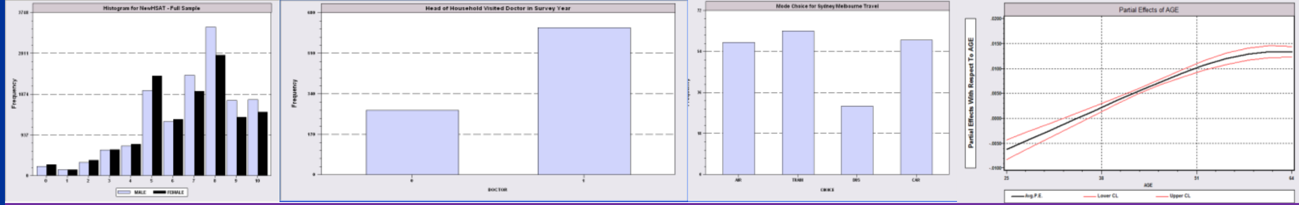

Discrete Choice Modeling

William Greene
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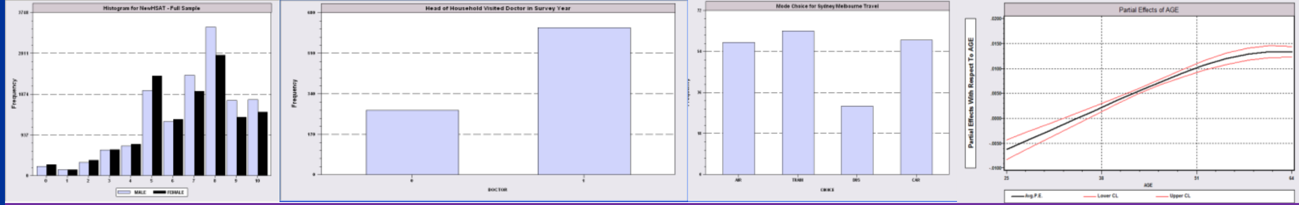
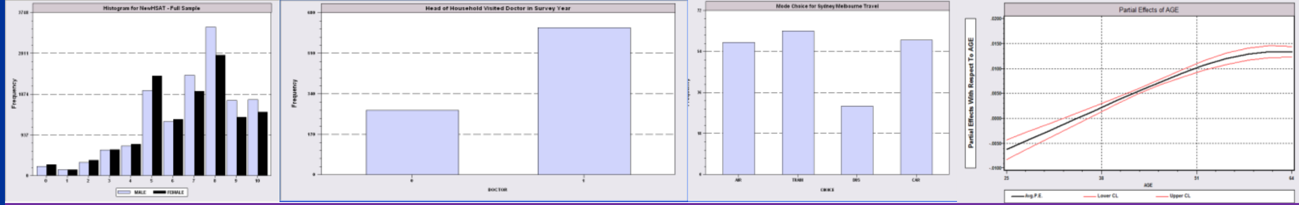


Aggregate Data and Multinomial Choice: The Model of Berry, Levinsohn and Pakes

ECONOMETRICA
JOURNAL OF THE ECONOMETRIC SOCIETY

Automobile Prices in Market Equilibrium
Author(s): Steven Berry, James Levinsohn and Ariel Pakes
Source: *Econometrica*, Vol. 63, No. 4 (Jul., 1995), pp. 841-890
Published by: The Econometric Society
Stable URL: http://www.jstor.org/stable/2171802
Accessed: 08/12/2014 22:40



Resources

- Automobile Prices in Market Equilibrium, S. Berry, J. Levinsohn, A. Pakes, *Econometrica, 63, 4, 1995, 841-890. (BLP)*

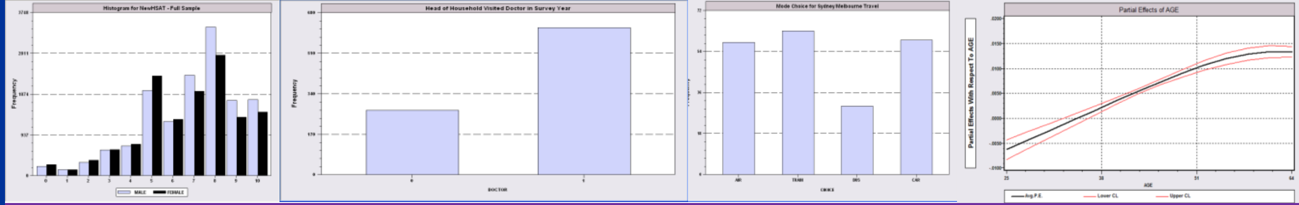
  http://people.stern.nyu.edu/wgreene/Econometrics/BLP.pdf

- A Practitioner’s Guide to Estimation of Random-Coefficients Logit Models of Demand, A. Nevo, *Journal of Economics and Management Strategy, 9, 4, 2000, 513-548*

  http://people.stern.nyu.edu/wgreene/Econometrics/Nevo-BLP.pdf

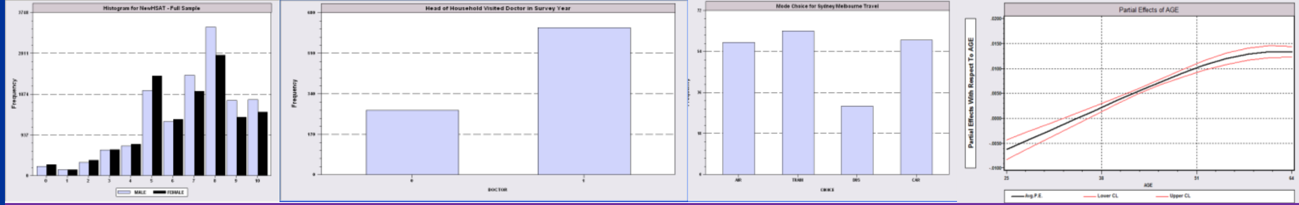
- A New Computational Algorithm for Random Coefficients Model with Aggregate-level Data, Jinyoung Lee, UCLA Economics, Dissertation, 2011

  http://people.stern.nyu.edu/wgreene/Econometrics/Lee-BLP.pdf



Theoretical Foundation

- Consumer market for J differentiated brands of a good
  - j =1,…, J_t brands or types
  - i = 1,…, N consumers
  - t = i,…,T "markets" (like panel data)
- Consumer i's utility for brand j (in market t) depends on
  - p = price
  - **x** = observable attributes
  - f = unobserved attributes
  - w = unobserved heterogeneity across consumers
  - ε = idiosyncratic aspects of consumer preferences
- Observed data consist of aggregate choices, prices and features of the brands.



BLP Automobile Market

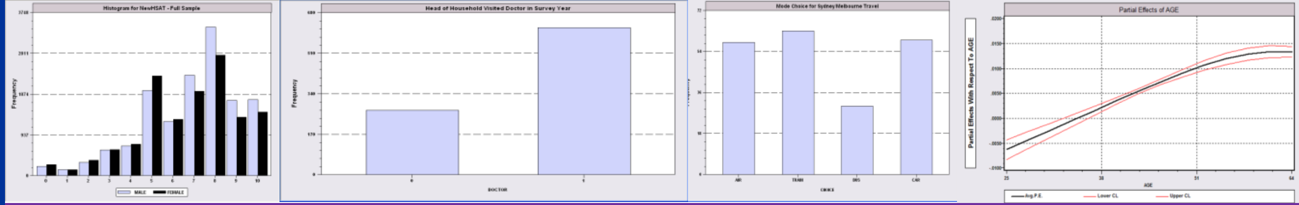
J_t

TABLE 1
DESCRIPTIVE STATISTICS

t

Year	No. of Models	Quantity	Price	Domestic	Japan	European	HP/Wt	Size	Air	MPG	MP$
1971	92	86.892	7.868	0.866	0.057	0.077	0.490	1.496	0.000	1.662	1.850
1972	89	91.763	7.979	0.892	0.042	0.066	0.391	1.510	0.014	1.619	1.875
1973	86	92.785	7.535	0.932	0.040	0.028	0.364	1.529	0.022	1.589	1.819
1974	72	105.119	7.506	0.887	0.050	0.064	0.347	1.510	0.026	1.568	1.453
1975	93	84.775	7.821	0.853	0.083	0.064	0.337	1.479	0.054	1.584	1.503
1976	99	93.382	7.787	0.876	0.081	0.043	0.338	1.508	0.059	1.759	1.696
1977	95	97.727	7.651	0.837	0.112	0.051	0.340	1.467	0.032	1.947	1.835
1978	95	99.444	7.645	0.855	0.107	0.039	0.346	1.405	0.034	1.982	1.929
1979	102	82.742	7.599	0.803	0.158	0.038	0.348	1.343	0.047	2.061	1.657
1980	103	71.567	7.718	0.773	0.191	0.036	0.350	1.296	0.078	2.215	1.466
1981	116	62.030	8.349	0.741	0.213	0.046	0.349	1.286	0.094	2.363	1.559
1982	110	61.893	8.831	0.714	0.235	0.051	0.347	1.277	0.134	2.440	1.817
1983	115	67.878	8.821	0.734	0.215	0.051	0.351	1.276	0.126	2.601	2.087
1984	113	85.933	8.870	0.783	0.179	0.038	0.361	1.293	0.129	2.469	2.117
1985	136	78.143	8.938	0.761	0.191	0.048	0.372	1.265	0.140	2.261	2.024
1986	130	83.756	9.382	0.733	0.216	0.050	0.379	1.249	0.176	2.416	2.856
1987	143	67.667	9.965	0.702	0.245	0.052	0.395	1.246	0.229	2.327	2.789
1988	150	67.078	10.069	0.717	0.237	0.045	0.396	1.251	0.237	2.334	2.919
1989	147	62.914	10.321	0.690	0.261	0.049	0.406	1.259	0.289	2.310	2.806
1990	131	66.377	10.337	0.682	0.276	0.043	0.419	1.270	0.308	2.270	2.852
All	2217	78.804	8.604	0.790	0.161	0.049	0.372	1.357	0.116	2.099	2.086

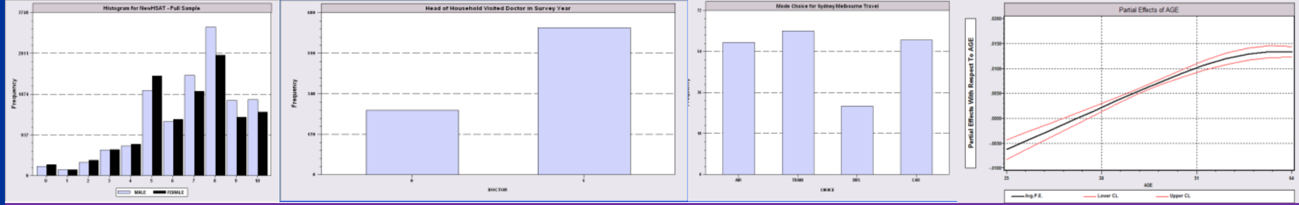
Note: The entry in each cell of the last nine columns is the sales weighted mean.



Random Utility Model

- Utility: $U_{ijt}=U(w_i,p_{jt},\mathbf{x}_{jt},f_{jt}|\theta)$, i = 1,…,(large)N, j=1,…,J
  - w_i = individual heterogeneity; time (market) invariant. w has a continuous distribution across the population.
  - p_{jt}, $\mathbf{x}_{jt}$, f_{jt}, = price, observed attributes, unobserved features of brand j; all may vary through time (across markets)
- Revealed Preference: Choice j provides maximum utility
- Across the population, given market t, set of prices $\mathbf{p}_t$ and features $(\mathbf{X}_t,\mathbf{f}_t)$, there is a set of values of w_i that induces choice j, for each j=1,…,J_t; then, $s_j(\mathbf{p}_t,\mathbf{X}_t,\mathbf{f}_t|\theta)$ is the market share of brand j in market t.
- There is an outside good that attracts a nonnegligible market share, j=0. Therefore,

$$\sum_{j=1}^{J_t} s_j(\mathbf{p}_t,\mathbf{X}_t,\mathbf{f}_t|\boldsymbol{\theta}) < 1$$

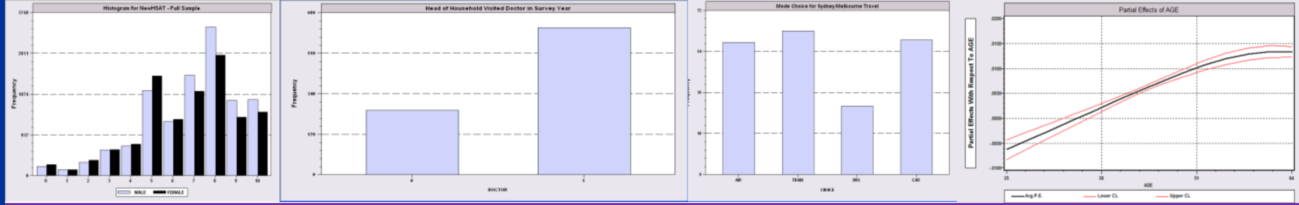


Functional Form

- (Assume one market for now so drop "'t.")
  $U_{ij}=U(w_i,p_j,\mathbf{x}_j,f_j|\theta)= \mathbf{x}_j'\boldsymbol{\beta} - \alpha p_j + f_j + \varepsilon_{ij}$
  $= \delta_j + \varepsilon_{ij}$
- $E_{\text{consumers } i}[\varepsilon_{ij}] = 0$, δ_j is E[Utility].

$$\text{Market Share}_j = E_\varepsilon\left[\prod_{q\neq j} \text{Prob}(\delta_j - \delta_q + \varepsilon)\right]$$

- Will assume logit form to make integration unnecessary. The expectation has a closed form.

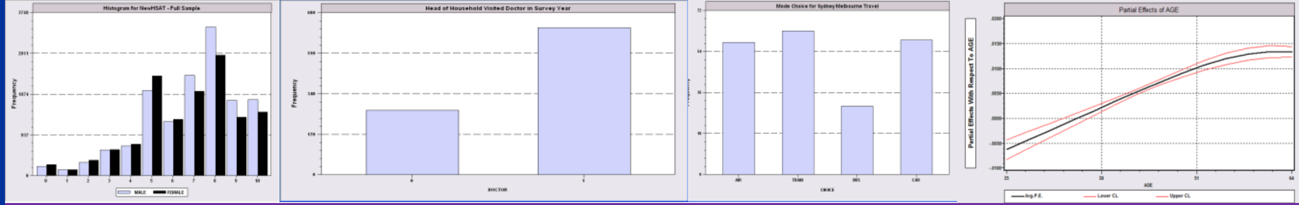


Heterogeneity

- Assumptions so far imply IIA. Cross price elasticities depend only on market shares.
- Individual heterogeneity: Random parameters
- $U_{ij}=U(w_i,p_j,\mathbf{x}_j,f_j|\theta_i)= \mathbf{x}_j'\boldsymbol{\beta}_i - \alpha p_j + f_j + \varepsilon_{ij}$

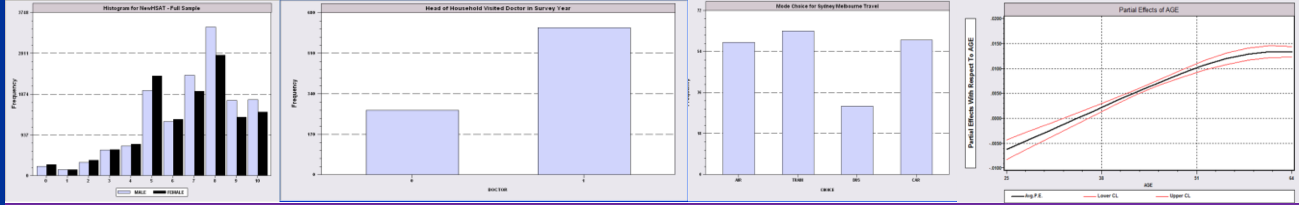
$$\beta_{ik} = \beta_k + \sigma_k v_{ik}.$$

- The mixed model only imposes IIA for a particular consumer, but not for the market as a whole.



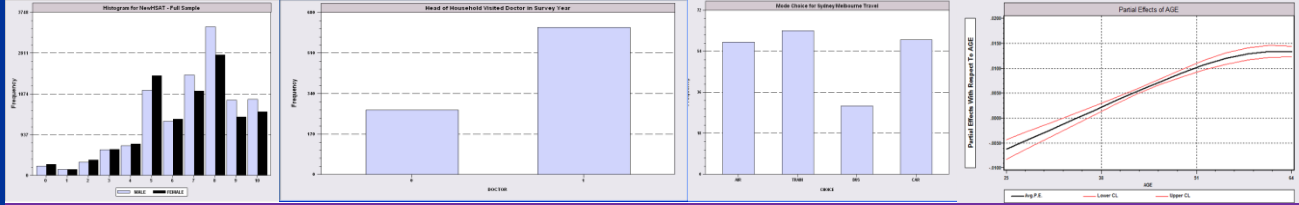
Endogenous Prices: Demand side

- $U_{ij}=U(w_i,p_j,\mathbf{x}_j,f_j|\theta)= \mathbf{x}_j'\boldsymbol{\beta}_i - \alpha p_j + f_j + \varepsilon_{ij}$
- f_j is unobserved
- Utility responds to the unobserved f_j
- Price p_j is partly determined by features f_j.
- In a choice model based on observables, price is correlated with the unobservables that determine the observed choices.



Endogenous Price: Supply Side

- There are a small number of competitors in this market
- Price is determined by firms that maximize profits given the features of its products and its competitors.
- mc_j = g(observed cost characteristics **c**,
  unobserved cost characteristics **h**)
- At equilibrium, for a profit maximizing firm that produces one product,
  $s_j + (p_j - mc_j)\partial s_j / \partial p_j = 0$
- Market share depends on unobserved cost characteristics as well as unobserved demand characteristics, and price is correlated with both.



Instrumental Variables
(ξ and ω are our h and f.)

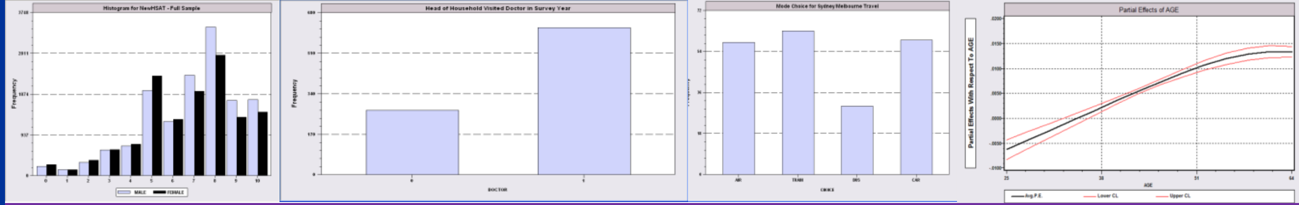
4. INSTRUMENTS

We need to specify instruments for both the demand and pricing equations. Any factors that are correlated with specific functions of the observed data, but are not correlated with the demand or supply disturbances, ξ and ω, will be appropriate instruments. Our procedure is to specify a list of variables that are mean independent of ξ and ω and use the logic of the estimation procedure to derive appropriate instruments.

Our mean independence assumption is that the supply and demand unobservables are mean independent of both observed product characteristics and cost shifters. Formally, if $z_j = [x_j, w_j]$ and $z = [z_1, \ldots, z_J]$, then

$$E[\xi_j | z] = E[\omega_j | z] = 0. \tag{4.1}$$

Note first that we do not include price or quantity in the conditioning vector, z. This is because our model implies that price and quantity are determined in part by ξ and ω. In contrast, we do not model the determination of product characteristics and cost shifters.



Econometrics: Essential Components

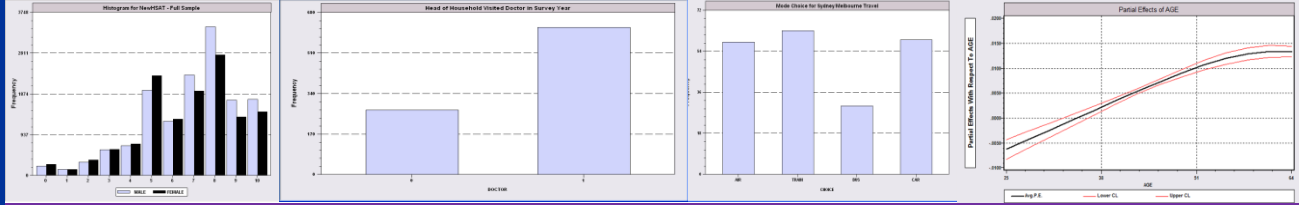
$$U_{ijt} = \mathbf{x}'_{jt}\beta_i + f_{jt} + \varepsilon_{ijt}$$

$$U_{i0t} = \varepsilon_{i0t} \quad \text{(Outside good)}$$

$$\beta_i = \beta + \Gamma v_i, \quad \Gamma = \text{ diagonal}(\sigma_1, \ldots)$$

$$\varepsilon_{ijt} \sim \text{ Type I extreme value, IID across all choices}$$

$$\text{Market shares: } s_j(\mathbf{X}_t, \mathbf{f}_t : \beta_i) = \frac{\exp(\mathbf{x}'_{jt}\beta_i + f_{jt})}{1 + \sum_{m=1}^{J} \exp(\mathbf{x}'_{mt}\beta_i + f_{mt})}, j = 1, \ldots, J_t$$



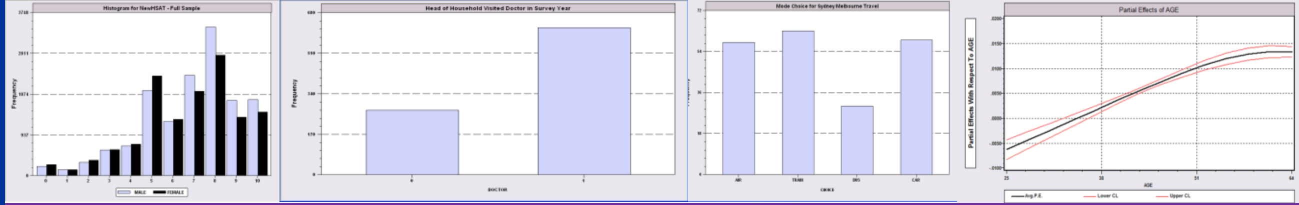
Econometrics

Market Shares: $s_j(\mathbf{X}_t, \mathbf{f}_t : \beta_i) = \dfrac{\exp(\mathbf{x}'_{jt}\beta_i + f_{jt})}{1 + \sum_{m=1}^{J} \exp(\mathbf{x}'_{mt}\beta_i + f_{mt})}, j = 1, \ldots, J_t$

Expected Share: $E[s_j(\mathbf{X}_t, \mathbf{f}_t : \beta, \Sigma)] = \displaystyle\int_{\beta_i} \dfrac{\exp(\mathbf{x}'_{jt}\beta_i + f_{jt})}{1 + \sum_{m=1}^{J} \exp(\mathbf{x}'_{mt}\beta_i + f_{mt})} dF(\beta_i)$

Expected Shares are estimated using simulation:

$$\hat{s}_j(\mathbf{X}_t, \mathbf{f}_t : \beta, \Gamma) = \frac{1}{R}\sum_{r=1}^{R} \frac{\exp[\mathbf{x}'_{jt}(\beta + \Gamma v_{ir}) + f_{jt}]}{1 + \sum_{m=1}^{J} \exp[\mathbf{x}'_{mt}(\beta + \Gamma v_{ir}) + f_{mt}]}$$



GMM Estimation Strategy - 1

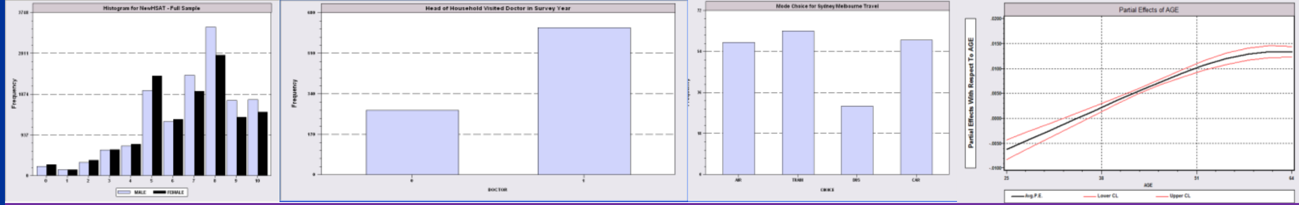
$$\hat{s}_{jt}(\mathbf{X}_t, \mathbf{f}_t : \beta, \Gamma) = \frac{1}{R}\sum_{r=1}^{R} \frac{\exp[\mathbf{x}'_{jt}(\beta + \Gamma v_{ir}) + f_{jt}]}{1 + \sum_{m=1}^{J} \exp[\mathbf{x}'_{mt}(\beta + \Gamma v_{ir}) + f_{mt}]}$$

We have instruments $\mathbf{z}_{jt}$ such that

$$E[f_{jt}(\beta, \Gamma)\mathbf{z}_{jt}] = 0$$

f_{jt} is obtained from an inverse mapping by equating the fitted market shares, $\hat{\mathbf{s}}_t$, to the observed market shares, $\mathbf{S}_t$.

$$\hat{\mathbf{s}}_t(\mathbf{X}_t, \mathbf{f}_t : \beta, \Gamma) = \mathbf{S}_t \;\; \text{so} \;\; \hat{\mathbf{f}}_t = \hat{\mathbf{s}}_t^{-1}(\mathbf{X}_t, \mathbf{S}_t : \beta, \Gamma).$$



GMM Estimation Strategy - 2

We have instruments $\mathbf{z}_{jt}$ such that

$$E[f_{jt}(\beta,\Gamma)\mathbf{z}_{jt}] = 0$$

$$\hat{\mathbf{s}}_t(\mathbf{X}_t, \mathbf{f}_t : \beta, \Gamma) = \mathbf{S}_t \quad \text{so} \quad \hat{\mathbf{f}}_t = \hat{\mathbf{s}}_t^{-1}(\mathbf{X}_t, \mathbf{S}_t : \beta, \Gamma).$$

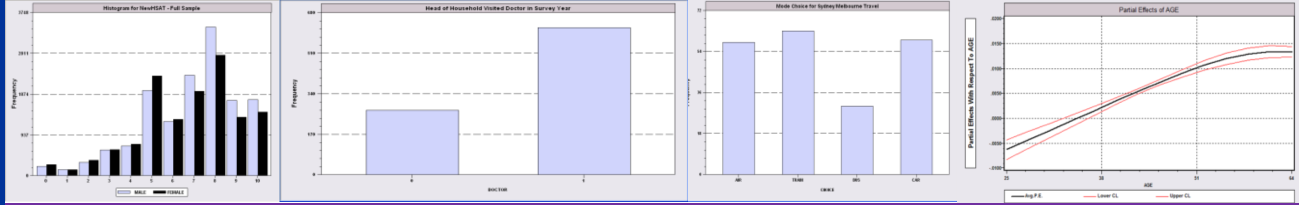
Define $\hat{\bar{\mathbf{g}}}_t = \frac{1}{J_t}\sum_{j=1}^{J_t} \hat{f}_{jt}\mathbf{z}_{jt}$

GMM Criterion would be $\hat{Q}_t(\beta,\Sigma) = \hat{\bar{\mathbf{g}}}_t{}'\mathbf{W}\hat{\bar{\mathbf{g}}}_t$

where $\mathbf{W}$ = the weighting matrix for minimum distance estimation.

For the entire sample, the GMM estimator is built on

$$\hat{\bar{\mathbf{g}}} = \frac{1}{T}\sum_{t=1}^{T} \frac{1}{J_t}\sum_{j=1}^{J_t} \hat{f}_{jt}\mathbf{z}_{jt} \text{ and } Q(\beta,\Sigma) = \hat{\bar{\mathbf{g}}}'\mathbf{W}\hat{\bar{\mathbf{g}}}$$



BLP Iteration

Begin with starting values for $\mathbf{f}_t = \hat{\mathbf{f}}_t^{(0)}$ and starting values for structural parameters β and Σ.

Compute predicted shares $\hat{\mathbf{s}}_t^{(M-1)}(\mathbf{X}_t, \hat{\mathbf{f}}_t^{(M-1)} : \hat{\beta}^{(M-1)}, \hat{\Sigma}^{(M-1)})$.

INNER (Contraction Mapping) Find a fixed point for

$$\hat{\mathbf{f}}_t^{(M)} = \hat{\mathbf{f}}_t^{(M-1)} + \log(\mathbf{S}_t) - \log[\hat{\mathbf{s}}_t^{(M-1)}(\mathbf{X}_t, \hat{\mathbf{f}}_t^{(M-1)} : \hat{\beta}^{(M-1)}, \hat{\Sigma}^{(M-1)})] \rightarrow \hat{\mathbf{f}}_t^{(M)}(\hat{\mathbf{f}}_t^{(M-1)}, \hat{\beta}^{(M-1)}, \hat{\Sigma}^{(M-1)})$$

OUTER (GMM Step) With $\hat{\mathbf{f}}_t^{(M)}$ in hand, use GMM to (re)estimate $\hat{\beta}^{(M)}, \hat{\Sigma}^{(M)}$.

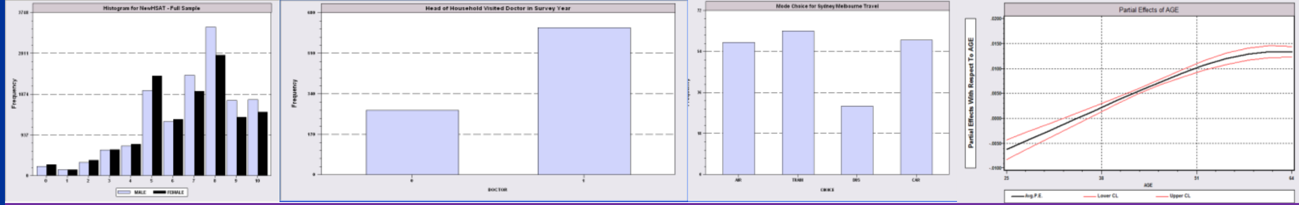
Return to **INNER** step or exit if $\hat{\mathbf{f}}_t^{(M)}$ - $\hat{\mathbf{f}}_t^{(M-1)}$ is sufficiently small.

GMM step is straightforward - concave function (quadratic form) of a concave function (logit probability).

Solving the **INNER** step is time consuming and very complicated.

Recent research has produced several alternative algorithms.

Overall complication: The estimates $\hat{\mathbf{f}}_t^{(M)}$ can diverge.



ABLP Iteration

A New Computational Algorithm for Random Coefficients Model with Aggregate-level Data, Jinyoung Lee, UCLA, Dissertation, 2011
http://people.stern.nyu.edu/wgreene/Econometrics/Lee-BLP.pdf

ξ_t is our $\mathbf{f}_t$.
θ is our (β,Σ)

No superscript is our (M); superscript 0 is our (M-1).

We take a different approach from DFS to avoid the time-consuming contraction mapping. Instead, we use a first-order approximation to the market share function. The advantage of approximation is that we can easily invert out ξ_t, the unobserved product characteristic in market t, using the analytic solution in (11) (which requires matrix inversions).

The first-order approximation of the log market share function, $\ln s(\xi_t;\theta)$, around a point of approximation, ξ_t^0, is[8]

$$\ln s(\xi_t;\theta) \approx \ln s\left(\xi_t^0;\theta\right) + \frac{\partial \ln s\left(\xi_t^0;\theta\right)}{\partial \xi_t'}\left(\xi_t - \xi_t^0\right),$$

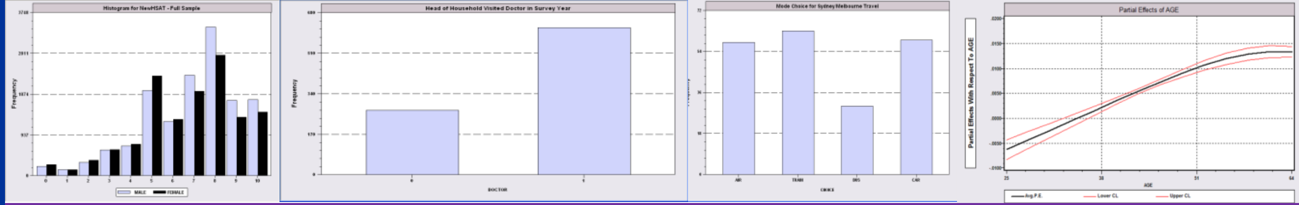
where $\approx$ denotes the first-order Taylor series expansion, $\ln s = (\ln s_1, ..., \ln s_J)'$, $\frac{\partial \ln s}{\partial \xi_t'} = \left(\frac{\partial \ln s}{\partial \xi_{1t}}, ..., \frac{\partial \ln s}{\partial \xi_{Jt}}\right)$, $\xi_t = (\xi_{1t}, ..., \xi_{Jt})'$, and $\xi_t^0 = \left(\xi_{1t}^0, ..., \xi_{Jt}^0\right)'$. Let $\ln s^A(\xi_t;\theta)$ denote the first-order approximation of the log market share function:

$$\ln s^A(\xi_t;\theta) \equiv \ln s\left(\xi_t^0;\theta\right) + \frac{\partial \ln s\left(\xi_t^0;\theta\right)}{\partial \xi_t'}\left(\xi_t - \xi_t^0\right). \tag{10}$$

Instead of using the (exact) market share equations, $\ln S_t = \ln s(\xi_t;\theta)$, we use the approximate market share equations, $\ln S_t = \ln s^A(\xi_t;\theta)$, to establish the following relationship:

$$\xi_t = \Phi_t\left(\theta, \xi_t^0\right) \equiv \xi_t^0 + \left[\frac{\partial \ln s\left(\xi_t^0;\theta\right)}{\partial \xi_t'}\right]^{-1}\left[\ln S_t - \ln s\left(\xi_t^0;\theta\right)\right] \tag{11}$$

for $t = 1, ..., T$. We call the mapping $\Phi_t\left(\theta, \xi_t^0\right)$ the ABLP inversion of unobserved product characteristic, ξ_t, at market t given θ with a point of approximation, ξ_t^0.



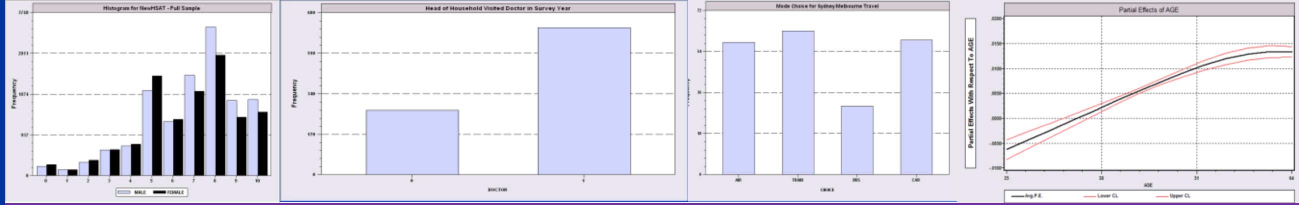
Side Results

$\frac{\partial \ln s(\theta,\xi)}{\partial \xi'} \left(= \frac{\partial \ln s(\theta,\delta)}{\partial \delta'}\right)$, where $\ln s = (\ln s_1, ..., \ln s_J)'$, and $\frac{\partial \ln s}{\partial \xi'} = \left(\frac{\partial \ln s}{\partial \xi_1}, ..., \frac{\partial \ln s}{\partial \xi_J}\right)$.

$$\frac{\partial \ln s\,(\xi;\theta)}{\partial \xi'} = \begin{bmatrix} \frac{\partial s_1}{s_1 \partial \xi_1} & \frac{\partial s_1}{s_1 \partial \xi_2} & \cdots & \frac{\partial s_1}{s_1 \partial \xi_J} \\ \frac{\partial s_2}{s_2 \partial \xi_1} & \frac{\partial s_2}{s_2 \partial \xi_2} & \cdots & \frac{\partial s_2}{s_2 \partial \xi_J} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial s_J}{s_J \partial \xi_1} & \frac{\partial s_J}{s_J \partial \xi_2} & \cdots & \frac{\partial s_J}{s_J \partial \xi_J} \end{bmatrix}_{\theta,\ \xi}$$

$$= \begin{bmatrix} \frac{1}{s_1}\frac{1}{I}\sum_{i=1}^{I}\left(s_{i1} - s_{i1}^2\right) & \frac{1}{s_1}\frac{1}{I}\sum_{i=1}^{I}\left(-s_{i1}s_{i2}\right) & \cdots & \frac{1}{s_1}\frac{1}{I}\sum_{i=1}^{I}\left(-s_{i1}s_{iJ}\right) \\ \frac{1}{s_2}\frac{1}{I}\sum_{i=1}^{I}\left(-s_{i2}s_{i1}\right) & \frac{1}{s_2}\frac{1}{I}\sum_{i=1}^{I}\left(s_{i2} - s_{i2}^2\right) & \cdots & \frac{1}{s_2}\frac{1}{I}\sum_{i=1}^{I}\left(-s_{i2}s_{iJ}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{s_J}\frac{1}{I}\sum_{i=1}^{I}\left(-s_{iJ}s_{i1}\right) & \frac{1}{s_J}\frac{1}{I}\sum_{i=1}^{I}\left(-s_{iJ}s_{i2}\right) & \cdots & \frac{1}{s_J}\frac{1}{I}\sum_{i=1}^{I}\left(s_{iJ} - s_{iJ}^2\right) \end{bmatrix}_{\theta,\ \xi}$$

where $s_{ij} = \frac{\exp\left(X_j\beta_i + \xi_j\right)}{1+\sum_{j'=1}^{J}\exp\left(X_{j'}\beta_i + \xi_{j'}\right)}$; $s_j = \frac{1}{I}\sum_{i=1}^{I} s_{ij}$; $\frac{\partial s_j}{\partial \xi_j} = \frac{1}{I}\sum_{i=1}^{I}\left(s_{ij} - s_{ij}^2\right)$ for $j = 1, ..., J$; and $\frac{\partial s_j}{\partial \xi_k} = \frac{1}{I}\sum_{i=1}^{I}\left(-s_{ij}s_{ik}\right)$ for $j \neq k$ and $k = 1, ..., J$.



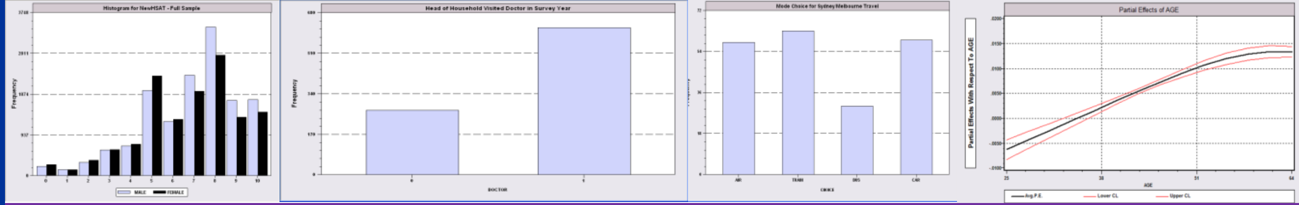
ABLP Iterative Estimator

The ABLP minimization problem starts with an initial point of approximation, ξ^0, for the unobserved product characteristics. Then we can obtain a GMM estimate of θ as

$$\theta^1 = \arg\min_{\theta \in \Theta} \Phi\left(\theta, \xi^0\right)' Z \widehat{W}_{JT} Z' \Phi\left(\theta, \xi^0\right), \tag{12}$$

where

$$\Phi_t\left(\theta, \xi_t^0\right) = \xi_t^0 + \left[\frac{\partial \ln s\left(\xi_t^0; \theta\right)}{\partial \xi_t'}\right]^{-1} \left[\ln S_t - \ln s\left(\xi_t^0; \theta\right)\right] \tag{13}$$



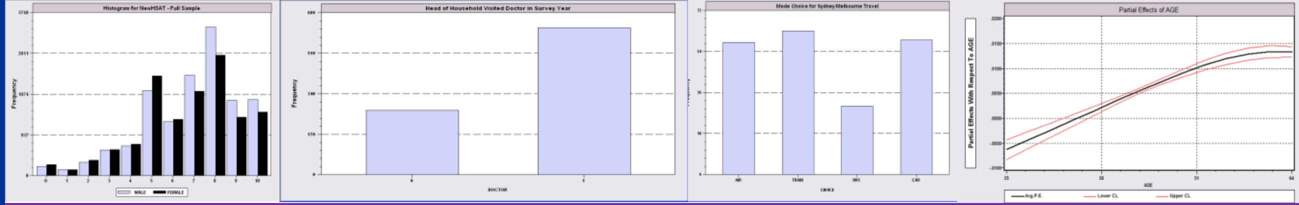
BLP Design Data

TABLE II

THE RANGE OF CONTINUOUS DEMAND CHARACTERISTICS (AND ASSOCIATED MODELS)

	Percentile				
Variable	0	25	50	75	100
Price	90 Yugo 3.393	79 Mercury Capri 6.711	87 Buick Skylark 8.728	71 Ford T-Bird 13.074	89 Porsche 911 Cabriolet 68.597
Sales	73 Toyota 1600CR .049	72 Porsche Rdstr 15.479	77 Plym. Arrow 47.345	82 Buick LeSabre 109.002	71 Chevy Impala 577.313
HP/Wt.	85 Plym. Gran Fury 0.170	85 Suburu DH 0.337	86 Plym. Caravelle 0.375	89 Toyota Camry 0.428	89 Porsche 911 Turbo 0.948
Size	73 Honda Civic 0.756	77 Renault GTL 1.131	89 Hyundai Sonata 1.270	81 Pontiac F-Bird 1.453	73 Imperial 1.888
MP$	74 Cad. Eldorado 8.46	78 Buick Skyhawk 15.57	82 Mazda 626 20.10	84 Pontiac 2000 24.86	89 Geo Metro 64.37
MPG	74 Cad. Eldorado 9	79 BMW 528i 17	81 Dodge Challenger 20	75 Suburu DL 25	89 Geo Metro 53

Notes: The top entry for each cell gives the model name and the number directly below it gives the value of the variable for this model.



The first set of results are based on the simplest logit specification for the utility function. They are obtained from an ordinary least squares regression of $\ln(s_j) - \ln(s_0)$ on product characteristics and price (see (5.5)).

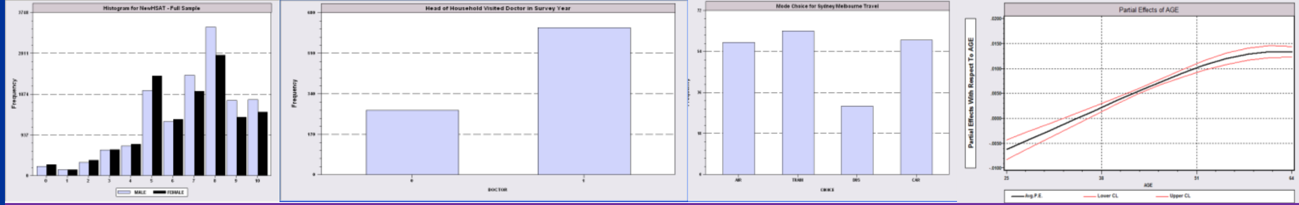
Exogenous price and nonrandom parameters

TABLE III

RESULTS WITH LOGIT DEMAND AND MARGINAL COST PRICING (2217 OBSERVATIONS)

Variable	OLS Logit Demand	IV Logit Demand	OLS ln (*price*) on *w*
Constant	−10.068	−9.273	1.882
	(0.253)	(0.493)	(0.119)
*HP/Weight**	−0.121	1.965	0.520
	(0.277)	(0.909)	(0.035)
Air	−0.035	1.289	0.680
	(0.073)	(0.248)	(0.019)
MP$	0.263	0.052	—
	(0.043)	(0.086)	
*MPG**	—	—	−0.471
			(0.049)
*Size**	2.341	2.355	0.125
	(0.125)	(0.247)	(0.063)
Trend	—	—	0.013
			(0.002)
Price	−0.089	−0.216	—
	(0.004)	(0.123)	
No. Inelastic Demands	1494	22	*n.a.*
(+/−2 s.e.'s)	(1429–1617)	(7–101)	
R^2	0.387	*n.a.*	.656

Notes: The standard errors are reported in parentheses.
*The continuous product characteristics—hp/weight, size, and fuel efficiency (*MP$* or *MPG*)—enter the demand equations in levels, but enter the column 3 price regression in natural logs.



IV Estimation

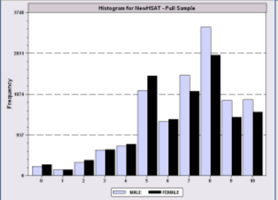
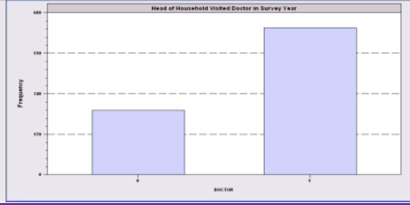
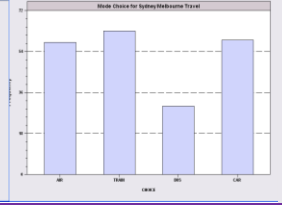
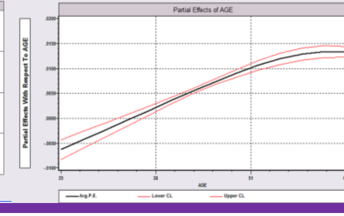
TABLE III

RESULTS WITH LOGIT DEMAND AND MARGINAL COST PRICING
(2217 OBSERVATIONS)

Variable	OLS Logit Demand	IV Logit Demand	OLS ln (*price*) on *w*
Constant	−10.068	−9.273	1.882
	(0.253)	(0.493)	(0.119)
*HP / Weight**	−0.121	1.965	0.520
	(0.277)	(0.909)	(0.035)
Air	−0.035	1.289	0.680
	(0.073)	(0.248)	(0.019)
MP$	0.263	0.052	—
	(0.043)	(0.086)	
*MPG**	—	—	−0.471
			(0.049)
*Size**	2.341	2.355	0.125
	(0.125)	(0.247)	(0.063)
Trend	—	—	0.013
			(0.002)
Price	−0.089	−0.216	—
	(0.004)	(0.123)	
No. Inelastic Demands	1494	22	*n.a.*
(+/−2 *s.e.'s*)	(1429–1617)	(7–101)	
R^2	0.387	*n.a.*	.656

Notes: The standard errors are reported in parentheses.
* The continuous product characteristics—hp/weight, size, and fuel efficiency (*MP$* or *MPG*)—enter the demand equations in levels, but enter the column 3 price regression in natural logs.

In the second column of Table III, we re-estimate the logit utility specification, this time allowing for unobservable product attributes that are known to the market participants (and hence can be used to set prices), but not to the econometrician. To account for the possible correlation between the price variable and the unobserved characteristics, we use an instrumental variable estimation technique, using the instruments discussed at the end of Section 5.1.

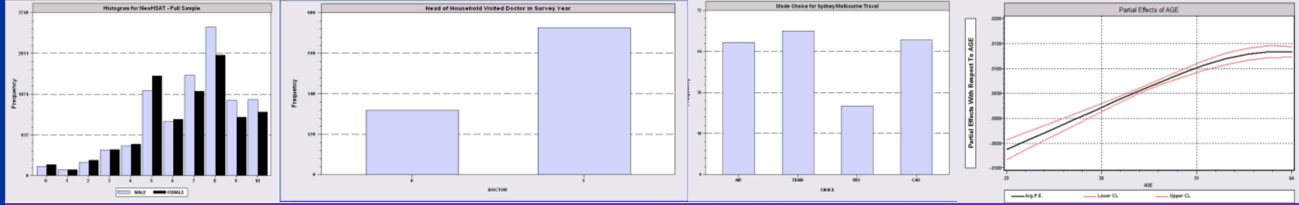





Full Model

TABLE IV

ESTIMATED PARAMETERS OF THE DEMAND AND PRICING EQUATIONS: BLP SPECIFICATION, 2217 OBSERVATIONS

Demand Side Parameters	Variable	Parameter Estimate	Standard Error	Parameter Estimate	Standard Error
Means ($\overline{\beta}$'s)	*Constant*	−7.061	0.941	~~−7.304~~	~~0.746~~
	HP/Weight	2.883	2.019	~~2.185~~	~~0.896~~
	Air	1.521	0.891	~~0.579~~	~~0.632~~
	MP$	−0.122	0.320	~~−0.049~~	~~0.164~~
	Size	3.460	0.610	~~2.604~~	~~0.285~~
Std. Deviations (σ_β's)	*Constant*	3.612	1.485	~~2.009~~	~~1.017~~
	HP/Weight	4.628	1.885	~~1.586~~	~~1.186~~
	Air	1.818	1.695	~~1.215~~	~~1.149~~
	MP$	1.050	0.272	~~0.670~~	~~0.168~~
	Size	2.056	0.585	~~1.510~~	~~0.297~~
Term on Price (α)	$\ln(y-p)$	43.501	6.427	~~23.710~~	~~4.079~~
Cost Side Parameters					
	Constant	0.952	0.194	0.726	0.285
	ln(*HP/Weight*)	0.477	0.056	0.313	0.071
	Air	0.619	0.038	0.290	0.052
	ln(*MPG*)	−0.415	0.055	0.293	0.091
	ln(*Size*)	−0.046	0.081	1.499	0.139
	Trend	0.019	0.002	0.026	0.004
	$\ln(q)$			−0.387	0.029



Some Elasticities

TABLE V

A SAMPLE FROM 1990 OF ESTIMATED DEMAND ELASTICITIES WITH RESPECT TO ATTRIBUTES AND PRICE (BASED ON TABLE IV (CRTS) ESTIMATES)

Model	Value of Attribute/Price Elasticity of demand with respect to: *HP/Weight*	*Air*	*MP $*	*Size*	*Price*
Mazda323	0.366	0.000	3.645	1.075	5.049
	0.458	0.000	1.010	1.338	6.358
Sentra	0.391	0.000	3.645	1.092	5.661
	0.440	0.000	0.905	1.194	6.528
Escort	0.401	0.000	4.022	1.116	5.663
	0.449	0.000	1.132	1.176	6.031
Cavalier	0.385	0.000	3.142	1.179	5.797
	0.423	0.000	0.524	1.360	6.433
Accord	0.457	0.000	3.016	1.255	9.292
	0.282	0.000	0.126	0.873	4.798
Taurus	0.304	0.000	2.262	1.334	9.671
	0.180	0.000	−0.139	1.304	4.220
Century	0.387	1.000	2.890	1.312	10.138
	0.326	0.701	0.077	1.123	6.755
Maxima	0.518	1.000	2.513	1.300	13.695
	0.322	0.396	−0.136	0.932	4.845
Legend	0.510	1.000	2.388	1.292	18.944
	0.167	0.237	−0.070	0.596	4.134
TownCar	0.373	1.000	2.136	1.720	21.412
	0.089	0.211	−0.122	0.883	4.320
Seville	0.517	1.000	2.011	1.374	24.353
	0.092	0.116	−0.053	0.416	3.973
LS400	0.665	1.000	2.262	1.410	27.544
	0.073	0.037	−0.007	0.149	3.085
BMW 735i	0.542	1.000	1.885	1.403	37.490
	0.061	0.011	−0.016	0.174	3.515

Notes: The value of the attribute or, in the case of the last column, price, is the top number and the number below it is the elasticity of demand with respect to the attribute (or, in the last column, price.)