}$$

$$U_{i,3,t} = \beta_{F,i}\,\text{Fashion}_{i,3,t} + \beta_Q \text{Quality}_{i,3,t} + \beta_{P,i}\,\text{Price}_{i,3,t} + \lambda_{Brand} W_{i,Brand} + \varepsilon_{i,3,t}$$

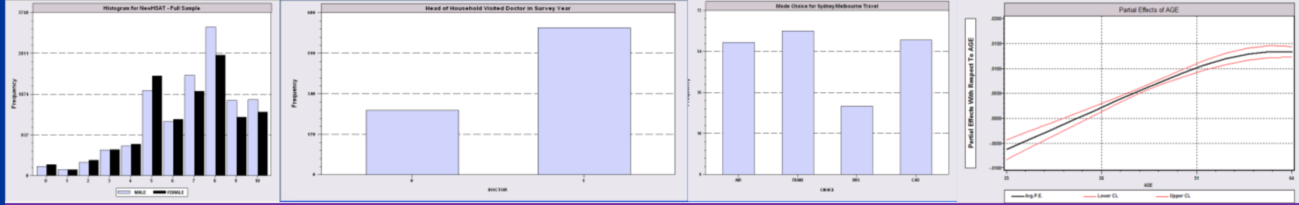
$$U_{i,NONE,t} = \alpha_{NONE} + \lambda_{NONE} W_{i,NONE} + \varepsilon_{i,NONE,t}$$

$$\beta_{F,i} = \overline{\beta}_F + \delta_F \text{Sex}_i + [\sigma_F \exp(\gamma_{F1}\text{AgeL25}_i + \gamma_{F2}\text{Age2539}_i)]\, w_{F,i};\; w_{F,i} \sim N[0,1]$$

$$\beta_{P,i} = \overline{\beta}_P + \delta_P \text{Sex}_i + [\sigma_P \exp(\gamma_{P1}\text{AgeL25}_i + \gamma_{P2}\text{Age2539}_i)]\; w_{P,i};\; w_{P,i} \sim N[0,1]$$

$$W_{Brand,i} \sim N[0,1]$$

$$W_{NONE,i} \sim N[0,1]$$



Extending the Basic MNL Model

Random Utility

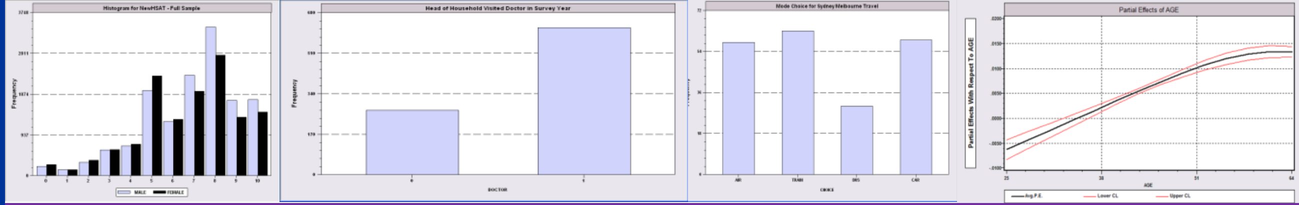
$$
\begin{aligned}
U_{i,1,t} &= \beta_{F,i}\,\text{Fashion}_{i,1,t} + \beta_Q\,\text{Quality}_{i,1,t} + \beta_{P,i}\,\text{Price}_{i,1,t} + \lambda_{Brand} W_{i,Brand} + \varepsilon_{i,1,t}\\
U_{i,2,t} &= \beta_{F,i}\,\text{Fashion}_{i,2,t} + \beta_Q\,\text{Quality}_{i,2,t} + \beta_{P,i}\,\text{Price}_{i,2,t} + \lambda_{Brand} W_{i,Brand} + \varepsilon_{i,2,t}\\
U_{i,3,t} &= \beta_{F,i}\,\text{Fashion}_{i,3,t} + \beta_Q\,\text{Quality}_{i,3,t} + \beta_{P,i}\,\text{Price}_{i,3,t} + \lambda_{Brand} W_{i,Brand} + \varepsilon_{i,3,t}\\
U_{i,NONE,t} &= \alpha_{NONE} + \lambda_{NONE} W_{i,NONE} + \varepsilon_{i,NONE,t}
\end{aligned}
$$

$$\beta_{F,i} = \overline{\beta}_F + \delta_F \text{Sex}_i + [\sigma_F \exp(\gamma_{F1}\text{AgeL25}_i + \gamma_{F2}\text{Age2539}_i)]\, w_{F,i};\ w_{F,i} \sim N[0,1]$$

$$\beta_{P,i} = \overline{\beta}_P + \delta_P \text{Sex}_i + [\sigma_P \exp(\gamma_{P1}\text{AgeL25}_i + \gamma_{P2}\text{Age2539}_i)]\ w_{P,i};\ w_{P,i} \sim N[0,1]$$

$$W_{Brand,i} \sim N[0,1]$$

$$W_{NONE,i} \sim N[0,1]$$



Error Components Logit Model

Error Components

$$U_{i,1,t} = \beta_{F,i}\text{Fashion}_{i,1,t} + \beta_Q\text{Quality}_{i,1,t} + \beta_{P,i}\text{Price}_{i,1,t} + \lambda_{Brand}W_{i,Brand} + \varepsilon_{i,1,t}$$

$$U_{i,2,t} = \beta_{F,i}\text{Fashion}_{i,2,t} + \beta_Q\text{Quality}_{i,2,t} + \beta_{P,i}\text{Price}_{i,2,t} + \lambda_{Brand}W_{i,Brand} + \varepsilon_{i,2,t}$$

$$U_{i,3,t} = \beta_{F,i}\text{Fashion}_{i,3,t} + \beta_Q\text{Quality}_{i,3,t} + \beta_{P,i}\text{Price}_{i,3,t} + \lambda_{Brand}W_{i,Brand} + \varepsilon_{i,3,t}$$

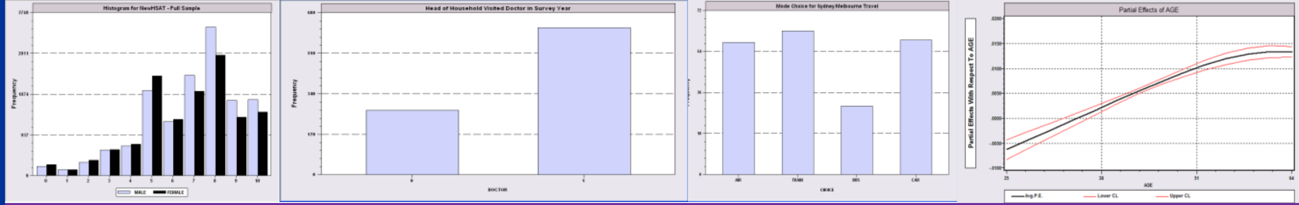
$$U_{i,NONE,t} = \alpha_{NONE} + \lambda_{NONE}W_{i,NONE} + \varepsilon_{i,NONE,t}$$

$$\beta_{F,i} = \overline{\beta}_F + \delta_F\text{Sex}_i + [\sigma_F\exp(\gamma_{F1}\text{AgeL25}_i + \gamma_{F2}\text{Age2539}_i)]\, w_{F,i};\ w_{F,i} \sim N[0,1]$$

$$\beta_{P,i} = \overline{\beta}_P + \delta_P\text{Sex}_i + [\sigma_P\exp(\gamma_{P1}\text{AgeL25}_i + \gamma_{P2}\text{Age2539}_i)]\, w_{P,i};\ w_{P,i} \sim N[0,1]$$

$$W_{Brand,i} \sim N[0,1]$$

$$W_{NONE,i} \sim N[0,1]$$



Random Parameters Model

$$U_{i,1,t} = \beta_{F,i}\,\text{Fashion}_{i,1,t} + \beta_Q\text{Quality}_{i,1,t} + \beta_{P,i}\,\text{Price}_{i,1,t} + \lambda_{\text{Brand}}W_{i,\text{Brand}} + \varepsilon_{i,1,t}$$

$$U_{i,2,t} = \beta_{F,i}\,\text{Fashion}_{i,2,t} + \beta_Q\text{Quality}_{i,2,t} + \beta_{P,i}\,\text{Price}_{i,2,t} + \lambda_{\text{Brand}}W_{i,\text{Brand}} + \varepsilon_{i,2,t}$$

$$U_{i,3,t} = \beta_{F,i}\,\text{Fashion}_{i,3,t} + \beta_Q\text{Quality}_{i,3,t} + \beta_{P,i}\,\text{Price}_{i,3,t} + \lambda_{\text{Brand}}W_{i,\text{Brand}} + \varepsilon_{i,3,t}$$

$$U_{i,\text{NONE},t} = \alpha_{\text{NONE}} + \lambda_{\text{NONE}}W_{i,\text{NONE}} + \varepsilon_{i,\text{NONE},t}$$

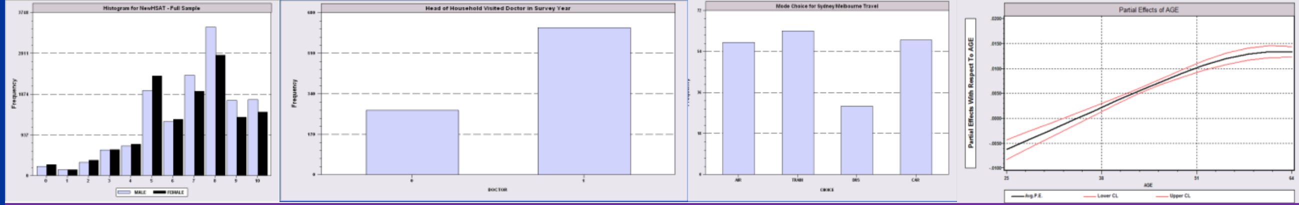
$$\beta_{F,i} = \bar{\beta}_F + \delta_F\text{Sex}_i + [\sigma_F\exp(\gamma_{F1}\text{AgeL25}_i + \gamma_{F2}\text{Age2539}_i)]\,w_{F,i};\; w_{F,i} \sim N[0,1]$$

$$\beta_{P,i} = \bar{\beta}_P + \delta_P\text{Sex}_i + [\sigma_P\exp(\gamma_{P1}\text{AgeL25}_i + \gamma_{P2}\text{Age2539}_i)]\,w_{P,i};\; w_{P,i} \sim N[0,1]$$

$$W_{\text{Brand},i} \sim N[0,1]$$

$$W_{\text{NONE},i} \sim N[0,1]$$

Random Parameters



Heterogeneous (in the Means) Random Parameters Model

$$U_{i,1,t} = \beta_{F,i}\text{Fashion}_{i,1,t} + \beta_Q\text{Quality}_{i,1,t} + \beta_{P,i}\text{Price}_{i,1,t} + \lambda_{Brand}W_{i,Brand} + \varepsilon_{i,1,t}$$

$$U_{i,2,t} = \beta_{F,i}\text{Fashion}_{i,2,t} + \beta_Q\text{Quality}_{i,2,t} + \beta_{P,i}\text{Price}_{i,2,t} + \lambda_{Brand}W_{i,Brand} + \varepsilon_{i,2,t}$$

$$U_{i,3,t} = \beta_{F,i}\text{Fashion}_{i,3,t} + \beta_Q\text{Quality}_{i,3,t} + \beta_{P,i}\text{Price}_{i,3,t} + \lambda_{Brand}W_{i,Brand} + \varepsilon_{i,3,t}$$

$$U_{i,NONE,t} = \alpha_{NONE} + \lambda_{NONE}W_{i,NONE} + \varepsilon_{i,NONE,t}$$

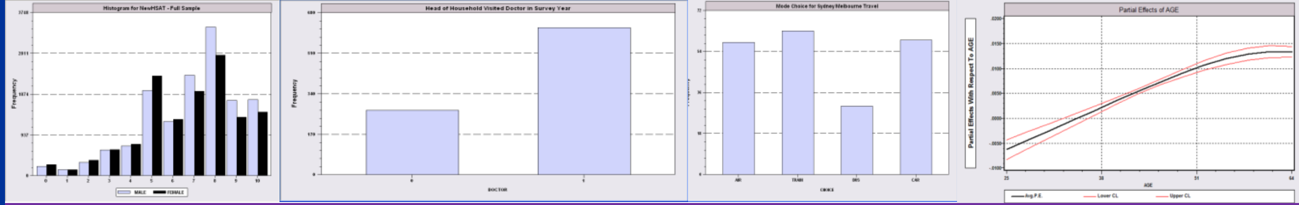
Heterogeneity in Means

$$\boxed{\beta_{F,i}} = \bar{\beta}_F + \boxed{\delta_F \text{Sex}_i} + [\sigma_F \exp(\gamma_{F1}\text{AgeL25}_i + \gamma_{F2}\text{Age2539}_i)]\, w_{F,i};\ w_{F,i} \sim N[0,1]$$

$$\boxed{\beta_{P,i}} = \bar{\beta}_P + \boxed{\delta_P \text{Sex}_i} + [\sigma_P \exp(\gamma_{P1}\text{AgeL25}_i + \gamma_{P2}\text{Age2539}_i)]\, w_{P,i};\ w_{P,i} \sim N[0,1]$$

$$W_{Brand,i} \sim N[0,1]$$

$$W_{NONE,i} \sim N[0,1]$$



Heterogeneity in Both Means and Variances

$$U_{i,1,t} = \beta_{F,i}\,\text{Fashion}_{i,1,t} + \beta_Q\,\text{Quality}_{i,1,t} + \beta_{P,i}\,\text{Price}_{i,1,t} + \lambda_{Brand} W_{i,Brand} + \varepsilon_{i,1,t}$$

$$U_{i,2,t} = \beta_{F,i}\,\text{Fashion}_{i,2,t} + \beta_Q\,\text{Quality}_{i,2,t} + \beta_{P,i}\,\text{Price}_{i,2,t} + \lambda_{Brand} W_{i,Brand} + \varepsilon_{i,2,t}$$

$$U_{i,3,t} = \beta_{F,i}\,\text{Fashion}_{i,3,t} + \beta_Q\,\text{Quality}_{i,3,t} + \beta_{P,i}\,\text{Price}_{i,3,t} + \lambda_{Brand} W_{i,Brand} + \varepsilon_{i,3,t}$$

$$U_{i,NONE,t} = \alpha_{NONE} + \lambda_{NONE} W_{i,NONE} + \varepsilon_{i,NONE,t}$$

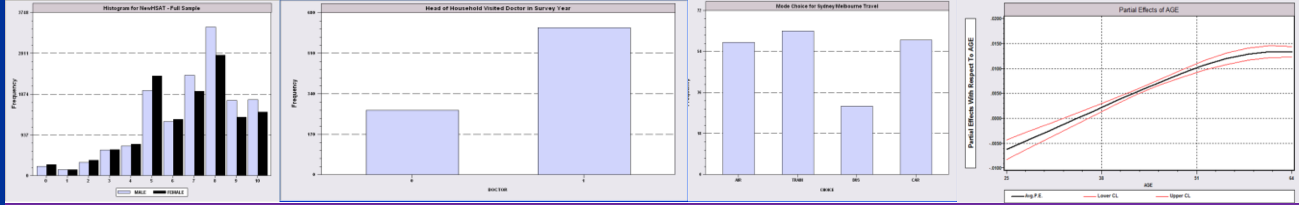
$$\boxed{\beta_{F,i}} = \bar{\beta}_F + \boxed{\delta_F \text{Sex}} + \boxed{[\sigma_F \exp(\gamma_{F1}\text{AgeL25}_i + \gamma_{F2}\text{Age2539}_i)]}\, w_{F,i};\; w_{F,i} \sim N[0,1]$$

$$\boxed{\beta_{P,i}} = \bar{\beta}_P + \boxed{\delta_P \text{Sex}} + \boxed{[\sigma_P \exp(\gamma_{P1}\text{AgeL25}_i + \gamma_{P2}\text{Age2539}_i)]}\, w_{P,i};\; w_{P,i} \sim N[0,1]$$

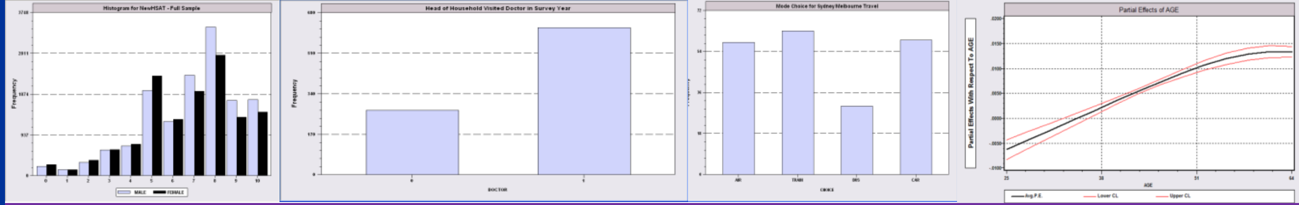
$$W_{Brand,i} \sim N[0,1]$$

$$W_{NONE,i} \sim N[0,1]$$

Heteroscedasticity




```
----------------------------------------------------------------
Random Parms/Error Comps. Logit Model
Dependent variable                    CHOICE
Log likelihood function          -4019.23544  (-4158.50286 for MNL)
Restricted log likelihood        -4436.14196  (Chi squared = 278.5)
Chi squared [  12 d.f.]           833.81303
Significance level                   .00000
McFadden Pseudo R-squared          .0939795
Estimation based on N =   3200, K =   12
Information Criteria: Normalization=1/N
              Normalized   Unnormalized
AIC              2.51952     8062.47089
R2=1-LogL/LogL* Log-L fncn R-sqrd R2Adj
No coefficients -4436.1420  .0940 .0928
Constants only  -4391.1804  .0847 .0836
At start values -4158.5029  .0335 .0323
Response data are given as ind. choices
Replications for simulated probs. =   50
Halton sequences used for simulations
RPL model with panel has     400 groups
Fixed number of obsrvs./group=         8
Number of obs.=  3200, skipped    0 obs
----------------------------------------------------------------
```

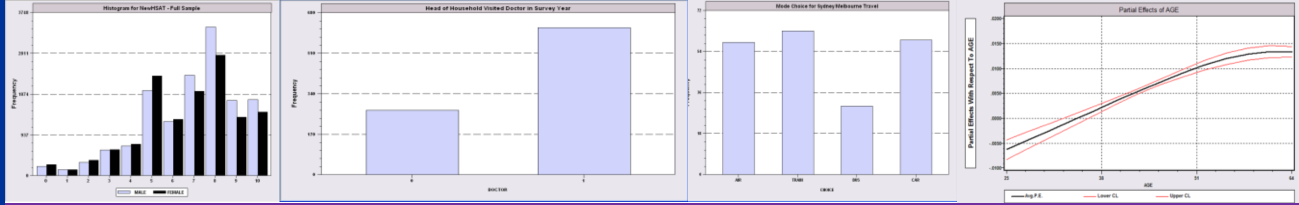


Estimated RP/ECL Model

```
--------+--------------------------------------------------
Variable| Coefficient    Standard Error  b/St.Er. P[|Z|>z]
--------+--------------------------------------------------
        |Random parameters in utility functions
    FASH|      .62768***       .13498        4.650   .0000
   PRICE|    -7.60651***      1.08418       -7.016   .0000
        |Nonrandom parameters in utility functions
    QUAL|     1.07127***       .06732       15.913   .0000
    ASC4|      .03874          .09017         .430   .6675
        |Heterogeneity in mean, Parameter:Variable
FASH:AGE|     1.73176***       .15372       11.266   .0000
FAS0:AGE|      .71872***       .18592        3.866   .0001
PRIC:AGE|    -9.38055***      1.07578       -8.720   .0000
PRI0:AGE|    -4.33586***      1.20681       -3.593   .0003
        |Distns. of RPs. Std.Devs or limits of triangular
  NsFASH|      .88760***       .07976       11.128   .0000
 NsPRICE|     1.23440         1.95780         .631   .5284
        |Heterogeneity in standard deviations
        |(cF1, cF2, cP1, cP2 omitted...)
        |Standard deviations of latent random effects
SigmaE01|      .23165          .40495         .572   .5673
SigmaE02|      .51260**        .23002        2.228   .0258
--------+--------------------------------------------------
Note: ***, **, * = Significance at 1%, 5%, 10% level.
-----------------------------------------------------------
```

```
Random Effects Logit
Model Appearance of
Latent Random Effects
in Utilities
 Alternative    E01 E02
+-------------+---+---+
|  BRAND1     | * |   |
+-------------+---+---+
|  BRAND2     | * |   |
+-------------+---+---+
|  BRAND3     | * |   |
+-------------+---+---+
|  NONE       |   | * |
+-------------+---+---+

Heterogeneity  in  Means.
Delta:     2 rows, 2 cols.
             AGE25      AGE39
FASH      1.73176     .71872
PRICE   -9.38055   -4.33586
```

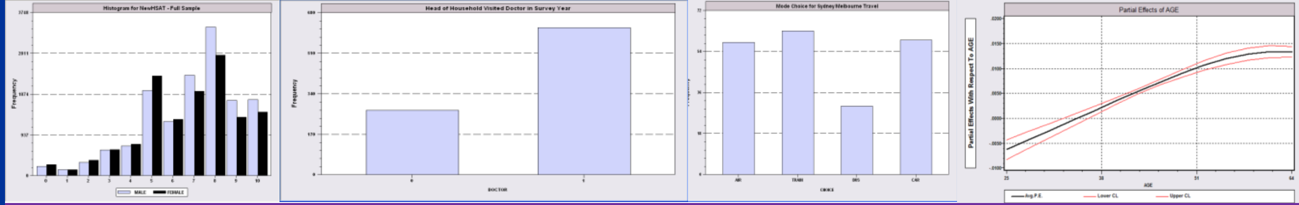


Estimated Elasticities

```
+---------------------------------------------------+
| Elasticity              averaged over observations.|
| Attribute is PRICE     in choice BRAND1            |
| Effects on probabilities of all choices in model:  |
| * = Direct Elasticity effect of the attribute.     |
|                                   Mean     St.Dev  |
| *     Choice=BRAND1              -.9210     .4661  |
|       Choice=BRAND2               .2773     .3053  |
|       Choice=BRAND3               .2971     .3370  |
|       Choice=NONE                 .2781     .2804  |
+---------------------------------------------------+
| Attribute is PRICE     in choice BRAND2            |
|       Choice=BRAND1               .3055     .1911  |
| *     Choice=BRAND2             -1.2692     .6179  |
|       Choice=BRAND3               .3195     .2127  |
|       Choice=NONE                 .2934     .1711  |
+---------------------------------------------------+
| Attribute is PRICE     in choice BRAND3            |
|       Choice=BRAND1               .3737     .2939  |
|       Choice=BRAND2               .3881     .3047  |
| *     Choice=BRAND3              -.7549     .4015  |
|       Choice=NONE                 .3488     .2670  |
+---------------------------------------------------+
```

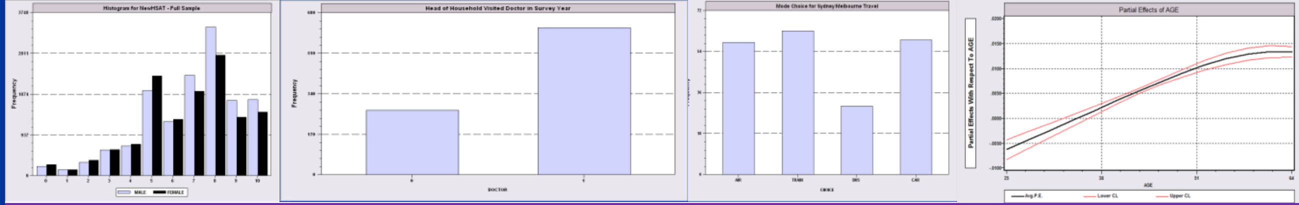
Multinomial Logit

```
+---------------------------+
| Effects on probabilities  |
| * = Direct effect te.     |
|              Mean St.Dev  |
| PRICE  in choice BRAND1   |
| * BRAND1    -.8895  .3647 |
|   BRAND2     .2907  .2631 |
|   BRAND3     .2907  .2631 |
|   NONE       .2907  .2631 |
+---------------------------+
| PRICE  in choice BRAND2   |
|   BRAND1     .3127  .1371 |
| * BRAND2   -1.2216  .3135 |
|   BRAND3     .3127  .1371 |
|   NONE       .3127  .1371 |
+---------------------------+
| PRICE  in choice BRAND3   |
|   BRAND1     .3664  .2233 |
|   BRAND2     .3664  .2233 |
| * BRAND3    -.7548  .3363 |
|   NONE       .3664  .2233 |
+---------------------------+
```



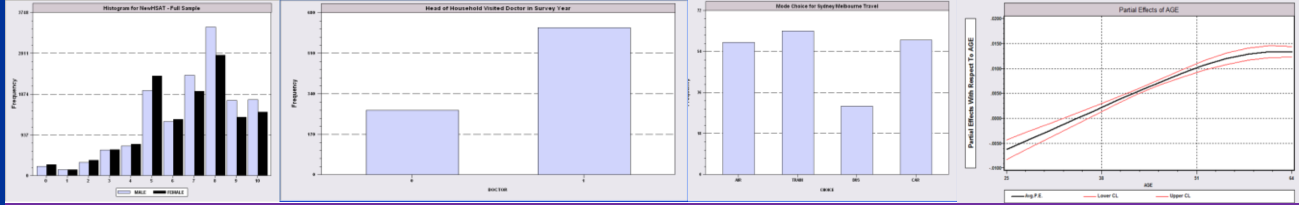
Estimating Individual Distributions

- Form posterior estimates of $E[\beta_i|data_i]$
- Use the same methodology to estimate $E[\beta_i^2|data_i]$ and $Var[\beta_i|data_i]$
- Plot individual “confidence intervals” (assuming near normality)
- Sample from the distribution and plot kernel density estimates

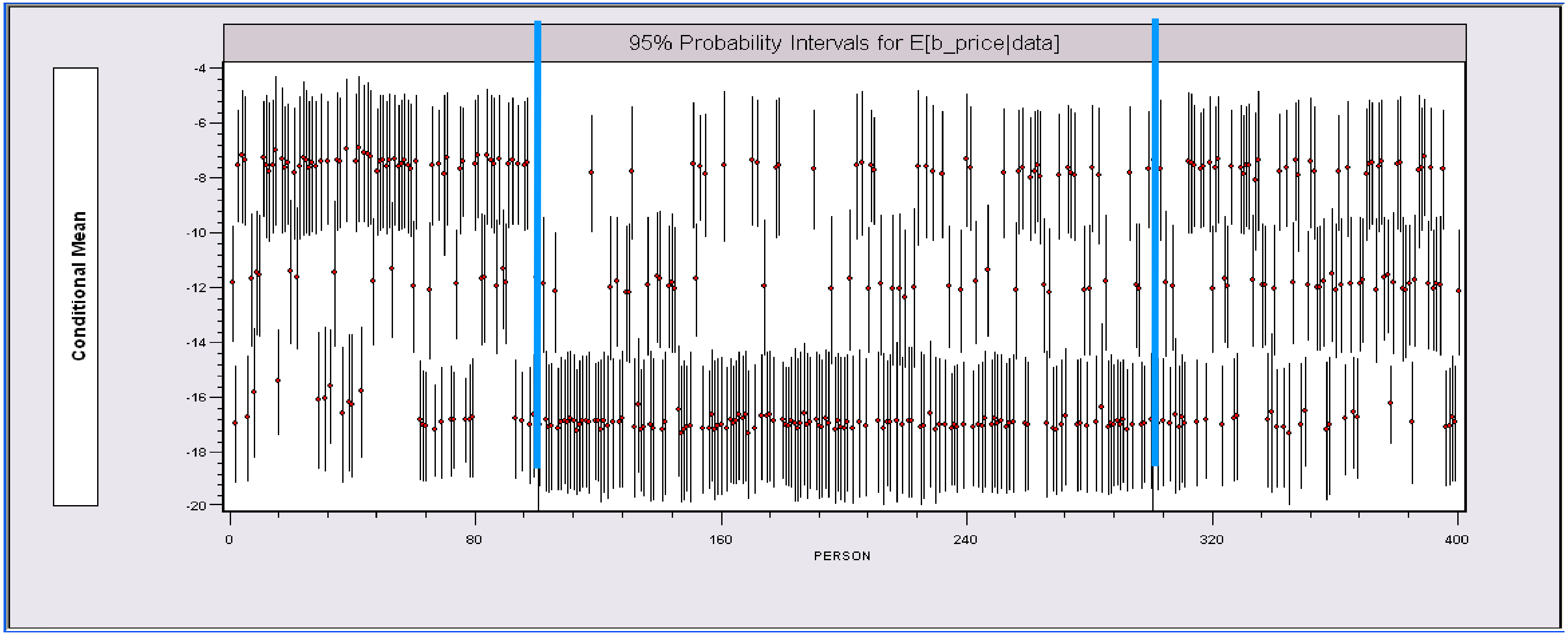


What is the 'Individual Estimate?'

- Point estimate of mean, variance and range of random variable β_i | data$_i$.
- Value is NOT an estimate of β_i ; it is an estimate of E[β_i | data$_i$]
- This would be the best estimate of the actual realization β_i|data$_i$
- An interval estimate would account for the sampling 'variation' in the estimator of $\mathbf{\Omega}$.
- Bayesian counterpart to the preceding: Posterior mean and variance. Same kind of plot could be done.



Individual E[β_i|data$_i$] Estimates*



The random parameters model is uncovering the latent class feature of the data.

***The intervals could be made wider to account for the sampling variability of the underlying (classical) parameter estimators.**

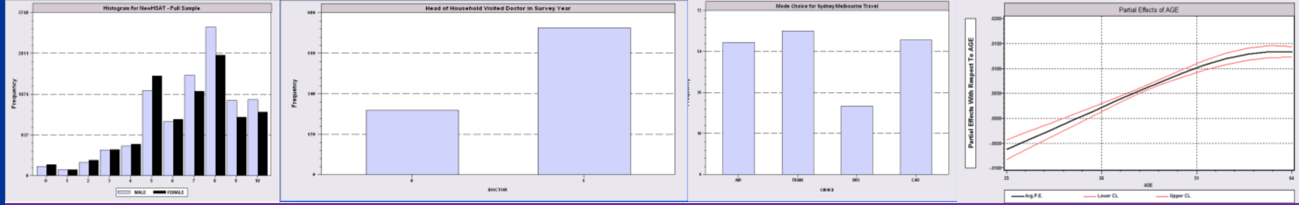
WTP Application (Value of Time Saved)

Estimating Willingness to Pay for Increments to an Attribute in a Discrete Choice Model

$$\boxed{\text{WTP} = \text{MU(attribute)} / \text{MU(Income)}}$$

Random

$$\text{WTP} = -\frac{\beta_{\text{attribute},i}}{\beta_{\text{cost}}}$$

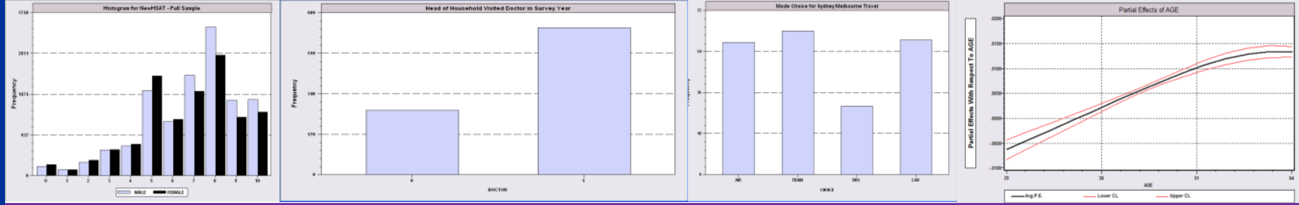


Extending the RP Model to WTP

- Use the model to estimate conditional distributions for any function of parameters
- Willingness to pay = $-\beta_{i,time} / \beta_{i,cost}$
- Use simulation method

$$\hat{E}[WTP_i \mid data_i] = \frac{(1/R)\Sigma_{r=1}^{R} WTP_{ir}\Pi_{t=1}^{T} P_{ijt}(\hat{\beta}_{ir} \mid \hat{\Omega}, data_{it})}{(1/R)\Sigma_{r=1}^{R} \Pi_{t=1}^{T} P_{ijt}(\hat{\beta}_{ir} \mid \hat{\Omega}, data_{it})}$$

$$= \frac{1}{R}\sum_{r=1}^{R} \hat{w}_{i,r} WTP_{ir}$$

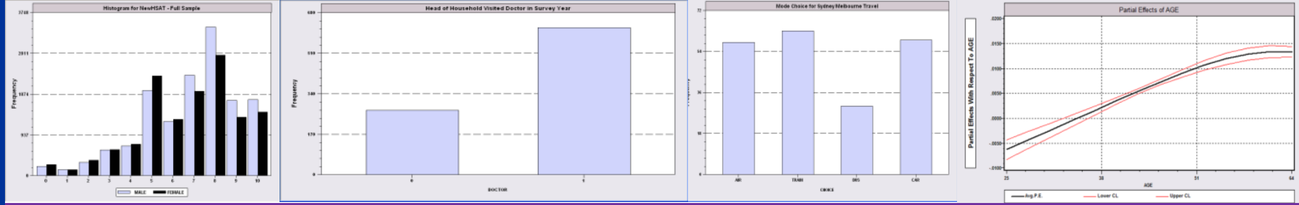


Simulation of WTP from β_i

$$\text{WTP}_i = E\left[\frac{-\beta_{i,\text{Attribute}}}{\beta_{i,\text{Cost}}} \,|\, \boldsymbol{\beta},\boldsymbol{\Delta},\boldsymbol{\Gamma},\mathbf{y}_i,\mathbf{X}_i,\mathbf{z}_i\right]$$

$$= \frac{\int_{\beta_i} \frac{-\beta_{i,\text{Attribute}}}{\beta_{i,\text{Cost}}}\left[\prod_{t=1}^{T} P(\text{choice } j \,|\, \mathbf{X}_i,\boldsymbol{\beta}_i) g(\boldsymbol{\beta}_i \,|\, \boldsymbol{\beta},\boldsymbol{\Delta},\boldsymbol{\Gamma},\mathbf{y}_i,\mathbf{X}_i,\mathbf{z}_i)\right] d\boldsymbol{\beta}_i}{\int_{\beta_i}\left[\prod_{t=1}^{T} P(\text{choice } j \,|\, \mathbf{X}_i,\boldsymbol{\beta}_i) g(\boldsymbol{\beta}_i \,|\, \boldsymbol{\beta},\boldsymbol{\Delta},\boldsymbol{\Gamma},\mathbf{y}_i,\mathbf{X}_i,\mathbf{z}_i)\right] d\boldsymbol{\beta}_i}$$

$$\text{WTP}_i = \frac{\frac{1}{R}\sum_{r=1}^{R} \frac{-\hat{\beta}_{i,\text{Attribute}}}{\hat{\beta}_{i,\text{Cost}}}\left[\prod_{t=1}^{T} P(\text{choice } j \,|\, \mathbf{X}_i,\hat{\boldsymbol{\beta}}_{ir})\right]}{\frac{1}{R}\sum_{r=1}^{R}\left[\prod_{t=1}^{T} P(\text{choice } j \,|\, X_i,\hat{\boldsymbol{\beta}}_{ir})\right]}, \quad \hat{\boldsymbol{\beta}}_{ir} = \hat{\boldsymbol{\beta}} + \hat{\boldsymbol{\Delta}}\mathbf{z}_i + \hat{\boldsymbol{\Gamma}}\mathbf{w}_{ir}$$



Untitled 1 *

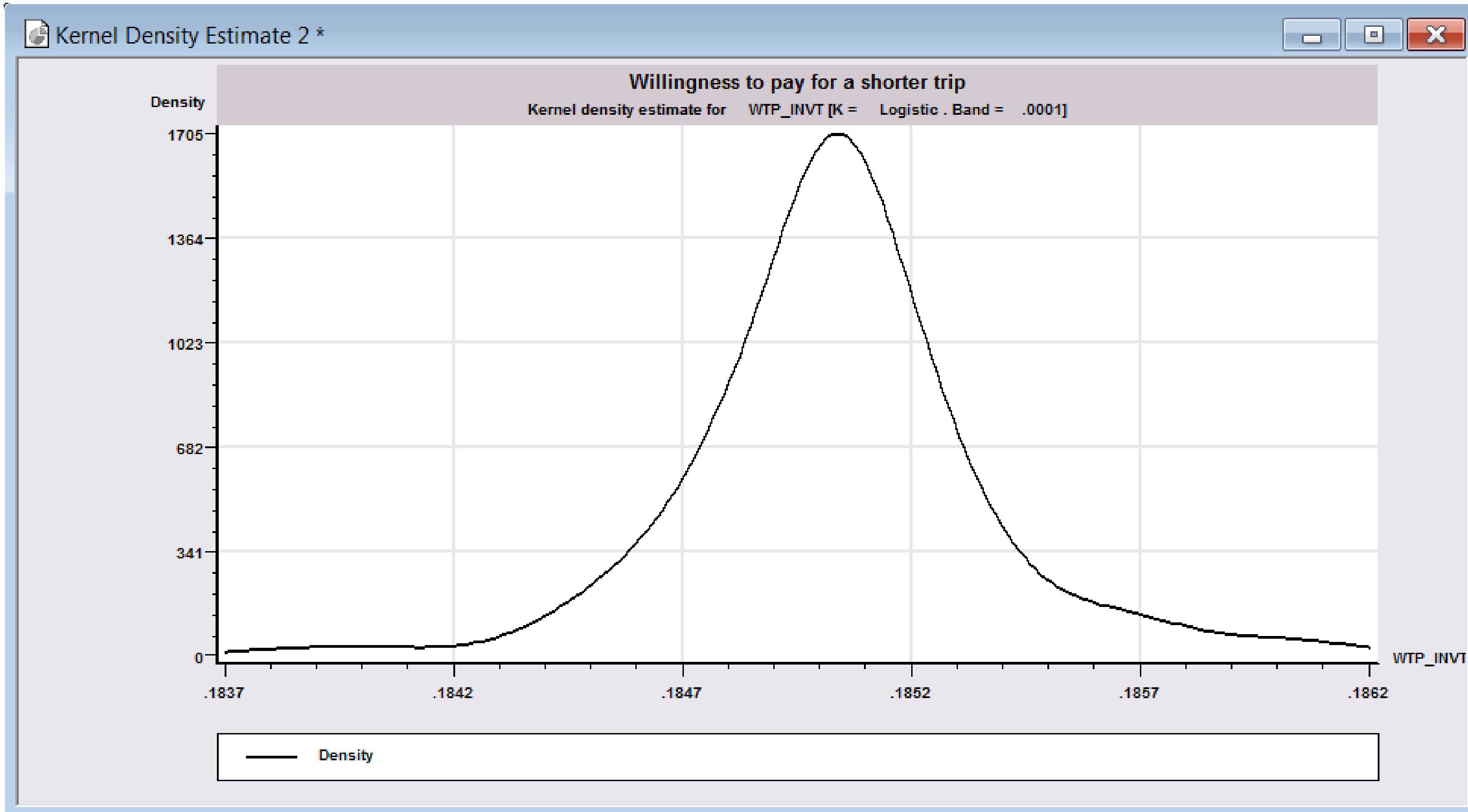
Insert Name:

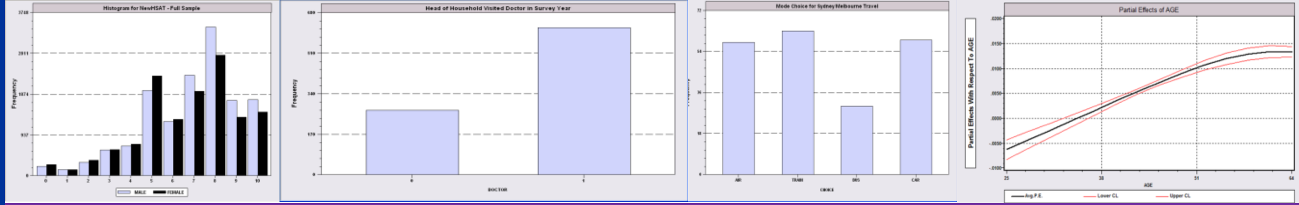
```
RPLOGIT;lhs=mode;choices=air,train,bus,car
;Rhs = gc,ttme,invc,invt;rh2=one,hinc
;wtp=invt/gc
;fcn=invt(n);pts=100;HALTON $

KERNEL;RHS=WTP_I[1,WTP_INVT]
;Title=Willingness to pay for a shorter trip
;grid$
```

MODE	Coefficient	Standard Error	z	Prob. |z|>Z*	95% Confidence Interval	
	Random parameters in utility functions					
INVT	-.01407***	.00316	-4.45	.0000	-.02027	-.00788
	Nonrandom parameters in utility functions					
GC	.07605***	.01940	3.92	.0001	.03803	.11407
TTME	-.10301***	.01147	-8.98	.0000	-.12550	-.08053
INVC	-.08087***	.02209	-3.66	.0003	-.12417	-.03756
A_AIR	4.35972***	1.07977	4.04	.0001	2.24341	6.47604
AIR_HIN1	.00454	.01426	.32	.7501	-.02341	.03250
A_TRAIN	5.92497***	.73159	8.10	.0000	4.49108	7.35887
TRA_HIN2	-.05905***	.01473	-4.01	.0001	-.08792	-.03019
A_BUS	4.47131***	.74812	5.98	.0000	3.00501	5.93760
BUS_HIN3	-.02292	.01595	-1.44	.1506	-.05418	.00833
	Distns. of RPs. Std.Devs or limits of triangular					
NsINVT	.00034	.00370	.09	.9270	-.00692	.00759

WTP

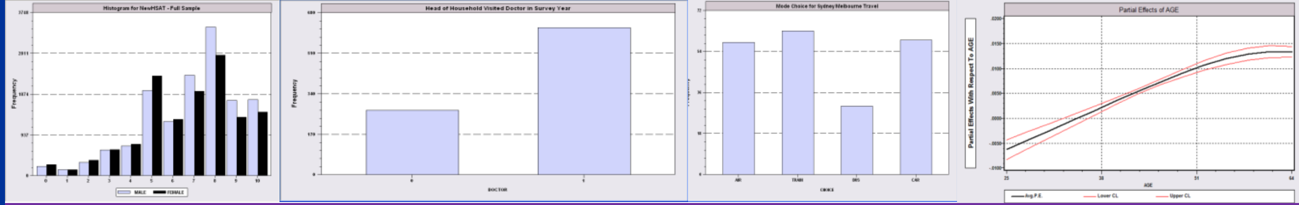

Kernel Density Estimate 2 *
Willingness to pay for a shorter trip
Kernel density estimate for WTP_INVT [K = Logistic . Band = .0001]
Density
1705
1364
1023
682
341
0
.1837
.1842
.1847
.1852
.1857
.1862
WTP_INVT
Density



Stated Choice Experiment: Travel Mode by Sydney Commuters

Table 3 Trip Attributes in Stated Choice Design

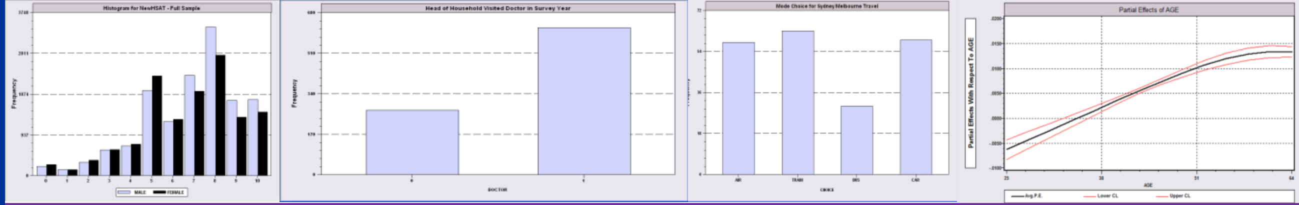
For existing public transport modes	For new public transport modes	For the existing car mode
Fare (one-way)	Fare (one-way)	Running Cost
In-vehicle travel time	In-vehicle travel time	In-vehicle Travel time
Waiting time	Waiting time	Toll Cost (One way)
Access Mode: Walk time	Transfer waiting time	Daily Parking Cost
Car time	Access Mode: Walk time	Egress time
Bus time	Car time	
Bus fare	Bus time	
Egress time	Access Mode Fare (one-way)	
	Bus fare	
	Egress time	



Would You Use a New Mode?

Table 4 Base times and costs for new public transport modes

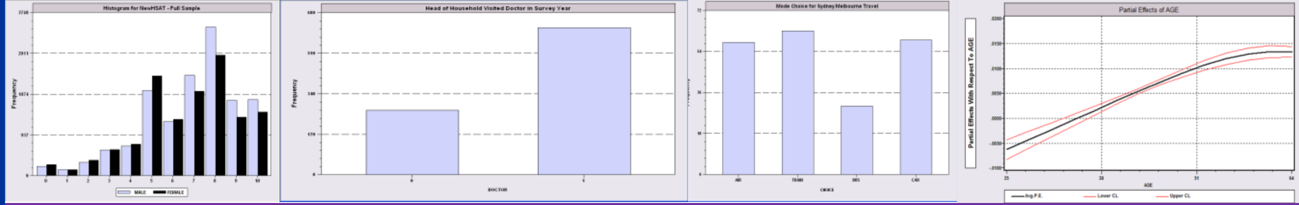
	Dollars	Busway	Heavy rail	Light Rail
	$	min	min	min
Mungerie Park	1.8	33	22	33
Burns Road	1	27	18	27
Norwest Business Park	1	22.5	15	22.5
Hills Centre	1	18	12	18
Castle Hill	0.2	13.5	9	13.5
Franklin Road	0.2	7.5	5	7.5



Value of Travel Time Saved

Behavioural Values of travel Time savings ($/person hour) based on individual parameters: mixed logit model, commuter trips (mean gross personal income per hour = $32.05)

Willingness to Pay Attribute	VTTS ($/person Hour) Mean and range
Main mode in-vehicle time – car	28.2 (6.37-39.8)
Egress time - car	31.61 (15.8-75.6)
Main mode in-vehicle time – public transport	15.71 (2.6-31.5)
Waiting time – all bus	19.1 (9.9-40.3)
Access time – all bus	17.1 (8.7-31.7)
Access plus wait time – all rail	10.50 96.6-22.9)
Egress time – all public transport	3.4 (2.0 – 7.4)



A Generalized Mixed Logit Model

$$U(i,t,j) = \boldsymbol{\beta}_i'\mathbf{x}_{i,t,j} + \text{Common effects} + \varepsilon_{i,t,j}$$

Random Parameters

$$\boldsymbol{\beta}_i = \sigma_i[\boldsymbol{\beta} + \boldsymbol{\Delta}\mathbf{z}_i] + [\gamma + \sigma_i(1-\gamma)]\boldsymbol{\Gamma}_i\mathbf{v}_i$$

$$\boldsymbol{\Gamma}_i = \boldsymbol{\Lambda}\boldsymbol{\Sigma}_i$$

$\boldsymbol{\Lambda}$ is a lower triangular matrix

with 1s on the diagonal (Cholesky matrix)

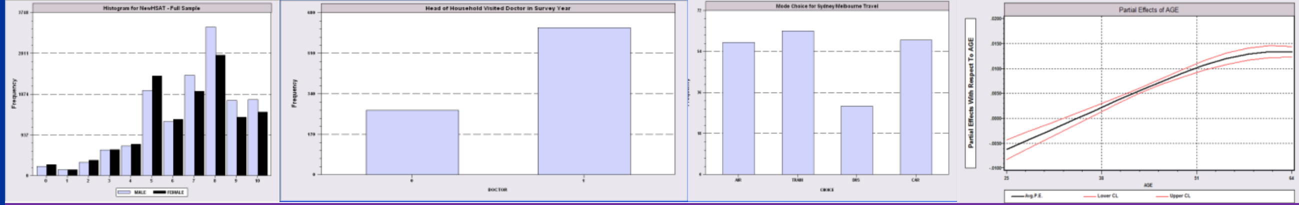
$\boldsymbol{\Sigma}_i$ is a diagonal matrix with $\varphi_k \exp(\boldsymbol{\psi}_k'\mathbf{h}_i)$

Overall preference scaling

$$\sigma_i = \sigma\exp(-\tau_i^2/2 + \tau_i^2 w_i + \boldsymbol{\theta}'\mathbf{h}_i]$$

$$\tau_i = \exp(\boldsymbol{\lambda}'\mathbf{r}_i)$$

$$0 < \gamma < 1$$



Generalized Multinomial Choice Model

$$U_{i,1,t} = \beta_{F,i}\,\text{Fashion}_{i,1,t} + \beta_Q\text{Quality}_{i,1,t} + \beta_{P,i}\,\text{Price}_{i,1,t} + \lambda_{Brand}W_{i,Brand} + \varepsilon_{i,1,t}$$

$$U_{i,2,t} = \beta_{F,i}\,\text{Fashion}_{i,2,t} + \beta_Q\text{Quality}_{i,2,t} + \beta_{P,i}\,\text{Price}_{i,2,t} + \lambda_{Brand}W_{i,Brand} + \varepsilon_{i,2,t}$$

$$U_{i,3,t} = \beta_{F,i}\,\text{Fashion}_{i,3,t} + \beta_Q\text{Quality}_{i,3,t} + \beta_{P,i}\,\text{Price}_{i,3,t} + \lambda_{Brand}W_{i,Brand} + \varepsilon_{i,3,t}$$

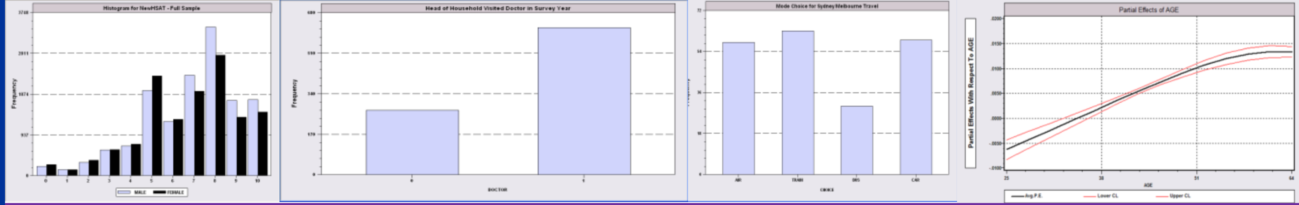
$$U_{i,NONE,t} = \alpha_{NONE} + \lambda_{NONE}W_{i,NONE} + \varepsilon_{i,NONE,t}$$

$$\beta_{F,i} = \overline{\beta}_F + \delta_F\text{Sex}_i + [\sigma_F\exp(\gamma_{F1}\text{AgeL25}_i + \gamma_{F2}\text{Age2539}_i)]\,w_{F,i};\ w_{F,i} \sim N[0,1]$$

$$\beta_{P,i} = \overline{\beta}_P + \delta_P\text{Sex}_i + [\sigma_P\exp(\gamma_{P1}\text{AgeL25}_i + \gamma_{P2}\text{Age2539}_i)]\,w_{P,i};\ w_{P,i} \sim N[0,1]$$

$$W_{Brand,i} \sim N[0,1]$$

$$W_{NONE,i} \sim N[0,1]$$



Estimation in Willingness to Pay Space

Both parameters in the WTP calculation are random.

$$U_{i,1,t} = \beta_{P,i}\left(\theta_{F,i}\,\text{Fashion}_{i,1,t} + \theta_Q\,\text{Quality}_{i,1,t} + \text{Price}_{i,1,t}\right) + \lambda_{Brand} W_{i,Brand} + \varepsilon_{i,1,t}$$

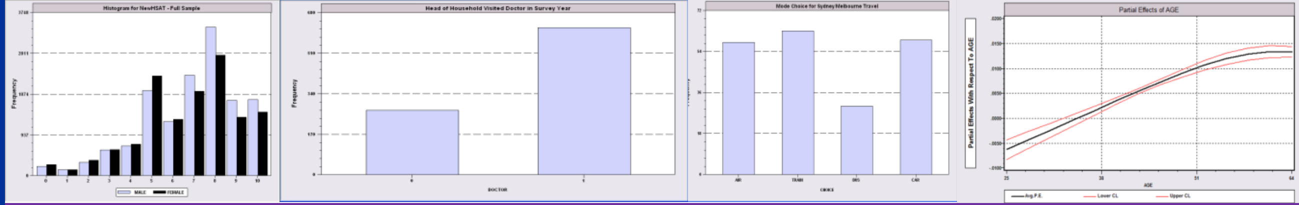
$$U_{i,2,t} = \beta_{P,i}\left(\theta_{F,i}\,\text{Fashion}_{i,2,t} + \theta_Q\,\text{Quality}_{i,2,t} + \text{Price}_{i,2,t}\right) + \lambda_{Brand} W_{i,Brand} + \varepsilon_{i,2,t}$$

$$U_{i,3,t} = \beta_{P,i}\left(\theta_{F,i}\,\text{Fashion}_{i,3,t} + \theta_Q\,\text{Quality}_{i,3,t} + \text{Price}_{i,3,t}\right) + \lambda_{Brand} W_{i,Brand} + \varepsilon_{i,3,t}$$

$$U_{i,NONE,t} = \alpha_{NONE} + \lambda_{NONE} W_{i,NONE} + \varepsilon_{i,NONE,t}$$

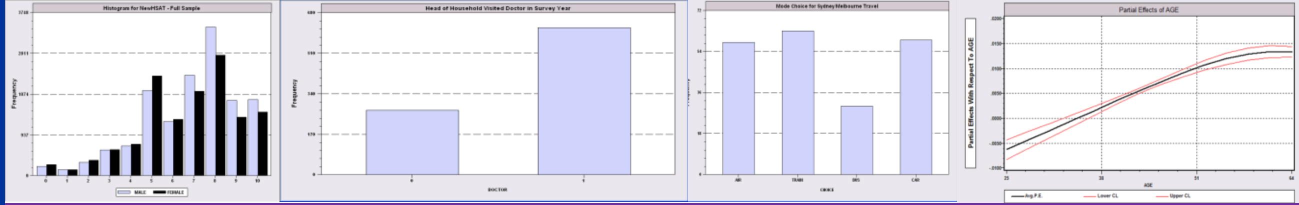
$$W_{Brand,i} \sim N[0,1] \quad W_{NONE,i} \sim N[0,1]$$

$$\begin{pmatrix}\theta_{F,i}\\ \beta_{P,i}\end{pmatrix} = \sigma_i \begin{bmatrix}\overline{\theta}_F + \delta_F \text{Sex}_i\\ \overline{\beta}_P + \delta_P \text{Sex}_i\end{bmatrix} + [\sigma_i\gamma + (1-\gamma)]\begin{bmatrix}\sigma_F & 0\\ \lambda_{PF} & \sigma_P\end{bmatrix}\begin{pmatrix}w_{F,i}\\ w_{P,i}\end{pmatrix} \quad \begin{matrix} w_{F,i} \sim N[0,1]\\ w_{P,i} \sim N[0,1]\end{matrix}$$

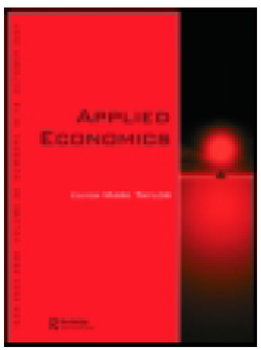


Estimated Model for WTP

```
--------+--------------------------------------------------
Variable| Coefficient    Standard Error  b/St.Er. P[|Z|>z]
--------+--------------------------------------------------
        |Random parameters in utility functions
    QUAL|    -.32668***        .04302       -7.593   .0000     1.01373  renormalized
   PRICE|    1.00000         ......(Fixed Parameter)......   -11.80230  renormalized
        |Nonrandom parameters in utility functions
    FASH|    1.14527***        .05788       19.787   .0000      1.4789  not rescaled
    ASC4|     .84364***        .05554       15.189   .0000       .0368  not rescaled
        |Heterogeneity in mean, Parameter:Variable
QUAL:AGE|     .05843           .04836        1.208   .2270      interaction terms
QUA0:AGE|    -.11620           .13911        -.835   .4035
PRIC:AGE|     .23958           .25730         .931   .3518
PRI0:AGE|    1.13921           .76279        1.493   .1353
        |Diagonal values in Cholesky matrix, L.
  NsQUAL|     .13234***        .04125        3.208   .0013      correlated parameters
 CsPRICE|      .000          ......(Fixed Parameter)......      but coefficient is fixed
        |Below diagonal values in L matrix. V = L*Lt
PRIC:QUA|      .000          ......(Fixed Parameter)......
        |Heteroscedasticity in GMX scale factor
  sdMALE|     .23110           .14685        1.574   .1156      heteroscedasticity
        |Variance parameter tau in GMX scale parameter
TauScale|    1.71455***        .19047        9.002   .0000      overall scaling, tau
        |Weighting parameter gamma in GMX model
GammaMXL|      .000          ......(Fixed Parameter)......
        |Coefficient on PRICE    in WTP space form
Beta0WTP|   -3.71641***        .55428       -6.705   .0000      new price coefficient
S_b0_WTP|     .03926           .40549         .097   .9229      standard deviation
        |  Sample Mean     Sample Std.Dev.
Sigma(i)|     .70246          1.11141         .632   .5274      overall scaling
        |Standard deviations of parameter distributions
  sdQUAL|     .13234***        .04125        3.208   .0013
 sdPRICE|      .000          ......(Fixed Parameter)......
--------+--------------------------------------------------
```



Latent Class Mixed Logit



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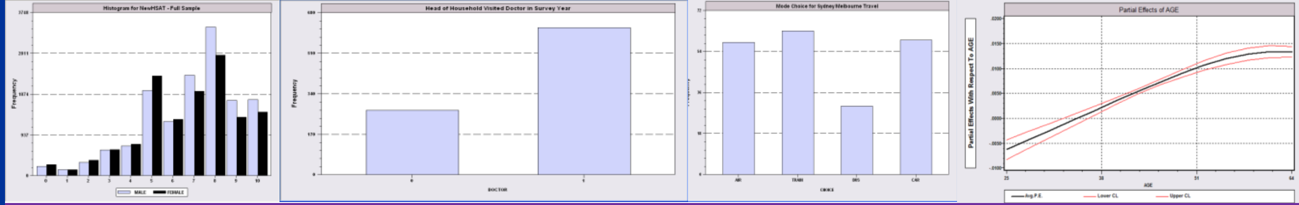
Revealing additional dimensions of preference heterogeneity in a latent class mixed multinomial logit model

William H. Greene [a] & David A. Hensher [b]

References

Bujosa, A., Riera, A. and Hicks, R. (2010) Combining discrete and continuous representation of preference heterogeneity: a latent class approach, *Environment and Resource Economics*, 47, 477–93.



$$f(y_i|\mathbf{x}_i, \text{class} = q) = g(y_i|\mathbf{x}_i, \boldsymbol{\beta}_q) \tag{1}$$

$$\text{Prob}(\text{class} = q) = \pi_q(\theta), \quad q = 1, \ldots, Q \tag{2}$$

The within-class heterogeneity is structured as

$$\boldsymbol{\beta}_i|q = \boldsymbol{\beta}_q + \mathbf{w}_{i|q} \tag{6}$$

$$\mathbf{w}_{i|q} \sim E[\mathbf{w}_{i|q}|\mathbf{X}] = \mathbf{0}, \quad \text{Var}[\mathbf{w}_{i|q}|\mathbf{X}] = \boldsymbol{\Sigma}_q \tag{7}$$

choice situations, where $T_i \geq 1$. The generic model is given in Equation 8,

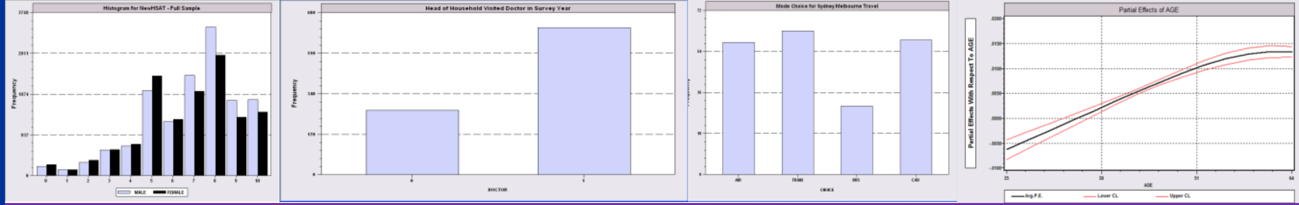
$$f(\mathbf{y}_i|\mathbf{X}_i, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_Q, \theta, \boldsymbol{\Sigma}_1, \ldots, \boldsymbol{\Sigma}_Q) = \sum_{q=1}^{Q} \pi_q(\theta) \int_{\mathbf{w}_i} \prod_{t=1}^{T_i} f[\,\mathbf{y}_{it}|(\boldsymbol{\beta}_q + \mathbf{w}_i), \mathbf{X}_{it}]h(\mathbf{w}_i|\boldsymbol{\Sigma}_q)d\mathbf{w}_i \tag{8}$$

We parameterize the class probabilities using a multinomial logit formulation to impose the adding up and positivity restrictions on $\pi_q(\theta)$. Thus,

$$\pi_q(\theta) = \frac{\exp(\theta_q)}{\Sigma_{q=1}^{Q} \exp(\theta_q)}, \quad q = 1, \ldots, Q; \;\; \theta_Q = 0 \tag{9}$$

A useful refinement of the class probabilities model is to allow the probabilities to be dependent on individual data, such as demographics. The class probability model becomes

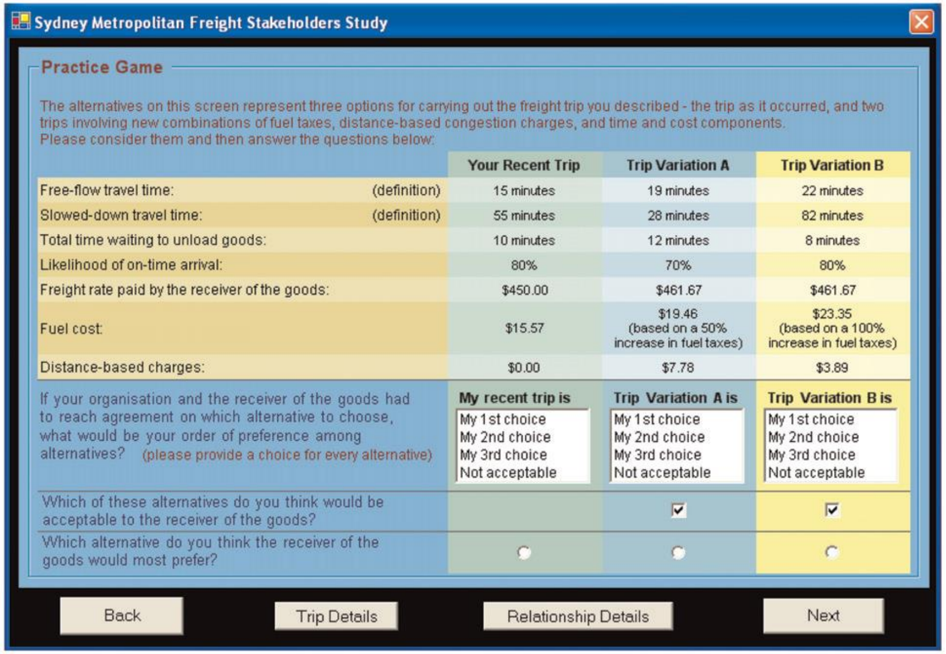
$$\pi_{iq}(\mathbf{z}_i, \theta) = \frac{\exp(\theta_q' \mathbf{z}_i)}{\Sigma_{q=1}^{Q} \exp(\theta_q' \mathbf{z}_i)}, \quad q = 1, \ldots, Q; \;\; \theta_Q = 0 \tag{10}$$



To illustrate the application of the model, we have selected a Stated Choice (SC) framework (Louviere *et al.*, 2000) within which a freight transporter defined a recent reference trip in terms of its time and cost attributes, treating fuel as a separate cost item to the Variable User Charge (VUC), with the latter being zero at present. An SC design of two alternatives using

charging policy or direct functions of such a policy. The levels of the attributes are expressed as deviations from the reference level, which is the exact value specified in the corresponding non-SC questions, unless noted:

(1) Free-flow time: −50%, −25%, 0, +25%, +50%.
(2) Congested time: −50%, −25%, 0, +25%, +50%.
(3) Waiting time at destination: −50%, −25%, 0, +25%, +50%.
(4) Probability of on-time arrival: −50%, −25%, 0, +25%, +50%, with the resulting value rounded to the nearest 5%.
(5) Fuel cost: −50%, −25%, 0, +25%, +50% (representing changes in fuel taxes of −100%, −50%, 0, +50%, +100%).
(6) Distance-based charges: Pivot base equals 0.5 ∗ (reference fuel cost), to reflect the amount of fuel taxes paid in the reference alternative. Variations around the pivot base are: −50%, −25%, 0, +25%, +50%.



Sydney Metropolitan Freight Stakeholders Study

Practice Game

The alternatives on this screen represent three options for carrying out the freight trip you described - the trip as it occurred, and two trips involving new combinations of fuel taxes, distance-based congestion charges, and time and cost components. Please consider them and then answer the questions below:

	Your Recent Trip	Trip Variation A	Trip Variation B
Free-flow travel time: (definition)	15 minutes	19 minutes	22 minutes
Slowed-down travel time: (definition)	55 minutes	28 minutes	82 minutes
Total time waiting to unload goods:	10 minutes	12 minutes	8 minutes
Likelihood of on-time arrival:	80%	70%	80%
Freight rate paid by the receiver of the goods:	$450.00	$461.67	$461.67
Fuel cost:	$15.57	$19.46 (based on a 50% increase in fuel taxes)	$23.35 (based on a 100% increase in fuel taxes)
Distance-based charges:	$0.00	$7.78	$3.89
If your organisation and the receiver of the goods had to reach agreement on which alternative to choose, what would be your order of preference among alternatives? (please provide a choice for every alternative)	My recent trip is: My 1st choice / My 2nd choice / My 3rd choice / Not acceptable	Trip Variation A is: My 1st choice / My 2nd choice / My 3rd choice / Not acceptable	Trip Variation B is: My 1st choice / My 2nd choice / My 3rd choice / Not acceptable
Which of these alternatives do you think would be acceptable to the receiver of the goods?		☑	☑
Which alternative do you think the receiver of the goods would most prefer?	○	○	○

Back | Trip Details | Relationship Details | Next

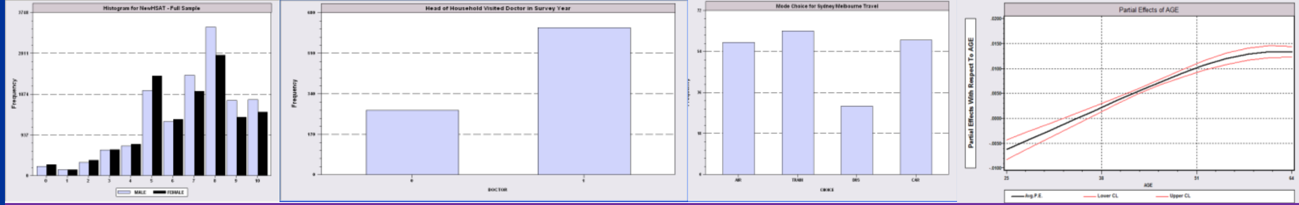


Table 1. Summary of models

	MNL (Model M1)	Mixed logit (Model M2)	Latent class: Fixed parameters (M3)		Latent class: Random parameters (M4)	
Attributes			Class 1	Class 2	Class 1	Class 2
Total time (min)	−0.0056	−0.0053	−0.00177	−0.0451	−0.0082	0.0008
	(−3.44)	(−2.97)$^{\mathrm{RP}}$	(−3.82)	(−1.73)	(1.95)	(0.25)
On time delivery (%)	0.03161	0.0360	0.0296	−0.0296	−0.0005$^{\mathrm{RP}}$	0.02713$^{\mathrm{RP}}$
	(4.75)	(4.51)$^{\mathrm{RP}}$	(3.35)	(−0.55)	(−0.03)	(2.53)
Total cost ($)	−0.0030	−0.0039	−0.0051	−0.0134	−0.0135	−0.0052
	(−4.15)	(−3.74)$^{\mathrm{RP}}$	(−3.95)	(−2.09)	(−2.13)	(−3.33)
No variable charge dummy (1,0)	0.9406	0.8798	0.7716	4.2819	1.5933	−1.2945
	(5.71)	(4.84)	(2.35)	(2.18)	(4.48)	(−2.1)
Class membership probability			0.7137	0.2853	0.574	0.4264
			(14.2)	(4.04)	(6.9)	(5.14)
BIC	1.8777	1.8651	1.8857		1.6650	
Log-likelihood	−393.44	−390.71	−380.01		−350.63	

Latent class with decomposition of class membership probability

	Fixed parameters (M5)		Random parameters (M6)	
Attributes	Class 1	Class 2	Class 1	Class 2
Total time (min)	0.0056	−0.00015	−0.0069	0.0015
	(0.69)	(−0.04)	(−1.96)	(0.56)
On time delivery (%)	−0.0062	0.0279	−0.00056$^{\mathrm{RP}}$	0.0276$^{\mathrm{RP}}$
	(−0.21)	(2.34)	(−0.040	(2.86)
Total cost ($)	−0.0474	−0.0043	−0.0157	−0.0055
	(−3.30)	(−1.91)	(−3.14)	(−4.26)
No variable charge dummy (1,0)	1.4220	−2.1567	1.4109	−1.3472
	(2.84)	(−1.34)	(4.25)	(−2.95)
Class membership probability	0.616	0.384	0.575	0.425
Constant	1.1741 (3.30)	0	1.8200 (2.79)	0
Freight rate ($)	−0.00076 (−2.32)	0	−0.0016 (−2.54)	0
BIC	1.7724		1.6498	
Log-likelihood	−372.85		−346.36	

Notes: Elasticities are not informative for this model given that the alternatives are unlabelled. BIC – Bayesian Information Criterion. There are 432 observations given. For mixed logit we used constrained *t*-distributions and 500 Halton draws. *t*-ratios are given within parentheses.
$^{\mathrm{RP}}$ Denotes random parameter.