**Chapter 11**

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**Models for Panel Data**

1. The pooled least squares estimator is

= ‑.747476 + 1.058959*x*, **e′e** = 120.6687

(.95595) (.058656)

The fixed effects regression can be computed just by including the three dummy variables since the sample sizes are quite small. The results are

= ‑ 1.4684*i*1 ‑ 2.8362*i*2 + .12166*i*3 + 1.102192*x* **e′e** = 79.183.

(.050719)

The *F* statistic for testing the hypothesis that the constant terms are all the same is

*F*[26,2] = [(120.6687 ‑ 79.183)/2]/[79.183/26] = 6.811.

The critical value from the *F* table is 19.458, so the hypothesis is not rejected.

In order to estimate the random effects model, we need some additional parameter estimates. The group means are ** 

Group 1 15.502 14.962

Group 2 15.415 16.559

Group 3 14.373 12.930

In the group means regression using these three observations, we obtain

*i.* = 10.665 + .29909*i.* with **e**\*\***′e**\*\* = .19747.

There is only one degree of freedom, so this is the candidate for estimation of σε2/*T* + σ*u*2. In the least squares dummy variable (fixed effects) regression, we have an estimate of σε2 of 79.183/26 = 3.045. Therefore, our estimate of σ*u*2 is = .19747/1 ‑ 3.045/10 = ‑.6703. Obviously, this won't do. Before abandoning the random effects model, we consider an alternative consistent estimator of the constant and slope, the pooled ordinary least squares estimator. Using the group means above, we find

Σ[*i.* ‑ (‑.747476) ‑ 1.058959*i.*]2 = 3.9273.

One ought to proceed with some caution at this point, but it is difficult to place much faith in the group means regression with but a single degree of freedom, so this is probably a preferable estimator in any event. (The true model underlying these data ‑‑ using a random number generator ‑‑ has a slope, β of 1.000 and a true constant of zero. Of course, this would not be known to the analyst in a real world situation.) Continuing, we now use = 3.9273 ‑ 3.045/10 = 3.6227 as the estimator. (The true value of ρ = σ*u*2/(σ*u*2+σε2) is .5.) This leads to θ = 1 ‑ [3.04551/2/(10(3.6227) + 3.045)1/2] = .721524. Finally, the FGLS estimator computed according to (16‑48) is = ‑1.3415(.786) + 1.0987 (.028998)*x*.

For the LM test, we return to the pooled ordinary least squares regression. The necessary quantities are **e′e** = 120.6687, Σ*t e*1*t* = ‑.55314, Σ*t e*2*t* = ‑13.72824, Σ*t e*3*t* = 14.28138. Therefore,

*LM* = {[3(10)]/[2(9)]}{[(‑.55314)2 + (13.72824)2 + (14.28138)2]/120.687 ‑ 1}2 = 8.4683

The statistic has one degree of freedom. The critical value from the chi‑squared distribution is 3.84, so the hypothesis of no random effect is rejected. Finally, for the Hausman test, we compare the FGLS and least squares dummy variable estimators. The statistic is χ2 = [(1.0987 ‑ 1.058959)2]/[(.058656)2 ‑ (.05060)2] = 1.794373. This is relatively small and argues (once again) in favor of the random effects model.

2. There is no effect on the coefficients of the other variables. For the dummy variable coefficients, with the full set of *n* dummy variables, each coefficient is

*i* \*= mean residual for the *i*th group in the regression of *y* on the *x*s omitting the dummy variables.

(We use the partitioned regression results of Chapter 6.) If an overall constant term and *n*‑1 dummy variables (say the last *n*‑1) are used, instead, the coefficient on the *i*th dummy variable is simply *i*\*- **1\*while the constant term is still**1\*For a full proof of these results, see the solution to Exercise 5 of Chapter 8 earlier in this book.

3. (a) The pooled OLS estimator will be where **X**i and **y**i have *Ti* observations. It remains true that **y***i* = **X***i***β** + **ε***i* + *ui***i**, where Var[**ε***i* + *ui***i**|Xi] = Var[**w***i*|**X***i*] = σε2**I** + σu2**ii′** and, maintaining the assumptions, both εi and *ui* are uncorrelated with **X**i. Substituting the expression for yi into that of b and collecting terms, we have

.

Unbiasedness follows immediately as long as E[**w**i|**X**i] equals zero, which it does by assumption. Consistency, as mentioned in Section 11.5.1, is covered in the discussion of Chapter 4. We would need for the matrix **Q** = to converge to a matrix of constants, or not to degenerate to a matrix of zeros. The requirements for the large sample behavior of the vector in the second set of brackets is quite the same as in our earlier discussions of consistency. The vector  has mean zero. We would require the conditions of the Lindeberg-Feller version of the central theorem to apply, which could be expected.

(b) We seek to establish consistency, not unbiasedness. As such, we will ignore the degrees of freedom correction, -K, in (11-40). Use n(T-1) as the denominator. Thus, the question is whether

plim

If so, then the estimator in (11-40) will be consistent. Using (11-36) and *eit* - , it follows that . Summing the squares in (11-40), we find that the estimator in (11-40)



The second term will converge to zero as the center matrix converges to a constant Q and the vectors converge to zero as b converges to β. (We use the Slutsky theorem.) The third term will converge to zero as both the leading vector converges to zero and the covariance vector between the regressors and the disturbances converges to zero. That leaves the first term, which is the average of the estimators in (11-37). The terms in the average are independent. Each has expected value exactly equal to σε2. So, if each estimator has finite variance, then the average will converge to its expectation. Appendix D discusses various different conditions underwhich a sample average will converge to its expectation. For example, finite fouth moment of εit would be sufficient here (though weaker conditions would also suffice). Note that this derivation follows through for any consistent estimator of **β**, not just for **b**.

4**.** To find plim(1/*n*)LM = plim [*T*/(2(*T*-1))]{[Σ*i*(Σ*teit*)2]/[Σ*i*Σ*teit*2] - 1}2 we can concentrate on the sums inside the curled brackets. First, Σ*i*(Σ*teit*)2 = *nT*2{(1/*n*)Σ*i*[(1/*T*)Σ*teit*]2} and Σ*i*Σ*teit*2 = *nT*(1/(*nT*))Σ*i*Σ*teit*2. The ratio equals [Σ*i*(Σ*teit*)2]/[Σ*i*Σ*teit*2] = *T*{(1/*n*)Σ*i*[(1/*T*)Σ*teit*]2}/{(1/(*nT*))Σ*i*Σ*teit*2}. Using the argument used in Exercise 8 to establish consistency of the variance estimator, the limiting behavior of this statistic is the same as that which is computed using the true disturbances since the OLS coefficient estimator is consistent. Using the true disturbances, the numerator may be written (1/*n*)Σ*i*[(1/*T*)Σ*t*ε*it*]2 = (1/n)Σ*i* Since *E*[] = 0, plim(1/*n*)Σ*i* = Var[] = σε2*T* + σ*u*2The denominator is simply the usual variance estimator, so plim(1/(*nT*))Σ*i*Σ*t*ε*it*2 = Var[ε*it*] = σε2+ σ*u*2Therefore, inserting these results in the expression for LM, we find that plim (1/*n*)LM = [*T*/(2(*T*-1))]{[*T*(σε2*T* + σ*u*2)]/[σε2+ σ*u*2] - 1}2. Under the null hypothesis that σ*u*2 = 0, this equals 0. By expanding the inner term then collecting terms, we find that under the alternative hypothesis that σ*u*2 is not equal to 0, plim (1/*n*)LM = [*T*(*T*-1)/2][ σ*u*2/(σε2+σ*u*2)]2. Within group *i*, Corr2[ε*it*,ε*is*] = ρ2 = σ*u*2/(σ*u*2+ σε2) so plim (1/*n*)LM = [*T*(*T*‑1)/2](ρ2)2. It is worth noting what is obtained if we do not divide the LM statistic by *n* at the outset. Under the null hypothesis, the limiting distribution of LM is chi‑squared with one degree of freedom. This is a random variable with mean 1 and variance 2, so the statistic, itself, does not converge to a constant; it converges to a random variable. Under the alternative, the LM statistic has mean and variance of order *n* (as we see above) and hence, explodes. It is this latter attribute which makes the test a consistent one. As the sample size increases, the power of the LM test must go to 1.

5. The ordinary least squares regression results are

*R*2 = .92803, **e′e** = 146.761, 40 observations

**Variable Coefficient Standard Error**

*X*1 .446845 .07887

*X*2 1.83915 .1534

Constant 3.60568 2.555

Period 1 ‑3.57906 1.723

Period 2 ‑1.49784 1.716

Period 3 2.00677 1.760

Period 4 ‑3.03206 1.731

Period 5 ‑5.58937 1.768

Period 6 ‑1.49474 1.714

Period 7 1.52021 1.714

Period 8 ‑2.25414 1.737

Period 9 ‑3.29360 1.722

Group 1 ‑.339998 1.135

Group 2 4.39271 1.183

Group 3 5.00207 1.125

**Estimated covariance matrix for the slopes:**

β1β2

β1 .0062209

β2 .00030947 .023523

For testing the hypotheses that the sets of dummy variable coefficients are zero, we will require the sums of squared residuals from the restrictions. These are

**Regression Sum of squares**

All variables included 146.761

Period variables omitted 318.503

Group variables omitted 369.356

Period and group variables omitted 585.622

The *F* statistics are therefore,

(1) *F*[9,25] = [(318.503 ‑ 146.761)/9]/[146.761/25] = 3.251

(2) *F*[3,25] = [(369.356 ‑ 146.761)/3]/[146.761/25] = 12.639

(3) *F*[12,25] = [(585.622 ‑ 146.761)/12]/[146.761/25] = 6.23

The critical values for the three distributions are 2.283, 2.992, and 2.165, respectively. All sample statistics are larger than the table value, so all of the hypotheses are rejected.

6. The covariance matrix would be

****

7. The two separate regressions are as follows:

**Sample 1 Sample 2**

*b* = **x′y/x′x** 4/5 = .8 6/10 = .6

**e′e** = **y′y** ‑ *b***x′y** 20 ‑ 4(4/5) = 84/5 10 ‑ 6(6/10) = 64/10

*R*2 = 1 ‑ **e′e**/**y′y** 1 ‑ (84/5)/20 = .16 1 ‑ (64/10)/10 = .36

*s*2 = **e′e**/(*n*‑1) (84/5)/19 = .88421 (64/10)/19 = .33684

Est.Var[*b*] = *s*2/**x′x** .88421/5 = .17684 .33684/10 = .033684

To carry out a Lagrange multiplier test of the hypothesis of equal variances, we require the separate and common variance estimators based on the restricted slope estimator. This, in turn, is the pooled least squares estimator. For the combined sample, we obtain

*b* = [**x**1**′y**1 + **x**2**′y**2]/[**x**1**′x**1 + **x**2**′x**2] = (4 + 6) / (5 + 10) = 2/3.

Then, the variance estimators are based on this estimate. For the hypothesized common variance,

**e′e** = (**y**1**′y**1 + **y**2**′y**2) ‑ *b*(**x**1**′y**1 + **x**2**′y**2) = (20 + 10) ‑ (2/3)(4 + 6) = 70/3,

so the estimate of the common variance is **e′e**/40 = (70/3)/40 = .58333. Note that the divisor is 40, not 39, because we are comptuting maximum likelihood estimators. The individual estimators are

**e**1**′e**1/20 = (**y**1**′y**1 ‑ 2*b*(**x**1**′y**1) + *b*2(**x**1**′x**1))/20 = (20 ‑ 2(2/3)4 + (2/3)25)/20 = .84444

and  **e**2**′e**2/20 = (**y**2**′y**2 ‑ 2*b*(**x**2**′y**2) + *b*2(**x**2**′x**2))/20 = (10 ‑ 2(2/3)6 + (2/3)210)/20 = .32222.

The LM statistic is given in Section 14.9.2.a.

*LM* = (*T*/2)[(*s*12/*s*2 ‑ 1)2 + (*s*22/s2 ‑ 1)2] = 10[(.84444/.58333 ‑ 1)2 + (.32222/.58333 ‑ 1)2] = 4.007.

This has one degree of freedom for the single restriction. The critical value from the chi‑squared table is 3.84, so we would reject the hypothesis.

In order to compute a two step GLS estimate, we can use either the original variance estimates based on the separate least squares estimates or those obtained above in doing the LM test. Since both pairs are consistent, both FGLS estimators will have all of the desirable asymptotic properties. For our estimator, we used  = **e***j***′e***j*/*T* from the original regressions. Thus,  = .84 and  = .32. The GLS estimator is

= [(1/ )**x**1**′y**1 + (1/)**x**2**′y**2]/[ (1/ )**x**1**′x**1 + (1/)**x**2**′x**2] = [4/.84 + 6/.32]/[5/.84 + 10/.32] = .632.

The estimated sampling variance is 1/[ (1/ )**x**1**′x**1 + (1/)**x**2**′x**2] = .02688. This implies an asymptotic standard error of (.02688)2 = .16395. To test the hypothesis that β = 1, we would refer z = (.632 ‑ 1) / .16395 = ‑2.245 to a standard normal table. This is reasonably large, and at the usual significance levels, would lead to rejection of the hypothesis.

The Wald test is based on the unrestricted variance estimates. Using *b* = .632, the variance estimators are  = [**y**1**′y**1 ‑ 2*b*(**x**1**′y**1) + *b*2(**x**1**′x**1)]/20 = .847056

and = [**y**2**′y**2 ‑ 2*b*(**x**2**′y**2) + *b*2(**x**2**′x**2)]/20 = .320512

while the pooled estimator would be = [**y′y** ‑ 2*b*(**x′y**) + *b*2(**x′x**)]/40 = .583784. The statistic is given at the end of Example 16.3, *W* = (*T*/2)[(  /  ‑ 1)2 + (/ ‑ 1)2]

= 10[(.583784/.847056 ‑ 1)2 + (.583784/.320512 ‑ 1)2] = 7.713.

We reach the same conclusion as before.

To compute the maximum likelihood estimators, we begin our iterations from the two separate ordinary least squares estimates of *b* which produce estimates  = .84 and  = .32. The iterations are

Iteration   

0 .840000 .320000 .632000

1 .847056 .320512 .631819

2 .847071 .320506 .631818

3 .847071 .320506 converged

Now, to compute the likelihood ratio statistic for a likelihood ratio test of the hypothesis of equal variances, we refer χ2 = 40ln.58333 ‑ 20ln.847071 ‑ 20ln.320506 to the chi‑squared table. (Under the null hypothesis, the pooled least squares estimator is maximum likelihood.) Thus, χ2 = 4.5164, which is roughly equal to the LM statistic and leads once again to rejection of the null hypothesis.

Finally, we allow for cross sectional correlation of the disturbances. Our initial estimate of *b* is the pooled least squares estimator, 2/3. The estimates of the two variances are .84444 and .32222 as before while the cross sectional covariance estimate is

**e1′e2**/20 = [**y**1**′y**2 ‑ *b*(**x**1**′y**2 + **x**2**′y**1) + *b*2(**x**1**′x**2)]/20 = .14444.

Before proceeding, we note, the estimated squared correlation of the two disturbances is

*r* = .14444 / [(.84444)(.32222)]1/2 = .277,

which is not particularly large. The LM test statistic given in (9-37) is 1.533, which is well under the critical value of 3.84. Thus, we would not reject the hypothesis of zero cross section correlation. Nonetheless, we proceed. The estimator is shown in (9-35). The two step FGLS and iterated maximum likelihood estimates appear below. Iteration 12 22 12 

0 .84444 .32222 .14444 .5791338

1 .8521955 .3202177 .1597994 .5731058

2 .8528702 .3203616 .1609133 .5727069

3 .8529155 .3203725 .1609873 .5726805

4 .8529185 .3203732 .1609921 .5726788

5 .8529187 .3203732 .1609925 converged

Because the correlation is relatively low, the effect on the previous estimate is relatively minor.

8. If all of the regressor matrices are the same, the estimator in (9-35) reduces to

= (**X′X**)-1 Σ {(1/σ*i*2)/[Σ  (1/σ*j*2)]}**X′y***i* = Σ *wi***b***i*

a weighted average of the ordinary least squares estimators, **b***i* = (**X′X**)-1**X′y***i* with weights

*wi* = (1/σ*i*2)/[Σ (1/σ*j*2)]. If it were necessary to estimate the weights, a simple two step estimator could be based on individual variance estimators. Either of *si*2 = **e***i***′e***i*/*T* based on separate least squares regressions (with different estimators of β) or based on residuals computed from a common pooled ordinary least squares slope estimator could be used.

9. The various least squares estimators of the parameters are

**Sample 1 Sample 2 Sample 3 Pooled**

*a* 11.6644 5.42213 1.41116 8.06392

(9.658) (10.46) (7.328)

*b* .926881 1.06410 1.46885 1.05413

(.4328) (.4756) (.3590)

**e′e** 452.206 673.409 125.281

(464.288) (732.560) (171.240) (1368.088)

(Values of **e′e** in parentheses above are based on the pooled slope estimator.) The FGLS estimator and its estimated asymptotic covariance matrix are

**b** = , Est.Asy.Var[**b**] = 

Note that the FGLS estimator of the slope is closer to the 1.46885 of sample 3 (the highest of the three OLS estimates). This is to be expected since the third group has the smallest residual variance. The LM test statistic is based on the pooled regression,

*LM* = (10/2){[(464.288/10)/(1368.088/30) ‑ 1]2 + ...} = 3.7901

To compute the Wald statistic, we require the unrestricted regression. The parameter estimates are given above. The sums of squares are 465.708, 785.399, and 145.055 for i = 1, 2, and 3, respectively. For the common estimate of σ2, we use the total sum of squared GLS residuals, 1396.162. Then,

*W* = (10/2){[(1396.162/30)/(465.708/10) ‑ 1]2 + ...} = 25.21.

The Wald statistic is far larger than the *LM* statistic. Since there are two restrictions, at significance levels of 95% or 99% with critical values of 5.99 or 9.21, the two tests lead to different conclusions. The likelihood ratio statistic based on the FGLS estimates is χ2 = 30ln(1396.162/30) ‑ 10ln(465.708/10) ... = 6.42

which is between the previous two and between the 95% and 99% critical values.

**Applications**

As usual, the applications below require econometric software. The computations can be done with any modern software package, so no specific program is recommended.

--> read $

Last observation read from data file was 200

End of data listing in edit window was reached

--> REGRESS ; Lhs = I ; Rhs = F,C,one $

+----------------------------------------------------+

| Ordinary least squares regression |

| LHS=I Mean = 145.9582 |

| Standard deviation = 216.8753 |

| WTS=none Number of observs. = 200 |

| Model size Parameters = 3 |

| Degrees of freedom = 197 |

| Residuals Sum of squares = 1755850. |

| Standard error of e = 94.40840 |

| Fit R-squared = .8124080 |

| Adjusted R-squared = .8105035 |

| Model test F[ 2, 197] (prob) = 426.58 (.0000) |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

F | .11556216 .00583571 19.803 .0000 1081.68110

C | .23067849 .02547580 9.055 .0000 276.017150

Constant| -42.7143694 9.51167603 -4.491 .0000

--> CALC ; R0=Rsqrd $

--> REGRESS ; Lhs = I ; Rhs = F,C,one; Cluster = 20 $

+----------------------------------------------------+

| Ordinary least squares regression |

| LHS=I Mean = 145.9582 |

| Standard deviation = 216.8753 |

| WTS=none Number of observs. = 200 |

| Model size Parameters = 3 |

| Degrees of freedom = 197 |

| Residuals Sum of squares = 1755850. |

| Standard error of e = 94.40840 |

| Fit R-squared = .8124080 |

| Adjusted R-squared = .8105035 |

| Model test F[ 2, 197] (prob) = 426.58 (.0000) |

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+---------------------------------------------------------------------+

| Covariance matrix for the model is adjusted for data clustering. |

| Sample of 200 observations contained 10 clusters defined by |

| 20 observations (fixed number) in each cluster. |

| Sample of 200 observations contained 1 strata defined by |

| 200 observations (fixed number) in each stratum. |

+---------------------------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

F | .11556216 .01589434 7.271 .0000 1081.68110

C | .23067849 .08496711 2.715 .0072 276.017150

Constant| -42.7143694 20.4252029 -2.091 .0378

The standard errors increase substantially. This is at least suggestive that there is correlation across observations within the groups. A formal test would be based on one of the panel models below. When the random effects model is fit by maximum likelihood, for example, the log likelihood function is -1095.257. The log likelihood function for the pooled model is -1191.802. Thus, the correlation is highly significant. The Lagrange multiplier statistic reported below is 798.16, which is far larger than the critical value of 3.84. Once again, these results do suggest within groups correlation.

--> REGRESS ; Lhs = I ; Rhs = F,C,one ; Panel ; Pds=20 ; Fixed $

+----------------------------------------------------+

| Least Squares with Group Dummy Variables |

| Ordinary least squares regression |

| LHS=I Mean = 145.9583 |

| Standard deviation = 216.8753 |

| WTS=none Number of observs. = 200 |

| Model size Parameters = 12 |

| Degrees of freedom = 188 |

| Residuals Sum of squares = 523478.1 |

| Standard error of e = 52.76797 |

| Fit R-squared = .9440725 |

| Adjusted R-squared = .9408002 |

| Model test F[ 11, 188] (prob) = 288.50 (.0000) |

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| Panel:Groups Empty 0, Valid data 10 |

| Smallest 20, Largest 20 |

| Average group size 20.00 |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

F | .11012380 .01185669 9.288 .0000 1081.68110

C | .31006534 .01735450 17.867 .0000 276.017150

+--------------------------------------------------------------------+

| Test Statistics for the Classical Model |

+--------------------------------------------------------------------+

| Model Log-Likelihood Sum of Squares R-squared |

|(1) Constant term only -1359.15096 .9359943929D+07 .0000000 |

|(2) Group effects only -1216.34872 .2244352274D+07 .7602173 |

|(3) X - variables only -1191.80236 .1755850484D+07 .8124080 |

|(4) X and group effects -1070.78103 .5234781474D+06 .9440725 |

+--------------------------------------------------------------------+

| Hypothesis Tests |

| Likelihood Ratio Test F Tests |

| Chi-squared d.f. Prob. F num. denom. P value |

|(2) vs (1) 285.604 9 .00000 66.932 9 190 .00000 |

|(3) vs (1) 334.697 2 .00000 426.576 2 197 .00000 |

|(4) vs (1) 576.740 11 .00000 288.500 11 188 .00000 |

|(4) vs (2) 291.135 2 .00000 309.014 2 188 .00000 |

|(4) vs (3) 242.043 9 .00000 49.177 9 188 .00000 |

+--------------------------------------------------------------------+

--> CALC ; R1 = Rsqrd $

--> MATRIX ; bf = b(1:2) ; vf = varb(1:2,1:2) $

--> CALC ; List ; Fstat=((R1-R0)/9)/((1-R1)/(n-2-10))

; FC=Ftb(.95,9,(n-2-10)) $

+------------------------------------+

| Listed Calculator Results |

+------------------------------------+

FSTAT = 49.176625

FC = 1.929957

The F statistic of 49.18 is far larger than the critical value, so the hypothesis of equal constant terms is rejected.

--> REGRESS ; Lhs = I ; Rhs = F,C,one

; Panel ; Pds=20 ; Random $

+--------------------------------------------------+

| Random Effects Model: v(i,t) = e(i,t) + u(i) |

| Estimates: Var[e] = .278446D+04 |

| Var[u] = .612849D+04 |

| Corr[v(i,t),v(i,s)] = .687594 |

| Lagrange Multiplier Test vs. Model (3) = 798.16 |

| ( 1 df, prob value = .000000) |

| (High values of LM favor FEM/REM over CR model.) |

| Sum of Squares .184029D+07 |

| R-squared .803387D+00 |

+--------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

F | .10974919 .01031952 10.635 .0000 1081.68110

C | .30780890 .01715154 17.946 .0000 276.017150

Constant| -57.7159079 27.1118671 -2.129 .0333

The LM statistic, as noted earlier, is very large, so the hypothesis of no effects is rejected.

--> MATRIX ; br = b(1:2) ; vr = varb(1:2,1:2) $

--> MATRIX ; db = bf-br ; vdb = vf-vr ; List ; Hausman=db'<vdb>db $

1

+--------------

1| 2.45500

--> CALC ; List ; Ctb(.95,2) $

+------------------------------------+

| Listed Calculator Results |

+------------------------------------+

Result = 5.991465

The Hausman statistic is quite small, which suggests that the random effects approach is consistent with the data.

2.

create ; logc=log(cost/pfuel)

; logp1=log(pmtl/pfuel)

; logp2=log(peqpt/pfuel)

; logp3=log(plabor/pfuel)

; logp4=log(pprop/pfuel)

; logp5=log(kprice/pfuel)

; logq=log(output)

; logq2=.5\*logq^2 $

Namelist ; cd = logp1,logp2,logp3,logp4,logp5 $

create

; p11=.5\* logp1^2

; p22=.5\* logp2^2

; p33=.5\* logp3^2

; p44=.5\* logp4^2

; p55=.5\* logp5^2

; p12=logp1\*logp2

; p13=logp1\*logp3

; p14=logp1\*logp4

; p15=logp1\*logp5

; p23=logp2\*logp3

; p24=logp2\*logp4

; p25=logp2\*logp5

; p34=logp3\*logp4

; p35=logp3\*logp5

; p45=logp4\*logp5 $

Namelist ; tl = p11,p12,p13,p14,p15,p22,p23,p24,p25,p33,p34,p35,p44,p45,p55$

Namelist ; z = loadfctr,stage,points $

regress;lhs=logc;rhs=one,logq,logq2,cd,z $

+----------------------------------------------------+

| Ordinary least squares regression |

| LHS=LOGC Mean = .7723984 |

| Standard deviation = 1.074424 |

| WTS=none Number of observs. = 256 |

| Model size Parameters = 11 |

| Degrees of freedom = 245 |

| Residuals Sum of squares = 2.965806 |

| Standard error of e = .1100242 |

| Fit R-squared = .9899249 |

| Adjusted R-squared = .9895136 |

| Model test F[ 10, 245] (prob) =2407.23 (.0000) |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

Constant| 20.3856176 22.8643711 .892 .3735

LOGQ | .95227889 .01832119 51.977 .0000 -1.11237037

LOGQ2 | .06568531 .01060839 6.192 .0000 1.45687077

LOGP1 | -.32662031 1.17956412 -.277 .7821 .37999226

LOGP2 | -.28619766 .56614750 -.506 .6136 -.25308254

LOGP3 | .16012937 .08634095 1.855 .0649 .66688211

LOGP4 | -.00519153 .07328859 -.071 .9436 -2.14504306

LOGP5 | 1.43718160 1.78896723 .803 .4225 -12.6860637

LOADFCTR| -.94688632 .18441822 -5.134 .0000 .54786115

STAGE | -.00021794 .402227D-04 -5.418 .0000 507.879666

POINTS | .00199712 .00031682 6.304 .0000 72.9843750

?

? Turns out the translog model cannot be computed with the firm

? dummy variables. I'll use the Cobb Douglas form.

?

regress;lhs=logc;rhs= one,logq,logq2,cd ; panel ; pds=ti $

+----------------------------------------------------+

| OLS Without Group Dummy Variables |

| Ordinary least squares regression |

| LHS=LOGC Mean = .7723984 |

| Standard deviation = 1.074424 |

| WTS=none Number of observs. = 256 |

| Model size Parameters = 8 |

| Degrees of freedom = 248 |

| Residuals Sum of squares = 4.190133 |

| Standard error of e = .1299834 |

| Fit R-squared = .9857657 |

| Adjusted R-squared = .9853639 |

| Model test F[ 7, 248] (prob) =2453.53 (.0000) |

+----------------------------------------------------+

+----------------------------------------------------+

| Panel Data Analysis of LOGC [ONE way] |

| Unconditional ANOVA (No regressors) |

| Source Variation Deg. Free. Mean Square |

| Between 272.013 24. 11.3339 |

| Residual 22.3551 231. .967752E-01 |

| Total 294.368 255. 1.15439 |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

LOGQ | .93708702 .01772733 52.861 .0000 -1.11237037

LOGQ2 | .07754607 .01211431 6.401 .0000 1.45687077

LOGP1 | -.94586281 1.38855410 -.681 .4964 .37999226

LOGP2 | -.79081045 .66530892 -1.189 .2357 -.25308254

LOGP3 | .01998606 .09963618 .201 .8412 .66688211

LOGP4 | .08893118 .08543313 1.041 .2989 -2.14504306

LOGP5 | 2.63118115 2.10504302 1.250 .2125 -12.6860637

Constant| 35.4178566 26.9017806 1.317 .1892

+----------------------------------------------------+

| Least Squares with Group Dummy Variables |

| Ordinary least squares regression |

| LHS=LOGC Mean = .7723984 |

| Standard deviation = 1.074424 |

| WTS=none Number of observs. = 256 |

| Model size Parameters = 32 |

| Degrees of freedom = 224 |

| Residuals Sum of squares = .9373686 |

| Standard error of e = .6468911E-01 |

| Fit R-squared = .9968157 |

| Adjusted R-squared = .9963750 |

| Model test F[ 31, 224] (prob) =2261.94 (.0000) |

+----------------------------------------------------+

+----------------------------------------------------+

| Panel:Groups Empty 0, Valid data 25 |

| Smallest 2, Largest 15 |

| Average group size 10.24 |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

LOGQ | .66448665 .03580894 18.556 .0000 -1.11237037

LOGQ2 | -.00955723 .01280811 -.746 .4563 1.45687077

LOGP1 | 1.84750938 .76113884 2.427 .0159 .37999226

LOGP2 | .73986763 .37612716 1.967 .0503 -.25308254

LOGP3 | -.05323942 .06396335 -.832 .4060 .66688211

LOGP4 | .22763995 .04625120 4.922 .0000 -2.14504306

LOGP5 | -1.83738098 1.16995945 -1.570 .1176 -12.6860637

+--------------------------------------------------------------------+

| Test Statistics for the Classical Model |

+--------------------------------------------------------------------+

| Model Log-Likelihood Sum of Squares R-squared |

|(1) Constant term only -381.12407 .2943684435D+03 .0000000 |

|(2) Group effects only -51.16832 .2235506489D+02 .9240575 |

|(3) X - variables only 163.14470 .4190132631D+01 .9857657 |

|(4) X and group effects 354.81332 .9373685874D+00 .9968157 |

+--------------------------------------------------------------------+

| Hypothesis Tests |

| Likelihood Ratio Test F Tests |

| Chi-squared d.f. Prob. F num. denom. P value |

|(2) vs (1) 659.911 24 .00000 117.116 24 231 .00000 |

|(3) vs (1) 1088.538 7 .00000 2453.527 7 248 .00000 |

|(4) vs (1) 1471.875 31 .00000 2261.945 31 224 .00000 |

|(4) vs (2) 811.963 7 .00000 731.160 7 224 .00000 |

|(4) vs (3) 383.337 24 .00000 32.388 24 224 .00000 |

+--------------------------------------------------------------------+

+--------------------------------------------------+

| Random Effects Model: v(i,t) = e(i,t) + u(i) |

| Estimates: Var[e] = .418468D-02 |

| Var[u] = .127110D-01 |

| Corr[v(i,t),v(i,s)] = .752323 |

| Lagrange Multiplier Test vs. Model (3) = 479.37 |

| ( 1 df, prob value = .000000) |

| (High values of LM favor FEM/REM over CR model.) |

| Baltagi-Li form of LM Statistic = 174.85 |

| Fixed vs. Random Effects (Hausman) = 40.99 |

| ( 7 df, prob value = .000001) |

| (High (low) values of H favor FEM (REM).) |

| Sum of Squares .648771D+01 |

| R-squared .978056D+00 |

+--------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

LOGQ | .79769706 .02494671 31.976 .0000 -1.11237037

LOGQ2 | .02011534 .01130089 1.780 .0751 1.45687077

LOGP1 | 1.11671466 .74579390 1.497 .1343 .37999226

LOGP2 | .27128619 .36294718 .747 .4548 -.25308254

LOGP3 | -.10761385 .06138583 -1.753 .0796 .66688211

LOGP4 | .18385724 .04550246 4.041 .0001 -2.14504306

LOGP5 | -.49374865 1.13625272 -.435 .6639 -12.6860637

Constant| -4.53328730 14.5229534 -.312 .7549

regress;lhs=logc;rhs=z,one,logq,logq2,cd ; panel ; pds=ti $

+----------------------------------------------------+

| OLS Without Group Dummy Variables |

| Ordinary least squares regression |

| LHS=LOGC Mean = .7723984 |

| Standard deviation = 1.074424 |

| WTS=none Number of observs. = 256 |

| Model size Parameters = 11 |

| Degrees of freedom = 245 |

| Residuals Sum of squares = 2.965806 |

| Standard error of e = .1100242 |

| Fit R-squared = .9899249 |

| Adjusted R-squared = .9895136 |

| Model test F[ 10, 245] (prob) =2407.23 (.0000) |

+----------------------------------------------------+

+----------------------------------------------------+

| Panel Data Analysis of LOGC [ONE way] |

| Unconditional ANOVA (No regressors) |

| Source Variation Deg. Free. Mean Square |

| Between 272.013 24. 11.3339 |

| Residual 22.3551 231. .967752E-01 |

| Total 294.368 255. 1.15439 |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

LOADFCTR| -.94688632 .18441823 -5.134 .0000 .54786115

STAGE | -.00021794 .402227D-04 -5.418 .0000 507.879666

POINTS | .00199712 .00031682 6.304 .0000 72.9843750

LOGQ | .95227889 .01832119 51.977 .0000 -1.11237037

LOGQ2 | .06568531 .01060839 6.192 .0000 1.45687077

LOGP1 | -.32662033 1.17956418 -.277 .7821 .37999226

LOGP2 | -.28619767 .56614753 -.506 .6136 -.25308254

LOGP3 | .16012937 .08634095 1.855 .0649 .66688211

LOGP4 | -.00519153 .07328859 -.071 .9436 -2.14504306

LOGP5 | 1.43718164 1.78896732 .803 .4225 -12.6860637

Constant| 20.3856181 22.8643723 .892 .3735

+----------------------------------------------------+

| Least Squares with Group Dummy Variables |

| Ordinary least squares regression |

| LHS=LOGC Mean = .7723984 |

| Standard deviation = 1.074424 |

| WTS=none Number of observs. = 256 |

| Model size Parameters = 35 |

| Degrees of freedom = 221 |

| Residuals Sum of squares = .7726037 |

| Standard error of e = .5912651E-01 |

| Fit R-squared = .9973754 |

| Adjusted R-squared = .9969716 |

| Model test F[ 34, 221] (prob) =2470.05 (.0000) |

+----------------------------------------------------+

+----------------------------------------------------+

| Panel:Groups Empty 0, Valid data 25 |

| Smallest 2, Largest 15 |

| Average group size 10.24 |

+----------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |t-ratio |P[|T|>t]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

LOADFCTR| -.89457348 .14242570 -6.281 .0000 .54786115

STAGE | -.00022827 .894260D-04 -2.553 .0113 507.879666

POINTS | .00010341 .00041551 .249 .8037 72.9843750

LOGQ | .75278467 .03923479 19.187 .0000 -1.11237037

LOGQ2 | -.00324835 .01306645 -.249 .8039 1.45687077

LOGP1 | 1.38217070 .72421015 1.909 .0575 .37999226

LOGP2 | .61609241 .35323609 1.744 .0824 -.25308254

LOGP3 | .00706546 .05918620 .119 .9051 .66688211

LOGP4 | .14433953 .04404683 3.277 .0012 -2.14504306

LOGP5 | -1.25331458 1.10477945 -1.134 .2577 -12.6860637

+--------------------------------------------------------------------+

| Test Statistics for the Classical Model |

+--------------------------------------------------------------------+

| Model Log-Likelihood Sum of Squares R-squared |

|(1) Constant term only -381.12407 .2943684435D+03 .0000000 |

|(2) Group effects only -51.16832 .2235506489D+02 .9240575 |

|(3) X - variables only 207.37940 .2965806000D+01 .9899249 |

|(4) X and group effects 379.55705 .7726036853D+00 .9973754 |

+--------------------------------------------------------------------+

| Hypothesis Tests |

| Likelihood Ratio Test F Tests |

| Chi-squared d.f. Prob. F num. denom. P value |

|(2) vs (1) 659.911 24 .00000 117.116 24 231 .00000 |

|(3) vs (1) 1177.007 10 .00000 2407.226 10 245 .00000 |

|(4) vs (1) 1521.362 34 .00000 2470.054 34 221 .00000 |

|(4) vs (2) 861.451 10 .00000 617.357 10 221 .00000 |

|(4) vs (3) 344.355 24 .00000 26.140 24 221 .00000 |

+--------------------------------------------------------------------+

+--------------------------------------------------+

| Random Effects Model: v(i,t) = e(i,t) + u(i) |

| Estimates: Var[e] = .349594D-02 |

| Var[u] = .860939D-02 |

| Corr[v(i,t),v(i,s)] = .711206 |

| Lagrange Multiplier Test vs. Model (3) = 466.36 |

| ( 1 df, prob value = .000000) |

| (High values of LM favor FEM/REM over CR model.) |

| Baltagi-Li form of LM Statistic = 170.10 |

| Fixed vs. Random Effects (Hausman) = 44.65 |

| (10 df, prob value = .000003) |

| (High (low) values of H favor FEM (REM).) |

| Sum of Squares .451094D+01 |

| R-squared .984812D+00 |

+--------------------------------------------------+

+--------+--------------+----------------+--------+--------+----------+

|Variable| Coefficient | Standard Error |b/St.Er.|P[|Z|>z]| Mean of X|

+--------+--------------+----------------+--------+--------+----------+

LOADFCTR| -1.07921018 .13264921 -8.136 .0000 .54786115

STAGE | -.00016415 .672354D-04 -2.441 .0146 507.879666

POINTS | .00044792 .00035950 1.246 .2128 72.9843750

LOGQ | .86611837 .02783747 31.113 .0000 -1.11237037

LOGQ2 | .02222380 .01102947 2.015 .0439 1.45687077

LOGP1 | .92719911 .70150544 1.322 .1863 .37999226

LOGP2 | .30782803 .33937387 .907 .3644 -.25308254

LOGP3 | -.02581955 .05671735 -.455 .6489 .66688211

LOGP4 | .09284095 .04277517 2.170 .0300 -2.14504306

LOGP5 | -.36595849 1.06514141 -.344 .7312 -12.6860637

Constant| -2.36774378 13.6315073 -.174 .8621

matrix ; List ; bz=b(1:3);vz=varb(1:3,1:3) ; wald = bz'<vz>bz $

Matrix WALD has 1 rows and 1 columns.

1

+--------------

1| 74.33957

**Chapter 12**

⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

**Estimation Frameworks in**

**Econometrics**

Exercise

1. A fully parametric model/estimator provides consistent, efficient, and comparatively precise results. The semiparametric model/estimator, by comparison, is relatively less precise in general terms. But, the payoff to this imprecision is that the semiparametric formulation is more likely to be robust to failures of the assumptions of the parametric model. Consider, for example, the binary probit model of Chapter 21, which makes a strong assumption of normality and homoscedasticity. If the assumptions are correct, the probit estimator is the most efficient use of the data. However, if the normality assumption or the homoscedasticity assumption are incorrect, then the probit estimator becomes inconsistent in an unknown fashion. Lewbel’s semiparametric estimator for the binary choice model, in contrast, is not very precise in comparison to the probit model. But, it will remain consistent if the normality assumption is violated, and it is even robust to certain kinds of heteroscedasticity.

**Applications**

1. Using the gasoline market data in Appendix Table F2.2, use the partially linear regression method in Section 16.3.3 to fit an equation of the form

ln(*G*/*Pop*) = *β*1ln(*Income*) + *β*2ln*Pnew cars* + *β*3ln*Pused cars* + *g*(ln*Pgasoline*) + *ε*

crea;gp=lg;ip=ly;ncp=lpnc;upp=lpuc;pgp=lpg$

sort;lhs=pgp;rhs=gp,ip,ncp,upp$

crea;dgp=.809\*gp - .5\*gp[-1] - .309\*gp[-2]$

crea;dip=.809\*ip - .5\*ip[-1] - .309\*ip[-2]$

crea;dnc=.809\*ncp -.5\*ncp[-1]-.309\*ncp[-2]$

crea;duc=.809\*upp -.5\*upp[-1]-.309\*upp[-2]$

samp;3-36$

regr;lhs=dgp;rhs=dip,dnc,duc;res=e$

+-----------------------------------------------------------------------+

| Ordinary least squares regression Weighting variable = none |

| Dep. var. = DGP Mean= .9708646870E-02, S.D.= .4738748109E-01 |

| Model size: Observations = 34, Parameters = 3, Deg.Fr.= 31 |

| Residuals: Sum of squares= .1485994289E-01, Std.Dev.= .02189 |

| Fit: R-squared= .799472, Adjusted R-squared = .78653 |

| Model test: F[ 2, 31] = 61.80, Prob value = .00000 |

| Diagnostic: Log-L = 83.2587, Restricted(b=0) Log-L = 55.9431 |

| LogAmemiyaPrCrt.= -7.559, Akaike Info. Crt.= -4.721 |

| Model does not contain ONE. R-squared and F can be negative! |

| Autocorrel: Durbin-Watson Statistic = 1.34659, Rho = .32671 |

+-----------------------------------------------------------------------+

+---------+--------------+----------------+--------+---------+----------+

|Variable | Coefficient | Standard Error |t-ratio |P[|T|>t] | Mean of X|

+---------+--------------+----------------+--------+---------+----------+

DIP .9629902959 .11631885 8.279 .0000 .14504254E-01

DNC -.1010972781 .87755182E-01 -1.152 .2581 .20153536E-01

DUC -.3197058148E-01 .51875022E-01 -.616 .5422 .35656776E-01

--> matr;varpl={1+1/(2\*2)}\*varb$

--> matr;stat(b,varpl)$

+---------------------------------------------------+

|Number of observations in current sample = 34 |

|Number of parameters computed here = 3 |

|Number of degrees of freedom = 31 |

+---------------------------------------------------+

+---------+--------------+----------------+--------+---------+

|Variable | Coefficient | Standard Error |b/St.Er.|P[|Z|>z] |

+---------+--------------+----------------+--------+---------+

B\_1 .9629902959 .13004843 7.405 .0000

B\_2 -.1010972781 .98113277E-01 -1.030 .3028

B\_3 -.3197058148E-01 .57998037E-01 -.551 .5815

2.

+---------------------------------------+

| Nonparametric Regression for G |

| Observations = 36 |

| Points plotted = 36 |

| Bandwidth = .468092 |

| Statistics for abscissa values---- |

| Mean = 2.316611 |

| Standard Deviation = 1.251735 |

| Minimum = .914000 |

| Maximum = 4.109000 |

| ---------------------------------- |

| Kernel Function = Logistic |

| Cross val. M.S.E. = 121.084982 |

| Results matrix = KERNEL |

+---------------------------------------+



3. A. Using the probit model and the Klein and Spady semiparametric models, the two sets of coefficient estimates are somewhat similar.

+---------------------------------------------+

| Binomial Probit Model |

| Maximum Likelihood Estimates |

| Model estimated: Jul 31, 2002 at 05:16:40PM.|

| Dependent variable P |

| Weighting variable None |

| Number of observations 601 |

| Iterations completed 5 |

| Log likelihood function -307.2955 |

| Restricted log likelihood -337.6885 |

| Chi squared 60.78608 |

| Degrees of freedom 5 |

| Prob[ChiSqd > value] = .0000000 |

| Hosmer-Lemeshow chi-squared = 5.74742 |

| P-value= .67550 with deg.fr. = 8 |

+---------------------------------------------+

+---------+--------------+----------------+--------+---------+----------+

|Variable | Coefficient | Standard Error |b/St.Er.|P[|Z|>z] | Mean of X|

+---------+--------------+----------------+--------+---------+----------+

Index function for probability

Z2 -.2202376072E-01 .10177371E-01 -2.164 .0305 32.487521

Z3 .5990084920E-01 .17086004E-01 3.506 .0005 8.1776955

Z5 -.1836462412 .51493239E-01 -3.566 .0004 3.1164725

Z7 .3751312008E-01 .32844576E-01 1.142 .2534 4.1946755

Z8 -.2729824396 .52473295E-01 -5.202 .0000 3.9317804

Constant .9766647244 .36104809 2.705 .0068

+---------------------------------------------+

| Seimparametric Binary Choice Model |

| Maximum Likelihood Estimates |

| Model estimated: Jul 31, 2002 at 11:01:24PM.|

| Dependent variable P |

| Weighting variable None |

| Number of observations 601 |

| Iterations completed 13 |

| Log likelihood function -334.7367 |

| Restricted log likelihood -337.6885 |

| Chi squared 5.903551 |

| Degrees of freedom 4 |

| Prob[ChiSqd > value] = .2064679 |

| Hosmer-Lemeshow chi-squared = 118.69649 |

| P-value= .00000 with deg.fr. = 8 |

| Logistic kernel fn. Bandwidth = .34423 |

+---------------------------------------------+

+---------+--------------+----------------+--------+---------+----------+

|Variable | Coefficient | Standard Error |b/St.Er.|P[|Z|>z] | Mean of X|

+---------+--------------+----------------+--------+---------+----------+

Characteristics in numerator of Prob[Y = 1]

Z2 -.3284308221E-01 .52254249E-01 -.629 .5297 32.487521

Z3 .1089817386 .86483083E-01 1.260 .2076 8.1776955

Z5 -.2384951835 .23320058 -1.023 .3064 3.1164725

Z7 -.1026067037 .17130225 -.599 .5492 4.1946755

Z8 -.1892263132 .21598982 -.876 .3810 3.9317804

Constant .0000000000 ........(Fixed Parameter)........

The probit model produces a set of marginal effects, as discussed in the text. These cannot be computed for the Klein and Spady estimator.

+-------------------------------------------+

| Partial derivatives of E[y] = F[\*] with |

| respect to the vector of characteristics. |

| They are computed at the means of the Xs. |

| Observations used for means are All Obs. |

+-------------------------------------------+

+---------+--------------+----------------+--------+---------+----------+

|Variable | Coefficient | Standard Error |b/St.Er.|P[|Z|>z] | Mean of X|

+---------+--------------+----------------+--------+---------+----------+

Index function for probability

Z2 -.6695300413E-02 .30909282E-02 -2.166 .0303 32.487521

Z3 .1821006800E-01 .51704684E-02 3.522 .0004 8.1776955

Z5 -.5582910069E-01 .15568275E-01 -3.586 .0003 3.1164725

Z7 .1140411992E-01 .99845393E-02 1.142 .2534 4.1946755

Z8 -.8298761795E-01 .15933104E-01 -5.209 .0000 3.9317804

Constant .2969094977 .11108860 2.673 .0075

These are the various fit measures for the probit model

+----------------------------------------+

| Fit Measures for Binomial Choice Model |

| Probit model for variable P |

+----------------------------------------+

| Proportions P0= .750416 P1= .249584 |

| N = 601 N0= 451 N1= 150 |

| LogL = -307.29545 LogL0 = -337.6885 |

| Estrella = 1-(L/L0)^(-2L0/n) = .10056 |

+----------------------------------------+

| Efron | McFadden | Ben./Lerman |

| .10905 | .09000 | .66451 |

| Cramer | Veall/Zim. | Rsqrd\_ML |

| .10486 | .17359 | .09619 |

+----------------------------------------+

| Information Akaike I.C. Schwarz I.C. |

| Criteria 1.04258 652.98248 |

+----------------------------------------+

Frequencies of actual & predicted outcomes

Predicted outcome has maximum probability.

Threshold value for predicting Y=1 = .5000

Predicted

------ ---------- + -----

Actual 0 1 | Total

------ ---------- + -----

0 437 14 | 451

1 130 20 | 150

------ ---------- + -----

Total 567 34 | 601

These are the fit measures for the probabilities computed for the Klein and Spady model. The probit model fits better by all measures computed.

+----------------------------------------+

| Fit Measures for Binomial Choice Model |

| Observed = P Fitted = KSPROBS |

+----------------------------------------+

| Proportions P0= .750416 P1= .249584 |

| N = 601 N0= 451 N1= 150 |

| LogL = -320.37513 LogL0 = -337.6885 |

| Estrella = 1-(L/L0)^(-2L0/n) = .05743 |

+----------------------------------------+

| Efron | McFadden | Ben./Lerman |

| .05686 | .05127 | .64117 |

| Cramer | Veall/Zim. | Rsqrd\_ML |

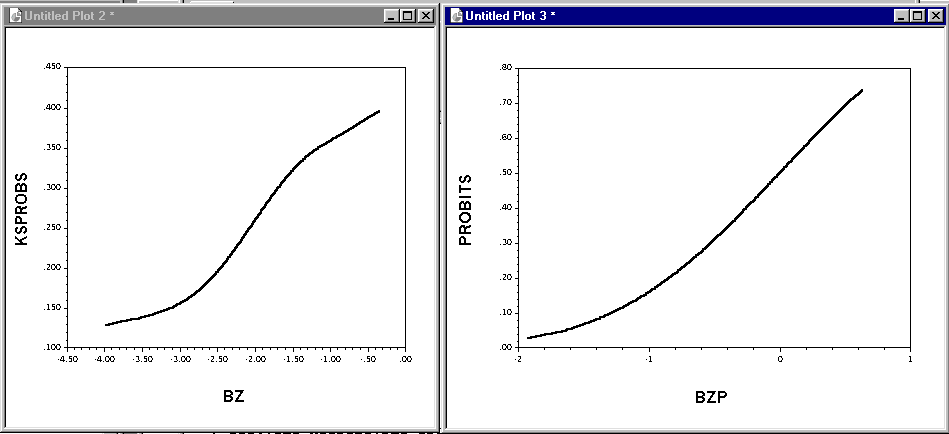
| .03897 | .10295 | .05599 |

+----------------------------------------+

The first figure below plots the probit probabilities against the Klein and Spady probabilities. The models are obviously similar, though there is substantial difference in the fitted values.



Finally, these two figures plot the predicted probabilities from the two models against the respective index functions, **b’x**. Note that the two plots are based on different coefficient vectors, so it is not possible to merge the two figures.



**Chapter 13**

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**Minimum Distance Estimation and The Generalized Method of Moments**

**Exercises**

1. The elements of **J** are





Using the formula given for the moments, we obtain, μ2 = σ2, μ3 = 0, μ4 = 3σ4. Insert these in the derivatives above to obtain



Since the rows of J are orthogonal, we know that the off diagonal term in **JVJ′** will be zero, which simplifies things a bit. Taking the parts directly, we can see that the asymptotic variance of will be σ-6 Asy.Var[m3], which will be

Asy.Var[] = σ-6(μ6 - μ32 + 9μ23 - 3μ2μ4 - 3μ2μ4).

The parts needed, using the general result given earlier, are μ6 = 15σ6, μ3 = 0, μ2 = σ2, μ4 = 3σ4. Inserting these in the parentheses and multiplying it out and collecting terms produces the upper left element of JVJ′ equal to 6, which is the desired result. The lower right element will be

Asy.Var[b2] = 36σ-4 Asy.Var[m2] + σ-8Asy.Var[m4] - 2(6)σ-6Asy.Cov[m2,m4].

The needed parts are

Asy.Var[m2] = 2σ4

Asy.Var[m4] = μ8 - μ42 = 105σ8 - (3σ4)2

Asy.Cov[m2,m4] = μ6 - μ2μ4 = 15σ6 - σ2(3σ4).

Inserting these parts in the expansion, multiplying it out and collecting terms produces the lower right element equal to 24, as expected.

2. The necessary data are given in Examples 13.5. The two moments are =31.278 and =1453.96. Based on the theoretical results m1′ = P/λ and m2′ = P(P+1)/λ2, the solutions are P = μ1′2/(μ2′ - μ1′2) and λ = μ1′/(μ2′ - μ1′2). Using the sample moments produces estimates P = 2.05682 and λ = 0.065759. The matrix of derivatives is



The covariance matrix for the moments is given in Example 18.7;



3. a. The log likelihood for sampling from the normal distribution is

logL = (-1/2)[nlog2π + nlogσ2 + (1/σ2)Σi (xi - μ)2]

write the summation in the last term as Σxi2 + nμ2 - 2μΣixi. Thus, it is clear that the log likelihood is of the form for an exponential family, and the sufficient statistics are the sum and sum of squares of the observations.

b. The log of the density for the Weibull distribution is

logf(x) = logα + logβ + (β-1)logxi - αΣi xiβ.

The log likelihood is found by summing these functions. The third term does not factor in the fashion needed to produce an exponential family. There are no sufficient statistics for this distribution.

c. The log of the density for the mixture distribution is

logf(x,y) = logθ - (β+θ)yi + xilogβ + xilogyi - log(x!)

This is an exponential family; the sufficient statistics are Σiyi and Σixi..

4. The question is (deliberately) misleading. We showed in Chapter 9 and in this chapter that in the classical regression model with heteroscedasticity, the OLS estimator is the GMM estimator. The asymptotic covariance matrix of the OLS estimator is given in Section 9.2.2. The estimator of the asymptotic covariance matrices are s2(**X′X**)-1 for OLS and the White estimator for GMM.

5. The GMM estimator would be chosen to minimize the criterion

q = n **m′Wm**

where **W** is the weighting matrix and **m** is the empirical moment,

**m** = (1/*n*)Σi (*yi* - Φ(**x**i′**β**))**x**i

For the first pass, we’ll use **W** = **I** and just minimize the sumof squares. This provides an initial set of estimates that can be used to compute the optimal weighting matrix. With this first round estimate, we compute

**W** = [(1/n2) Σi (yi - Φ(**x**i′**β**))2 **x**i **x**i**′]-1**

then return to the optimization problem to find the optimal estimator. The asymptotic covariance matrix is computed from the first order conditions for the optimization. The matrix of derivatives is

**G** = ∂**m**/∂**β′** = (1/n)Σi -φ(**x**i′**β**)**x**i**x**i**′**

The estimator of the asymptotic covariance matrix will be

**V** = (1/*n*)[**G′WG**]-1

6. This is the comparison between (13-12) and (13-11). The proof can be done by comparing the inverses of the two covariance matrices. Thus, if the claim is correct, the matrix in (13-11) is larger than that in (13-12) or its inverse is smaller. We can ignore the (1/n) as well. We require, then, that

