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A New Baseline Model for Estimating Willingness to Pay from Discrete Choice Models

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Calculating WTP from discrete choice models

- Contingent valuation
  - Marginal rates of substitution (implicit prices, WTP)
    - Ratios of utility function parameters at constant utility level
- A simple example

ANS	Coefficient	Standard Error	z	Prob. |z|>Z*	95% Confidence Interval	
B1	.56023***	.09336	6.00	.0000	.37726	.74320
COST	.00715***	.00084	8.47	.0000	.00550	.00881

Note: ***, **, * ==> Significance at 1%, 5%, 10% level.

- True parameters – WTP (MRS) = ratio
- Uncertainty – parameters are random variables

Calculating WTP from discrete choice models

- Usual practice for calculating mean (median) WTP
  - Ratio of coefficients
- Usual practice for calculating confidence interval of WTP:
  - Delta method
  - Krinsky and Robb parametric bootstraping
- But...
  - Algebra of random variables – regular algebraic rules do not apply
  - Maximum likelihood estimation – parameters asymptotically normal
  - Distribution of WTP = ratio (quotient) distribution
    - Shape of the distribution is not normal (not even symmetrical)
    - Undefined moments (mean, standard deviation)
    - Median not necessarily equal to the ratio of means
- Demonstrate problems with the delta / K&R method
- Propose alternative model specification to overcome these problems

Ratio distribution of two normally distributed variables

- B, C – random variables following some known distributions, with joint distribution function $f(b,c)$
- $W = B/C$ – random variable following some (ratio) distribution, with PDF given by:

$$w(u) = \int_{-\infty}^{+\infty} |c| f(uc, c)\, dc$$

- For B and C following bivariate normal density:

$$w_{BC}(b,c;\mu_B,\mu_C;\sigma_B,\sigma_C;\rho) =$$

$$= \frac{1}{2\pi\sigma_B\sigma_C\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2}\frac{1}{1-\rho^2}\left(\left(\frac{b-\mu_B}{\sigma_B}\right)^2 - 2\rho\left(\frac{b-\mu_B}{\sigma_B}\right)\left(\frac{c-\mu_C}{\sigma_C}\right) + \left(\frac{c-\mu_C}{\sigma_C}\right)^2 \right)\right)$$

Ratio distribution of two normally distributed variables

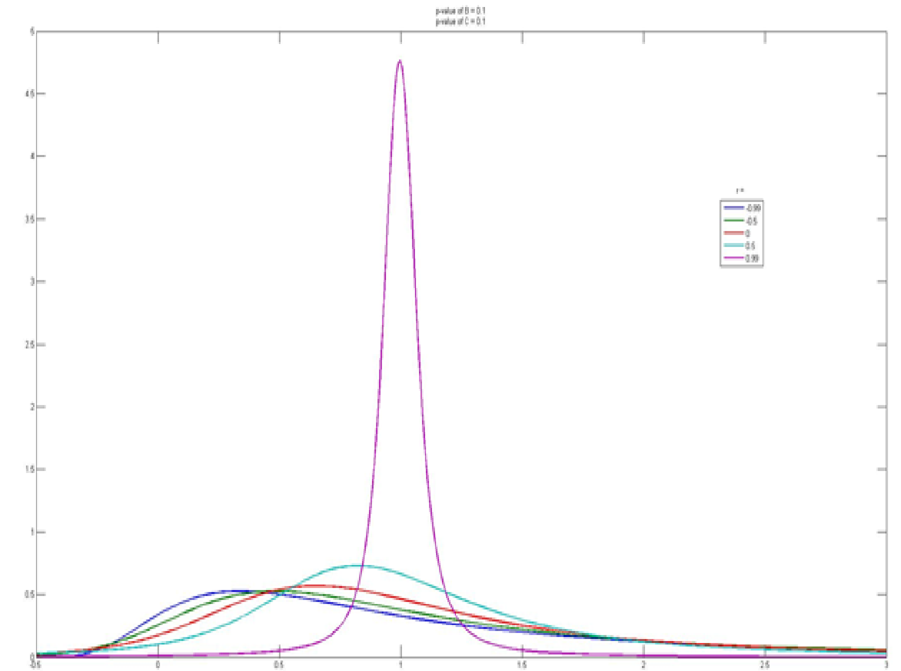
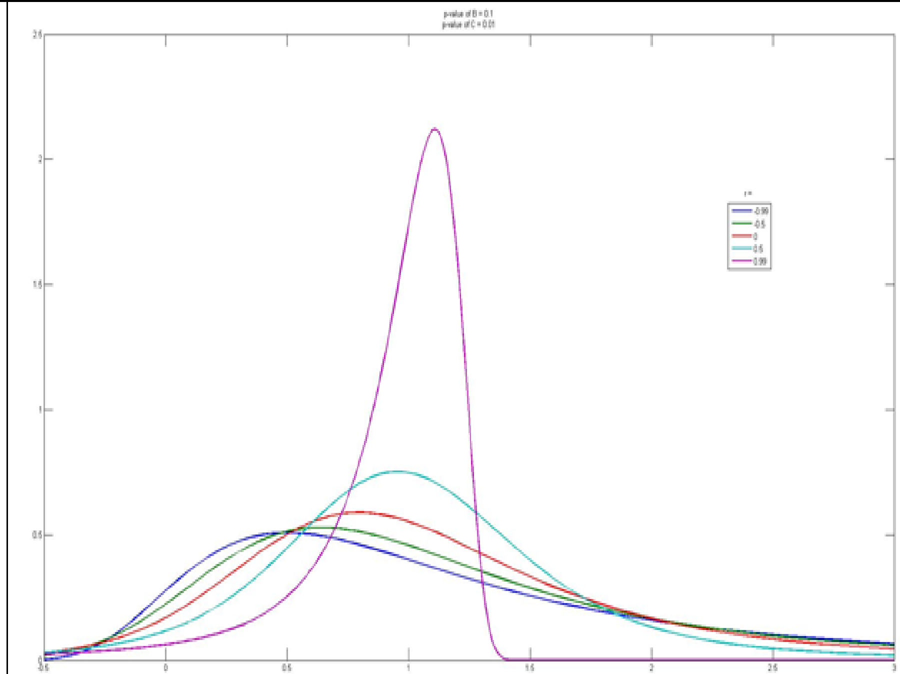
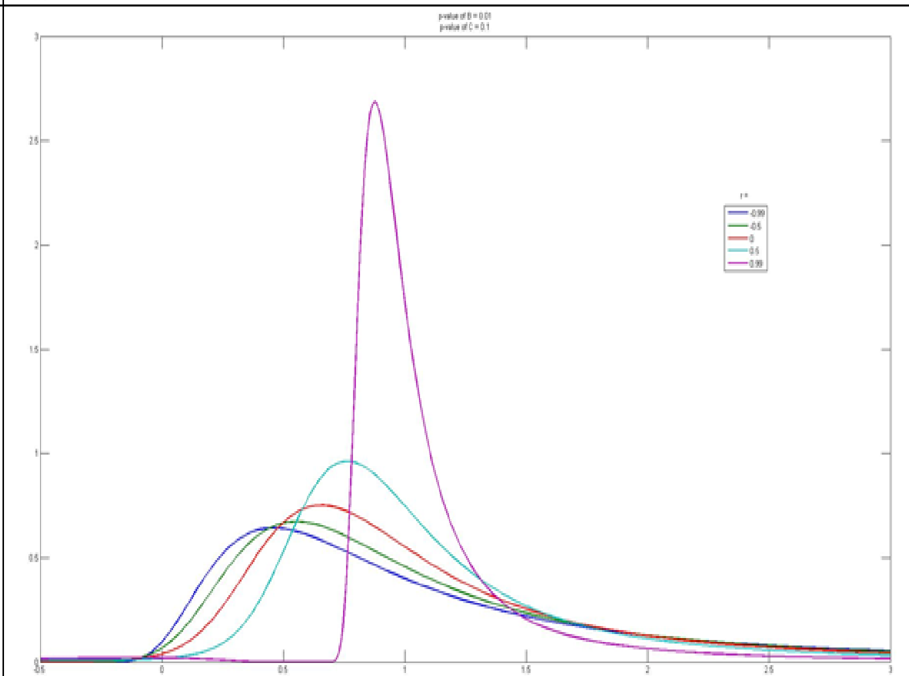
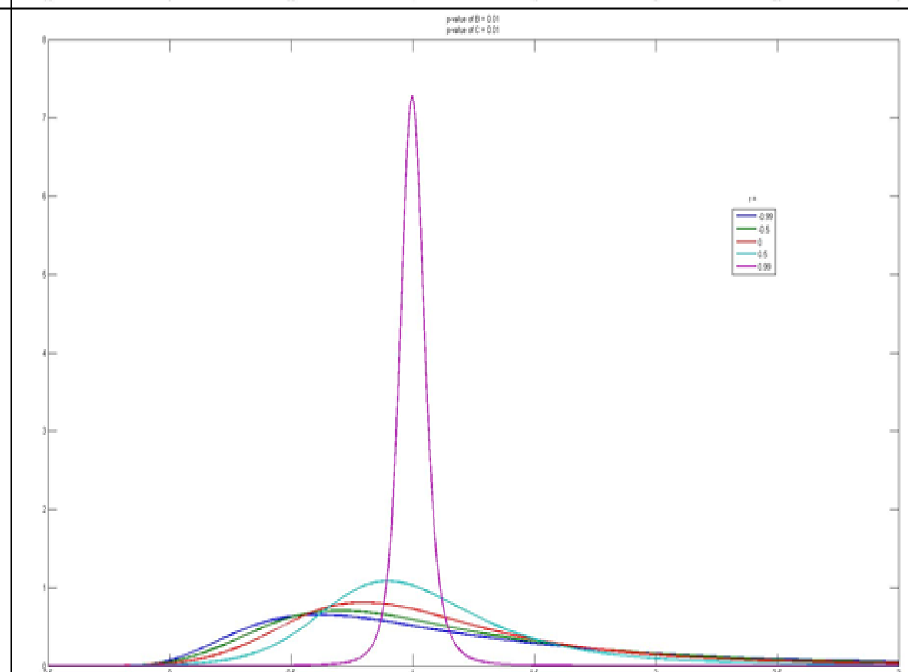
- PDF of $W = B/C$ becomes:

$$w(u) = \frac{\sigma_C \sigma_B \sqrt{1-\rho^2}}{\pi\left(\sigma_B^2 - 2\rho\sigma_B\sigma_C u + \sigma_C^2 u^2\right)} \exp\left(-\frac{1}{2}\frac{1}{1-\rho^2}\left(\frac{\mu_B^2}{\sigma_B^2} - 2\rho\frac{\mu_B\mu_C}{\sigma_B\sigma_C} + \frac{\mu_C^2}{\sigma_C^2}\right)\right) +$$

$$\exp\left(-\frac{1}{2}\frac{\left(\mu_B - \mu_C u\right)^2}{\sigma_B^2 - 2\rho\sigma_B\sigma_C q + \sigma_C^2 u^2}\right)\frac{\sigma_B\left(\rho\mu_B\sigma_C - \mu_C\sigma_B\right) + \sigma_C\left(\rho\mu_C\sigma_B - \mu_B\sigma_C\right)u}{\pi\left(\sigma_B^2 - 2\rho\sigma_B\sigma_C u + \sigma_C^2 u^2\right)^{\frac{3}{2}}} \times$$

$$\int_0^{\frac{\sigma_B\left(\rho\mu_B\sigma_C - \mu_C\sigma_B\right) + \sigma_C\left(\rho\mu_C\sigma_B - \mu_B\sigma_C\right)u}{\sigma_B\sigma_C\left(\left(1-\rho^2\right)\left(\sigma_B^2 - 2\rho\sigma_B\sigma_C u + \sigma_C^2 u^2\right)\right)^{\frac{1}{2}}}} \exp\left(-\frac{1}{2}q^2\right) dq$$

- $\mu_B = \mu_C = 0$ => Cauchy distribution
- Moments are infinite if C has non-zero density at $u \leq 0$

	p-value of C = 0.1	p-value of C = 0.01
p-value of B = 0.1		
p-value of B = 0.01		

p-value of B	p-value of C	corr. coef.	B/C	pseudo-mean (K&R)	median (K&R)	pseudo-st.dev. (K&R)	pseudo-st.dev. (delta)
0.10	0.10	-0.90	1.00	0.67	0.77	0.57	1.19
0.10	0.10	-0.50	1.00	0.82	0.77	0.74	1.05
0.10	0.10	0.00	1.00	1.00	0.83	0.94	0.86
0.10	0.10	0.50	1.00	1.18	0.91	1.13	0.61
0.10	0.10	0.90	1.00	1.33	0.98	1.29	0.27
0.10	0.01	-0.90	1.00	0.79	0.86	0.44	0.97
0.10	0.01	-0.50	1.00	0.88	0.86	0.59	0.87
0.10	0.01	0.00	1.00	1.00	0.90	0.76	0.72
0.10	0.01	0.50	1.00	1.12	0.95	0.91	0.53
0.10	0.01	0.90	1.00	1.21	0.99	1.02	0.31
0.01	0.10	-0.90	1.00	0.79	0.86	0.44	0.97
0.01	0.10	-0.50	1.00	0.88	0.86	0.59	0.87
0.01	0.10	0.00	1.00	1.00	0.90	0.76	0.72
0.01	0.10	0.50	1.00	1.12	0.95	0.91	0.53
0.01	0.10	0.90	1.00	1.21	0.99	1.02	0.31
0.01	0.01	-0.90	1.00	0.86	0.89	0.27	0.76
0.01	0.01	-0.50	1.00	0.92	0.90	0.42	0.67
0.01	0.01	0.00	1.00	1.00	0.93	0.57	0.55
0.01	0.01	0.50	1.00	1.08	0.96	0.69	0.39
0.01	0.01	0.90	1.00	1.14	0.99	0.78	0.17

Ratio distribution of two normally distributed variables

- In summary
  - Distribution is not normal (and not even symmetrical)
  - Ratio of coefficients is not equal to the median
  - … and not equal to the mean (which does not exist)
  - … so does standard deviation
- Using Krinsky and Robb parametric bootstrapping is wrong
  - Not enough draws mask infinite mean and standard deviation of the distribution
  - Usually standard deviation explodes more easily than mean
- Using Gaussian approximation (delta method) to derive standard deviation and confidence intervals is wrong
  - Cauchy tails generated by positive density on both side of zero (Khuri, Casella, 2002, *American Statistician*)
  - Gleser, Hwang (1987, *The Annals of Statistics*) – impossible to construct finite confidence intervals for ratio variables that have positive density around zero

Behavior of delta and Krinsky Robb estimates for ratio-based WTP

Method	Draws	Mean	Standard Deviation	Median	95% c.i. / quantile range
Analytical	–	Undefined	Undefined	0.97	(-0.52,4.28)
Delta	–	1*	0.51	1*	(0.00,2.00)
Krinsky and Robb	100	1.31	2.14	0.91	(0.06,9.05)
	1000	1.00	3.94	0.93	(0.00,3.98)
	10,000	0.65	58.32	0.96	(-0.45,4.13)
	100,000	0.90	53.30	0.97	(-0.50,4.26)
	1,000,000	0.93	155.20	0.97	(-0.52,4.27)
	10,000,000	-26.42	86104.39	0.97	(-0.52,4.29)
Fieller	–	1*	–	1*	$(-\infty,-2.21e13)\cup(0.00,+\infty)$

The problem with the ratio

- The problem not unique to choice modeling
  - ML estimator of the ratio is inconsistent: Bergstrom (1962, Econometrica), Zellner (1978, Journal of Econometrics)
    - Ratio undefined
  - Distributed lagged models (Lianos and Rausser, 1972, *Journal of the American Statistical Association*)
  - Reduce rank regression used in tests of cointegration (Phillips 1994, *Econometrica*)
  - Instrumental variables (Woglom 2001, *Econometrica*)
  - Travel cost demand models (Adamowicz, Fletcher, Graham-Tomasi, 1989, AJAE)
    - …but not seen as a serious problem due to Dorfman, Kling, Sexton (1990, AJAE)
  - Fieller bounds (Hirschberg, Lye, 2010, *The American Statistician*)

Alternative specification for the MNL model

- The typical specification for the MNL model
  - Cost enters linearly:

$$U_i\left(Alternative = j\right) = U_{ij} = \boldsymbol{\beta}'\mathbf{x}_{ij} + \gamma z_{ij} + \varepsilon_{ij}$$

- The alternative specification
  - Cost enters exponentially:

$$U_i\left(Alternative = j\right) = U_{ij} = \boldsymbol{\beta}'\mathbf{x}_{ij} + \exp\left(\gamma\right) z_{ij} + \varepsilon_{ij}$$

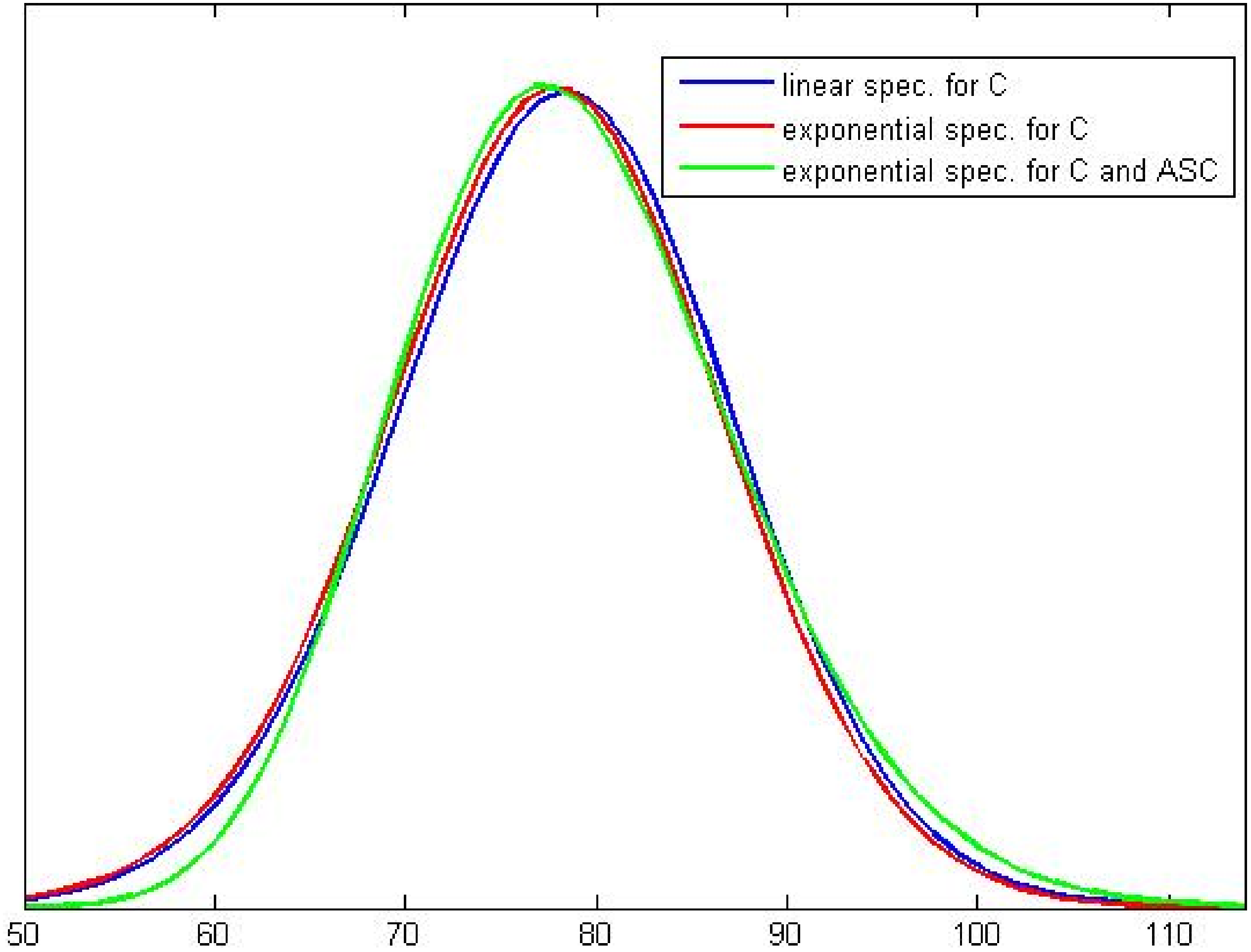
  - Estimation proceeds in the usual way
  - Cost parameter will now be strictly positive
  - Daly Hess Train (2012) show this parameterization assures finite moments
  - Consistent with economic theory

p-value of B	p-value of C	corr. coefficient	B/C	Linear specification			Exponential specification		
				pseudo-mean	pseudo-std.dev.	median	mean	std.dev.	median
0.1	0.1	-0.99	1	4.73	3365	0.86	2.23	3.46	1.14
0.1	0.1	-0.50	1	0.38	1094	0.88	1.89	2.67	1.08
0.1	0.1	0.00	1	-0.21	784	0.90	1.53	1.83	1.02
0.1	0.1	0.50	1	2.51	1550	0.94	1.17	1.17	0.96
0.1	0.1	0.99	1	1.00	22	1.00	0.83	0.95	1.00
0.1	0.01	-0.99	1	1.04	824	0.99	1.62	1.81	1.10
0.1	0.01	-0.50	1	1.70	224	0.99	1.43	1.44	1.05
0.1	0.01	0.00	1	1.36	95	0.99	1.24	1.09	1.02
0.1	0.01	0.50	1	0.39	610	1.00	1.05	0.77	0.98
0.1	0.01	0.99	1	0.86	74	1.00	0.87	0.54	1.02
0.01	0.1	-0.99	1	-0.14	978	0.88	1.98	2.68	1.15
0.01	0.1	-0.50	1	1.38	385	0.90	1.76	2.14	1.11
0.01	0.1	0.00	1	-0.52	1115	0.92	1.53	1.59	1.07
0.01	0.1	0.50	1	0.53	593	0.94	1.30	1.00	1.04
0.01	0.1	0.99	1	1.16	220	0.97	1.08	0.42	1.11
0.01	0.01	-0.99	1	1.60	145	0.99	1.48	1.38	1.10
0.01	0.01	-0.50	1	1.55	163	0.99	1.36	1.13	1.07
0.01	0.01	0.00	1	1.33	122	0.99	1.24	0.86	1.04
0.01	0.01	0.50	1	1.18	68	1.00	1.12	0.58	1.02
0.01	0.01	0.99	1	1.00	3	1.00	1.00	0.21	1.04

Empirical illustration 1a – Oil spill prevention in California's central coast

	MNL typical specification (cost enters linearly)	MNL alternative specification 1 (cost enters exponentially)	MNL alternative specification 2 (ASC and cost enter exponentially)
B – ASC associated with introducing the scenario	0.5602*** (0.0941)	0.5602*** (0.0941)	-0.5794*** (0.1680)
C – cost associated with introducing the scenario	7.1523*** (0.8361)	1.9674*** (0.1169)	1.9674*** (0.1169)
Ratio of coefficients	\$78.32	\$78.34	\$78.35
Median WTP – K&R	\$78.32	\$77.81	\$78.35
E(WTP) – K&R	\$78.25 (undefined)	\$77.73	\$78.85
Std. err. E(WTP) – delta	\$8.83 (undefined)	\$8.83	\$8.83
Std. err. E(WTP) – K&R	\$9.03 (undefined)	\$9.00	\$8.91
95% c.i. E(WTP) – delta	\$60.63 – \$96.01	\$60.70 – \$95.98	\$60.88 – \$95.81
95% c.i. E(WTP) – Fieller	\$60.22 – \$95.83	\$60.25 – \$95.84	\$60.26 – \$95.85
95% c.i. E(WTP) – K&R (quantile range)	\$60.25 – \$95.82	\$59.71 – \$95.21	\$62.83 – \$97.71
Log-likelihood	-712.7737	-712.7737	-712.7737
AIC/n	1.3180	1.3180	1.3180
n (observations)	1085	1085	1085

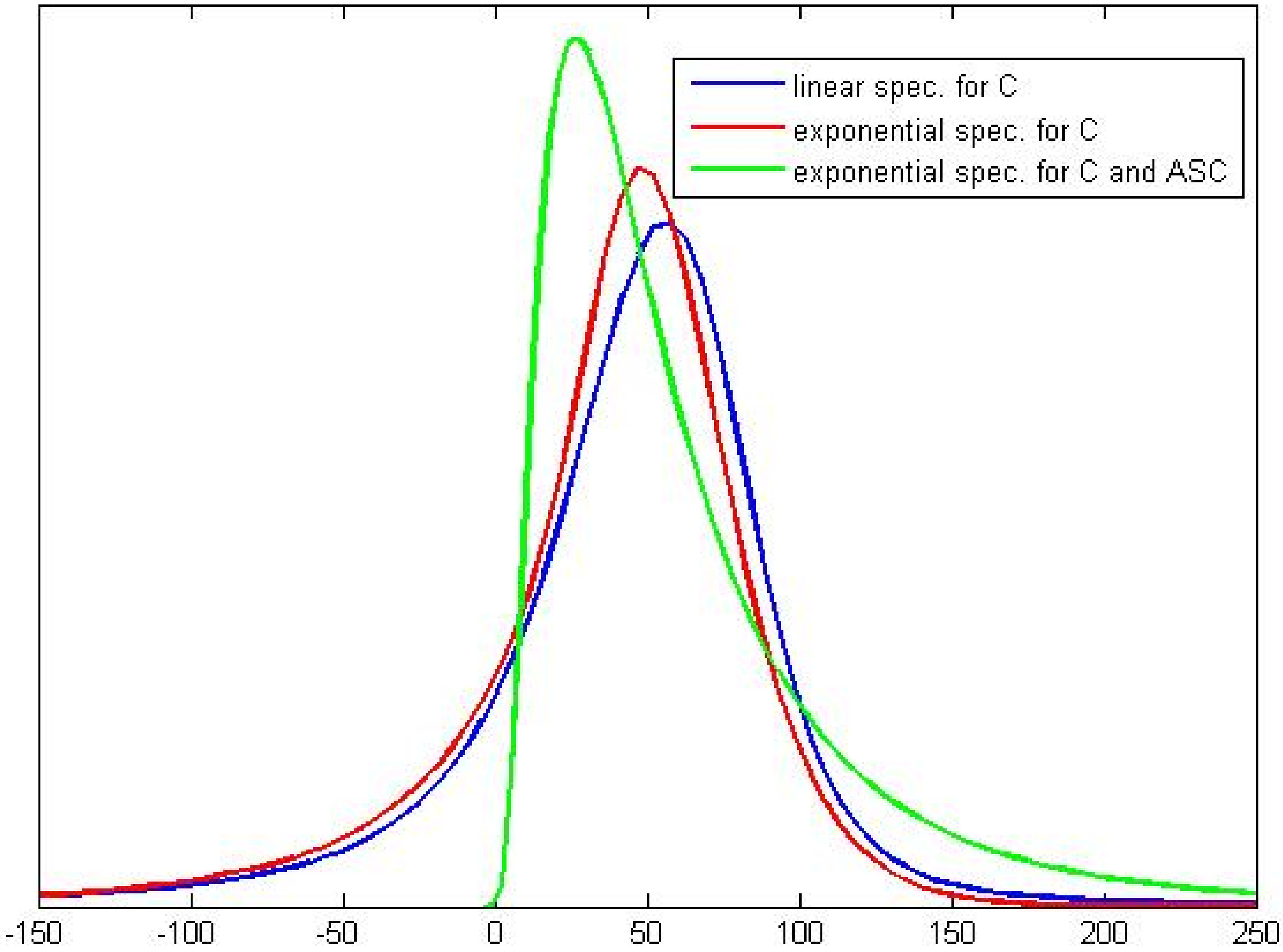
Empirical distribution of WTP (n=1085)



Empirical illustration 1b random subsample of *n*=100

	MNL typical specification (cost enters linearly)	MNL alternative specification 1 (cost enters exponentially)	MNL alternative specification 2 (ASC and cost enter exponentially)
B – ASC associated with introducing the scenario	0.2842 (0.2988)	0.2842 (0.2988)	-1.2579 (1.0513)
C – cost associated with introducing the scenario	5.8786** (2.8110)	1.7713*** (0.4782)	1.7713*** (0.4782)
Ratio of coefficients	$48.35	$48.37	$48.38
Median WTP – K&R	$49.48	$43.71	$48.38
E(WTP) – K&R	$37.49 (undefined)	$34.43	$65.37
Std. err. E(WTP) – delta	$37.55 (undefined)	$37.52	$37.52
Std. err. E(WTP) – K&R	$1.59*10^5 (undefined)	$56.23	$59.42
95% c.i. E(WTP) – delta	-$3.11*10^5 – $3.11*10^5	-$61.83 – $158.57	-$68.08 – $164.84
95% c.i. E(WTP) – Fieller	-$567.43 – $139.11	-$565.00 – $139.12	-$564.38 – $139.14
95% c.i. E(WTP) – K&R (quantile range)	-$151.99 – $152.64	-$106.30 – $113.71	$10.58 – $221.35
Log-likelihood	-66.8654	-66.8654	-66.8654
AIC/n	1.3770	1.3770	1.3770
n (observations)	100	100	100

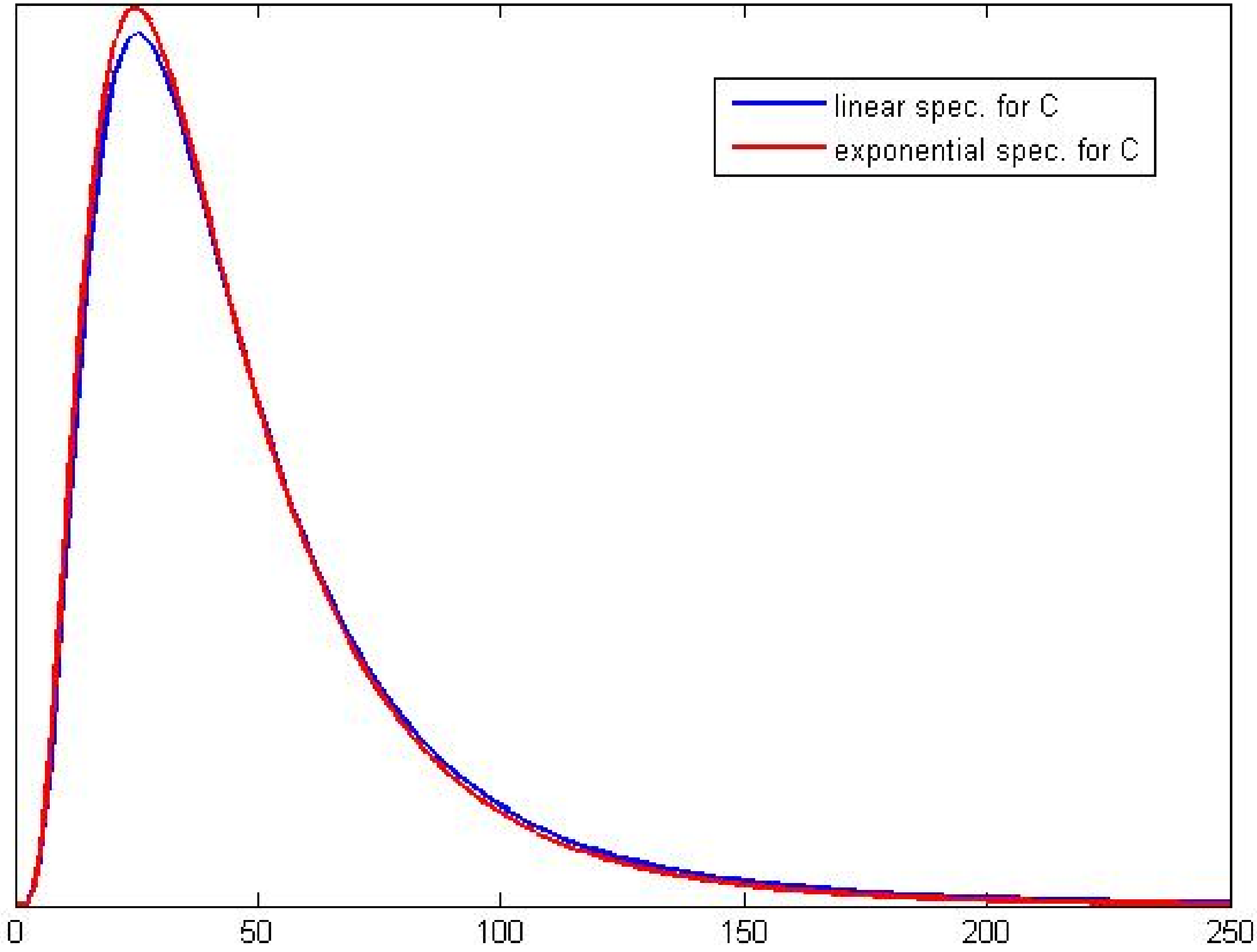
Empirical distribution of WTP (n=100)



Empirical illustration 2
Alternative Fuels Vehicle DCE

	RPL – typical specification (cost parameters enter linearly)		RPL – alternative specification (cost parameters enter exponentially)	
	Means	Standard deviations	Means	Standard deviations
range **(log-normally distributed)**	-0.9007 (0.6439)	-0.6041*** (0.2342)	-0.9007 (0.6439)	-0.6041*** (0.2342)
electric **(normally distributed)**	-1.5392*** (0.3961)	-0.7544** (0.3248)	-1.5393*** (0.3961)	-0.7543** (0.3249)
hybrid **(normally distributed)**	0.3839** (0.1635)	0.7306*** (0.1216)	0.3840** (0.1635)	0.7306*** (0.1216)
p_medium **(normally distributed)**	0.1197 (0.1016)	0.2074 (0.2337)	0.1197 (0.1016)	0.2075 (0.2337)
p_high **(normally distributed)**	0.4548*** (0.1115)	0.4671*** (0.1546)	0.4548*** (0.1115)	0.4672*** (0.1546)
c_purchase **(fixed)**	0.4472*** (0.0295)	–	-0.8046*** (0.0659)	–
c_operate **(fixed)**	0.0132*** (0.0035)	–	-4.3297*** (0.2641)	–
Ratio of coefficients	$22.82		$22.81	
Median WTP – K&R	$41.73		$39.42	
E(WTP) – K&R	$49.96 (undefined)		$47.19	
Std. err. E(WTP) – delta	$28.53 (undefined)		$28.51	
Std. err. E(WTP) – K&R	$715.76 (undefined)		$34.78	
95% c.i. E(WTP) – delta	-$33.09 – $78.73		-$33.08 – $78.69	
95% c.i. E(WTP) – Fieller	-$15.61 – $118.35		-$15.60 – $118.27	
95% c.i. E(WTP) – K&R (quantile range)	$11.00 – $151.97		$10.58 – $138.01	
Log-likelihood	-1358.0826		-1358.0826	
AIC/n	1.8465		1.8465	
n (observations)	1484		1484	

Empirical distribution of WTP (*range / c_operate*)



Conclusions

- The alternative specification does not change the model fit or shape of WTP distribution much
- It allows to calculate moments of the WTP distribution
  - Ratio of coefficients *may* be close to the median (especially if *B* and *C* far from 0 and highly correlated)
- New reference statistical model for discrete choice experiment contingent valuation
  - Monetary attribute has support in positive (negative) values only
  - Possible to calculate moments / apply K&R and delta
  - Possible to apply the same approach to other parameters