**Chapter 17**

**Discrete Choice**

**Exercises**

1. The log‑likelihood is

ln*L* = Σ0,0lnProb[*y*=0,*d*=0] + Σ0,1lnProb[*y*=0,*d*=1] + Σ1,0lnProb[*y*=1,*d*=0] + Σ1,1lnProb[*y*=1,*d*=1]

where Σi,j indicates the sum over observations for which *y* = *i* and *d* = *j*. Since there are no other regressors, this reduces to ln*L* = 24ln(1 ‑ *F*(α)) + 32ln(1 ‑ *F*(δ)) + 28ln*F*(α) + 16ln*F*(δ). Although it is straightforward to maximize the log-likelihood directly in terms of α and δ, an alternative, convenient approach is to estimate *F*(α) and *F*(δ). These functions can then be inverted to estimate the original parameters. The invariance of maximum likelihood estimators to transformation will justify this approach. One virtue of this approach is that the same procedure is used for both probit and logit models. Let *A* = *F*(α) and *D* = *F*(δ). Then, the log likelihood is simply ln*L* = 24ln(1 - *A*) + 32ln(1 - *D*) + 28ln*A* + 16ln*D.* The necessary conditions are

∂ln*L*/∂*A* = ‑24/(1 - *A*) + 28/*A* = 0

∂ln*L*/∂*D* = ‑32/(1 ‑ *D*) + 16/*D* = 0.

Simple manipulations produce the two solutions *A* = 28/(24+28) = .539 and *D* = 16/(32+16) = .333. Then, these functions can be inverted to produce the MLEs of α and β. Thus, = *F*-1(*A*) and  = *F*-1(*D*) - . The two inverse functions are Φ-1(A) for the probit model, which must be approximated, and ln[F/(1-F)] for the logit model. The estimates are,

Probit Logit

α .098 .156

δ ‑.431 ‑.694

β ‑.529 ‑.850

(Notice the proportionality relationship, .156/.098 = 1.592 and ‑.848/‑.529 = 1.607.)

We will compute the asymptotic covariance matrix forand directly using (17‑22) for the probit model and (17-21) for the logit model. We will require *h*i = ∂2ln*L*/∂(α+β*d*)2 for the four cells. For the computation, we will require φ(*c*)/Φ(*c*) and -φ(*c*)/[1-Φ(*c*)]. These are listed in the table below.

λ1 λ0

*y* *d* α+β*d* Φ φ φ/Φ -φ/(1-Φ) λ0λ1

0 0 .098 .539 .397 .737 ‑.861 -.636

1 0 .098 .539 .397 .737 -.861 -.636

0 1 ‑.431 .333 .364 1.093 ‑.546 ‑.597

1 1 ‑.431 .333 .364 1.093 -.546 -.597

The estimated asymptotic covariance matrix is the inverse of the estimate of -*E*[**H**].

 Then,

. The asymptotic standard errors are the square roots of the diagonal elements, which are .1739 and .2552, respectively. To test the hypothesis that β = 0, we would refer *z* = ‑.529 / .2552 = -2.073 to the standard normal table. This is larger than the 1.96 critical value, so we would reject the hypothesis. To compute the likelihood ratio statistic, we will require the two log‑likelihoods. The restricted log‑likelihood (for both the probit and logit models) is given in (17-29). This would be

ln*L*0 = 100[.44ln.44 + .56ln.56] = ‑68.593. Let the predicted values above be denoted

P00 = Prob[y=0,d=0] = .461 (i.e., 1 - .539)

P10 = Prob[y=1,d=0] = .539

P01 = Prob[y=0,d=1] = .667

P11 = Prob[y=0,d=1] = .333

and let *nij* equal the number of observations in each cell Then, the unrestricted log‑likelihood is

ln*L* = 24ln.461 + 28ln.539 + 32ln.667 + 16ln.333 = ‑66.442. The likelihood ratio statistic would be

λ = ‑2(‑66.6442 ‑ (‑68.593)) = 4.302. The critical value from the chi‑squared distribution with one degree of freedom is 3.84, so once again, the test statistic is slightly larger than the table value.

We now compute the Hessian for the logit model. The predicted probabilities are

Prob[y = 0 , d = 0] = *P*00 = 1/(1 + e.156) = .462

Prob[y = 1 , d = 0] = *P*10 = 1 - *P*00 = .538

Prob[y = 0 , d = 1] = *P*01 = 1/(1 + e-.431) = .667

Prob[y = 1 , d = 1] = *P*11 = 1 - *P*01 = .333.

Notice that in spite of the quite different coefficients, these are identical to the results for the probit model. Remember that we originally estimated the probabilities, not the parameters, and these were independent of the distribution. Then, the Hessian is computed in the same manner as for the probit model using

*hij* = *Fij*(1‑*Fij*) instead of λ0λ1 in each cell. The asymptotic covariance matrix is the inverse of

(28+24)(.462)(.538)+(32+16)(.667)(.333). The standard errors are .2782 and .4137. For testing the hypothesis that β equals zero, the t‑statistic is *z* = ‑.850/.4137 = ‑2.055, which is almost the same as that for the probit model. The unrestricted log‑likelihood is ln*L* = 24ln.4285 + ... + 16ln.3635 = ‑66.442 (again). The chi‑squared statistic is 4.302, as before.

2. Using the usual regression statistics, we would have *a*= , .

For data in which *y* is a binary variable, we can decompose the numerator somewhat further. First, divide both numerator and denominator by the sample size. Second, since only one variable need be in deviation form, drop the deviation in *x*. That leaves . The denominator is the sample variance of *x*. Since *yi* is only 0s and 1s, is the proportion of 1s in the sample, *P*. Thus, the numerator is

(1/*n*)Σ*i xiyi* - (1/*n*)Σ*i**xi*= (1/*n*)Σ1*xi* - *P* = (*n*1/*n*)  - *P*[*P*  + (1-*P*) ] = *P*(1 - *P*)(  - ).

Therefore, the regression is essentially measuring how much the mean of *x* varies across the two groups of observations. The constant term does not simplify into any intuitively useful form.

3. The model was estimated using Newton's method as described in the text. The estimated coefficients and their standard are shown below: **= -.51274 + .15964X

(1.042) (.202)

Log‑likelihood = ‑6.403 Restricted log‑likelihood = ‑6.9315.

The t‑ratio for testing the hypothesis is .15964/.202 = .79. The chi‑squared for the likelihood ratio test is 1.057. Neither is large enough to lead to rejection of the hypothesis.

4. The derivatives of the log‑likelihood are given in (17-18) and (17-21). If all coefficients except the constant term are zero, then the first order condition for maximizing the log‑likelihood would be

∂ln*L*/∂β = Σ*i*(*yi* ‑ λ)(1) = 0 since with no regressors, λ*i* will not vary with *i*. This leads to the constrained maximum **= Σi *y*i/*n* = *P*, which might be expected. Thus, we estimate the constant term such that *P* = , or = logit(*P*). The LM statistic based on the BHHH estimator of the covariance matrix of the first derivatives would be LM = [Σ*i***g***i*]**′**[Σ*i***g***i***g***i***′**]-1[Σ*i***g***i*] where **g***i* = Σ*i*(*yi* ‑ *P*)**x***i*. In full, the statistic is

LM = [Σ*i*(*yi* ‑ *P*)**x***i*]**′**[Σ*i*(*yi* ‑ *P*)2**x***i***x***i***′**]-1[Σ*i*(*yi* ‑ *P*)**x***i*].

The actual (and expected) Hessian can be used instead by replacing (*yi* ‑ *P*)2 with *P*(1 ‑ *P*) in the inverse matrix. The statistic could then be written

LM = [**X′**(**y** ‑ *P***i**)]**′**[(**X′X**)-1][**X′**(**y** ‑ *P***i**)]/*P*(1 ‑ *P*) = **e′X**(**X′X**)-1**X′e**/*P*(1 ‑ *P*)

In the preceding, **e′e** = Σ*i*(*yi* ‑ *P*)2 = *nP*(1 ‑ *P*). Therefore, LM = *n*[**e′X**(**X′X**)-1**X′e**/**e′e**], which establishes the result. To compute the statistic, we regress (*yi* ‑ *P*) on the **x**s, then carry *nR*2 into the chi‑squared table.

5. (The section on grouped data for binary choice modeling does not appear in the 7th edition of the text.) To estimate the coefficients, we will use a two step FGLS procedure. Ordinary least squares estimates based on the moment equation Pi = Φ( + Ti) are consistent, but inefficient. The OLS regression produces

Φ-1(*P*i) = = ‑2.18098 + .0098898*T*

(.7404) (.002883).

The predicted values from this regression can then be used to compute weights  (see Section 21.4.6 from the 5th edition) . The weighted least squares regression produces

= -2.3116 + .010646*T*

(.8103) (.003322)

In order to achieve a predicted proportion of 95%, we will require *z*i = 1.645. The *T* required to achieve this is

*T* = (1.645 + 2.3116) / .010646 = 372.

The *z*i which corresponds to 90% is 1.282. Doing the same calculation as above, this requires *T* = 338 trucks. At $20,000 per truck, this requires $6.751 million, so the budget is inadequate.

The marginal effect is ∂Φi/∂*T* = .010646φ(*z*i). At *T* = 300, *z*i = .8822, so φ(*z*i) = .2703 and the marginal effect is .00288.

6**.** This is similar to Exercise 1. It is simplest to prove it in that framework. Since the model has only a dummy variable, we can use the same log likelihood as in Exercise 1. But, in this exercise, there are no observations in the cell (*y*=1,*x*=0). The resulting log likelihood is, therefore,

ln*L* = Σ0,0lnProb[*y*=0,*x*=0] + Σ0,1lnProb[*y*=0,*x*=1] + Σ1,1lnProb[*y*=1,*x*=1]

or ln*L* = *n*3lnProb[*y*=0,*x*=0] + *n*2lnProb[*y*=0,*x*=1] + *n*1lnProb[*y*=1,*x*=1].

Now, let δ = α + β. The log likelihood function is ln*L* = *n*3ln(1 ‑ *F*(α)) + *n*2ln(1 ‑ *F*(δ)) + *n*1ln*F*(δ). For estimation, let *A* = *F*(α) and *D* = *F*(δ). We can estimate *A* and *D*, then α = *F*-1(*A*) and β = *F*-1(*D*) - α. The first order condition for estimation of A is ∂ln*L*/∂*A* = -*n*3/(1 - *A*) = 0, which obviously has no solution. If *A* cannot be estimated then α cannot either, nor, in turn, can β. This applies to both probit and logit models.

7. We’ll do this more generally for any model F(α). Since the ‘model’ contains only a constant, the log likelihood is logL = Σ0log[1-F(α)] + Σ1logF(α) = n0log[1-F(α)]+n1logF(α) . The likelihood equation is ∂logL/∂α = Σ0[-f(α)/[1-F(α)] + Σ1f(α)/F(α) = 0 where f(α) is the density (derivative of F(α) so that at the solution, n0f(α)/[1-F(α)] = n1f(α)/F(α). Divide both sides of this equation by f(α) and solve it for F(α) = n1/(n0+n1), as might be expected. You can then insert this solution for F(α) back into the log likelihood, and (17-29) follows immediately.

8. Look at the two cases. Neither case has an estimator which is consistent in both cases. In both cases, the unconditional fixed effects effects estimator is inconsistent, so the rest of the analysis falls apart. This is the incidental parameters problem at work. Note that the fixed effects estimator is inconsistent because in both models, the estimator of the constant terms is a function of 1/T. Certainly in both cases, if the fixed effects model is appropriate, then the random effects estimator is inconsistent, whereas if the random effects model is appropriate, the maximum likelihood random effects estimator is both consistent and efficient. Thus, in this instance, the random effects satisfies the requirements of the test. In fact, there does exist a consistent estimator for the logit model with fixed effects - see the text. However, this estimator must be based on a restricted sample observations with the sum of the ys equal to zero or T muust be discarded, so the mechanics of the Hausman test are problematic. This does not fall into the template of computations for the Hausman test.

**Applications**

1. Binary Choice for Extramarital Affairs using Redbook data

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? Application 23.1

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Create ; A = (Yrb > 0) $

Namelist ; X = one,v1,v2,v5,v6 $

Probit ; Lhs = A ; Rhs = X ; marginal Effects $

Logit ; Lhs = A ; Rhs = X ; marginal Effects $

+---------------------------------------------+

| Binomial Probit Model |

| Maximum Likelihood Estimates |

| Dependent variable A |

| Number of observations 6366 |

| Log likelihood function -3547.865 |

| Number of parameters 5 |

| Info. Criterion: AIC = 1.11620 |

| Info. Criterion: BIC = 1.12151 |

| Restricted log likelihood -4002.530 |

+---------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

---------+Index function for probability

Constant| 1.43453507 .15493583 9.259 .0000

V1 | -.42595261 .01807583 -23.565 .0000 4.10964499

V2 | .02797013 .00254409 10.994 .0000 29.0828621

V5 | -.20942202 .02015534 -10.390 .0000 2.42617028

V6 | -.03522668 .00801808 -4.393 .0000 14.2098649

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| Partial derivatives of E[y] = F[\*] with |

| respect to the vector of characteristics. |

| They are computed at the means of the Xs. |

| Observations used for means are All Obs. |

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+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]|Elasticity|

+--------+--------------+----------------+--------+--------+----------+

---------+

Constant| .27876593 .01081795 25.769 .0000

V1 | -.14911732 .00634679 -23.495 .0000 -2.01181601

V2 | .00979177 .00088860 11.019 .0000 .93487672

V5 | -.07331438 .00703451 -10.422 .0000 -.58393740

V6 | -.01233214 .00280535 -4.396 .0000 -.57528664

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| Binary Logit Model for Binary Choice |

| Maximum Likelihood Estimates |

| Dependent variable A |

| Number of observations 6366 |

| Log likelihood function -3549.741 |

| Number of parameters 5 |

| Info. Criterion: AIC = 1.11679 |

| Info. Criterion: BIC = 1.12210 |

| Restricted log likelihood -4002.530 |

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+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

---------+Characteristics in numerator of Prob[Y = 1]

Constant| 2.41622262 .26160831 9.236 .0000

V1 | -.70802698 .03091557 -22.902 .0000 4.10964499

V2 | .04624150 .00426119 10.852 .0000 29.0828621

V5 | -.35139771 .03413337 -10.295 .0000 2.42617028

V6 | -.05899324 .01354756 -4.355 .0000 14.2098649

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| Partial derivatives of probabilities with |

| respect to the vector of characteristics. |

| They are computed at the means of the Xs. |

| Observations used are All Obs. |

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+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]|Elasticity|

+--------+--------------+----------------+--------+--------+----------+

---------+Marginal effect for variable in probability

Constant| .50898166 .05554126 9.164 .0000

V1 | -.14914716 .00650799 -22.918 .0000 -2.03205673

V2 | .00974086 .00089378 10.898 .0000 .93918419

V5 | -.07402256 .00714156 -10.365 .0000 -.59539053

V6 | -.01242703 .00285019 -4.360 .0000 -.58542862

**Chapter 18**

**Discrete Choices and Event Counts**

1. Since there is no regressor, we may write the log-likelihood as

lnL = 50lnΦ(-α) + 40ln[Φ(μ1-α) - Φ(-α)] + 45ln[Φ(μ2-α) - Φ(μ1-α)] +

80ln[Φ(μ3-α) - Φ(μ2-α)] + 35ln[1 - Φ(μ3-α)].

There are four unknown parameters to estimate and four free probabilities. Suppose, then, we treat Φ(-α), Φ(μ1-α), Φ(μ2-α), and Φ(μ3-α) as the unknown parameters, π0, π1, π2, and π3, respectively. If we can find estimators of these, we can solve for the underlying parameters. We may write the log-likelihood as

ln*L* = 50lnπ0 + 40ln(π1 - π0) + 45ln(π2 - π1) + 80ln(π3 - π2) + 35ln(1 - π3).

The necessary conditions are

∂ln*L*/∂π0 = 50/π0 - 40/(π1-π0) = 0

∂ln*L*/∂π1 = 40/(π1 - π0) - 45/(π2 - π1) = 0

∂ln*L*/∂π2 = 45/(π2 - π1) - 80/(π3 - π2) = 0

∂ln*L*/∂π3 = 80/(π3 - π2) - 35/(1 - π3) = 0.

By a simple rearrangement, these can be recast as a set of linear equations. Thus,

90π0 - 50π1 = 0

45π0 - 85π1 + 40π2 = 0

80π1 - 125π2 + 45π3 = 0

- 35π2 + 115π3 = 80

The solution (as might be expected) is

π0 = .2 (50/250)

π1 = .36 ((50+40)/250)

π2 = .54 ((50+40+45)/250)

π3 = .86 ((50+40+45+80)/250).

Now, we can solve for the underlying parameters.

-α = Φ-1(.2) = -.841, so α = .841.

μ1-α = Φ-1(.36) = -.358, so μ1 = .483

μ2-α = Φ-1(.54) = .101, so μ2 = .942

μ3-α = Φ-1(.86) = 1.081, so μ3 = 1.922.

2. a. Conditional variance in the ZIP model. The essential ingredients that are needed for this derivation are

 = Ei\*

and



[See, e.g., Winkelmann (2003, pp. 33-34).]. We found the conditional mean in the text to be

E[yi|xi,wi] =  = Fi Ei\*

To obtain the variance, we will use the variance decomposition,

Var[yi|xi,wi] = Ez[Var[yi|xi,z]] + Varz[E[yi|xi,z]].

The expectation of the conditional variance is

Ez[Var[yi|xi,z]] = (1 – Fi)×0 + Fi× = Fi × Ei\* × Vi\*

The variance of the conditional mean is

(1 – Fi) × + Fi = Fi(1-Fi)

= Fi(1 – Fi)Ei\*2.

The unconditional variance is thus, Fi Ei\* [Vi\* + (1 – Fi)Ei\*]. To obtain τi we divide by the conditional mean, which is Fi Ei\*, so τi = [Vi\* + (1 – Fi)Ei\*]. Is this greater than Ei\*? Not necessarily. The figure below plots Fi(1 – Fi)Ei\*2for Fi = .9 and various values of λ from .1 to about 12. There is a large range over which the function is less than one.



b. Partial Effects. The mean is Fi Ei\*. We suppose that wi and xi are the same for the moment.

∂Ei/∂xi = Ei\*∂Fi/∂xi + Fi ∂Ei\*/∂xi.

The first term is Ei\*×fi×γ. The second term is Fi ∂Ei\*/∂λi λiβ. The missing element is

∂Ei\*/∂λi = λi/[1-exp(-λi)] × [1 – exp(-λi)/[1-exp(-λi)].

Comnbining terms produces the marginal effects.

3. Let y\* denote the unobserved random variable that is distributed as Poisson with probability

Prob(y\* = j|x) = P(j) = exp(-λ)λj/j!.

The observed random variable before the censoring is is y = y\*|y\*>0. The probabilities are

Prob(y = j|x) = P(j)/[1 – P(0)].

Let yc = the censored random variable. Then, yc = y for y = 1,2,3,4. yc = 5 when y > 5. The probabilities associated with the observed yc are

Prob(yc = 1|x) = Prob(y = 1|x) = P(1)/[1-P(0)]

Prob(yc = 2|x) = Prob(y = 2|x) = P(2)/[1-P(0)]

Prob(yc = 3|x) = Prob(y = 3|x) = P(3)/[1-P(0)]

Prob(yc = 4|x) = Prob(y = 4|x) = P(4)/[1-P(0)]

Prob(yc = 5|x) = Prob(y = 5|x) + Prob(y = 6|x) + Prob(y = 7|x) + ...

The last term is an infinite sum. But,

Prob(y = 5|x) + Prob(y = 6|x) + Prob(y = 7|x) + ...

= 1 - Prob(y = 1|x) - Prob(y = 2|x) - Prob(y = 3|x) - Prob(y = 4|x)

Therefore,

Prob(yc = 5|x) = [1 – P(1) – P(2) – P(3) – P(4)]/[1 – P(0)].

These are the probabilities used to construct the log likelihood function for the observed values of yc, 1,2,3,4,5.

**Applications**

Application 18.1 is the same as Application 17.1.

2. Ordered Choice For Self Reported Marriage Rating

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| Ordered Probability Model |

| Maximum Likelihood Estimates |

| Dependent variable MARRIAGE |

| Weighting variable None |

| Number of observations 6366 |

| Iterations completed 15 |

| Log likelihood function -7720.145 |

| Number of parameters 12 |

| Info. Criterion: AIC = 2.42920 |

| Info. Criterion: BIC = 2.44194 |

| Restricted log likelihood -7926.487 |

| Underlying probabilities based on Normal |

+---------------------------------------------+

+---------------------------------------------+

| Ordered Probability Model |

| Cell frequencies for outcomes |

| Y Count Freq Y Count Freq Y Count Freq |

| 0 99 .015 1 348 .054 2 993 .155 |

| 3 2242 .352 4 2684 .421 |

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+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

---------+Index function for probability

Constant| 1.87997564 .12760529 14.733 .0000

YRB | -.09669427 .00649907 -14.878 .0000 .70537389

V2 | -.00624520 .00471646 -1.324 .1855 29.0828621

V3 | -.00952932 .00506534 -1.881 .0599 9.00942507

V4 | -.05879586 .01520251 -3.868 .0001 1.39687402

V5 | .10524384 .01624338 6.479 .0000 2.42617028

V6 | .02526318 .00727002 3.475 .0005 14.2098649

V7 | .02069865 .01614318 1.282 .1998 3.42412818

V8 | .02725715 .01072244 2.542 .0110 3.85014138

---------+Threshold parameters for index

Mu(1) | .71088354 .02219910 32.023 .0000

Mu(2) | 1.47186849 .01737814 84.697 .0000

Mu(3) | 2.46392113 .01923976 128.064 .0000

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| Summary of Marginal Effects for Ordered Probability Model (probit) |

+-------------------------------------------------------------------------+

Variable| Y=00 Y=01 Y=02 Y=03 Y=04 Y=05 Y=06 Y=07 |

--------------------------------------------------------------------------+

YRB .0031 .0087 .0167 .0093 -.0377

V2 .0002 .0006 .0011 .0006 -.0024

V3 .0003 .0009 .0016 .0009 -.0037

V4 .0019 .0053 .0101 .0056 -.0229

V5 -.0033 -.0095 -.0182 -.0101 .0411

V6 -.0008 -.0023 -.0044 -.0024 .0099

V7 -.0007 -.0019 -.0036 -.0020 .0081

V8 -.0009 -.0025 -.0047 -.0026 .0106

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| Cross tabulation of predictions. Row is actual, column is predicted. |

| Model = Probit . Prediction is number of the most probable cell. |

+-------+-------+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+

| Actual|Row Sum| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

+-------+-------+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+

| 0| 99| 0| 0| 0| 68| 31|

| 1| 348| 2| 0| 5| 170| 171|

| 2| 993| 7| 0| 7| 453| 526|

| 3| 2242| 3| 0| 10| 674| 1555|

| 4| 2684| 2| 0| 5| 593| 2084|

+-------+-------+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+

|Col Sum| 6366| 14| 0| 27| 1958| 4367| 0| 0| 0| 0| 0|

+-------+-------+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+

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? Application 18.3

?===================================================

Namelist ;x = age,educ,hhninc,hsat $

Poisson ; Lhs = HospVis ; Rhs = One,X

; Marginal effects $

Calc ; Lp = logl $

Regress ; Lhs = HospVis ; Rhs = One,X $

Negbin ; Lhs = HospVis ; Rhs = One,X

; Marginal effects $

Calc ; Ln = logl $

Calc ; List ; LRstat = 2\*(ln - lp) $

?===================================================

? Application 18.4

?===================================================

Sample ; All $

Regress ; Lhs = one ; Rhs = one ; Str = ID ; Panel $

Poisson ; Lhs = HospVis ; Rhs = One,X

; Marginal effects

; Pds = \_Groupti $

Poisson ; Lhs = HospVis ; Rhs = One,X

; Marginal effects

; Pds = \_Groupti ; Random $

+---------------------------------------------+

| Poisson Regression |

| Maximum Likelihood Estimates |

| Dependent variable HOSPVIS |

| Weighting variable None |

| Number of observations 27326 |

| Iterations completed 9 |

| Log likelihood function -12636.40 |

| Number of parameters 5 |

| Info. Criterion: AIC = .92523 |

| Info. Criterion: BIC = .92673 |

| Restricted log likelihood -13433.21 |

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+---------------------------------------------+

| Poisson Regression |

| Chi- squared =124476.35621 RsqP= .1947 |

| G - squared = 20025.66932 RsqD= .0737 |

| Overdispersion tests: g=mu(i) : 5.279 |

| Overdispersion tests: g=mu(i)^2: 5.468 |

+---------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| .12613692 .12567036 1.004 .3155

AGE | -.00340754 .00149685 -2.276 .0228 43.5256898

EDUC | -.05295428 .00834958 -6.342 .0000 11.3206310

HHNINC | .39889043 .08982355 4.441 .0000 .35208362

HSAT | -.24901310 .00634000 -39.277 .0000 6.78542607

+-------------------------------------------+

| Partial derivatives of expected val. with |

| respect to the vector of characteristics. |

| Effects are averaged over individuals. |

| Observations used for means are All Obs. |

| Conditional Mean at Sample Point .1383 |

| Scale Factor for Marginal Effects .1383 |

+-------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| .01743926 .02183573 .799 .4245

AGE | -.00047111 .00025979 -1.813 .0698 43.5256898

EDUC | -.00732128 .00149415 -4.900 .0000 11.3206310

HHNINC | .05514924 .01579375 3.492 .0005 .35208362

HSAT | -.03442771 .00220148 -15.638 .0000 6.78542607

+----------------------------------------------------+

| Ordinary least squares regression |

| LHS=HOSPVIS Mean = .1382566 |

| Standard deviation = .8843390 |

| WTS=none Number of observs. = 27326 |

| Model size Parameters = 5 |

| Degrees of freedom = 27321 |

| Residuals Sum of squares = 21121.96 |

| Standard error of e = .8792630 |

| Fit R-squared = .1159150E-01 |

| Adjusted R-squared = .1144679E-01 |

| Model test F[ 4, 27321] (prob) = 80.10 (.0000) |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| .49839670 .04097910 12.162 .0000

AGE | -.00064393 .00048945 -1.316 .1883 43.5256898

EDUC | -.00619390 .00241633 -2.563 .0104 11.3206310

HHNINC | .04936160 .03122845 1.581 .1140 .35208362

HSAT | -.04117251 .00240443 -17.124 .0000 6.78542607

+---------------------------------------------+

| Negative Binomial Regression |

| Dependent variable HOSPVIS |

| Number of observations 27326 |

| Iterations completed 9 |

| Log likelihood function -10044.46 |

| Number of parameters 6 |

| Info. Criterion: AIC = .73560 |

| Info. Criterion: BIC = .73740 |

| Restricted log likelihood -12636.40 |

+---------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| .10394982 .12631220 .823 .4105

AGE | -.00369348 .00143149 -2.580 .0099 43.5256898

EDUC | -.05795593 .00826247 -7.014 .0000 11.3206310

HHNINC | .38542430 .09259876 4.162 .0000 .35208362

HSAT | -.23323713 .00651715 -35.788 .0000 6.78542607

---------+Dispersion parameter for count data model

Alpha | 6.70461029 .17537071 38.231 .0000

+-------------------------------------------+

| Partial derivatives of expected val. with |

| respect to the vector of characteristics. |

| Effects are averaged over individuals. |

| Observations used for means are All Obs. |

| Conditional Mean at Sample Point .1367 |

| Scale Factor for Marginal Effects .1367 |

+-------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| .01421398 .02120646 .670 .5027

AGE | -.00050504 .00024071 -2.098 .0359 43.5256898

EDUC | -.00792483 .00146645 -5.404 .0000 11.3206310

HHNINC | .05270247 .01588312 3.318 .0009 .35208362

HSAT | -.03189257 .00226820 -14.061 .0000 6.78542607

+------------------------------------+

| Listed Calculator Results |

+------------------------------------+

LRSTAT = 5183.862874

2.

+---------------------------------------------+

| Panel Model with Group Effects |

| Dependent variable HOSPVIS |

| Weighting variable None |

| Number of observations 27326 |

| Log likelihood function -4198.145 |

| Number of parameters 4 |

| Info. Criterion: AIC = .30756 |

| Info. Criterion: BIC = .30876 |

| Unbalanced panel has 7293 individuals. |

| Missing or sumY=0, Skipped 5640 groups. |

| Poisson Regression -- Fixed Effects |

+---------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

AGE | -.00020613 .00705126 -.029 .9767 43.5256898

EDUC | -.04033708 .09220144 -.437 .6618 11.3206310

HHNINC | .49927712 .18484588 2.701 .0069 .35208362

HSAT | -.16686419 .01027579 -16.239 .0000 6.78542607

+-------------------------------------------+

| Partial derivatives of expected val. with |

| respect to the vector of characteristics. |

| They are computed at the means of the Xs. |

| Observations used for means are All Obs. |

| Conditional Mean at Sample Point .1383 |

| Scale Factor for Marginal Effects .1383 |

+-------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

AGE | -.284995D-04 .00097488 -.029 .9767 1.00000000

EDUC | -.00557687 .01274746 -.437 .6618 43.5256898

HHNINC | .06902836 .02555616 2.701 .0069 11.3206310

HSAT | -.02307008 .00142070 -16.239 .0000 .35208362

+---------------------------------------------+

| Panel Model with Group Effects |

| Dependent variable HOSPVIS |

| Number of observations 27326 |

| Log likelihood function -10200.91 |

| Number of parameters 6 |

| Info. Criterion: AIC = .74705 |

| Info. Criterion: BIC = .74885 |

| Unbalanced panel has 7293 individuals. |

| Poisson Regression -- Random Effects |

+---------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| -.22178663 .13617622 -1.629 .1034

AGE | -.00170639 .00145901 -1.170 .2422 43.5256898

EDUC | -.05399730 .01001912 -5.389 .0000 11.3206310

HHNINC | .40499179 .06938275 5.837 .0000 .35208362

HSAT | -.20075292 .00400154 -50.169 .0000 6.78542607

Alpha | 3.59227655 .11685254 30.742 .0000

+-------------------------------------------+

| Partial derivatives of expected val. with |

| respect to the vector of characteristics. |

| They are computed at the means of the Xs. |

| Observations used for means are All Obs. |

| Conditional Mean at Sample Point .1383 |

| Scale Factor for Marginal Effects .1383 |

+-------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| -.03066347 .01882726 -1.629 .1034

AGE | -.00023592 .00020172 -1.170 .2422 43.5256898

EDUC | -.00746548 .00138521 -5.389 .0000 11.3206310

HHNINC | .05599279 .00959262 5.837 .0000 .35208362

HSAT | -.02775542 .00055324 -50.169 .0000 6.78542607

18.5. Ship Accidents

Create ; logmth = log(months) $

Name ; X=logmth,one,ta,tb,tc,td,t6064,t6569,t7074,o6074$

Reject ; acc < 0 $

Pois ; lhs = acc ; Rhs = x $

Pois ; lhs = acc ; Rhs = x ; Rst = 1,9\_b $

Negb ; lhs = acc ; Rhs = x ; Rst = 1,9\_b,alpha $

+---------------------------------------------+

| Poisson Regression |

| Dependent variable ACC |

| Number of observations 34 |

| Log likelihood function -67.99930 |

| Number of parameters 10 |

| Info. Criterion: AIC = 4.58819 |

| Info. Criterion: BIC = 5.03712 |

| Restricted log likelihood -356.2029 |

+---------------------------------------------+

+---------------------------------------------+

| Poisson Regression |

| Chi- squared = 39.70580 RsqP= .9491 |

| G - squared = 38.13211 RsqD= .9380 |

| Overdispersion tests: g=mu(i) : .853 |

| Overdispersion tests: g=mu(i)^2: -.760 |

+---------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

LOGMTH | .90617018 .10174566 8.906 .0000 7.04925451

Constant| -4.61752968 .72938865 -6.331 .0000

TA | -.26966656 .24189066 -1.115 .2649 .20588235

TB | -.62826604 .32582681 -1.928 .0538 .20588235

TC | -1.03179604 .34039236 -3.031 .0024 .20588235

TD | -.40106977 .30540945 -1.313 .1891 .20588235

T6064 | -.36146212 .24726698 -1.462 .1438 .23529412

T6569 | .30035782 .21325393 1.408 .1590 .29411765

T7074 | .39874282 .20053445 1.988 .0468 .29411765

O6074 | -.36986273 .11821010 -3.129 .0018 .41176471

+---------------------------------------------+

| Poisson Regression |

| Maximum Likelihood Estimates |

| Dependent variable ACC |

| Number of observations 34 |

| Log likelihood function -68.41456 |

| Number of parameters 9 |

| Info. Criterion: AIC = 4.55380 |

| Info. Criterion: BIC = 4.95783 |

| Restricted log likelihood -356.2029 |

+---------------------------------------------+

+---------------------------------------------+

| Poisson Regression |

| Chi- squared = 42.44145 RsqP= .9456 |

| G - squared = 38.96262 RsqD= .9366 |

| Overdispersion tests: g=mu(i) : .934 |

| Overdispersion tests: g=mu(i)^2: -.613 |

+---------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

LOGMTH | 1.00000000 ......(Fixed Parameter).......

Constant| -5.25351861 .24642858 -21.319 .0000

TA | -.32052881 .23575203 -1.360 .1740 .20588235

TB | -.86524026 .19852119 -4.358 .0000 .20588235

TC | -1.00929327 .33950071 -2.973 .0030 .20588235

TD | -.39483795 .30680184 -1.287 .1981 .20588235

T6064 | -.44497064 .23323916 -1.908 .0564 .23529412

T6569 | .25087485 .20875483 1.202 .2295 .29411765

T7074 | .37248476 .19930193 1.869 .0616 .29411765

O6074 | -.38385913 .11826046 -3.246 .0012 .41176471

There is no evidence of overdispersion. The tests from the Poisson model are both insignificant, and the estimate of α in the negative binomial model is essentially zero.

+---------------------------------------------+

| Negative Binomial Regression |

| Dependent variable ACC |

| Weighting variable None |

| Number of observations 34 |

| Log likelihood function -68.42007 |

| Number of parameters 10 |

| Info. Criterion: AIC = 4.61295 |

| Finite Sample: AIC = 4.89428 |

| Info. Criterion: BIC = 5.06188 |

| Info. Criterion:HQIC = 4.76604 |

| NegBin form 2; Psi(i) = theta |

+---------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

LOGMTH | 1.00000000 ......(Fixed Parameter).......

Constant| -5.25074235 .26830333 -19.570 .0000

TA | -.32296435 .39695609 -.814 .4159 .20588235

TB | -.86731524 .20092395 -4.317 .0000 .20588235

TC | -1.01171406 .24980570 -4.050 .0001 .20588235

TD | -.39875463 .23889734 -1.669 .0951 .20588235

T6064 | -.44585250 .31679943 -1.407 .1593 .23529412

T6569 | .25060358 .27552926 .910 .3631 .29411765

T7074 | .37073607 .25504806 1.454 .1461 .29411765

O6074 | -.38364155 .15800844 -2.428 .0152 .41176471

---------+Dispersion parameter for count data model

Alpha | .648724D-04 .02406424 .003 .9978

**Chapter 19**

**Limited Dependent Variables – Truncation, Censoring and Sample**

**Selection**

**Exercises**

1. The sample mean of all 20 observations is 4.18222. For the 14 nonzero observations, the mean is (20/14)4.18222 = 5.9746. Both of these should overestimate μ. In the first case, all negative values have been transformed to zeroes. Therefore, if we had had the original data, our estimator would include the negative values as well as the positive ones. Since we have only the zeroes, instead, our estimator includes, for every negative *y*\* a number which is larger than the true *y*\*. This will inflate the estimate. Likewise, for the truncated mean, whereas a complete sample might include some negative values, the observed one will not. Once again, this will serve to inflate the estimator of the mean.

2. The log‑likelihood for the Tobit model is given in (19‑13). With only a constant term, this is

ln*L* = (-*n*1/2)[ln(2π) + lnσ2] ‑ (1/(2σ2))Σ1(*yi* - μ)2 + Σ0lnΦ(‑μ/σ)

In terms of γ and θ, this is ln*L* = (-*n*1/2)[ln(2π) - lnθ2] ‑ (1/2)Σ1(θ*yi* - γ)2 + Σ0lnΦ(‑γ)

= (-*n*1/2)ln(2π) + *n*1lnθ ‑ (1/2)Σ1(θ*yi* - γ)2 + Σ0lnΦ(‑γ).

The necessary conditions for maximizing this with respect to γ and θ are

∂ln*L*/∂γ = Σ1(θ*yi* ‑ γ) ‑ Σ0φ(‑γ)/Φ(‑γ) = θΣ1*yi* ‑ *n*1γ ‑ *n*0[φ(‑γ)/Φ(γ)] = 0

∂ln*L*/∂θ = *n*1/θ ‑ Σ1*yi*(θ*yi* ‑ γ) = *n*1/θ ‑ θΣ1*yi*2 + γΣ1*yi* = 0.

There are a few different ways one might solve these two equations. A grid search over the values of γ and θ is a possibility. A direct maximum likelihood estimator for the tobit model is the simpler choice if one is available. The model with only a constant term is otherwise the same as the usual model. Using the data above, the tobit maximum likelihood estimates are = 3.2731,  = 5.0303.

3. The log‑likelihood for the truncated regression with only a constant term is

ln*L* = (-*n*/2)[ln(2π) + lnσ2] ‑ (1/(2σ2))Σ1(y*i* - μ)2 ‑ Σ*i*lnΦ(μ/σ)

Once again transforming to γ and σ, this is

ln*L* = ‑(*n*/2)ln(2π) + *n*lnθ ‑ (1/2)Σ*i*(θ*yi* ‑ γ)2 ‑ nlnΦ(γ).

The necessary conditions for maximizing this are

∂ln*L*/∂γ = Σi(θ*yi* ‑ γ) ‑ *n*φ(γ)/Φ(γ) = 0

∂ln*L*/∂θ = *n*/θ ‑ Σ*iyi*(θ*yi* ‑ γ)

The first of the two equations can be = γ/θ + λ/θ, where λ = φ(γ)/Φ(γ). Now, reverting back to μ and σ, this is = μ + σλ which is (19‑6). The second equation can be manipulated to produce Σ*y*i2/*n* ‑ μ = σ2. Once again, trial and error could be used to find a solution. As before, estimating the model as a truncated regression with only a constant term will also produce a solution. The solution by this method is = 3.3439, = 5.6368. With the data of the first problem, we would have the following: Estimated Prob[*y*\* > 0] = 14/20 = .7. This is an estimate of Φ(μ/σ), so we would have μ/σ = Φ-1(.7) = .525 or μ = .525σ. Now, we can use the relationship *E*[*y*|*y* > 0] = μ + σφ(μ/σ)/Φ(μ/σ) = μ + σλ. Since μ/σ is now known, we have λ = φ(.525) / Φ(.525) = .496 so a second equation is 5.9746 = μ + .496σ. The joint solution is = 3.0697, = 5.8470. The three solutions are surprisingly close.

4. Using Theorem 19.5, we have 1 ‑ Φ(αz) = 14/35 = .4, αz = Φ-1(.6) = .253, λ(αz) = .9659,

δ(αz) = .6886. The two moment equations are based on the mean and variance of *y* in the observed data, 5.9746 and 9.869, respectively. The equations would be 5.9746 = μ + σ(.7)(.9659) and 9.869 = σ2(1 ‑ .72(.6886)). The joint solution is = 3.3651, = 3.8594.

5. The conditional mean function is E[y|**x**] = Φ(**β′x**i/σi)**β′xi** + σiΦ(**β′x**i/σi) using the equation before (19-12). Suppose that σi = σexp(**α′x**i) for the same vector **x**i. (We’ll relax that assumption shortly.) Now, differentiate this expression with respect to **x**. We differentiate the two parts, first with respect to **β′x** then with respect to σi.



After collecting the terms, we obtain ∂E[yi|**x**i]/∂**x**i = Φ(ai)**β** + σiφ(ai)**α** where ai = **β′x**i/σi. Thus, the marginal effect has two parts. one for **β** and one for α. Now, if a variable appears in σi but not in **x**i, then only the second term appears while if a variable appears only in **x**i and not in σi, then only the first term appears in the marginal effect.

6. The transformed log likelihood function is

logL = Σy > 0 (-1/2)[log2π - logθ2 + (θy - **x′γ**)2] + Σy=0 log[1-Φ(**x′γ**)]

It will be convenient to define ai = **x**i′**γ**. Note also that 1 - Φ(ai) = Φ(-ai). The first derivatives and Hessian in the transformed parameters are



The second derivatives can be collected in a matrix format:



where δi is the last scalar term in ∂2logL/∂δ∂**γ′**. By Theorem 19.2 (see (19-4)), we know that δi is negative. Thus, all three parts of the matrix are negative semidefinite. Assuming the data are not linearly dependent and there are more than K observations, the Hessian will have full rank and be negative definite.

**Applications**

1. Tobit model for Psychology Today Data

-----------------------------------------------------------------------------

Ordinary least squares regression ............

LHS=Y Mean = 1.45591

Standard deviation = 3.29876

No. of observations = 601 Degrees of freedom

Regression Sum of Squares = 860.129 8

Residual Sum of Squares = 5668.95 592

Total Sum of Squares = 6529.08 600

Standard error of e = 3.09450

Fit R-squared = .13174 R-bar squared = .12000

Model test F[ 8, 592] = 11.22774 Prob F > F\* = .00000

Diagnostic Log likelihood = -1527.15358 Akaike I.C. = 2.27411

Restricted (b=0) = -1569.60277

Chi squared [ 8] = 84.89838 Prob C2 > C2\* = .00000

--------+--------------------------------------------------------------------

| Standard Prob. 95% Confidence

Y| Coefficient Error z |z|>Z\* Interval

--------+--------------------------------------------------------------------

Constant| 5.87201\*\*\* 1.13750 5.16 .0000 3.64256 8.10146

Z1| .05409 .30049 .18 .8572 -.53486 .64303

Z2| -.05098\*\* .02262 -2.25 .0242 -.09531 -.00664

Z3| .16947\*\*\* .04122 4.11 .0000 .08867 .25027

Z4| -.14262 .35020 -.41 .6838 -.82900 .54375

Z5| -.47761\*\*\* .11173 -4.27 .0000 -.69660 -.25863

Z6| -.01375 .06414 -.21 .8303 -.13946 .11196

Z7| .10492 .08888 1.18 .2378 -.06929 .27912

Z8| -.71188\*\*\* .12001 -5.93 .0000 -.94709 -.47667

--------+--------------------------------------------------------------------

Note: \*\*\*, \*\*, \* ==> Significance at 1%, 5%, 10% level.

-----------------------------------------------------------------------------

-----------------------------------------------------------------------------

Limited Dependent Variable Model - CENSORED

Dependent variable Y

Log likelihood function -704.73107

Estimation based on N = 601, K = 10

Inf.Cr.AIC = 1429.462 AIC/N = 2.378

Model estimated: Mar 30, 2011, 00:10:58

Threshold values for the model:

Lower= .0000 Upper=+infinity

LM test [df] for tobit= 10.513[ 9]

Normality Test, LM = 3.338[ 2]

ANOVA based fit measure = .690931

DECOMP based fit measure = .454706

--------+--------------------------------------------------------------------

| Standard Prob. 95% Confidence

Y| Coefficient Error z |z|>Z\* Interval

--------+--------------------------------------------------------------------

|Primary Index Equation for Model

Constant| 7.60849\* 3.90599 1.95 .0514 -.04711 15.26408

Z1| .94579 1.06287 .89 .3735 -1.13739 3.02897

Z2| -.19270\*\* .08097 -2.38 .0173 -.35139 -.03400

Z3| .53319\*\*\* .14661 3.64 .0003 .24584 .82054

Z4| 1.01918 1.27958 .80 .4257 -1.48874 3.52710

Z5| -1.69900\*\*\* .40548 -4.19 .0000 -2.49373 -.90427

Z6| .02536 .22767 .11 .9113 -.42086 .47158

Z7| .21298 .32116 .66 .5072 -.41647 .84244

Z8| -2.27329\*\*\* .41541 -5.47 .0000 -3.08747 -1.45910

|Disturbance standard deviation

Sigma| 8.25844\*\*\* .55458 14.89 .0000 7.17148 9.34539

--------+--------------------------------------------------------------------

Note: \*\*\*, \*\*, \* ==> Significance at 1%, 5%, 10% level.

-----------------------------------------------------------------------------

-----------------------------------------------------------------------------

Partial derivatives of expected val. with

respect to the vector of characteristics.

They are computed at the means of the Xs.

Observations used for means are All Obs.

Conditional Mean at Sample Point 1.1181

Scale Factor for Marginal Effects .2322

--------+--------------------------------------------------------------------

| Partial Standard Prob. 95% Confidence

Y| Effect Error z |z|>Z\* Interval

--------+--------------------------------------------------------------------

Z1| .21962 .24630 .89 .3726 -.26311 .70235

Z2| -.04475\*\* .01874 -2.39 .0170 -.08148 -.00802

Z3| .12381\*\*\* .03390 3.65 .0003 .05737 .19025

Z4| .23666 .29626 .80 .4244 -.34399 .81732

Z5| -.39452\*\*\* .09315 -4.24 .0000 -.57709 -.21196

Z6| .00589 .05286 .11 .9113 -.09772 .10950

Z7| .04946 .07463 .66 .5075 -.09682 .19573

Z8| -.52788\*\*\* .09613 -5.49 .0000 -.71629 -.33946

--------+--------------------------------------------------------------------

Note: \*\*\*, \*\*, \* ==> Significance at 1%, 5%, 10% level.

-----------------------------------------------------------------------------

---------------------------------------------------------------------

Partial Effects Analysis for Tobit (Censored) Regression Function

---------------------------------------------------------------------

Effects on function with respect to Z1

Partial effects for binary var Z1 computed by first difference

---------------------------------------------------------------------

df/dZ1 Partial Standard

(Delta method) Effect Error |t| 95% Confidence Interval

---------------------------------------------------------------------

Partial effect .24202 .27248 .89 -.29203 .77606

---------------------------------------------------------------------

Effects on function with respect to Z2

Partial effects for continuous Z2 computed by differentiation

Effect is computed as derivative = df(.)/dx

---------------------------------------------------------------------

df/dZ2 Partial Standard

(Delta method) Effect Error |t| 95% Confidence Interval

---------------------------------------------------------------------

Partial effect -.04921 .02078 2.37 -.08993 -.00849

---------------------------------------------------------------------

Effects on function with respect to Z3

Partial effects for continuous Z3 computed by differentiation

Effect is computed as derivative = df(.)/dx

---------------------------------------------------------------------

df/dZ3 Partial Standard

(Delta method) Effect Error |t| 95% Confidence Interval

---------------------------------------------------------------------

Partial effect .13616 .03805 3.58 .06158 .21075

---------------------------------------------------------------------

Effects on function with respect to Z4

Partial effects for binary var Z4 computed by first difference

---------------------------------------------------------------------

df/dZ4 Partial Standard

(Delta method) Effect Error |t| 95% Confidence Interval

---------------------------------------------------------------------

Partial effect .25085 .30312 .83 -.34326 .84495

---------------------------------------------------------------------

Effects on function with respect to Z5

Partial effects for continuous Z5 computed by differentiation

Effect is computed as derivative = df(.)/dx

---------------------------------------------------------------------

df/dZ5 Partial Standard

(Delta method) Effect Error |t| 95% Confidence Interval

---------------------------------------------------------------------

Partial effect -.43388 .10561 4.11 -.64086 -.22690

---------------------------------------------------------------------

Effects on function with respect to Z6

Partial effects for continuous Z6 computed by differentiation

Effect is computed as derivative = df(.)/dx

---------------------------------------------------------------------

df/dZ6 Partial Standard

(Delta method) Effect Error |t| 95% Confidence Interval

---------------------------------------------------------------------

Partial effect .00648 .05814 .11 -.10747 .12043

---------------------------------------------------------------------

Effects on function with respect to Z7

Partial effects for continuous Z7 computed by differentiation

Effect is computed as derivative = df(.)/dx

---------------------------------------------------------------------

df/dZ7 Partial Standard

(Delta method) Effect Error |t| 95% Confidence Interval

---------------------------------------------------------------------

Partial effect .05439 .08209 .66 -.10651 .21529

---------------------------------------------------------------------

Effects on function with respect to Z8

Partial effects for continuous Z8 computed by differentiation

Effect is computed as derivative = df(.)/dx

---------------------------------------------------------------------

df/dZ8 Partial Standard

(Delta method) Effect Error |t| 95% Confidence Interval

---------------------------------------------------------------------

Partial effect -.58054 .10890 5.33 -.79397 -.36710

Application 19.2. Mroz labor supply data:

-----------------------------------------------------------------------------

Limited Dependent Variable Model - CENSORED

Dependent variable WHRS

Log likelihood function -3900.06588

Estimation based on N = 753, K = 9

Inf.Cr.AIC = 7818.132 AIC/N = 10.383

Model estimated: Mar 30, 2011, 00:20:47

Threshold values for the model:

Lower= .0000 Upper=+infinity

LM test [df] for tobit= 33.995[ 8]

Normality Test, LM = 8.212[ 2]

ANOVA based fit measure = .051845

DECOMP based fit measure = .167662

--------+--------------------------------------------------------------------

| Standard Prob. 95% Confidence

WHRS| Coefficient Error z |z|>Z\* Interval

--------+--------------------------------------------------------------------

|Primary Index Equation for Model

Constant| 1690.88\*\*\* 507.7730 3.33 .0009 695.66 2686.10

KL6| -1084.05\*\*\* 126.5730 -8.56 .0000 -1332.13 -835.97

K618| -126.952\*\*\* 42.53643 -2.98 .0028 -210.322 -43.582

WA| -26.0634\* 14.25488 -1.83 .0675 -54.0025 1.8756

WE| 138.029\*\*\* 28.37337 4.86 .0000 82.419 193.640

HA| -18.7077 14.00194 -1.34 .1815 -46.1510 8.7356

HE| -54.0294\*\* 21.65038 -2.50 .0126 -96.4634 -11.5955

CIT| -46.6573 109.4845 -.43 .6700 -261.2429 167.9283

|Disturbance standard deviation

Sigma| 1272.87\*\*\* 47.84181 26.61 .0000 1179.10 1366.63

--------+--------------------------------------------------------------------

Note: \*\*\*, \*\*, \* ==> Significance at 1%, 5%, 10% level.

-----------------------------------------------------------------------------

-----------------------------------------------------------------------------

Partial derivatives of expected val. with

respect to the vector of characteristics.

They are computed at the means of the Xs.

Observations used for means are All Obs.

Conditional Mean at Sample Point 671.6324

Scale Factor for Marginal Effects .5931

--------+--------------------------------------------------------------------

| Partial Standard Prob. 95% Confidence

WHRS| Effect Error z |z|>Z\* Interval

--------+--------------------------------------------------------------------

KL6| -642.905\*\*\* 73.70149 -8.72 .0000 -787.357 -498.453

K618| -75.2897\*\*\* 25.26098 -2.98 .0029 -124.8003 -25.7791

WA| -15.4571\* 8.44964 -1.83 .0674 -32.0181 1.1039

WE| 81.8593\*\*\* 16.73570 4.89 .0000 49.0579 114.6606

HA| -11.0947 8.30358 -1.34 .1815 -27.3694 5.1800

HE| -32.0425\*\* 12.83415 -2.50 .0125 -57.1970 -6.8880

CIT| -27.6705 64.93211 -.43 .6700 -154.9351 99.5941

--------+--------------------------------------------------------------------

Note: \*\*\*, \*\*, \* ==> Significance at 1%, 5%, 10% level.

-----------------------------------------------------------------------------

Application 19.3. We break the tobit model into a two part model

The first equation is for labor force participation. The second is for hours given LFP = 1. The two equations are a probit model for LFP and a truncated regression for positive hours. The chi squared statistic is 61.1 with 9 degrees of freedom. The critical value is 15.51, so the null ypothesis of the tobit model is rejected infavor of a model that provides for different participation and hours equations.

**--> calc ; lc = logl $**

**--> namelist ; x = one,kl6,k618,wa,we,ha,he,cit$**

**--> probit;lhs=lfp;rhs=x$**

Normal exit: 5 iterations. Status=0, F= 461.2908

-----------------------------------------------------------------------------

Binomial Probit Model

Dependent variable LFP

Log likelihood function -461.29077

Restricted log likelihood -514.87320

Chi squared [ 7 d.f.] 107.16486

Significance level .00000

McFadden Pseudo R-squared .1040692

Estimation based on N = 753, K = 8

Inf.Cr.AIC = 938.582 AIC/N = 1.246

Model estimated: Mar 30, 2011, 06:22:15

Hosmer-Lemeshow chi-squared = 15.79209

P-value= .04545 with deg.fr. = 8

--------+--------------------------------------------------------------------

| Standard Prob. 95% Confidence

LFP| Coefficient Error z |z|>Z\* Interval

--------+--------------------------------------------------------------------

|Index function for probability

Constant| .98816\*\* .50094 1.97 .0485 .00634 1.96998

KL6| -.90345\*\*\* .11423 -7.91 .0000 -1.12735 -.67956

K618| -.05390 .04025 -1.34 .1805 -.13279 .02499

WA| -.02554\* .01338 -1.91 .0562 -.05175 .00068

WE| .16084\*\*\* .02776 5.79 .0000 .10642 .21525

HA| -.01651 .01330 -1.24 .2142 -.04257 .00955

HE| -.05034\*\* .02074 -2.43 .0152 -.09099 -.00969

CIT| -.04387 .10467 -.42 .6751 -.24901 .16127

--------+--------------------------------------------------------------------

Note: \*\*\*, \*\*, \* ==> Significance at 1%, 5%, 10% level.

-----------------------------------------------------------------------------

**--> calc ; lp=logl$**

**--> reject ; lfp=0$**

**--> trunc ; lhs=whrs;rhs=x $**

Normal exit: 5 iterations. Status=0, F= 3407.819

-----------------------------------------------------------------------------

Limited Dependent Variable Model - TRUNCATE

Dependent variable WHRS

Log likelihood function -3407.81897

Estimation based on N = 428, K = 9

Inf.Cr.AIC = 6833.638 AIC/N = 15.966

Model estimated: Mar 30, 2011, 06:22:15

Threshold values for the model:

Lower= .0000 Upper=+infinity

Observations after truncation 428

--------+--------------------------------------------------------------------

| Standard Prob. 95% Confidence

WHRS| Coefficient Error z |z|>Z\* Interval

--------+--------------------------------------------------------------------

|Primary Index Equation for Model

Constant| 2524.20\*\*\* 502.0844 5.03 .0000 1540.13 3508.27

KL6| -576.991\*\*\* 170.1457 -3.39 .0007 -910.470 -243.511

K618| -180.278\*\*\* 47.06950 -3.83 .0001 -272.533 -88.023

WA| -2.53480 15.47448 -.16 .8699 -32.86423 27.79462

WE| -11.3288 28.86337 -.39 .6947 -67.8999 45.2424

HA| -11.8879 14.93989 -.80 .4262 -41.1695 17.3938

HE| -24.1512 22.07714 -1.09 .2740 -67.4216 19.1192

CIT| -33.8840 112.7479 -.30 .7638 -254.8659 187.0978

|Disturbance standard deviation

Sigma| 900.020\*\*\* 48.45086 18.58 .0000 805.058 994.982

--------+--------------------------------------------------------------------

Note: \*\*\*, \*\*, \* ==> Significance at 1%, 5%, 10% level.

-----------------------------------------------------------------------------

**--> calc ; lt=logl$**

**--> calc ; list ; lrtest = 2\*((lp+lt)-lc)$**

[CALC] LRTEST = 61.9122923

**--> calc ; list ; ctb(.95,kreg)$**

[CALC] \*Result\*= 15.5073131

=============================================================

Application 19.4

=============================================================

create ; lc = log(cost/pf)$

create ; lq = log(q) ; lq2 = lq\*lq $

create ; lpk=log(pk/pf) $

create ; lpl=log(pl/pf)$

sample ; 1-123 $

frontier ; lhs = lc ; cost ; rhs = one,lq,lq2,lpk,lpl ; eff=ucost $

dstat ; rhs = ucost $

kernel ; rhs = ucost $

plot ; lhs = q ; rhs = ucost ; grid $

-----------------------------------------------------------------------------

Limited Dependent Variable Model - FRONTIER

Dependent variable LC

Log likelihood function 66.86502

Estimation based on N = 123, K = 7

Inf.Cr.AIC = -119.730 AIC/N = -.973

Model estimated: Mar 30, 2011, 06:33:40

Variances: Sigma-squared(v)= .01185

Sigma-squared(u)= .02233

Sigma(v) = .10884

Sigma(u) = .14944

Sigma = Sqr[(s^2(u)+s^2(v)]= .18488

Stochastic Cost Frontier Model, e = v+u

--------+--------------------------------------------------------------------

| Standard Prob. 95% Confidence

LC| Coefficient Error z |z|>Z\* Interval

--------+--------------------------------------------------------------------

|Primary Index Equation for Model

Constant| -7.49421\*\*\* .32997 -22.71 .0000 -8.14095 -6.84747

LQ| .41098\*\*\* .03599 11.42 .0000 .34043 .48152

LQ2| .03029\*\*\* .00247 12.27 .0000 .02545 .03513

LPK| .05531 .06002 .92 .3567 -.06232 .17295

LPL| .26059\*\*\* .06554 3.98 .0001 .13212 .38905

|Variance parameters for compound error

Lambda| 1.37312\*\*\* .29718 4.62 .0000 .79066 1.95557

Sigma| .18488\*\*\* .00114 162.29 .0000 .18264 .18711

--------+--------------------------------------------------------------------

Note: \*\*\*, \*\*, \* ==> Significance at 1%, 5%, 10% level.

-----------------------------------------------------------------------------

**--> dstat ; rhs = ucost $**

Descriptive Statistics

--------+---------------------------------------------------------------------

Variable| Mean Std.Dev. Minimum Maximum Cases Missing

--------+---------------------------------------------------------------------

UCOST| .118840 .614993E-01 .298216E-01 .378595 123 0

--------+---------------------------------------------------------------------

**--> kernel ; rhs = ucost $**

+---------------------------------------+

| Kernel Density Estimator for UCOST |

| Observations = 123 |

| Points plotted = 123 |

| Bandwidth = .021141 |

| Statistics for abscissa values---- |

| Mean = .118840 |

| Standard Deviation = .061499 |

| Minimum = .029822 |

| Maximum = .378595 |

| ---------------------------------- |

| Kernel Function = Logistic |

| Cross val. M.S.E. = .000000 |

| Results matrix = KERNEL |

+---------------------------------------+





?============================================================

? Applications in Chapter 19.

? This is based on Fair’s Redbook data

?============================================================

? 1. Tobit, Scaled Tobit, Probit and Truncated Regression.

? In principle, all are estimating the same paramter.

? For consistency and convenience, we are going to use the

? sample with YRB <= 5 only.

?============================================================

Sample ; All $

Reject ; YRB > 5 $

Namelist ; X = one,v1,v2,v3,v4,v5$

Tobit ; Lhs = yrb ; Rhs = x ; marginal $

Matrix ; list ; scaled\_b = 1/s \* b $

Probit ; Lhs = a ; Rhs = x $

reject ; yrb <= 0 $

Truncation ; Lhs = yrb ; Rhs = x $

+---------------------------------------------+

| Limited Dependent Variable Model - CENSORED |

| Maximum Likelihood Estimates |

| Dependent variable YRB |

| Weighting variable None |

| Number of observations 6217 |

| Iterations completed 6 |

| Log likelihood function -6118.089 |

| Number of parameters 7 |

| Info. Criterion: AIC = 1.97043 |

| Finite Sample: AIC = 1.97044 |

| Info. Criterion: BIC = 1.97802 |

| Info. Criterion:HQIC = 1.97306 |

| Threshold values for the model: |

| Lower= .0000 Upper=+infinity |

| LM test [df] for tobit= 622.887[ 6] |

| Normality Test, LM = 150.850[ 2] |

| ANOVA based fit measure = .293201 |

| DECOMP based fit measure = .438743 |

+---------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

---------+Primary Index Equation for Model

Constant| 4.13828429 .31908252 12.969 .0000

V1 | -.80415431 .03782416 -21.260 .0000 4.12272800

V2 | -.06923599 .01229186 -5.633 .0000 29.1829661

V3 | .10402446 .01325380 7.849 .0000 9.12329098

V4 | -.02190617 .03898707 -.562 .5742 1.41499115

V5 | -.43110692 .04356398 -9.896 .0000 2.43670581

---------+Disturbance standard deviation

Sigma | 2.27697641 .04212836 54.049 .0000

+-------------------------------------------+

| Partial derivatives of expected val. with |

| respect to the vector of characteristics. |

| They are computed at the means of the Xs. |

| Observations used for means are All Obs. |

| Conditional Mean at Sample Point .3941 |

| Scale Factor for Marginal Effects .2796 |

+-------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| 1.15697490 .09110678 12.699 .0000

V1 | -.22482418 .01048093 -21.451 .0000 4.12272800

V2 | -.01935689 .00342807 -5.647 .0000 29.1829661

V3 | .02908299 .00367661 7.910 .0000 9.12329098

V4 | -.00612449 .01090115 -.562 .5742 1.41499115

V5 | -.12052818 .01207702 -9.980 .0000 2.43670581

Sigma | .000000 ......(Fixed Parameter).......

Matrix SCALED\_B has 6 rows and 1 columns.

1

+--------------

1| 1.81745

2| -.35317

3| -.03041

4| .04569

5| -.00962

6| -.18933

+---------------------------------------------+

| Binomial Probit Model |

| Maximum Likelihood Estimates |

| Dependent variable A |

| Weighting variable None |

| Number of observations 6217 |

| Iterations completed 5 |

| Log likelihood function -3310.310 |

| Number of parameters 6 |

| Info. Criterion: AIC = 1.06685 |

| Info. Criterion: BIC = 1.07335 |

| Restricted log likelihood -3830.126 |

+---------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

---------+Index function for probability

Constant| 2.03641060 .15678428 12.989 .0000

V1 | -.41449474 .01860450 -22.279 .0000 4.12272800

V2 | -.03568737 .00593540 -6.013 .0000 29.1829661

V3 | .07215336 .00640693 11.262 .0000 9.12329098

V4 | -.00241124 .01891503 -.127 .8986 1.41499115

V5 | -.21212886 .02089864 -10.150 .0000 2.43670581

+---------------------------------------------+

| Limited Dependent Variable Model - TRUNCATE |

| Maximum Likelihood Estimates |

| Dependent variable YRB |

| Weighting variable None |

| Number of observations 1904 |

| Iterations completed 8 |

| Log likelihood function -2437.473 |

| Number of parameters 7 |

| Info. Criterion: AIC = 2.56772 |

| Info. Criterion: BIC = 2.58813 |

| Threshold values for the model: |

| Lower= .0000 Upper=+infinity |

| Observations after truncation 1904 |

+---------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

---------+Primary Index Equation for Model

Constant| 5.22651388 .94010948 5.559 .0000

V1 | -.45753380 .10715203 -4.270 .0000 3.65388655

V2 | -.04779763 .03766086 -1.269 .2044 30.9776786

V3 | -.25376184 .04622853 -5.489 .0000 11.6919643

V4 | -.37961397 .12878071 -2.948 .0032 1.81407563

V5 | -.22780476 .13328147 -1.709 .0874 2.28308824

---------+Disturbance standard deviation

Sigma | 2.38479704 .13327563 17.894 .0000

4. Strikes. There are 9 years of data. The number of strikes is 8,6,11,3,3,2,19,2,9. The Poisson regression is shown below. It does appear that the number of strikes is significantly related to the PROD variable. However, with only 9 observations, use of the asymptotic distribution for the test is probably overly optimistic. The result is probably borderline.

+---------------------------------------------+

| Poisson Regression |

| Dependent variable \_GROUPTI |

| Weighting variable None |

| Number of observations 9 |

| Log likelihood function -28.99317 |

| Number of parameters 2 |

| Info. Criterion: AIC = 6.88737 |

| Info. Criterion: BIC = 6.93120 |

| Restricted log likelihood -31.19884 |

+---------------------------------------------+

+---------------------------------------------+

| Poisson Regression |

| Chi- squared = 25.08061 RsqP= .2317 |

| G - squared = 26.13767 RsqD= .1444 |

| Overdispersion tests: g=mu(i) : 1.954 |

| Overdispersion tests: g=mu(i)^2: 2.618 |

+---------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| 1.90854253 .12998621 14.683 .0000

PROD | 5.16576744 2.51306610 2.056 .0398 -.00302000