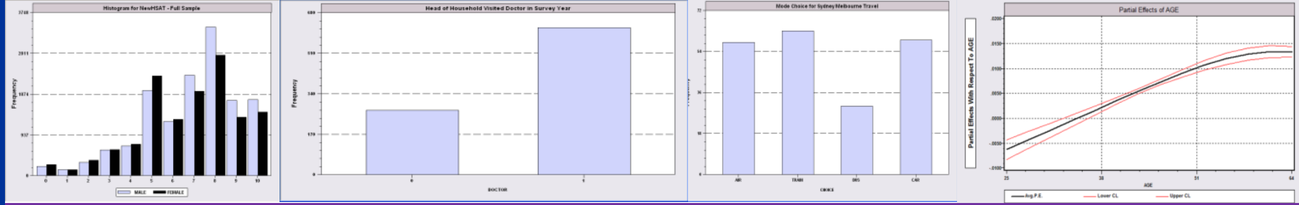
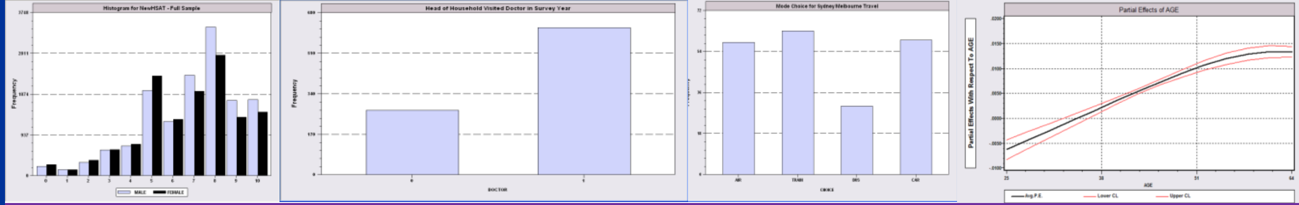


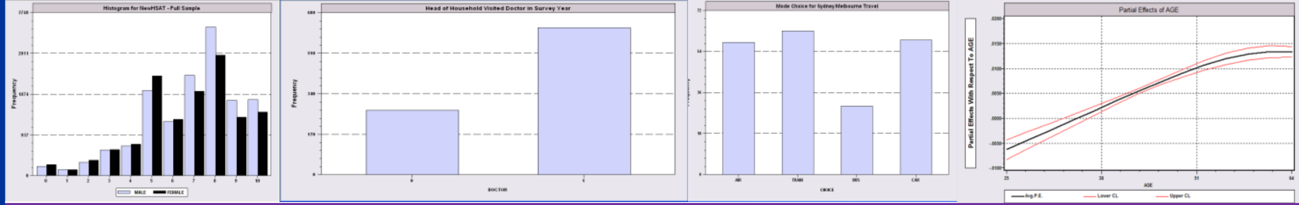
Discrete Choice Modeling

William Greene
Stern School of Business
New York University



What is a hybrid choice model?

- Incorporates **latent variables** in choice model
- Extends development of discrete choice model to incorporate other aspects of preference structure of the chooser
- Develops **endogeneity** of the preference structure.

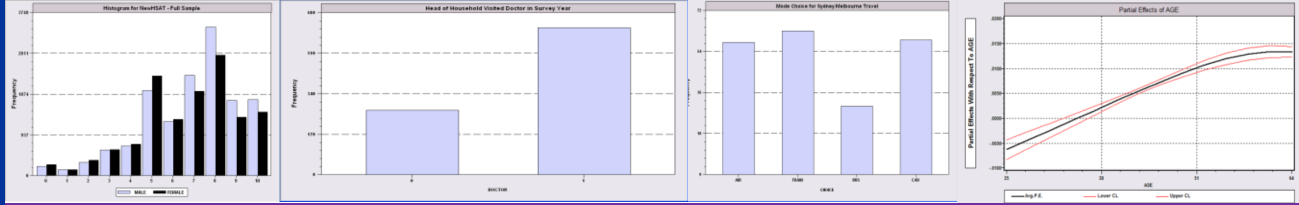


Endogeneity

- "Recent Progress on Endogeneity in Choice Modeling" with Jordan Louviere & Kenneth Train & Moshe Ben-Akiva & Chandra Bhat & David Brownstone & Trudy Cameron & Richard Carson & J. Deshazo & Denzil Fiebig & William Greene & David Hensher & Donald Waldman, 2005. *Marketing Letters* Springer, vol. 16(3), pages 255-265, December.

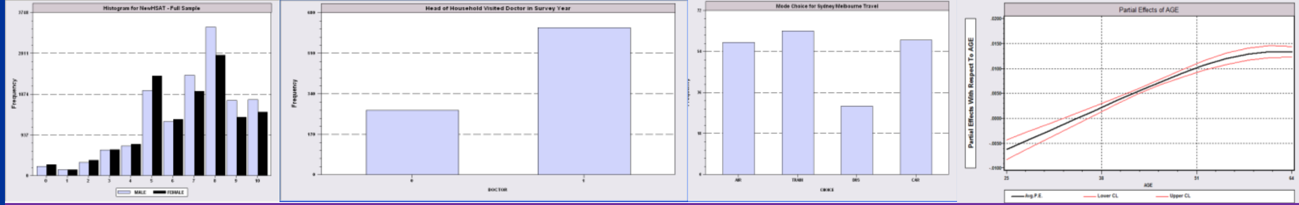
- **Narrow view**: $U(i,j) = \mathbf{b'x}(i,j) + \varepsilon(i,j)$, $\mathbf{x}(i,j)$ correlated with $\varepsilon(i,j)$ (Berry, Levinsohn, Pakes, brand choice for cars, endogenous price attribute.)
  Implications for estimators that assume it is.

- **Broader view**: Sounds like heterogeneity.
  - Preference structure: RUM vs. RRM
  - Heterogeneity in choice strategy – e.g., omitted attribute models
  - Heterogeneity in taste parameters: location and scaling
  - Heterogeneity in functional form: Possibly nonlinear utility functions



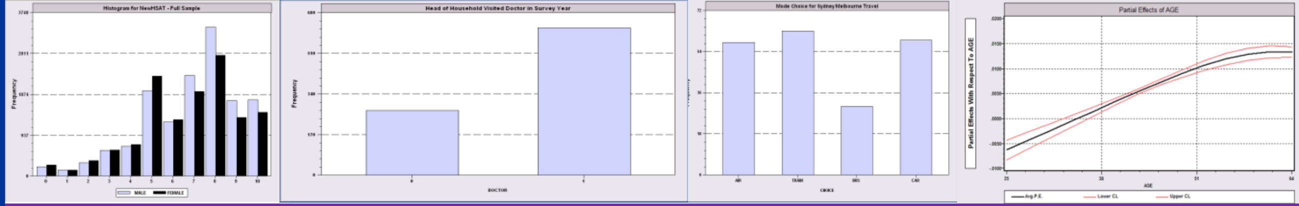
Heterogeneity

- Narrow view: Random variation in marginal utilities and scale
  - RPM, LCM
  - Scaling model
  - Generalized Mixed model
- Broader view: Heterogeneity in preference weights
  - RPM and LCM with exogenous variables
  - Scaling models with exogenous variables in variances
  - Looks like hierarchical models



Heterogeneity and the MNL Model

$$P[\text{choice } j \mid i] = \frac{\exp(\alpha_j + \boldsymbol{\beta}'\mathbf{x}_{ij})}{\sum_{j=1}^{J(i)} \exp(\alpha_j + \boldsymbol{\beta}'\mathbf{x}_{ij})}$$



Observable Heterogeneity in Preference Weights

Hierarchical model - Interaction terms

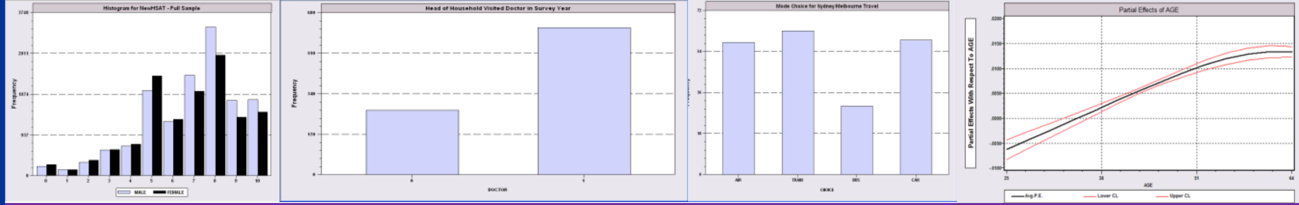
$$U_{ij} = \alpha_j + \boldsymbol{\beta}_i'\mathbf{x}_{ij} + \boldsymbol{\gamma}_j'\mathbf{z}_i + \varepsilon_{ij}$$

$$\boldsymbol{\beta}_i = \boldsymbol{\beta} + \boldsymbol{\Phi}\mathbf{h}_i$$

Parameter heterogeneity is observable.

Each parameter $\beta_{i,k} = \beta_k + \boldsymbol{\varphi}_k'\mathbf{h}_i$

$$\text{Prob[choice } j \mid i] = \frac{\exp(\alpha_j + \boldsymbol{\beta}_i'\mathbf{x}_{ij} + \boldsymbol{\gamma}_j'\mathbf{z}_i)}{\sum_{j=1}^{J_i}\exp(\alpha_j + \boldsymbol{\beta}_i'\mathbf{x}_{ij} + \boldsymbol{\gamma}'\mathbf{z}_i)}$$

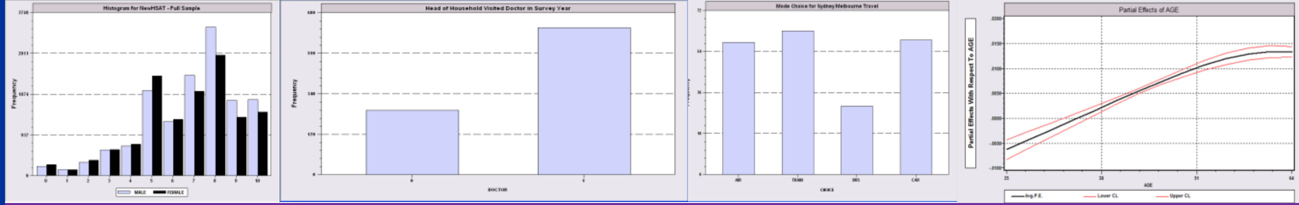


‘Quantifiable’ Heterogeneity in Scaling

$$U_{ij} = \alpha_j + \boldsymbol{\beta}_i'\mathbf{x}_{ij} + \boldsymbol{\gamma}_j'\mathbf{z}_i + \varepsilon_{ij}$$

$$\text{Var}[\varepsilon_{ij}] = \sigma_j^2 \exp(\boldsymbol{\delta}_j'\mathbf{w}_i), \sigma_1^2 = \pi^2/6$$

w_i = observable characteristics: age, sex, income, etc.



Unobserved Heterogeneity in Scaling

HEV formulation: $U_{ij} = \boldsymbol{\beta}'\mathbf{x}_{ij} + (1/\sigma_i)\varepsilon_{ij}$

Generalized model with $\gamma = 1$ and $\boldsymbol{\Gamma} = [\mathbf{0}]$.

Produces a scaled multinomial logit model with $\boldsymbol{\beta}_i = \sigma_i\boldsymbol{\beta}$

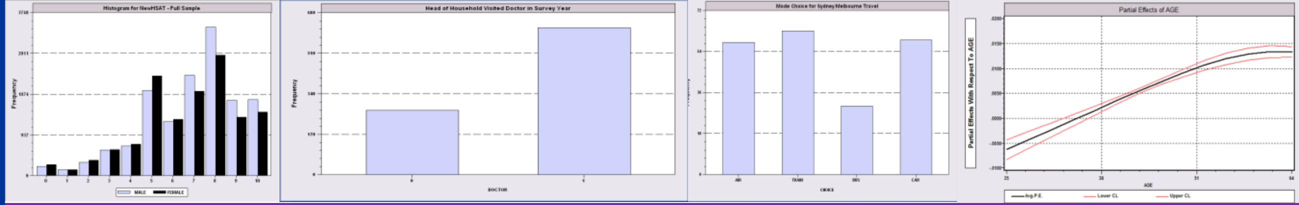
$$\text{Prob}(\text{choice}_i = \text{j}) = \frac{\exp(\boldsymbol{\beta}_i'\mathbf{x}_{ij})}{\sum_{j=1}^{J_i}\exp(\boldsymbol{\beta}_i'\mathbf{x}_{ij})}, i = 1,\ldots,N, j = 1,\ldots,J_i$$

The random variation in the scaling is

$$\sigma_i = \exp(-\tau^2/2 + \tau w_i)$$

The variation across individuals may also be observed, so that

$$\sigma_i = \exp(-\tau^2/2 + \tau w_i + \boldsymbol{\delta}'\mathbf{z}_i)$$



Generalized Mixed Logit Model

$$U(i,j) = \boldsymbol{\beta}_i'\mathbf{x}_{i,j} + \text{Common effects} + \varepsilon_{i,j}$$

Random Parameters

$$\boldsymbol{\beta}_i = \sigma_i[\boldsymbol{\beta} + \boldsymbol{\Delta}\mathbf{h}_i] + [\gamma + \sigma_i(1-\gamma)]\boldsymbol{\Gamma}_i\mathbf{v}_i$$

$$\boldsymbol{\Gamma}_i = \boldsymbol{\Lambda}\boldsymbol{\Sigma}_i$$

$\boldsymbol{\Lambda}$ is a lower triangular matrix

with 1s on the diagonal (Cholesky matrix)

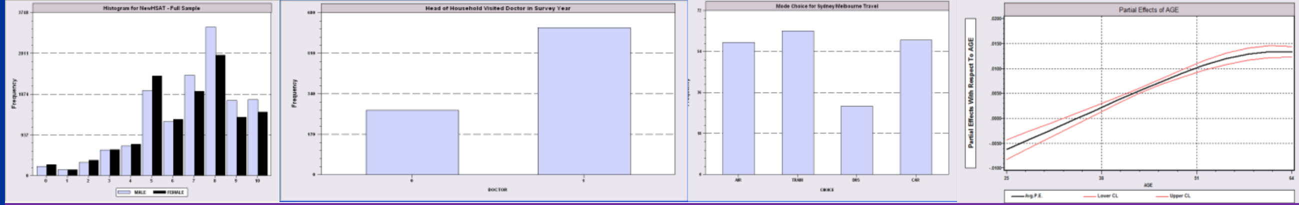
$\boldsymbol{\Sigma}_i$ is a diagonal matrix with $\varphi_k \exp(\boldsymbol{\psi}_k'\mathbf{h}_i)$

Overall preference scaling

$$\sigma_i = \exp(-\tau_i^2/2 + \tau_i w_i + \boldsymbol{\theta}'\mathbf{h}_i]$$

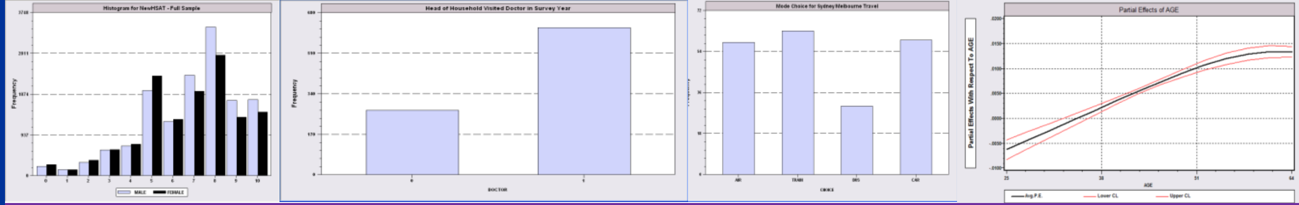
$$\tau_i = \exp(\boldsymbol{\lambda}'\mathbf{r}_i)$$

$$0 < \gamma < 1$$



A helpful way to view hybrid choice models

- Adding attitude variables to the choice model
- In some formulations, it makes them look like mixed parameter models
- “Interactions” is a less useful way to interpret



Observable Heterogeneity in Utility Levels

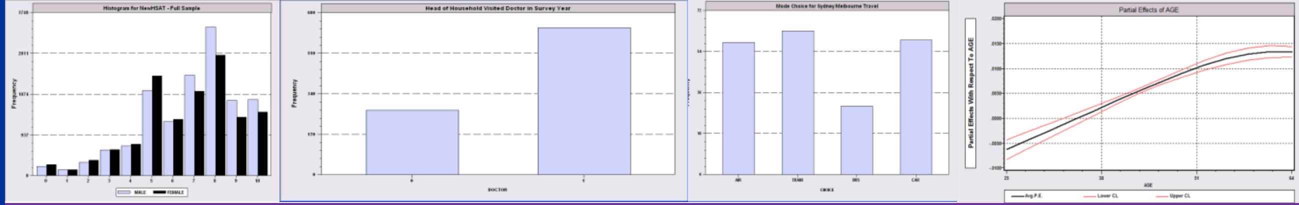
$$U_{ij} = \alpha_j + \boldsymbol{\beta}_i'\mathbf{x}_{ij} + \boldsymbol{\gamma}_j'\mathbf{z}_i + \varepsilon_{ij}$$

$$\text{Prob[choice } j \,|\, i] = \frac{\exp(\alpha_j + \boldsymbol{\beta}'\mathbf{x}_{ij} + \boldsymbol{\gamma}_j'\mathbf{z}_i)}{\sum_{j=1}^{J(i)} \exp(\alpha_j + \boldsymbol{\beta}'\mathbf{x}_{ij} + \boldsymbol{\gamma}'\mathbf{z}_i)}$$

Choice, e.g., among brands of cars

x_{itj} = attributes: price, features

z_{it} = observable characteristics: age, sex, income



Unbservable heterogeneity in utility levels and other preference indicators

$$\mathbf{z}_i = \mathbf{b}'\mathbf{w}_i + \varphi_i$$

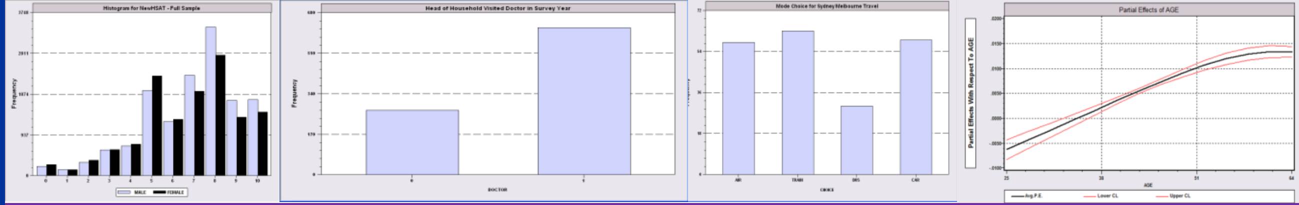
Multinomial Choice Model

$$U_{ij} = \alpha_j + \boldsymbol{\beta}_i'\mathbf{x}_{ij} + \boldsymbol{\gamma}_j'\mathbf{z}_i + \varepsilon_{ij}$$

$$\text{Prob}[\text{choice } j \mid i] = \frac{\exp(\alpha_j + \boldsymbol{\beta}'\mathbf{x}_{ij} + \boldsymbol{\gamma}_j'\mathbf{z}_i)}{\sum_{j=1}^{J_t(i)} \exp(\alpha_j + \boldsymbol{\beta}'\mathbf{x}_{ij} + \boldsymbol{\gamma}'\mathbf{z}_i)}$$

Indicators (Measurement) Model(s)

$$\text{Outcomes } \mathbf{y}_{im} = f_m(\mathbf{z}_i, v_{im})$$



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Measuring Consumer Preferences Using Hybrid Discrete Choice Models

David Palma, Juan de Dios Ortúzar, Gerard Casaubon, Luis I. Rizzi, and Eduardo Agosin

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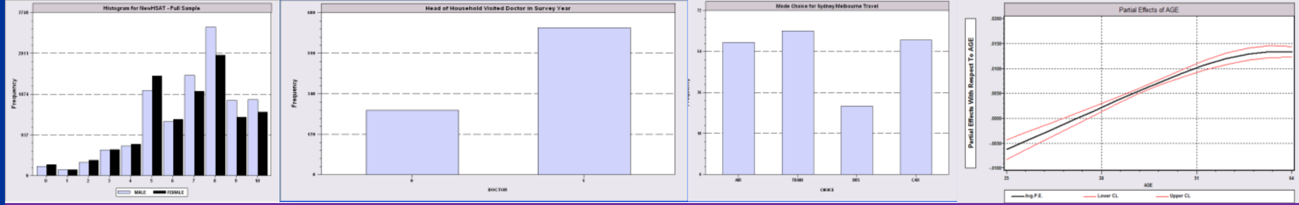


Discrete Choice Modeling
Hybrid Choice Models

Table 3 - Attribute importance ranking according to consumers

Ranking	Attribute	Average standard score
1	Taste	6.74
2	Grape variety	6.51
3	Aroma	6.36
4	Previous experience	6.31
5	Type of wine (reserve, great reserve, etc.)	6.19
6	Colour	5.7
7	Harvest year	5.63
8	Meal matching	5.52
9	Winery	5.51
10	Friend's advice	5.51
11	Place of origin	5.48
12	Price	5.36
13	Critic's advice	4.31
14	Prizes and medals	4.81

Figure 1 - Screen-shot from the web-based choice experiment

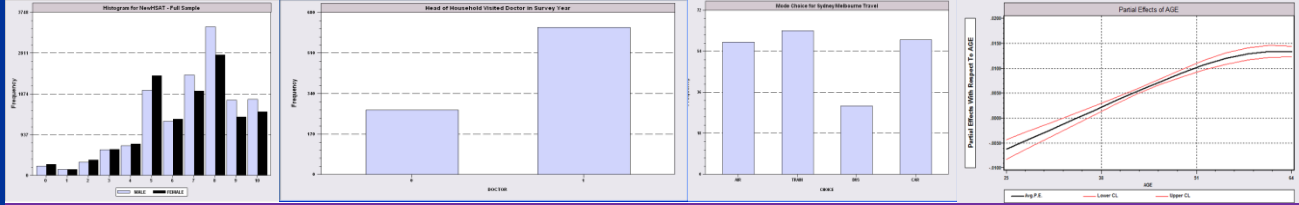
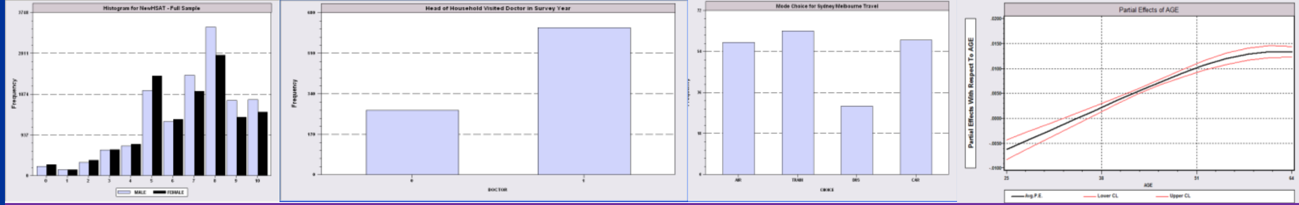


Table 4 - Levels considered for the six attributes selected

Level	Label Design	Grape Variety	Alcohol content	Advice	Price	Discount
1	Delicate	Cabernet Sauvignon	8.5° G.L.	None	100%	0%
2	Contrast	Merlot	11.0° G.L.	Salesman	120%	10%
3	Natural	Carménère	12.5° G.L.	Friend	130%	20%
4	-	Syrah	14.5° G.L.	Critic	160%	-



Observed x → Latent z* → Observed y

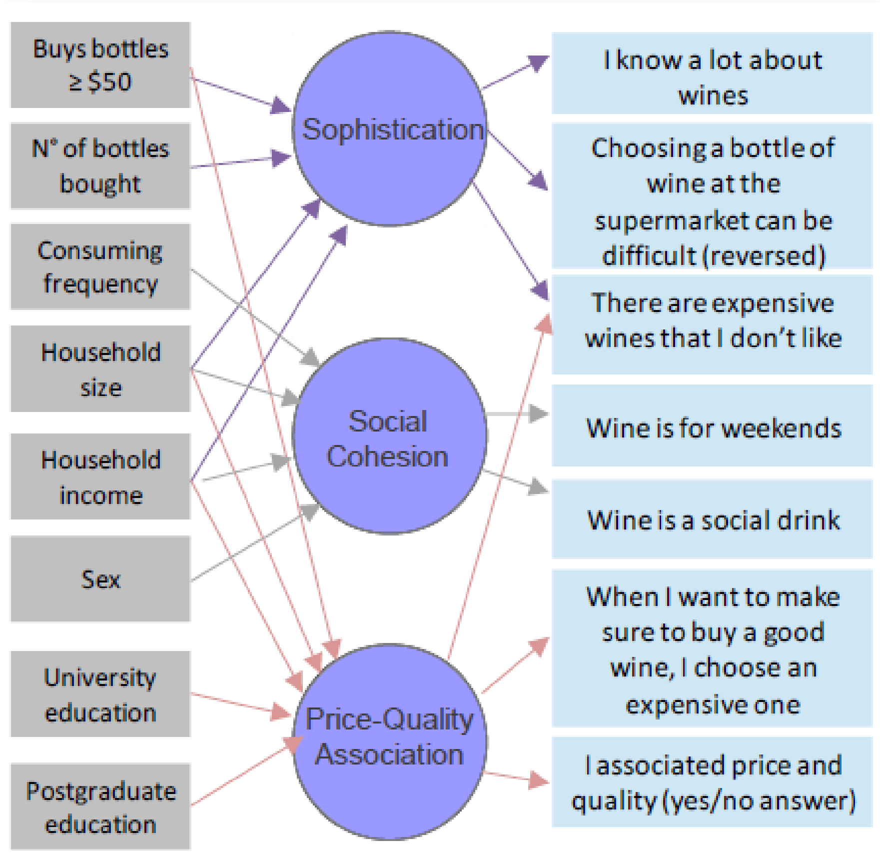
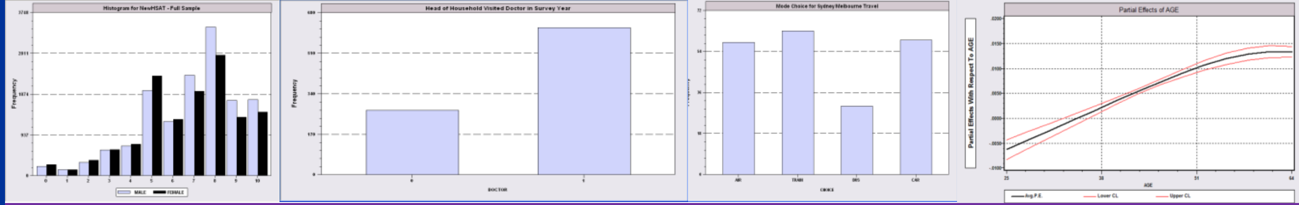
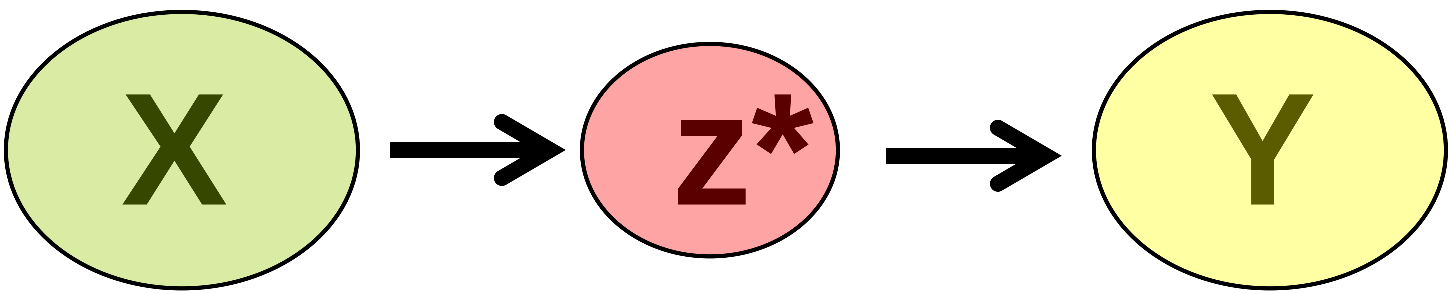


Figure 2 - Structure of the MIMIC model

$$z_1^* = \boldsymbol{\pi}_1'\mathbf{h}_1 + u_1$$
$$z_2^* = \boldsymbol{\pi}_2'\mathbf{h}_2 + u_2$$
$$z_3^* = \boldsymbol{\pi}_3'\mathbf{h}_3 + u_3$$
$$y_1 = g_1(z_1^*, \varepsilon_1)$$
$$y_2 = g_2(z_1^*, \varepsilon_1)$$
$$y_3 = g_3(z_1^*, z_2^*, \varepsilon_1)$$
$$y_4 = g_4(z_2^*, \varepsilon_2)$$
$$y_5 = g_5(z_2^*, \varepsilon_2)$$
$$y_6 = g_6(z_3^*, \varepsilon_3)$$
$$y_7 = g_7(z_3^*, \varepsilon_3)$$

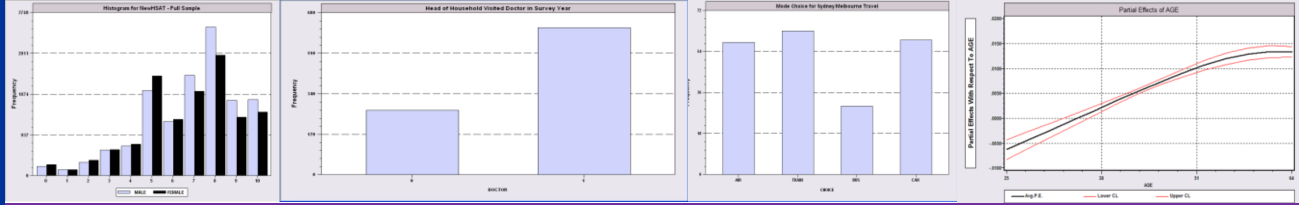


MIMIC Model
Multiple Causes and Multiple Indicators



$$\boldsymbol{\beta}'\mathbf{x} + w \rightarrow z^*$$

$$\begin{pmatrix} y_1 \\ y_2 \\ \cdots \\ y_M \end{pmatrix} = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \cdots \\ \gamma_M \end{pmatrix} z^* + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdots \\ \varepsilon_M \end{pmatrix}$$



4.2 HDC model

In the HDC model (Walker & Ben-Akiva 2002; Bolduc & Alvarez-Daziano 2010; Raveau *et al.* 2010) the wine attributes were interacted with three latent variables associated with the individuals: sophistication, sociability and price-quality association. The model was estimated sequentially using Python Biogeme (Bierlaire, 2003).

The utility function of any alternative can be written as follows:

$$U_{ij} = \sum_k \beta_k x_{ik} + \sum_l \sum_k \gamma_l x_k \eta_{jl} + \varepsilon_{ij}$$

should be γ_{kl} x_{ik}

Where:

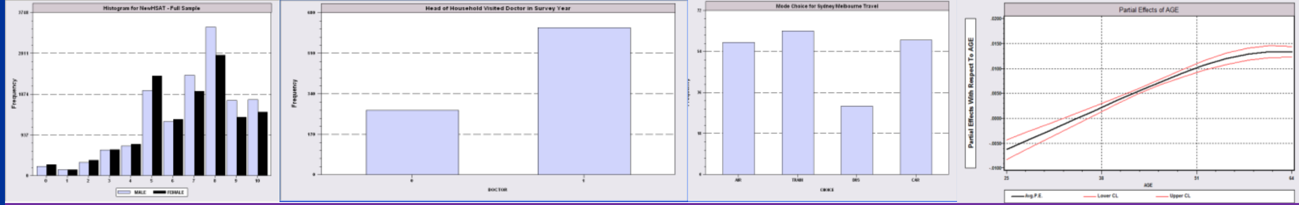
x_{ik} is attribute k of alternative i.

η_{jl} is the latent variable l of individual j

ε_{ij} is a logit error (Gumbel distributed)

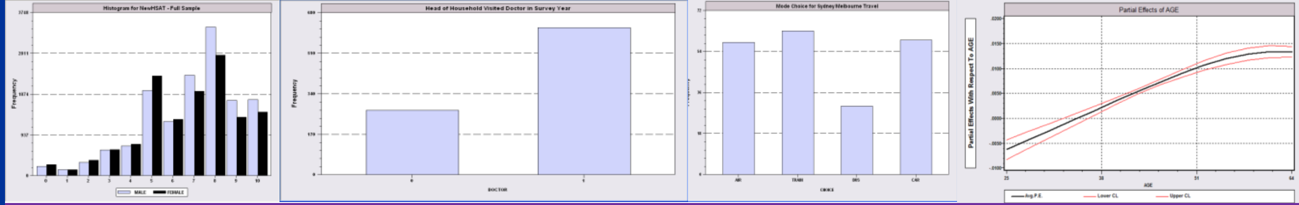
β_k and γ_l are parameters to be estimated.

Note. Alternative i, Individual j.



$$
\begin{aligned}
U_{ij} &= \sum_k \beta_k x_{ik} + \sum_l \sum_k \gamma_{kl} x_{ik} \eta_{jl} + \varepsilon_{ij} \\
&= \sum_k \beta_k x_{ik} + \sum_k \sum_l \gamma_{kl} \eta_{jl} x_{ik} + \varepsilon_{ij} \\
&= \sum_k \beta_k x_{ik} + \sum_k \left(\sum_l \gamma_{kl} \eta_{jl} \right) x_{ik} + \varepsilon_{ij} \\
&= \sum_k \beta_k x_{ik} + \sum_k \left(\Gamma^*_{kj} \right) x_{ik} + \varepsilon_{ij} \\
&= \sum_k \left(\beta_k + \Gamma^*_{kj} \right) x_{ik} + \varepsilon_{ij}
\end{aligned}
$$

This is a mixed logit model. The interesting extension is the source of the individual heterogeneity in the random parameters.



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Incorporating Latent Variables into Discrete Choice Models – A Simultaneous Estimation Approach Using SEM Software

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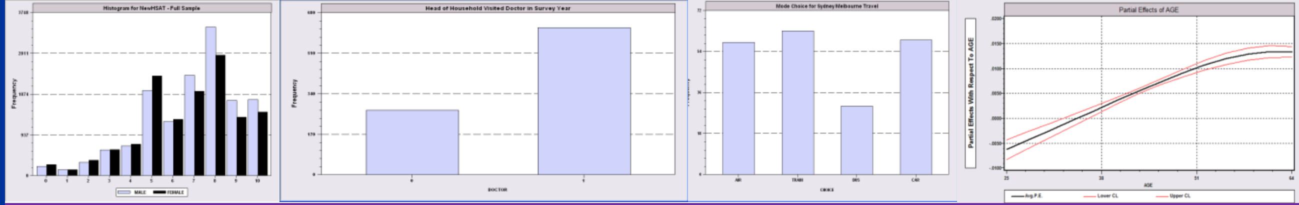
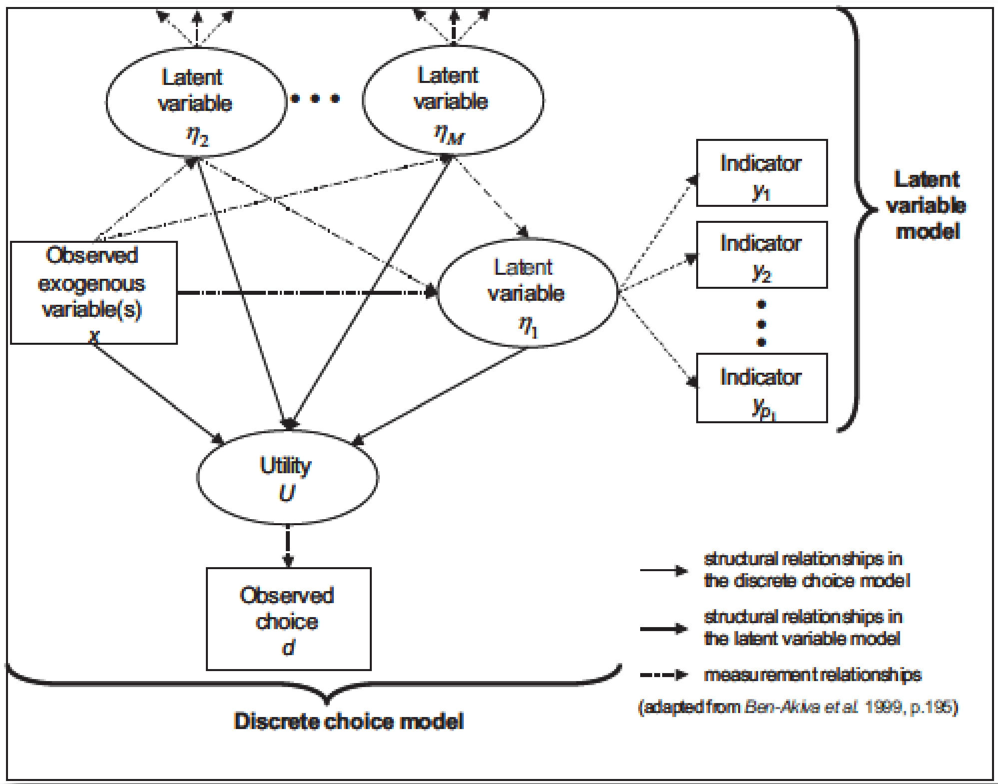
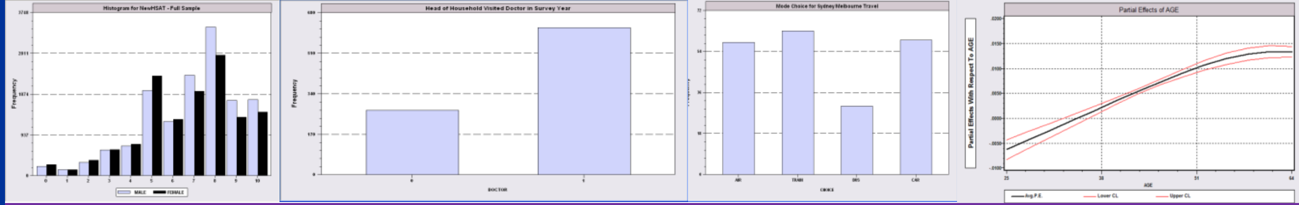


“Integrated Model”

Incorporate attitude measures in preference structure

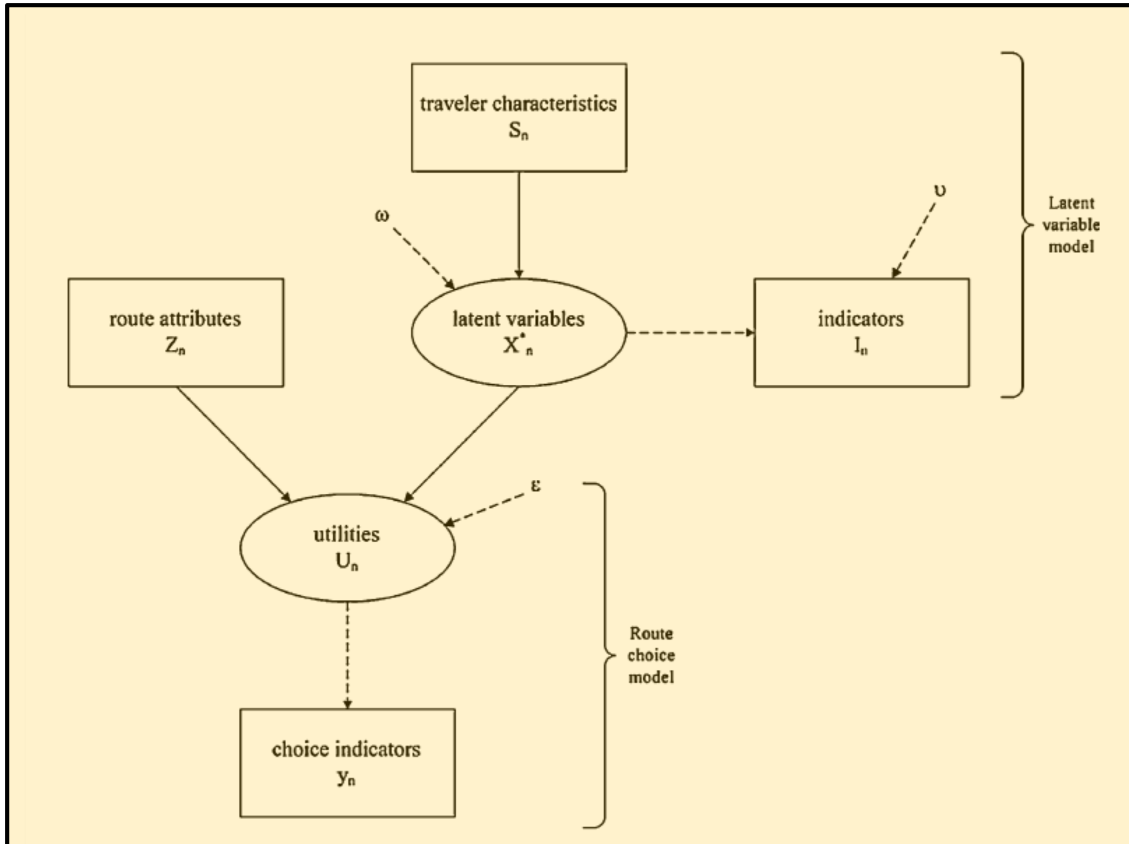


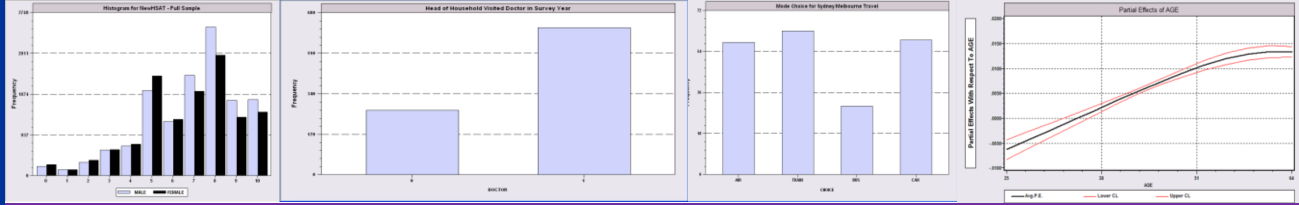


Transportation
DOI 10.1007/s11116-011-9344-y

Latent variables and route choice behavior

Carlo Giacomo Prato · Shlomo Bekhor · Cristina Pronello





On estimation of Hybrid Choice Models

by

Denis Bolduc[1], Ricardo Alvarez-Daziano

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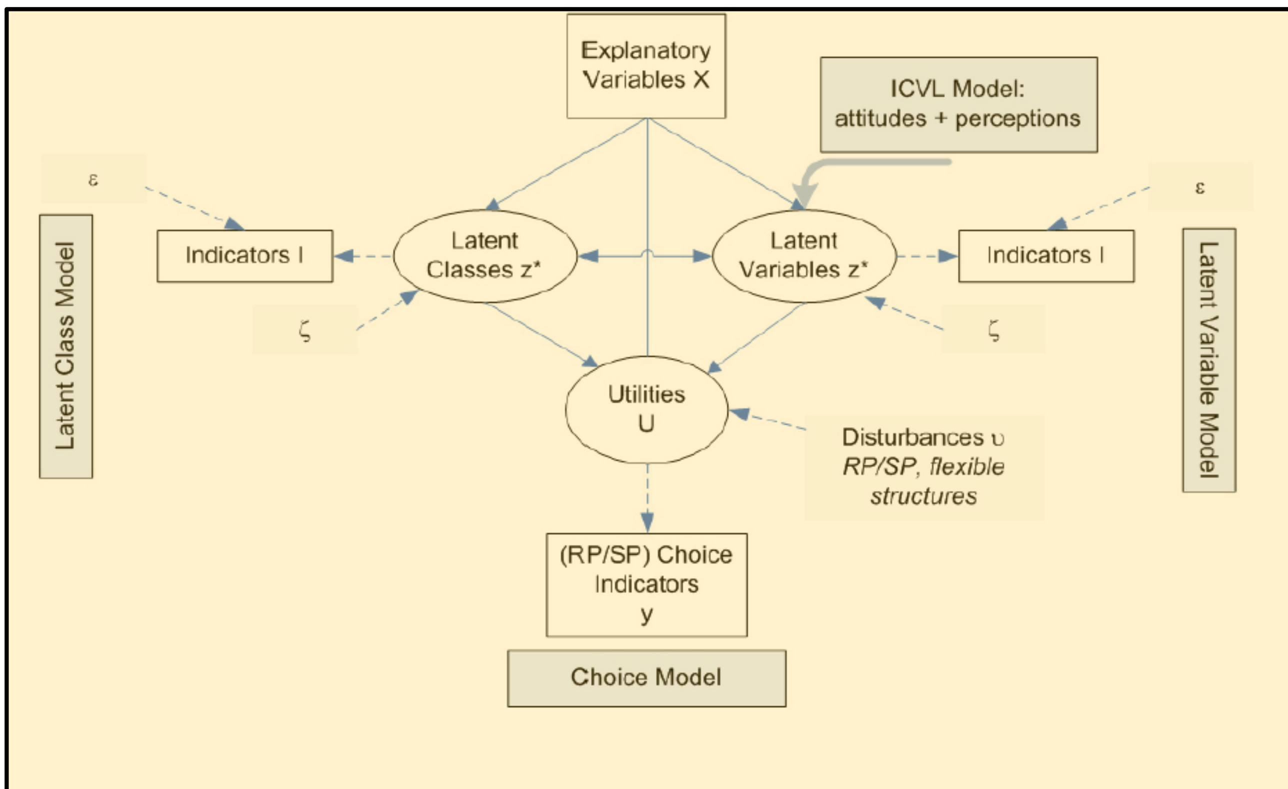
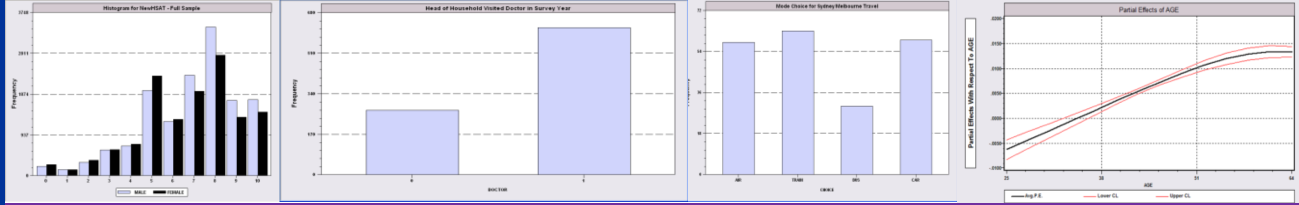


FIGURE 1 Hybrid Choice Model



$$U_{in} = X_{in}\beta + \upsilon_{in} \qquad (1)$$

$$y_{in} = \begin{cases} 1 & \text{if } U_{in} \geq U_{jn}, \forall j \in C_n, j \neq i \\ 0 & \text{otherwise,} \end{cases} \qquad (2)$$

Structural equations

$$z_n^* = \Pi z_n^* + Bw_n + \zeta_n = (I_L - \Pi)^{-1} Bw_n + (I_L - \Pi)^{-1}\zeta_n, \quad \zeta_n \sim N(0, \Psi) \qquad (3)$$

$$U_n = X_n\beta + Cz_n^* + \upsilon_n \qquad (4)$$

Measurement equations

$$I_n = \alpha + \Lambda z_n^* + \varepsilon_n, \quad \varepsilon_n \sim N(0, \Theta) \qquad (5)$$

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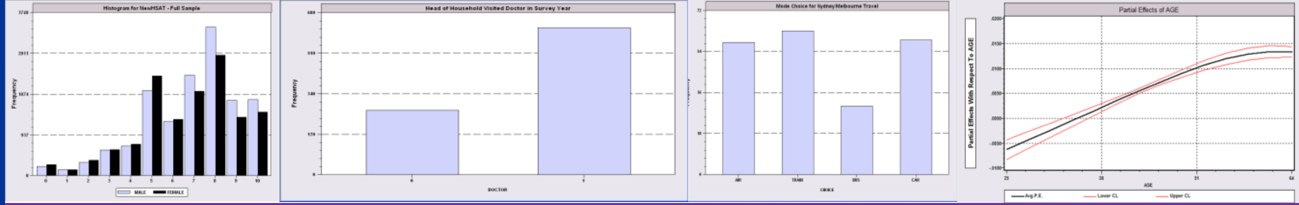
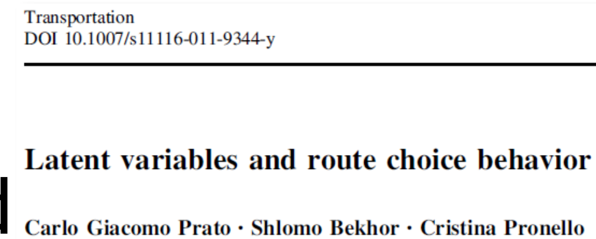


- Hybrid choice
- Equations of the MIMIC Mod



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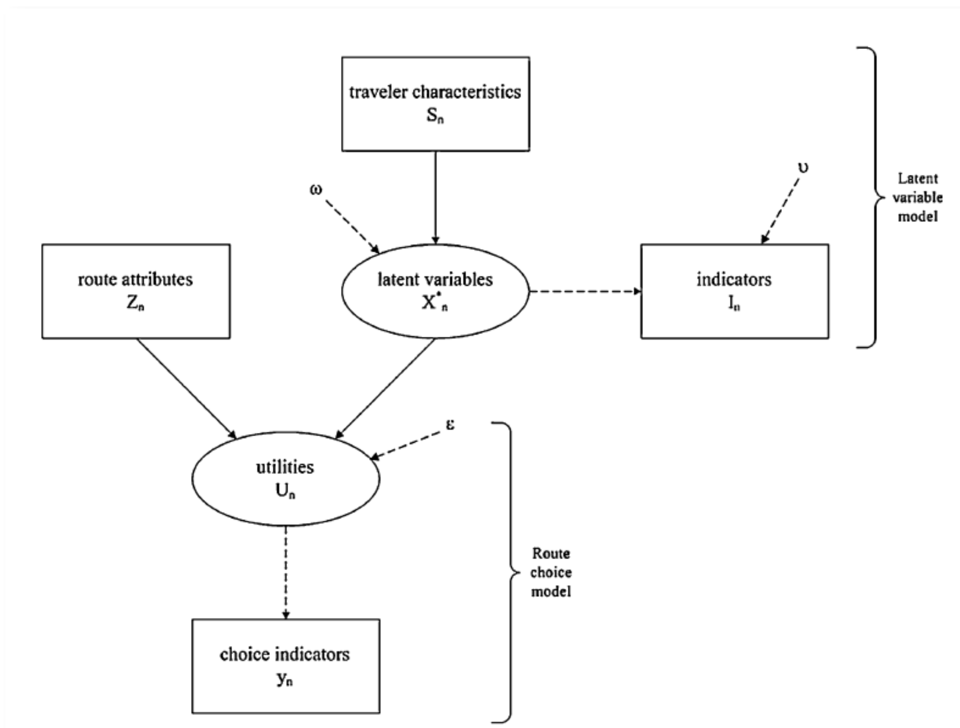
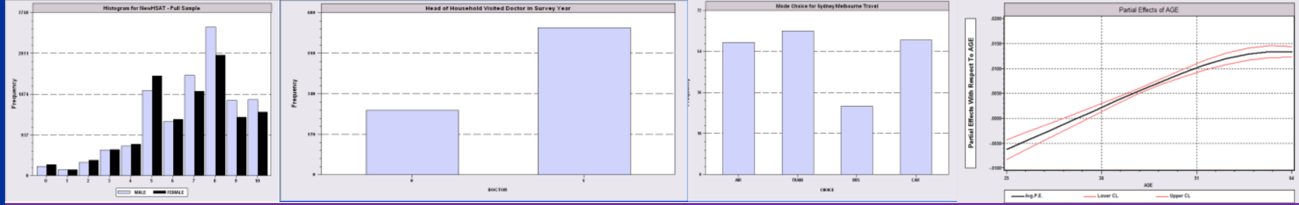
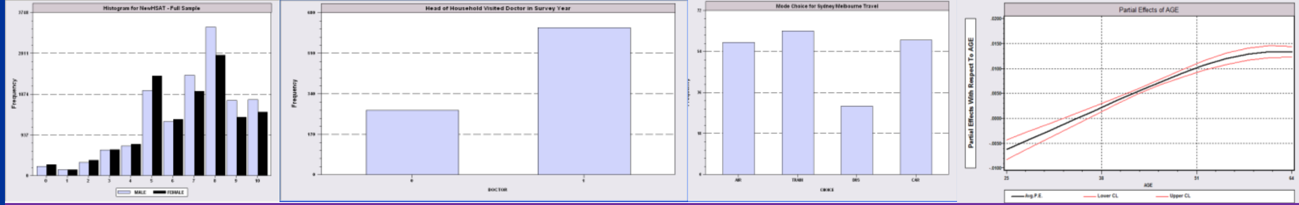




Identification Problems

- Identification of latent variable models with cross sections
- How to distinguish between different latent variable models. How many latent variables are there? More than 0. Less than or equal to the number of indicators.
- Parametric point identification



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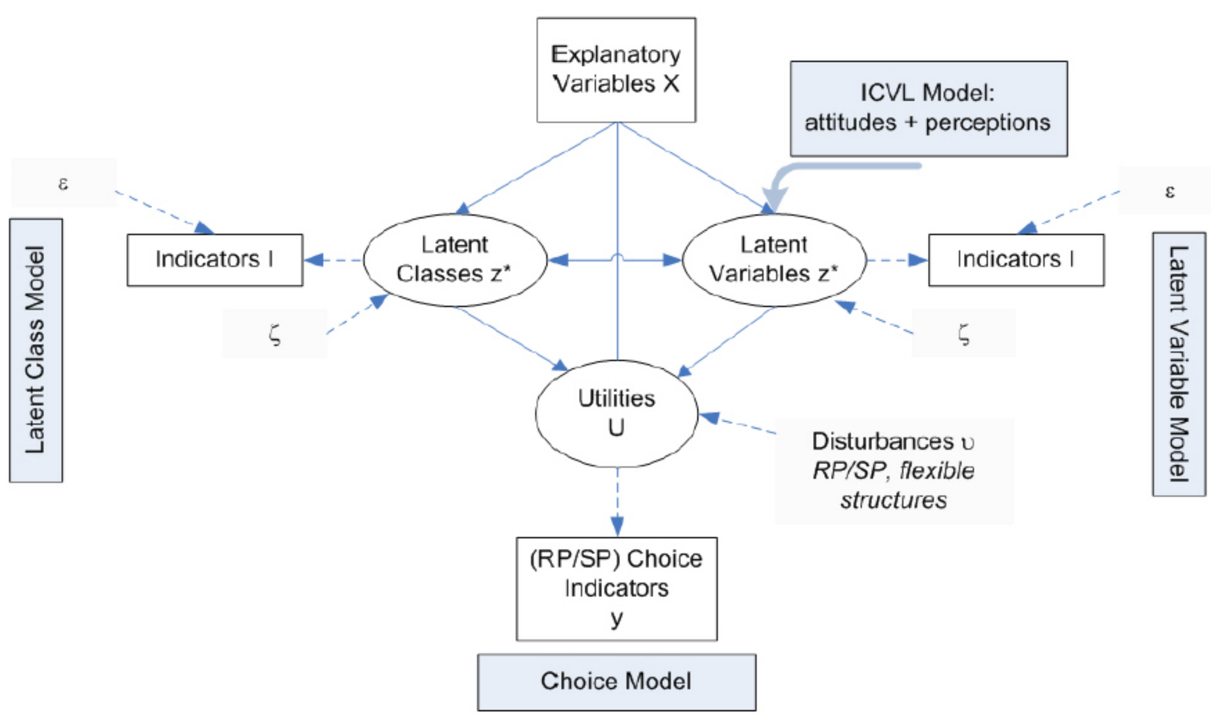
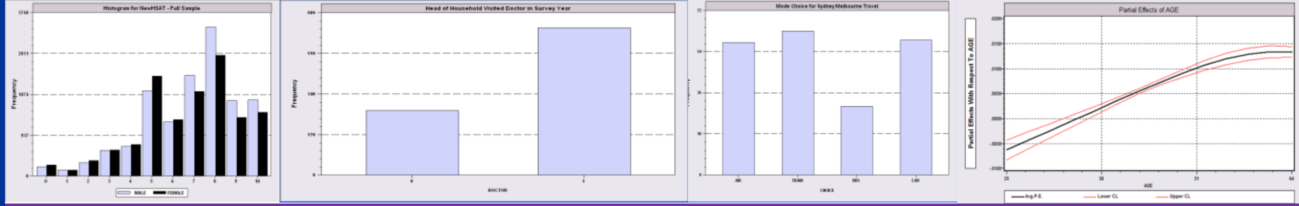


FIGURE 1 Hybrid Choice Model



$$U_{in} = X_{in}\beta + \upsilon_{in} \tag{1}$$

$$y_{in} = \begin{cases} 1 & \text{if } U_{in} \geq U_{jn}, \forall j \in C_n, j \neq i \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

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$$U_n = X_n\beta + C z_n^* + \upsilon_n \tag{4}$$

Measurement equations

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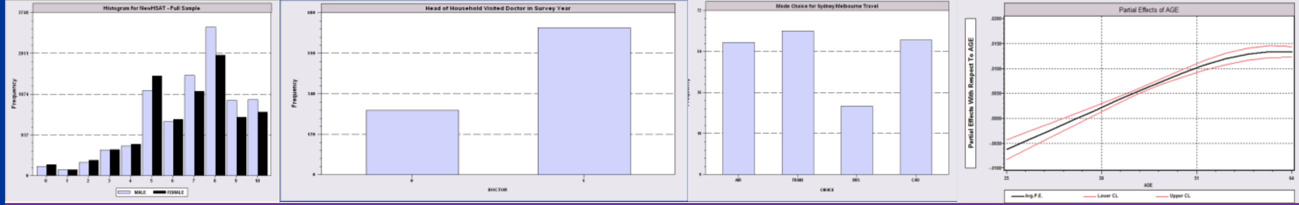

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Incorporating Latent Variables into Discrete Choice Models – A Simultaneous Estimation Approach Using SEM Software

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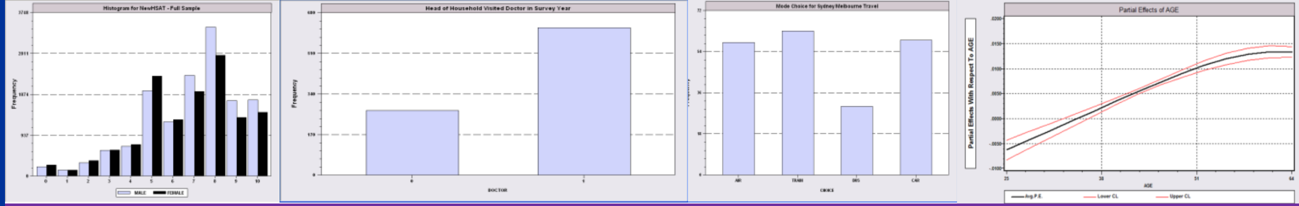

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